논리연구 26-2(2023) pp. 131-144

Implicational Partial Gaggle Logics and Matrix Semantics*

Eunsuk Yang

[Abstract] Implicational tonoid logics and their extensions with abstract Galois properties have been introduced by Yang and Dunn. They introduced matrix semantics for the implicational tonoid logics but did not do for the extensions. Here we provide such semantics for implicational partial gaggle logics as one sort of such extensions. To this end, first we discuss implicational partial gaggle logics in Hilbert-style. We next introduce one kind of matrix semantics based on Lindenbaum–Tarski matrices for the logics and show that those logics are complete with respect to the matrix semantics. Finally, we further introduce a slightly different kind of matrix semantics based on reduced models for the logics and show that those logics are complete with respect to this matrix semantics.
[Key Words] matrix semantics, implicational partial gaggle logic, implicational tonoid logic, gaggles.

Received: Mar. 23. 2023 Revised: May. 30, 2023 Accepted: Jun. 2, 2023.
* This research was supported by "Research Base Construction Fund Support Program" funded by Jeonbuk National University in 2023. I would like to thank the referees for their helpful comments.

1 Introduction

The class of implicational tonoid logics was first introduced by Yang–Dunn (2021a) as a subclass of both *tonoid logics*[1] (Dunn (1993), Dunn–Hardigree (2001), Bimbó–Dunn (2018)) and *weakly implicative logics*[2] (Cintula (2006), Cintula–Noguera (2011)). For the logics, they provided both matrix and relational semantics. Moreover, Yang–Dunn (2021b) extended those logics to implicational partial Galois logics and provided relational semantics for them. However, they did not establish matrix semantics for the logics. Then the following question arises naturally.

Is one capable of providing matrix semantics for implicational partial Galois logics?

In this paper, we answer positively the question by introducing matrix semantics for the logics. To this end, we discuss implicational partial gaggle logics as one kind of implicational partial Galois logics in Section 2. We then introduce matrix semantics for the logics and prove that the logics are complete with respect to (w.r.t.) matrix semantics in Sections 3 and 4. More exactly, we first introduce one kind of matrix semantics based on Lindenbaum–Tarski matrices for the logics and prove that those logics are complete w.r.t. the matrix

[1] *Gaggles*, the acronym of generalized Galois logics, were first introduced by Dunn (1991) as an algebraic structural class so as to provide a unified approach to the semantics for non-classical logics. Since then, Dunn (1993) introduced *partial* gaggles with an underlying partial order as a generalization of gaggles, which have a distributive lattice as an underlying structure, and introduced also *tonoids* as a generalization of partial gaggles. Bimbó–Dunn (2018) further developed gaggles, in particular tonoids.

[2] Cintula (2006) first introduced weakly implicative logics as a generalization of implicative logics for non-classical logics introduced by Rasiowa (1974). Contula–Noguera (2011) further developed those logics.

semantics in Section 3. We next introduce a slightly different kind of matrix semantics based on reduced models for the logics and prove that those logics are complete w.r.t. this matrix semantics in Section 4.

We finally note that most logical systems have the rule modus ponens and their corresponding deduction theorem. One important feature between the rule and the theorem is that they satisfy the so-called residuated connection property. If such systems have negations, they also satisfy the so-called Galois connection property. The work of this paper verifies that one can provide algebraic semantics (as well as relational semantics) for logics satisfying such properties.

2 Implicational partial gaggle logics

Here we recall some basic concepts related to implicational partial gaggle logics introduced in Yang–Dunn (2021a; 2021b). For the class of implicational partial gaggle logics, we first introduce a language for the logics. A propositional language $\mathscr{L}$, where the set of formulas is denoted by *FMR*, is constituted inductively by a set of countable propositional variables *VRL*, a set of propositional connectives *CON*, and an arity function **art** assigning each element of *CON* a natural number. By $(\#,n)$, we denote a part of $\mathscr{L}$ such that $\# \in CON$ and $\mathbf{art}(\#) = n$; by *$\mathscr{L}$-substitution*, a function $sb : FMR_{\mathscr{L}} \to FMR_{\mathscr{L}}$ satisfying $sb(\#(A_1, \ldots, A_n)) = \#(sb(A_1), \ldots, sb(A_n))$; and by $A, B, C, \ldots$, and $T, S, \ldots$, formulas and their sets, respectively.

