Modeling Mental Qualities

* Forthcoming in the *Philosophical Review*

Andrew Y. Lee | andrew.yuan.lee@gmail.com | http://andrewyuanlee.com/
Rice University, Department of Philosophy

Abstract: Conscious experiences are characterized by mental qualities, such as those involved in seeing red, feeling pain, or smelling cinnamon. The standard framework for modeling mental qualities represents them as points in multidimensional spaces, where distances between points inversely correspond to degrees of phenomenal similarity. This paper argues that the standard framework is structurally inadequate and develops a new framework that is more powerful and flexible. The core problem for the standard framework is that it cannot capture *precision structure*: for example, consider the phenomenal contrast between seeing an object as crimson in foveal vision versus merely as red in peripheral vision. The solution I favor is to model mental qualities using regions, rather than points. I explain how this seemingly simple formal innovation not only provides a natural way of modeling precision, but also yields a variety of further theoretical fruits: it enables us to formulated novel hypotheses about the space and structures of mental qualities, formally differentiate two dimensions of phenomenal similarity, generate a quantitative model of the phenomenal sorites, and deploy a new theoretical tool in the empirical investigation of consciousness. A noteworthy consequence of my new framework is that the structure of the mental qualities of conscious experiences is fundamentally different from the structure of the perceptible qualities of external objects.

KEYWORDS: quality space, mental quality, phenomenal precision, formal phenomenology, structure of experience, phenomenal sorites, phenomenal similarity, perceptual confidence

* This paper has been enriched by generous feedback from Ned Block, David Chalmers, and Referees 0 and 1 of the *Philosophical Review*, as well as by valuable comments from Ron Avni, David Builes, Rachel Denison, Gianni Galbiati, Daniel Hoek, Uriah Kriegel, Geoffrey Lee, Matthias Michel, Joshua Myers, Thomas Nagel, Chris Scambler, and James Walsh.

Introduction

Conscious experiences are characterized by mental qualities, such as those involved in feeling pain, seeing red, or smelling cinnamon. The standard approach to modeling mental qualities is to target a particular domain, such as color experience, and to develop a *quality-space model* for that domain. These models represent mental qualities via points in multidimensional spaces, where points that lie closer in the space correspond to mental qualities that are more similar to each other. For example, in the canonical three-dimensional model of color qualities, any particular color quality can be specified via its values along the hue, saturation and brightness dimensions, and color qualities that are more similar correspond to points closer in the space. The result is a systematic model of the structure of color experience.

This framework for modeling mental qualities is highly promising, since every domain of mental qualities is structured by similarity relations. While the model for color qualities is the most developed, in recent years quality-space models have also been proposed for pain, temporal experience, auditory experience, and olfactory experience. And even domains of experience that are difficult to empirically investigate, such as emotional or cognitive experience, arguably could be modeled in such a way if only we knew the relevant structural facts. If we wish to map the structures of conscious experiences, it seems a significant part of the project will consist in constructing quality-space models across different experiential domains.[1]

This paper (1) argues that this standard framework is structurally inadequate, and (2) develops a new framework that is more powerful and flexible. The core issue for the standard framework is that it cannot capture what I call *precision structure*. Consider, for example, the phenomenal contrast between seeing an object in foveal vision as crimson versus seeing an object in peripheral vision merely as red (rather than as any particular shade of red): in such a case, your foveal visual experience is more precise than your peripheral visual experience. The problem for the standard framework is that imprecise qualities do not correspond to individual points within the standard models. Though this may at first appear to be a minor

[1] See Churchland [2005] on color experience, Klincewicz [2011] on temporal experience, Kostic [2014] on pain, Renero [2014] on auditory experience, and Young, Keller, & Rosenthal [2014] on olfactory experience. See Clark [2000] and Rosenthal [2010, 2015] for general discussions of the quality-space model framework.

technical challenge, solving the problem has significant philosophical and methodological ramifications.

