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Relational Semantics for Fuzzy Extensions of R: Set-theoretic Approach*

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【Abstract】 This paper addresses a set-theoretic completeness based on a relational semantics for fuzzy extensions of two versions $\mathbf{R}^t$ and $\mathbf{R}^T$ of $\mathbf{R}$ (Relevance logic). To this end, two fuzzy logics $\mathbf{FR}^t$ and $\mathbf{FR}^T$ as extensions of $\mathbf{R}^t$ and $\mathbf{R}^T$, respectively, and the relational semantics, so called Routley-Meyer semantics, for them are first recalled. Next, on the semantics completeness results are provided for them using a set-theoretic way.

【Key Words】 Relational Semantics, Routley-Meyer Semantics, **R**, Fuzzy Logic, Relevance Logic.

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1. Introduction

The logic **R** is very famous as the logic of relevance. As Yang mentioned in Yang (2014), interestingly there exist three versions of **R**: one is the **R** without propositional constants, denoted by $\mathbf{R}^0$, another is the **R** with propositional constants **t**, **f**, denoted by $\mathbf{R}^\mathbf{t}$, and the other is the **R** with propositional constants **f**, **t**, **F**, **T**, denoted by $\mathbf{R}^\mathbf{T}$. The ternary relational semantics, called Routley-Meyer (RM for short) semantics, for **R** introduced in Dunn (1986) is for $\mathbf{R}^0$.[1] Similar semantics for the other two versions $\mathbf{R}^\mathbf{t}$, $\mathbf{R}^\mathbf{T}$ were introduced by Yang (2015).

The systems $\mathbf{FR}^\mathbf{t}$ and $\mathbf{FR}^\mathbf{T}$ were introduced as the least fuzzy extensions of $\mathbf{R}^\mathbf{t}$ and $\mathbf{R}^\mathbf{T}$, respectively, by Yang (2012; 2019). Especially, he recently provided RM semantics for the systems $\mathbf{FR}^\mathbf{t}$ and $\mathbf{FR}^\mathbf{T}$ in Yang (2022). This semantics is interesting in the sense that it is a different style relational semantics from the above semantics for $\mathbf{R}^0$, $\mathbf{R}^\mathbf{t}$, and $\mathbf{R}^\mathbf{T}$. The former semantics is based on order reversely considered definition of Fine's one ($Rxyz := z \geq (x * y)$, see Fine (1974)) and has the same structures as algebraic semantics for $\mathbf{FR}^\mathbf{t}$ and $\mathbf{FR}^\mathbf{T}$.

However, the completeness results of $\mathbf{FR}^\mathbf{t}$ and $\mathbf{FR}^\mathbf{T}$ in it are not interesting in that the completeness proof is established *indirectly*, i..e., their completeness is established based on the fact that those systems are algebraically complete and the RM semantics has the same structures as the algebraic semantics for

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1) The so-called Routley-Meyer semantics for relevance logics was first introduced by Routley & Routley (1972a; 1972b; 1973).

them. Note that the completeness results in Dunn (1986) and Yang (2015) were established *directly* without the help of algebraic completeness. Namely, those completeness proofs were provided set-theoretically. Then, the following natural question arises.

- Can we establish set-theoretic completeness results for $\mathbf{FR^t}$ and $\mathbf{FR^T}$ using the same RM semantics in Yang (2022)?

By providing set-theoretic completeness results for $\mathbf{FR^t}$ and $\mathbf{FR^T}$, we give its answer. The organization of the paper is as follows: As preliminaries, Section 2 discusses the systems $\mathbf{FR^t}$ and $\mathbf{FR^T}$ and their RM semantics introduced in Yang (2022). Section 3 proves the completeness of the systems set-theoretically and so verifies that $\mathbf{FR^t}$ and $\mathbf{FR^T}$ are set-theoretically complete with respect to (w.r.t.) the RM semantics.