A *consecution* in $\mathscr{L}$ means a relation $T \vdash_R A$ such that $T \cup \{R\} \subseteq FMR_{\mathscr{L}}$ and a pair $\langle T, A\rangle \in R$. A *logic* in $\mathscr{L}$ is a nonempty subset L of the set of all the consecutions satisfying the following three conditions:

(i) If $A \in T$, then $T \vdash_L A$.

(ii) If $T \vdash_L B$ for all $B \in S$ and $S \vdash_L A$, then $T \vdash_L A$.

(iii) If $T \vdash_L A$, then $sb(T) \vdash_L sb(A)$ for all $\mathscr{L}$-substitution sb.

A set of formulas T is said to be a *theory* of a logic L if $A \in T$ whenever $T \vdash_L A$. $Th(L)$ denotes the set of all theories of L.

Let a tonicity map **tm** be a function, which maps a connective # of arity $n > 0$ to its tonic type $\mathbf{tm}(\#) = (\sigma_1, \cdots, \sigma_n)$ such that each σ_i is isotone or antitone, where the isotonicity and antitonicity are denoted by $+$ and $-$, respectively. We define a *tonic language* as $\mathscr{L}$ with **tm**. Let a tonic language have a binary connective $\Rightarrow$ satisfying $(\Rightarrow, 2) \in \mathscr{L}$ and $\mathbf{tm}(\Rightarrow) = (-, +)$. We call a tonic language with $\Rightarrow$ *implicational*. Assume that $\vec{A}, B \in VRL$ and # is an n-ary connective. $\#^n(\vec{A}, B_i)$ indicates n arguments the application of # such that B is the i-th argument of # and $\vec{A}$ is the sequence of arguments of # excepting its i-th argument.

Henceforth, $\mathscr{L}$ and L denote an implicational tonic language and a logic in $\mathscr{L}$, respectively.

Definition 2.1. (Implicational tonoid logic, Yang–Dunn (2021a)) L is said to be an implicational *tonoid* logic in case L satisfies:

$(i) \vdash_L A \Rightarrow A$ (Ref).

(ii) $A \Rightarrow B, A \vdash_L B$ $(ModPon)$.

(iii) $A \Rightarrow B, B \Rightarrow C \vdash_L A \Rightarrow C$ $(Tran)$.

(iv) For each $(\#, n) \in \mathscr{L}$ and for each $i \leq n$,

- If $\mathbf{tm}(\#)(i) = +$, then $A \Rightarrow B \vdash_L \#^n(\vec{C}, A_i) \Rightarrow \#^n(\vec{C}, B_i)$, and
- If $\mathbf{tm}(\#)(i) = -$, then $A \Rightarrow B \vdash_L \#^n(\vec{C}, B_i) \Rightarrow \#^n(\vec{C}, A_i)$ $(Ton^i_\#)$.

Definition 2.2. (Implicational partial gaggle logic, Yang–Dunn (202-1b)) L is said to be an *implicational partial gaggle logic* if it is an implicational tonoid logic and each (f,n), $(g,n) \in \mathscr{L}$ satisfies one of the following: for each $i \leq n$,

(GC, Galois connection) $A \Rightarrow f^n(\vec{C},B_i) \dashv\vdash_L B \Rightarrow g^n(\vec{C},A_i)$;[3]

(dGC, dual GC) $f^n(\vec{C},B_i) \Rightarrow A \dashv\vdash_L g^n(\vec{C},A_i) \Rightarrow B$;