The basic idea behind my new framework is to model mental qualities using regions (rather than points) in multidimensional spaces. This seemingly simple innovation not only provides a natural way of capturing imprecise experiences, but also yields a variety of other philosophical fruits. In particular, my new framework enables us to formulate novel hypotheses about the space and structure of mental qualities, formally differentiate two dimensions of phenomenal similarity, generate a quantitative model of the phenomenal sorites, and deploy a new theoretical tool for the empirical investigation of consciousness.

A core lesson of this paper is that precision is much more significant for understanding the structure of experience than has been previously appreciated. On the picture I develop, precision is a structural feature of experience, akin to similarity and magnitude. A noteworthy consequence of my new framework is that the structure of the mental qualities of conscious experiences is fundamentally different from the structure of the perceptible qualities of external objects: only mental qualities have precision structure. This subverts the common assumption that the structure of the mental qualities of conscious experiences is isomorphic to the structure of the perceptible qualities of external objects.

Alongside my philosophical aims, a methodological goal of this paper is to illustrate the prospects for *formal phenomenology*, or the application of formal tools to the study of conscious experiences. Conscious experiences are richly structured, yet there is relatively little work that attempts to capture that structure using formal models.[2] This paper aims to exhibit why such an endeavor is a promising means of advancing consciousness research.

§1 explains what precision is and why it poses a problem for standard models of mental qualities; **§2** develops my new framework for modeling mental qualities, which I call the *regional framework*; **§3** discusses applications of the regional framework to issues concerning discriminatory grain, empirical methodology, and probabilistic interpretations of precision.

[2] For some other examples of recent work applying formal tools to consciousness research, see Tononi [2007], Prentner [2019], and Kleiner [2020].

§ 1 | The Standard Framework

Let me begin by providing a brief overview of quality-space models. After that, I will explain what imprecise qualities are and why they pose a problem for standard quality-space models.

Quality-Space Models

A *quality-space model* is a model of a set of qualities and the relations between them.[3] The standard approach to modeling mental qualities is to represent qualities via points in multidimensional spaces. Under this approach, a quality-space model aims to represent every quality of a given domain so that there is one-to-one correspondence between qualities of the domain and points in the model and so that qualities that are more similar are represented by points that are less distant in the model.[4] As an example, a quality-space model for colors represents particular colors via points in a three-dimensional space such that points that are closer in the space represent colors that are more similar to each other.

Quality-space models can be developed for either the mental qualities of conscious experiences (such as phenomenal red) or the perceptible qualities of external objects (such as red).[5] But the focus of this paper is solely on mental qualities. As I discuss later, precision is not a property of perceptible qualities, so the framework I develop is inapplicable to perceptible qualities. For the rest of the paper, whenever I use the term 'quality' without qualification I will mean mental qualities.

---

[3] There is often ambiguity between talking about the formal representation of a domain of qualities and the domain of qualities itself: for example, consider Clark [2000]'s characterization of a quality-space as an "ordering of the qualities presented by a sensory modality in which relative similarities among those qualities are represented by their relative distances." To disambiguate, I will always use 'quality-space' to mean the domain of qualities and 'quality-space model' to mean the formal representation of those qualities.

[4] Quality-space models often aim to also capture magnitude relations, such that higher magnitudes of quantities are represented by higher values along the dimensions of the model (e.g., if color quality *a* is brighter than color quality *b*, then *a* has a higher value along the brightness dimension than *b*). To simplify the exposition, I focus mainly on similarity.

[5] See Byrne [2011] for more on the distinctions between different kinds of qualities.

And for brevity, I will use terms such as 'hue' rather than 'phenomenal hue' to designate the dimensions of mental qualities.

I take for granted that conscious experiences are characterized by mental qualities, meaning that what it is like to undergo a conscious experience is partly constituted by which mental qualities are instantiated by that conscious experience. Otherwise, I will be neutral on most questions about the nature of mental qualities. In particular, I remain neutral on whether mental qualities can be instantiated even in the absence of consciousness, on issues about the metaphysical relationship between mental qualities and perceptible qualities (such as issues about which theory of perception is correct), and on whether mental qualities are physical in nature. Staying neutral on these issues increases the generality of my framework, in that the framework will be compatible with a wide variety of philosophical positions.