2. Logics and RM semantics

2.1 Logics

Here we first recall the fuzzy systems $\mathbf{FR^t}$ and $\mathbf{FR^T}$. The languages for them are provided as usual. More exactly, the language for $\mathbf{FR^t}$ consists of a countable propositional language equipped with the set of formulas *Fm* built inductively from a set of propositional variables p, q, r ⋯, propositional constant **f**, and binary connectives $\wedge$, $\vee$, $\rightarrow$, together with the defined constant and unary and binary connectives: $\mathbf{t} := \mathbf{f} \rightarrow \mathbf{f}$, $\sim X := X \rightarrow \mathbf{f}$, X

& Y := ~(X → ~Y), and X ↔ Y := (X → Y) ∧ (Y → X). For the language for $\mathbf{FR}^{\mathbf{T}}$, we add the constant **F** and the defined connective **T** := **F** → **F** to the language for $\mathbf{FR}^{\mathbf{t}}$.

For FR ∈ {$\mathbf{FR}^{\mathbf{t}}$, $\mathbf{FR}^{\mathbf{T}}$}, a consequence relation ⊢ is introduced as a logic in Hilbert style.

Definition 2.1 (i) (Yang (2012)) $\mathbf{FR}^{\mathbf{t}}$ consists of the below axioms and rules:

A1. X → X (SI, self-implication)

A2. (X → Y) → ((Y → Z) → (X → Z)) (SF, suffixing)

A3. (X → (X → Y)) → (X → Y) (CR, contraction)

A4. (X → (Y → Z)) ↔ (Y → (X → Z)) (PM, permutation)

A5. (X ∧ Y) → X, (X ∧ Y) → Y (∧-E, ∧-elimination)

A6. ((X→Y)∧(X→Z)) → (X→(Y∧Z)) (∧-I, ∧-introduction)

A7. ((X→Z)∧(Y→Z)) → ((X∨Y)→Z) (∨-E, ∨-elimination)

A8. X → (X ∨ Y), Y → (X ∨ Y) (∨-I, ∨-introduction)

A9. (X∧(Y∨Z))→((X∧Y)∨(X∧Z)) (D, distributivity)

A10. X ↔ (**t** → X) (PP, push and pop)

A11. ~~X → X (DNE, double negation elimination)

A12. ((X → Y) ∧ **t**) ∨ ((Y → X) ∧ **t**) ($PL_{\mathbf{t}}$, **t**-prelinearity)

X → Y, X ⊢ Y (modus ponens, mp)

X, Y ⊢ X ∧ Y (adjunction, adj).

(ii) (Yang (2022)) $\mathbf{FR}^{\mathbf{T}}$ is $\mathbf{FR}^{\mathbf{t}}$ plus constant **F** and the axiom:

A13. **F** → X.

Here we recall the two facts: First, the axiom A12 is for linearity as fuzzy logics are required. Second, dropping the axiom

A12 from $\mathbf{FR^t}$ and $\mathbf{FR^T}$ induces the two versions $\mathbf{R^t}$ and $\mathbf{R^T}$, respectively, of **R**.

Proposition 2.2 (Yang (2012; 2015; 2022)) $\mathbf{FR^t}$ proves:

(1) (X & (Y & Z)) ↔ ((X & Y) & Z) (&-ASS, &-associativity)
(2) (X ∧ Y) → (X & Y)
(3) (X & (Y ∧ Z)) ↔ ((X & Y) ∧ (X & Z))
(4) (X → (Y ∨ Z)) ↔ ((X → Y) ∨ (X → Z))
(5) ((X → (Y ∨ Z)) ∧ (Y → Z)) → (X → Z)
(6) (X & Y) → (Y & X) (&-C, &-commutativity)
(7) (X → (Y → Z)) ↔ ((X & Y) → Z) (RE, residuation)
(8) X → (X & X) (CTR. contraction)
(9) ~~X ↔ X (DN, double negation)
(10) (X → Y) → (~Y → ~X) (CP, contraposition)

One can define a *theory* Γ on FL ∈ {$\mathbf{FR^t}$, $\mathbf{FR^T}$} as a set of formulas. A *proof* in Γ on FL is a sequence of propositions where each member of the sequence is either a member of Γ, an axiom of FL, or derives from its preceding members by the rules in Definition 2.1. Using the notation Γ ⊢ X, more precisely Γ $\vdash_{FL}$ X, we mean that in Γ X is *provable* on FL, i.e., there exists an FL-proof of X in Γ. The relevant deduction theorem (RDT_t) for FL is that:

Proposition 2.3 (Dunn, Meyer, & Leblanc (1976)) Let Γ be a theory, and X, Y formulas.