(RC, residuated connection) $f^n(\vec{C},A_i) \Rightarrow B \dashv\vdash_L A \Rightarrow g^n(\vec{C},B_i)$;

(dRC, dual RC) $B \Rightarrow f^n(\vec{C},A_i) \dashv\vdash_L g^n(\vec{C},B_i) \Rightarrow A$,

where:

(1) in each (GC) and (dGC), f and g have the same tonic types, in particular, they have the antitonicity as the tonic type of their i-th arguments;

(2) in each (RC) and (dRC), f and g have the different tonic types of f and g from each other w.r.t. an argument distinct from i, in particular, they have the isotonicity as the tonic type of their i-th arguments.

Note that the definition for implicational partial gaggle logics drops the notations related to labels introduced in Yang-Dunn (2021b) because we need not introduce a labeled language for the logics w.r.t. matrix semantics. Note also that '$\Rightarrow$' is not a concrete implication connective. It is an abstract connective, which may denote any connectives satisfying (Ref), $(Tran)$, $(ModPon)$, and $(Ton^i_\#)$. The following are examples, which have two implications $\rightarrow$, $\rightsquigarrow$, two negations $\sim$, $\neg$, and one fusion $\circ$ for the connectives f and g, introduced in Yang-Dunn (2021b).

Example 2.3. (Yang-Dunn (2021b))

(i) (GC) $A \Rightarrow (B \rightarrow C) \dashv\vdash B \Rightarrow (A \rightsquigarrow C)$.

(ii) (dGC) $(C \trianglelefteq B) \Rightarrow A \dashv\vdash (C \trianglerighteq A) \Rightarrow B$,

$$\text{where } C \trianglelefteq B =_{df} \neg B \circ C \text{ and } C \trianglerighteq A =_{df} C \circ \sim A.$$

[3] We use "$A \dashv\vdash B$" as shorthand for $A \vdash B$ and $B \vdash A$.

(iii) (RC) $(C \circ A) \Rightarrow B \dashv\vdash A \Rightarrow (C \rightarrow B)$.
(iv) (dRC) $B \Rightarrow (A \oplus C) \dashv\vdash (B \trianglerighteq C) \Rightarrow A$,

$$\text{where } A \oplus C =_{df} \neg A \rightarrow C.$$

Define $A \Leftrightarrow B := \{A \Rightarrow B, B \Rightarrow A\}$. We can verify that implicational tonoid logics satisfy the congruence property, and so do implicational partial gaggle logics.

Theorem 2.4. *Let L be an implicational tonoid logic. L satisfies the following congruence property: for every $(\#, n)$ and every $i \leq n$,*

$$(Cong)\ A \Leftrightarrow B \vdash_L \#^n(\vec{C}, A_i) \Leftrightarrow \#^n(\vec{C}, B_i).$$

Proof. Let T be a theory T of L such that $A \Rightarrow B \in T$ and $B \Rightarrow A \in T$. First consider the case $\mathbf{tm}(\#)(i) = +$. $(Ton^i_\#)$ assures that $A \Rightarrow B \in T$ and $B \Rightarrow A \in T$ imply $\#^n(\vec{C}, A_i) \Rightarrow \#^n(\vec{C}, B_i) \in T$ and $\#^n(\vec{C}, B_i) \Rightarrow \#^n(\vec{C}, A_i) \in T$, respectively. Hence $(Cong)$ holds. Next consider the case $\mathbf{tm}(\#)(i) = -$. $(Ton^i_\#)$ similarly assures that $A \Rightarrow B \in T$ and $B \Rightarrow A \in T$ imply $\#^n(\vec{C}, B_i) \Rightarrow \#^n(\vec{C}, A_i) \in T$ and $\#^n(\vec{C}, A_i) \Rightarrow \#^n(\vec{C}, B_i) \in T$, respectively. Hence $(Cong)$ holds. Therefore, L satisfies $(Cong)$. □

Corollary 2.5. *Let L be an implicational partial gaggle logic. L satisfies the congruence property $(Cong)$.*

3 Matrix semantics I

This section introduces one well-known kind of *matrix semantics* based on Lindenbaum–Tarski matrices for implicational gaggle logics. To this end, we first define several related notions.