The target of this paper is the standard *framework* for modeling mental qualities, rather than any particular *model* within that framework. A model is a formal representation of a particular domain of qualities, whereas a framework is a general schema for developing models. In order to construct a model for any particular domain of experience (such as color qualities), we must empirically investigate the relevant domain. But in order to develop an adequate framework, we must specify what kind of formal structure is required for modeling any arbitrary domain of mental qualities, regardless of how any particular model is structured. I will eventually argue that in order to capture the precision structure of mental qualities, we must make fundamental changes to the whole quality-space model framework (rather than just revisions to particular models). In §3, I will discuss how empirical methods can be used to construct models within my new framework.

There is a diverse body of literature in both philosophy and cognitive science pertaining to the modeling of mental qualities. The relevant philosophical literature has focused mainly on questions about the relationship between mental qualities and perceptible qualities and on the individuation of mental qualities.[6] The relevant

[6] For some classic and contemporary philosophical texts addressing these questions, see Goodman [1954] and Clark [2000], Rosenthal [2000, 2015]. For discussion of the role of phenomenology in psychophysical theorizing, see Horst [2005]. For recent discussions of the nature of phenomenal qualities, see Coates & Coleman [2015].

cognitive science literature has focused mainly on issues concerning the psychophysical relations between physical stimuli and mental qualities and the challenges in measuring mental qualities.[7] However, in both disciplines, research that directly addresses the modeling of mental qualities tends to focus on similarity structure, leaving out precision structure.

This lacuna is likely due to the common assumption that models of mental qualities are isomorphic to models of perceptible qualities. The standard methodology for constructing a model of mental qualities is to first use data concerning perceptual discrimination judgments to construct a model of perceptible qualities, and to then extrapolate from that to a model of mental qualities. The justification is that mental qualities can be individuated by their perceptual roles: in particular, it seems that subjects make perceptual discriminations between physical stimuli only on the basis of being in mental states with different mental qualities. Since perceptible qualities (as opposed to physical stimuli)[8] are also individuated by subjects' perceptual discriminatory capacities, this suggests that the structure of mental quality-spaces is isomorphic to the structure of perceptible quality-spaces.[9] I will eventually argue that this isomorphism thesis is false: models of mental qualities require more structure than models of perceptible qualities.

For the rest of the paper, I will call the approach to modeling mental qualities outlined above the *standard framework*, and I will call any particular model within

---

[7] For a classic text in psychophysics, see Fechner [1860]. For more recent overviews of psychophysics, see Murray [1993] and Gescheider [1997]. For an overview of the application of measurement theory to psychological models, see Luce & Krumhansl [1988]. For discussion of models of color qualities in particular, see Logvinenko [2015]. For an approach to geometrically modeling concepts that shares some (though not all) formal features with my framework, see Gärdenfors [2014].

[8] Perceptible qualities (e.g., colors) should be distinguished from physical stimuli (e.g., specific wavelengths of light): only the former are held to be isomorphic to mental qualities. See Clark [2000] for more on this distinction.

[9] As examples, Sellars [1963] talks of an "isomorphism of acts of sense and material things," Churchland [2007] talks of a "homomorphism" from the space of objective colors to "the internal structure of human phenomenological color space," and Rosenthal [2016] talks of extrapolating "from the quality space of perceptual discriminations to an isomorphic quality space of the mental qualities that enable those discriminations."

that framework a *standard model*. More specifically, we can think of standard models as formally specifiable via a set of points (representing individual qualities) and a distance metric (where distances between points are inversely correlated with degrees of similarity between the qualities represented by those points). In what follows, I will explain why the standard framework is structurally inadequate.