(RDT_t) Γ ∪ {X} ⊢ Y if and only if (iff) Γ ⊢ (X ∧ **t**) → Y.

2.2 Semantics

Here we discuss operational RM semantics for FR ∈ {$\mathbf{FR^t}$, $\mathbf{FR^T}$}. We first deal with the definitions of *operational RM frames* for FR.

Definition 2.4 (i) (Yang (2020), RM frames) A relational structure **RMF** = (RMF, R, 1) is called an *RM frame* if R ⊆ RMF^3 and 1 is a special element in RMF. By *states of information*, we say the elements of **RMF**.

(ii) (Yang (2020), Operational RM frames) An RM frame **RMF** = (RMF, R, 1, ∗, ≤) is called *operational* if (RMF, 1, ∗) forms a unital groupoid and R has the definition and postulates below:

df0. x ≤ y := R1yx for all x, y ∈ RMF;

p_s. R1xy and R1yx imply x = y for all x, y ∈ RMF;

p_t. R1xy and R1yz imply R1xz for all x, y, z ∈ RMF.[2)]

(iii) (Linear operational RM frames) An operational RM frame is called *linear* if R further satisfies the postulate below:

p^l. R1xy or R1yx for all x, y ∈ RMF.

By oRM^l frames, we denote such frames for simplicity.[3)]

(iv) (Yang (2021), (Pointed, residuated) Fine oRM^l frames) An oRM^l frame is called a Fine oRM*l* (F-oRM^l for simplicity) frame if R is defined as follows: (df_F) Rxyz := z ≤ (x * y).[4)] An

2) The definition and postulates df0, p_s, p_t assure that (RMF, ≤) forms a partially ordered set.

3) The postulate p^l assures that (RMF, ≤) forms a linearly ordered set.

4) Note that Fine order-reversely considered this definition. That is, he defined R as follows: Rxyz := z ≥ x * y (see Fine (1974).

F-oRM^l frame is called pointed if it further has 0, an arbitrary element, ∈ RMF; residuated if it has a residuated implication →, which is defined as follows: x → y := sup{z: x * z ≤ y} for all x, y ∈ RMF.

(v) (Yang (2022), FR^1 frames) Define the negation operation ~ as follows: ~x := x → 0 for all x ∈ RMF. An *FR^1 frame* is a pointed, residuated F-oRM^l frame satisfying:

df1. R^2xyzw := (∃a)(Rxya∧Razw) for all x, y, z, w ∈ RMF;

df2. $R^2x(yz)w$:= (∃a)(Rxaw∧Ryza) for all x, y, z, w ∈ RMF;

df3. x → y := ~(x * ~y) for all x, y ∈ RMF;

p_e. Rxyz only if Ryxz for all x, y, z ∈ RMF;

p_a. R^2xyzw iff $R^2x(yz)w$ for all x, y, z, w ∈ RMF;

p_c. Rxxx for all x ∈ RMF;

p_{inv}. ~~x = x for all x ∈ RMF.

(vi) (Yang (2022), FR$^\top$ frames) A bounded FR^1 frame, denoted by *FR$^\top$ frame*, is an FR^1 frame with least and greatest elements ⊥, ⊤.

By FR frames, henceforth, both FR^1 and FR$^\top$ frames are denoted ambiguously if one does not have to distinguish them.

Over an FR^1 frame, a *forcing* relation ⊩ is defined as a relation between states of information and propositional variables, propositional constants, and formulas subject to: for all propositional variables p,

(max) the set {x ∈ RMF : x ⊩ p} has a maximum;

(AHC) if y ≤ x and x ⊩ p, then y ⊩ p;

for the proposition constants **t**, **f**,

(1) x $\Vdash$ **t** iff x $\leq$ 1;
(0) x $\Vdash$ **f** iff x $\leq$ 0;

for formulas X, Y,

(~) x $\Vdash$ ~X iff ~x $\nVdash$ X;
($\vee$) x $\Vdash$ X $\vee$ Y iff x $\Vdash$ X or x $\Vdash$ Y;
($\wedge$) x $\Vdash$ X $\wedge$ Y iff x $\Vdash$ X and x $\Vdash$ Y;
($\rightarrow$) x $\Vdash$ X $\rightarrow$ Y iff for all y, z $\in$ RMF, if Ryxz and y $\Vdash$ X, then z $\Vdash$ Y.