Let L be an implicational partial gaggle logic. A pair $\mathbf{P} = (\mathbf{E}, F)$ is said to be an $\mathscr{L}$*-matrix* if $\mathbf{E}$ is an $\mathscr{L}$-algebra as an algebra interpreting the formulas and $F \subseteq \mathbf{E}$ whose elements are called designated elements of $\mathbf{E}$. A homomorphism v from $\mathbf{FMR}_{\mathscr{L}}$ to $\mathbf{E}$ is said to be an $\mathbf{E}$-*evaluation* if $v : FMR_{\mathscr{L}} \to E$ satisfies:

$$v(\#(A_1, \ldots, A_n)) = \#^{\mathbf{E}}(v(A_1), \ldots, v(A_n))$$

for every $(\#, n) \in \mathscr{L}$ and every n-tuple of formulas $A_1, \ldots, A_n$. We in particular say $\mathbf{P}$-evaluation for a matrix $\mathbf{P} = (\mathbf{E}, F)$. A $\mathbf{P}$-evaluation v is said to be a $\mathbf{P}$-*model* of T, a theory in $\mathscr{L}$, if $v(A) \in F$ for every $A \in T$. The class of $\mathbf{P}$-models of T is denoted by $Mod(T, \mathbf{P})$. A formula A is said to be a *semantic consequence* of T for $\mathscr{K}$ if $Mod(T, \mathbf{P}) = Mod(T \cup \{A\}, \mathbf{P})$ for every $\mathbf{P} \in \mathscr{K}$. The semantic consequence is denoted by $T \models_{\mathscr{K}} A$. $\mathbf{P}$ is said to be an *L-matrix* if $\mathrm{L} \subseteq \models_{\{\mathbf{P}\}}$. The class of L-matrices is denote by *MOD(L)*, but in lieu of $T \models_{MOD(L)} A$, $T \models_L A$ is written.

We prove the following proposition.

Proposition 3.1. *Let L be an implicational partial gaggle logic and* $\mathbf{P} = (\mathbf{E}, F)$ *be an L-matrix. Define the relation* $\leq_{\mathbf{P}}$ *as follows:* $x \leq_{\mathbf{P}} y$ *iff* $x \Rightarrow y \in F$.

(i) $\leq_{\mathbf{P}}$ *forms a preorder.*

(ii) $x =_{\mathbf{P}} y$ *iff* $x \leq_{\mathbf{P}} y$ *and* $y \leq_{\mathbf{P}} x$ *forms a congruence on* $\mathbf{E}$.

(iii) F *forms an an upset on* $\leq_{\mathbf{P}}$*, i.e.,* $x \in F$ *and* $x \leq_{\mathbf{P}} y$ *imply* $y \in F$.

Proof. (i) (Ref) assures $x \Rightarrow x \in F$, and so $x \leq_{\mathbf{P}} x$. $(Tran)$ assures that $x \Rightarrow y \in F$ and $y \Rightarrow z \in F$ imply $x \Rightarrow z \in F$, and so $x \leq_{\mathbf{P}} y$ and $y \leq_{\mathbf{P}} z$ imply $x \leq_{\mathbf{P}} z$. Hence $\leq_{\mathbf{P}}$ is a preorder.

(ii) Corollary 2.5 assures that $x =_{\mathbf{P}} y$ implies $\#^{\mathbf{E}n}(\vec{z}, x_i) =_{\mathbf{P}} \#^{\mathbf{E}n}(\vec{z}, y_i)$. Thus $=_{\mathbf{P}}$ forms a congruence on $\mathbf{E}$.