Imprecise Experiences

Consider your color experience in foveal vision versus in peripheral vision. In foveal vision, you see an object as a specific shade of red, such as crimson. But in peripheral vision, you no longer see it as any specific shade of red, but instead just as red. It is not merely that you see the object as a different specific shade of red across the two cases. Instead, even if your peripheral color experience represents its object as having *some* specific shade of red or other, it leaves open *which* shade of red that might be, and it is compatible with your peripheral color experience that you are seeing *any* given shade of red within a certain range.[10] Speaking somewhat metaphorically, peripheral color experience is less sharp and crisp than foveal color experience. This difference in phenomenal character is what I call *precision*.[11]

Though I will focus on precision with respect to color experience across foveal and peripheral vision, there are other examples that may also be used to illustrate the phenomenon. For example, consider the contrast between your color experience of an object that is far away versus nearby, your spatial visual experience with vision correction lenses versus without vision correction, or your tactile experience touching a texture with your fingertips versus touching a texture with your back. In each case, the former experience is more precise than the latter.

Precision is different in kind from familiar phenomenal properties such as hue, loudness, or painfulness. Those phenomenal properties correspond to dimensions of quality-space models. But as we will see, precision cannot be captured in the same way (at least not without making substantive theoretical assumptions). As

---

[10] See Hansen, Prajecus, & Gegenfurtner [2009] for psychophysical evidence of this contrast. See Strasburger et al [2011] for a recent review of the science of peripheral vision.

[11] I take the term 'precision' from Block [2015]. A number of philosophers have used the terms 'determinacy' or 'determinability' for what I call 'precision', but I will later explain why 'precision' is better.

some initial evidence for this, consider how a visual experience could (arguably) be precise with respect to color yet imprecise with respect to shape or how qualities even across different modalities can be similar with respect to precision. Instead of thinking of precision as merely another phenomenal dimension, it is more apt to think of precision as a structural feature of experience (like similarity or magnitude). This hints at why modeling precision requires modifying the entire quality-space model framework rather than just patching up particular models.[12]

Questions about the nature of precision depend on more fundamental issues in the philosophy of perception. For representationalists, it is natural to think that precision is a matter of the granularity of the representational content of an experience. For naïve realists, it is natural to think that precision is a matter of being perceptually acquainted with more determinate properties of external objects. For qualia theorists, it is natural to think that precision is a structural property of phenomenal character somewhat akin to the resolution of an image. Since the quality-space model framework is largely theory neutral, the issues discussed in this paper will be relevant to theorists across the board. At times I will talk of mental qualities representing perceptible qualities, but my discussion could likewise be framed in terms of other relations (such as perceptual acquaintance or causal correspondence).[13]

There is a mix of literature across both philosophy and cognitive science pertaining to imprecise qualities. In philosophy, there has been recent work examining imprecise qualities in connection with philosophical theories of perception, generic phenomenology, and the representational contents of experience.[14] However, these discussions have not directly addressed how precision structure relates to similarity

[12] Notably, Block [2015] says that the notion of "phenomenal precision [is] obscure…we have a well-developed science of perception but very little science of the phenomenology of perception." This paper aims to help bridge this gap.

[13] For a general overview of theories of perception, see Crane & French [2017]. For argument against representationalism about precision, see Block [2015]. For argument against naïve realism about precision, see Cutter [2019].

[14] See Block [2015] and Cutter [2019] on precision's implications for the nature of perception, Fink [2015] and Fazekas & Overgaard [2018] on precision in relation to generic phenomenology and empirical methods for investigating consciousness, Stazicker [2011], Morrison [2016], Munton [2016], and Nanay [forthcoming] on the relation between precision and representation, and Hellie [2005] and Pelling [2008] on precision and perceptual discrimination.

structure or how to integrate imprecise qualities into the quality-space model framework. In cognitive science, this includes research on perceptual discrimination capacities (such as spatial resolution and tactile discrimination), on how attention affects perceptual discrimination, and on the neurophysiological properties underlying these differences.[15] However, these discussions tend to focus on perceptual capacities and their functional roles, rather than the structure of the mental qualities associated with those perceptual capacities.