Over an $FR^{\top}$ frame, a forcing relation $\Vdash$ has to further satisfy:

(min) $\bot$ $\Vdash$ p for all propositional variables p;
($\bot$) x $\Vdash$ **F** iff x = $\bot$ for the propositional constant **F**.

A pair (**RMF**, $\Vdash$) is an *FR model* if **RMF** is an FR frame and $\Vdash$ is a forcing relation over **RMF**. Let (**RMF**, $\Vdash$) be an FR model, x a state of information of **RMF** and X a formula. We say that x *forces X* whenever x $\Vdash$ X; X is *true* in (**RMF**, $\Vdash$) whenever 1 $\Vdash$ X; and X is *valid* in the frame **RMF** (expressed by **RMF** $\models$ X) whenver X is true in (**RMF**, $\Vdash$) for every forcing $\Vdash$ on **RMF**. An FR frame **RMF** is called an **FR** frame whenever every axiom of FR is valid in **RMF**. Moreover, if **RMF** is an **FR** frame, we say an **FR** model (**RMF**, $\Vdash$) as an ***FR*** *model*.

3. Soundness and completeness

We first recall the soundness of FR in Yang (2022).

Proposition 3.1 (Yang (2022)) (Soundness) If $\vdash_{FR}$ X, then X is valid in all FR frames.

We next establish set-theoretic completeness results for FR. We say that a theory Γ is *linear* in case $\Gamma \vdash X \to Y$ or $\Gamma \vdash Y \to X$ for each pair X, Y of formulas. By an **FR**-theory, A theory Γ closed under rules of FR and containing all of its theorems is henceforth meant.

Let Γ be a linear $\mathbf{FR^t}$-theory. The *canonical* $\boldsymbol{FR^t}$ *frame* determined by Γ is defined as a structure **RMF** = (RMF_{can}, 1_{can}, 0_{can}, $\leq_{can}$, $*_{can}$, $\sim_{can}$, R_{can}) so that RMF_{can} is the set of linear $\mathbf{FR^t}$-theories extending Γ, $1_{can} = \Gamma\cup\{X : \Gamma \vdash_{FR^t} \mathbf{t} \to X\}$, $0_{can} = \Gamma\cup\{X : \Gamma \vdash_{FR^t} \mathbf{f} \to X\}$, $\leq_{can}$ is $\supseteq$ over RMF_{can}, $*_{can}$ is defined as $x *_{can} y := \{X \,\&\, Y : \text{for some } X \in x, Y \in y\}$ satisfying monoid properties corresponding to $\mathbf{FR^t}$ frames on (RMF_{can}, 1_{can}, $\leq_{can}$), $\sim_{can}$ is defined as $\sim_{can}x := \{X : {\sim}X \notin x\}$, and R_{can} is defined as:

(△) $R_{can}xyz$ iff for all formulas X, Y, $X \in x$ and $Y \in y$ imply $Y \,\&\, X \in z$.

The *canonical* ***FR***^{T} *frame* determined by Γ is the **FR**^{t} frame with $\top_{can}$ and $\bot_{can}$, such that $\top_{can} = \Gamma \cup \{X : \Gamma \vdash_{FR^T} \mathbf{T} \to X\}$ and $\bot_{can} = \Gamma \cup \{X : \Gamma \vdash_{FR^T} \mathbf{F} \to X\}$.

Notice that we take the base Γ as the linear **FR**-theory excluding nontheorems of FR

Proposition 3.2 A canonical **FR** frame is linearly ordered.