(iii) (*ModPon*) assures that $x \in F$ and $x \Rightarrow y \in F$ imply $y \in F$, and so $x \in F$ and $x \leq_{\mathbf{P}} y$ imply $y \in F$. Hence F is the upset. □

Let L be an implicational partial gaggle logic. One is capable of providing Lindenbaum–Tarski matrices for L by Proposition 3.1 (ii). More precisely, the Lindenbaum–Tarski matrices can be defined as follows. Let T be a theory of L. Define $[A]_T$ and L_T as $\{B : T \vdash_L A \Leftrightarrow B\}$ and $\{[A]_T : A \in \mathbf{FMR}_{\mathscr{L}}\}$, respectively. The $\mathscr{L}$-matrix is said to be the *Lindenbaum–Tarski matrix* $\mathbf{LT}_T$ w.r.t. L and T if the designated set of $\mathscr{L}$-matrix is $\{[A]_T : A \in T\}$, $\mathscr{L}$-algebra has the domain L_T and operations $\#^{\mathbf{LT}_T}([A_1]_T, \ldots, [A_n]_T) = [\#(A_1, \ldots, A_n)]_T$.

Theorem 3.2. (*Strong completeness*) *Let L be an implicational partial gaggle logic, T be a theory of L and A be a formula. $T \vdash_L A$ iff $T \models_L A$.*

Proof. The left-to-right direction is clear. For the right-to-left direction, define $\mathbf{LT}_T$-evaluation v as follows: $v(B) = [B]_T$. We notice that Lemma 8 in Cintula (2006) assures that $\mathbf{LT}_T \in \mathbf{MOD}(L)$ and $v \in \mathbf{Mod}(T; \mathbf{LT}_T)$ for $\mathbf{LT}_T$-evaluation v. Then $T \models_L A$ entails that $[A]_T = v(A) \in F_{\mathbf{LT}_T}$. Hence $T \vdash_L A$, as required. □

4 Matrix semantics II

This section introduces a slightly different kind of *matrix semantics* based on reduced models for implicational gaggle logics. To this end, we first note that one is capable of providing completeness for reduced models. Similarly we introduce some related additional notions.

The Leibniz congruence $\Omega_{\mathbf{P}}(F)$ of $\mathbf{E}$ is defined as follows:

$$(\alpha)\ \ \langle x,y\rangle \in \Omega_{\mathbf{P}}(F) \text{ iff } x \Rightarrow y \in F \text{ and } y \Rightarrow x \in F.$$

A congruence $\mathbf{C}$ of $\mathbf{E}$ compatible with F such that for all $x,y \in E$, $x \in F$ and $\langle x,y\rangle \in \mathbf{C}$ imply $y \in F$ is said to be a *logical congruence* in a matrix $\mathbf{P} = (\mathbf{E},F)$. The Leibniz congruence $\Omega_{\mathbf{P}}(F)$ is well known as the largest logical congruence of $\mathbf{P}$. An L-matrix $\mathbf{P} = (\mathbf{E},F)$ is said to be *reduced* if $\Omega_{\mathbf{P}}(F)$ is the identity relation $=_{\mathbf{P}}$. The notation $\boldsymbol{MOD}^*(L)$ denotes the class of all reduced models of L. For an L-matrix $\mathbf{P} = (\mathbf{E},F)$, we define $[x]_F$ as $\{y \in E : \langle x,y\rangle \in \Omega_{\mathbf{P}}(F)\}$; $[F]$ as $\{[x]_F : x \in F\}$; and $\mathbf{P}^*$ as $(\mathbf{P}/\Omega_{\mathbf{P}}(F),[F])$.

Proposition 4.1. *Let L be an implicational partial gaggle logic and* $\boldsymbol{P} = (\mathbf{E},F) \in \boldsymbol{MOD}(L)$.

(i) $\boldsymbol{P}^* \in \boldsymbol{MOD}(L)$.