Precision vs. Determinability vs. Noise

Before explaining why precision poses a problem for the standard framework, I need to first address two conceptual traps in thinking about precision. In what follows, I explain why precision is conceptually distinct from two other phenomena: namely, determinability and noise.

Let us start with *determinability*, or the relation between determinates and determinables. A determinate is a way for a determinable to be instantiated, and determinables may be thought of as disjunctions of determinates. To see why precision and determinability are independent, consider the maximally determinate phenomenal property characterizing the particular peripheral color experience you are currently undergoing. That property is maximally determinate since there is only one way for that property to be instantiated, but it is also imprecise since it does not represent any specific shade of color. Conversely, consider the determinable phenomenal property that has as determinates the precise color phenomenal properties characterizing your foveal visual experiences when looking at a series of color chips in optimal conditions. That property is determinable since there are multiple ways for that property to be instantiated but each of its determinates is precise since they all represent specific shades of color. Since there are both maximally determinate

[15] See Intriligator & Cavanagh [2001] and Block [2012] on the relationship between visual resolution and visual attention, Anton-Erxleben & Carrasco [2013] on attention, spatial resolution and the role of receptor cell density for perceptual discrimination, Bruns et al [2014] on tactile spatial resolution, Debats et al [2012] on measuring haptic precision, Denison [2017] on precision in relation to perceptual uncertainty, and Gescheider [1997] for an overview of the role of perceptual discrimination thresholds in psychophysics.

imprecise phenomenal properties and determinable phenomenal properties with only precise determinates, precision and determinability are doubly dissociable.

Precision and determinability are liable to be confused because of systematic ambiguities in natural language. For example, 'phenomenal red' can mean either a determinable phenomenal property that has precise determinates (e.g., the disjunction of phenomenal crimson, phenomenal scarlet, etc.) or a determinate phenomenal property that is imprecise. The risk of confusing precision and determinability is also due to the fact that precision of mental qualities is inversely correlated with determinability of the perceptible qualities represented by those mental qualities: for example, the determinate crimson is represented by a precise red experience whereas the determinable red is represented by an imprecise red experience.[16]

These conceptual distinctions are worth highlighting, for taking precision to be merely a matter of determinability masks the importance of precision for understanding the structure of experience. Developing a model of determinable phenomenal properties would not be particularly interesting, since questions about which determinable properties there could be are somewhat analogous to questions about which disjunctive properties there could be. By contrast, we will soon address a variety of philosophically substantive questions about the space and structure of imprecise qualities, the implications of precision for phenomenal similarity, how precision relates to the nature of discriminability and the empirical investigation of consciousness, and how precision challenges the thesis that mental quality-spaces are isomorphic to perceptible quality-spaces.

At this point, some might suggest the rather radical hypothesis that imprecise qualities involve the instantiation of a determinable without the instantiation of any of its determinates.[17] But even if this view were correct about the metaphysics, it would still be important to conceptually distinguish precision from determinability. Otherwise, not only would there be systematic terminological ambiguities of the kind mentioned above, but we would not even have the concepts needed to resolve those ambiguities. Consider, for example, the claim that an experience instantiates the determinable phenomenal red. On this view, that claim could mean either that

---

[16] Note that representationalism does not undermine my point, since the property of representing a determinable property need not itself be a determinable property.

[17] See Wilson [2013] for a general defense of this metaphysical view.

the experience is precise and instantiates one amongst many determinate phenomenal properties or it could mean that the experience is imprecise and instantiates the determinable phenomenal red (without instantiating any of its determinates). This means that no matter what view one favors about the metaphysics of precision, one ought to be sensitive to the distinction between precision and determinability.[18]

Let us turn now to *noise*, or the degree of random variation within a psychophysical channel. A *psychophysical channel* is a causal pathway connecting a stimulus (such as a color chip) to a signal (such as a color experience). The noisier a channel, the less the channel's source determines its signal.[19] A hypothesis that some might find attractive is that precision is simply the phenomenal manifestation of psychophysical noise.[20] Though this is an interesting empirical hypothesis, it is important to appreciate why precision and noise are at least conceptually distinct.