Proof: First, it is clear that a **FR** frame canonically defined is partially ordered since $\leq_{can}$ is $\supseteq$, an order reversed subset relation, over RMF_{can}. Next, we prove that $\leq_{can}$ is connected and so linearly ordered, To this end, let $x \nleq_{can} y$ and $y \nleq_{can} x$ for contradiction. There should be X, Y such that $X \in y$, $X \notin x$, $Y \in x$, and $Y \notin y$. Note that $X \to Y \in \Gamma$ or $Y \to X \in \Gamma$ because Γ is a linear theory. Assume first $X \to Y \in \Gamma$ and so $X \to Y \in y$. One has $Y \in y$, a contradiction, by (mp). Assume next $Y \to X \in \Gamma$ and so $Y \to X \in x$. Analogously, one has $X \in x$, a contradiction. □

A canonical forcing relation $\Vdash_{can}$ is defined:

$$(\triangledown)\ x \Vdash_{can} X \text{ iff } X \in x.$$

Lemma 3.3 $1_{can} \Vdash_{can} X \to Y$ iff for every $x \in RMF_{can}$, $x \Vdash_{can} X$ implies $x \Vdash_{can} Y$.

Proof: By $(\triangledown)$, one can instead verify that $X \to Y \in 1_{can}$ iff for every $x \in RMF_{can}$, $X \in x$ implies $Y \in x$. (⇒) Let $X \to Y \in 1_{can}$ and $X \in x$. Using the definition of $*_{can}$, one obtains that $X \& (X \to Y) \in x *_{can} 1_{can} = x$. Moreover, by (mp), one further has $Y \in x$ because $(X \& (X \to Y)) \to Y \in \Gamma$ and thus $(X \& (X \to Y)) \to Y \in x$. (⇐) Suppose towards a contradiction that $X \to Y \notin 1_{can}$ and thus $X \to Y \notin \Gamma$. We set x_0 as the set $\{\chi$: there is $\phi \in \Gamma$ such that $\Gamma \vdash \phi \to (X \to \chi)\}$. Then, it is immediate that $x_0 \supseteq \Gamma$, $X \in x_0$, and $Y \notin x_0$ because otherwise $\Gamma \vdash \phi \to (X \to Y)$ and so $\Gamma \vdash X \to Y$, a contradiction, by (mp).

Finally note that the Linear Extension Property (Cintula, Horčík, & Noguera (2015), Theorem 12.9) assures that one can have a linear theory $x_0 \subseteq x$, where $Y \notin x$. Hence, $X \in x$ and $Y \notin x$. □

Lemma 3.4 The postulates for **FR** frames hold for the canonically defined relation R_{can}.

Proof: As examples, the postulates p_c and p_{inv} are considered here.

p_c: Let $R_{can}xxx$ and $X \in x$. We verify $X \& X \in x$. Using (CTR), one has $X \to (X \& X) \in \Gamma$ and so $X \to (X \& X) \in x$. By (mp), one further has $X \& X \in x$, as desired.

p_{inv}: We have to verify that $X \in x$ iff $X \in {\sim}{\sim}x$. Since ${\sim}{\sim}X \leftrightarrow X$, using the condition (~), one can easily prove it. (⇒) Let $X \in x$. Since ${\sim}{\sim}X \leftrightarrow X \in \Gamma$ and so ${\sim}{\sim}X \leftrightarrow X \in x$, it holds that ${\sim}{\sim}X \in x$. The condition (~) assures that ${\sim}{\sim}X \in x$ iff ${\sim}X$

$\not\in$ ~x iff X $\in$ ~~x, hence X $\in$ ~~x. ($\Leftarrow$) The proof is similar. □

Lemma 3.5 (Canonical forcing relation Lemma) $\Vdash_{can}$ is an forcing relation.

Proof: For (AHC), (min), ($\perp$), ($\wedge$), and ($\vee$), see Lemma 3.1 in Yang (2019). The condition (max) holds by Schmidt's theorem since FR is a finitary logic. Thus, here we consider the conditions (0), (1), (~), and ($\rightarrow$).

For (0), we verify that:

$$x \Vdash_{can} \mathbf{f} \text{ iff } x \leq_{can} 0_{can}.$$

By ($\triangledown$), we verify that $\mathbf{f} \in x$ and $0_{can} \subseteq x$. The claim follows from the definition of 0_{can} since 0_{can} is the least theory extending Γ with $\mathbf{f}$.

For (1), we verify that:

$$x \Vdash_{can} \mathbf{t} \text{ iff } x \leq_{can} 1_{can}.$$

The proof is analogous to that of (0).

For (~), we verify that:

$$x \Vdash_{can} {\sim}X \text{ iff } {\sim}_{can}x \nVdash_{can} X.$$

By ($\triangledown$), we verify that ~X $\in$ x iff X $\not\in$ $\sim_{can}$x. By the definition of $\sim_{can}$, the claim holds.