(ii) For all $x,y \in E$, $[x]_F \leq_{\boldsymbol{P}^*} [y]_F$ *iff* $x \Rightarrow^{\boldsymbol{P}} y \in F$.

(iii) $\boldsymbol{P}^* \in \boldsymbol{MOD}^*(L)$.

Proof. (i) It is clear that $[\cdot]_F$ is a surjective homomorphism from $\mathbf{P}$ to $\mathbf{P}/\Omega_{\mathbf{P}}(F)$. Then it suffices to verify that $[x]_F \in [F]$ entails $x \in F$, by Lemma 2.1.19 in Cintula–Noguera (2011). Let $[x]_F \in [F]$. Then $[x]_F = [y]_F$ for some $y \in F$, and so $\langle x,y\rangle \in \Omega_{\mathbf{P}}(F)$ by (α). Then $x \in F$ since $\Omega_{\mathbf{P}}(F)$ is a logical congruence.

(ii) For one direction, let $[x]_F \leq_{\mathbf{P}^*} [y]_F$ for all $x,y \in E$. Then $[x]_F \Rightarrow^{\mathbf{P}/\Omega_{\mathbf{P}}(F)} [y]_F \in [F]$, and so $x \Rightarrow^{\mathbf{P}} y \in F$. The proof for the other direction is analogous.

(iii) Let $[x]_F \leq_{\mathbf{P}^*} [y]_F$ and $[y]_F \leq_{\mathbf{P}^*} [x]_F$. This implies $\langle x,y\rangle \in \Omega_{\mathbf{P}}(F)$. Hence $[x]_F =_{\mathbf{P}^*} [y]_F$. □

Now we introduce related Lindenbaum–Tarski matrices for an implicational partial gaggle logic L. Let $T \in Th(L)$. Define $[A]_T$ and

L_T as $\{B : A \Leftrightarrow B \subseteq T\}$ and $\{[A]_T : A \in \mathbf{FMR}_{\mathscr{L}}\}$, respectively. The *Lindenbaum–Tarski matrix* $\mathbf{LT}_T$ w.r.t. L and T is defined as in Section 3.

Theorem 4.2. (*Strong completeness for reduced models*) *Let L be an implicational partial gaggle logic, T be a theory of L and A be a formula.* $T \vdash_L A$ *iff* $T \models_{\boldsymbol{MOD}^*(L)} A$.

Proof. The left-to-right direction is clear. For the right-to-left direction, as above, define $\mathbf{LT}_T$-evaluation v as follows: $v(B) = [B]_T$. Proposition 4.1 assures that $\mathbf{LT}_T \in \mathbf{MOD}^*(L)$. As above, define $\mathbf{LT}_T$-evaluation v as follows: $v(B) = [B]_T$. Then $T \models_L A$ entails that $[A]_T = v(A) \in [T]$. Hence $T \vdash_L A$, as desired. □

Remark 4.3. We in particular provided matrix semantics based on the Leibniz congruence in this section. Note that the Leibniz congruence assures that $(Cong)$ is needed for a logic L since the Leibniz congruence also satisfies $(Cong)$.

Remark 4.4. We established two matrix completeness results for implicational partial gaggle logics, using Lindenbaum–Tarski matrices. We notice that $(Cong)$ is needed for a logic L to have these kinds of semantic completeness. As the proof for Corollary 2.5 shows, in implicational gaggle logics one is capable of proving the $(Cong)$ using the tonicity rule $(Ton^i_\#)$. This shows that $(Ton^i_\#)$ plays a significant role in matrix semantics for implicational gaggle logics.

5 Conclusion

We introduced two matrix semantics for implicational partial gaggle logics and prove that the logics are complete w.r.t. those semantics.

More exactly, we first introduced one kind of matrix semantics based on Lindenbaum–Tarski matrices for the logics and proved that those logics are complete w.r.t. the matrix semantics in the sense that true sentences in implicational partial gaggle logics are provable in their corresponding logics. We next introduced a different kind of matrix semantics based on reduced models for the logics and proved that those logics are complete w.r.t. this matrix semantics.

Among implicational partial Galois logics, here we just deal with implicational partial gaggle logics. Note that implicational residuated partial gaggle logics and implicational dual residuated partial gaggle logics were also introduced as implicational partial Galois logics in Yang-Dunn (2021b). Thus the two sorts of matrix semantics have to be addressed for the other implicational partial Galois logics. Moreover, we may consider Lindenbaum matrices in place of Lindenbaum–Tarski matrices as matrix semantics. These are problems left in this paper.

References

Bimbó, K. and Dunn, J. M. (2018), *Generalized Galois Logics: Relational Semantics of Nonclassical Logical Calculi*, Stanford, CSLI Publications.

Cintula, P. (2006), "Weakly Implicative (Fuzzy) Logics I: Basic properties", *Archive for Mathematical Logic* 45, pp. 673-704.

Cintula, P. and Noguera, C. (2011), "A general framework for mathematical fuzzy logic", *Handbook of Mathematical Fuzzy Logic*, vol 1, P. Cintula, P. Hájek, and C. Noguera (Eds.), London, College publications, pp. 103-207.

Dunn, J. M. (1991), "Gaggle theory, an abstraction of galois connections and residuation, with applications to negation, implication, and various logical operators", *Logics in AI* (JELIA 1990, Amsterdam), Lecture Notes in AI, Van Eijck, J. (Ed.), Springer Verlag, pp. 31-51.

Dunn, J. M. (1993), Partial gaggles applied to logics with restricted structural rules, *Substructural Logics*, P.Schroeder-Heister and K. Došen (Eds.), Oxford, Clarendon, pp. 63–108.

Dunn, J. M. and Hardegree, G. (2001), *Algebraic Methods in Philosophical Logic*, Oxford, Oxford Univ Press.

Rasiowa, H. (1974), *An Algebraic Approach to Non-Classical Logics*. Amsterdam, North-Holland.

Yang, E. and Dunn, J. M. (2021a), "Implicational tonoid logics: algebraic and relational semantics", *Logica Universalis* 15, pp. 435-456.

Yang, E. and Dunn, J. M. (2021b), "Implicational Partial Galois Logics: Relational semantics", *Logica Universalis* 15, pp. 457–476.

전북대학교 철학과, 비판적사고와 논술연구소
Department of Philosophy & Institute of Critical Thinking and Writing, Jeonbuk National University
eunsyang@jbnu.ac.kr

함의적 부분 개글 논리와 행렬 의미론

양은석

함의적 토노이드 논리와 추상화된 갈로아 성질을 갖는 그것의 확장이 소개되어왔다. 이와 관련하여 함의적 토노이드 논리를 위한 행렬 의미론은 소개되었지만 그것의 확장을 위해서는 소개되지 않았다. 이 논문에서 우리는 그러한 확장 중 함의적 부분 개글 논리를 위한 행렬 의미론을 다룬다. 이를 위하여 먼저 함의적 부분 개글 논리를 소개한다. 다음으로 함의적 부분 개글 논리를 위한 린덴바움-타르스키 행렬에 기반한 행렬 의미론을 소개하고 함의적 부분 개글 논리가 이 의미론에 대해 완전하다는 것을 보인다. 마지막으로 함의적 부분 개글 논리를 위한 린덴바움-타르스키 행렬에 기반한 행렬 의미론을 소개하고 함의적 부분 개글 논리가 이 의미론에 대해 완전하다는 것을 보인다. 마지막으로 축소 모델에 기반한 행렬 의미론을 소개하고 함의적 부분 개글 논리가 이 의미론에 대해 완전하다는 것을 보인다.

주요어: 행렬 의미론, 함의적 부분 개글 논리, 함의적 토노이드 논리, 개글