For ($\rightarrow$), we verify

$$x \Vdash_{can} X \rightarrow Y \text{ iff for any } y, z \in RMF, R_{can}yxz \text{ and } y \Vdash_{can} X \text{ imply } z \Vdash_{can} Y.$$

By ($\triangledown$), we verify that $X \rightarrow Y \in x$ iff for all $y, z \in X_{can}$, $R_{can}xyz$ and $X \in y$ imply $Y \in z$. ($\Rightarrow$) Let $X \rightarrow Y \in x$ and $X \in y$. Using ($\triangledown$), one has that $X \& (X \rightarrow Y) \in z$. since $(X \& (X \rightarrow Y)) \rightarrow Y \in \Gamma$ and so $(X \& (X \rightarrow Y)) \rightarrow Y \in z$. This ensures that $Y \in z$ by Lemma 3.3. ($\Leftarrow$) Suppose towards a contradiction that $X \rightarrow Y \notin x$. We verify that there are linear theories y and z such that $X \in y$ and $Y \notin z$. Let y' be the least **FR**-theory extending Γ with $\{X\}$. Let z be $y' *_{can} x$ as the set $\{\chi$: there are $\alpha \in x$, $\psi \in y'$ such that $\Gamma \vdash (\psi \& \alpha) \rightarrow \chi\}$ and so $\{\chi$: there is $\alpha \in x$ such that $\Gamma \vdash (X \& \alpha) \rightarrow \chi\}$. It is clear that $\Gamma \subseteq y' \in U_{can}$ and so $\Gamma \subseteq z \in U_{can}$. Moreover, let z' be $\{Y\}$. One can ensure that (z, z') is an exclusive pair. (If not, there is $\alpha \in x$ such that $\Gamma \vdash (X \& \alpha) \rightarrow Y$ and so $\Gamma \vdash \alpha \rightarrow (X \rightarrow Y)$ and $1_{can} \vdash \alpha \rightarrow (X \rightarrow Y)$. Then, since $x \vdash \alpha$, one has $x \vdash X \rightarrow Y$, a contradiction, by Lemma 3.3). As above, the Linear extension Property assures that one is capable of having a linear theory y such that $y \supseteq y'$ and $y *_{can} x$ (= $\{\chi$: there are $\alpha \in x$, $\psi \in y$, and $\Gamma \vdash (\psi \& \alpha) \rightarrow \chi\}$). Hence, the definitions of y and z assure that $R_{can}xyz$, $X \in y$, but $Y \notin z$ because $Y \in z'$. □

Let a model **M** for FR be an **FR** model. Then, using Lemma 3.5 and the Linear Extension Property, one can prove the strong completeness of FR.

Theorem 3.6 (Strong completeness) FR is strongly complete w.r.t. the class of all **FR**-frames.

4. Concluding Remarks

The RM semantics for FR was first introduced in Yang (2022). However, in it the completeness of FR was provided indirectly based on the algebraic completeness of FR. Based on a set-theoretical method, this paper instead established direct completeness results for FR.

The RM semantics for FR has the same structures as its algebraic semantics. We note that the RM semantics for **R** in Dunn (1986) does not have the same structure as its algebraic semantics. To provide such semantics for FR is an open problem left in this paper.

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R의 퍼지 확장을 위한 관계 의미론: 집합-이론적 접근

양 은 석

이 글은 연관 논리 R의 두 버전 $\mathbf{R^t}$, $\mathbf{R^T}$의 퍼지 확장 체계 $\mathbf{FR^t}$, $\mathbf{FR^T}$를 위한 관계 의미론에 기초해 집합-이론적인 완전성을 다룬다. 먼저 $\mathbf{R^t}$, $\mathbf{R^T}$ 각각의 확장으로서 퍼지 논리 체계 $\mathbf{FR^t}$, $\mathbf{FR^T}$와 그것들의 관계 의미론 즉 루트리-마이어 의미론을 논한다. 다음으로 이러한 체계들이 주어진 의미론에서 완전하다는 것을 집합-이론적인 방식으로 증명한다.

주요어: 관계 의미론, 루트리 마이어 의미론, **R**, 퍼지 논리, 연관 논리