Suppose Aya's color perception system stochastically generates one of a number of precise color experiences upon detection of a color. If Aya looks at a scarlet color chip ten times, she might first have a scarlet experience, then a crimson experience, then a vermillion experience, and so forth. Aya's color experiences are precise yet result from noisy psychophysical processes. Conversely, suppose Beto lacks foveal vision but otherwise has an extremely reliable color perception system. If Beto looks at a scarlet color chip ten times, then he has the exact same color experience every single time, but that color experience is the same as the one you would have when looking at the scarlet chip via peripheral vision. Beto's color experiences are imprecise yet result from unnoisy psychophysical processes.

In psychophysics, measurement theory, and related fields, the term 'precision' refers to the reciprocal of the degree of variance in a set of measurements, which does in fact vary inversely with noise. But *statistical precision* (the statistical property of measurements just defined) is distinct from *phenomenal precision* (the

---

[18] Imprecision is also sometimes characterized as vagueness. For brevity, I will simply note that vagueness is typically understood as a property of terms or concepts whereas imprecision is a property of phenomenal properties or experiences, that terms and concepts for imprecise qualities can be sharp, and that none of the theories of precision mentioned previously appeal to vagueness. For an overview of vagueness, see Williamson [1994].

[19] See Gescheider [1997] for a classic text on psychophysics. See Dretske [1981] for a philosophical discussion of noise from the perspective of information theory.

[20] See Hellie [2005] for an example of this kind of view.

phenomenal property of mental qualities this paper focuses on). As an analogy, imagine a gun firing a round of bullets at a target: statistical precision corresponds to how closely clustered the bullet holes are to each other, whereas phenomenal precision (inversely) corresponds to the surface area of the bullet holes. These structural differences hint at the formal treatment of precision that will come later.

Although precision and noise are conceptually distinct, it is empirically possible that imprecise mental qualities result from noisy psychophysical processes. Suppose our cognitive systems are structured so that whenever a psychophysical channel is noisy (prior to the generation of an experience), it reliably generates an imprecise mental quality (rather than stochastically generates a precise mental quality). For example, suppose you see a scarlet color chip, but the noise in your color perception system renders it uncertain whether the chip is scarlet or vermillion or crimson, so your color perception system generates a red experience. If such a hypothesis is correct, then imprecise mental qualities would be the phenomenal manifestation of noise (though as a matter of empirical fact, rather than conceptual necessity).[21] For the purposes of this paper, I will stay neutral on this hypothesis. Though resolving the issue would provide us with a richer picture of the psychophysical processes resulting in imprecise experiences, it would still leave open how to best model imprecise mental qualities using the quality-space model framework.

The Problem of Precision

We are now in position to see why standard models cannot capture precision. The core problem is that in the standard framework, individual qualities are represented by individual points in quality-space models, but no individual points in such models are adequate for representing imprecise qualities. Putting it another way, the standard framework takes mental qualities to be specifiable by a single

[21] At first pass, it may seem that in such a scenario the source (i.e., the scarlet color chip) reliably determines the signal (i.e., the red mental quality), which suggests that the psychophysical process as a whole is unnoisy. But as we will see in the next section, there are many different red mental qualities, and which imprecise red experience is generated may depend on random variation in the psychophysical channel (rather than merely the source).

value along each of the dimensions of the model, but imprecise qualities seem to instead correspond to ranges of values along those dimensions.[22]

It may be tempting to attempt to solve the problem by simply adding an extra dimension (representing degree of precision) to existing models. But such a framework would still require assigning individual values along ordinary dimensions (such as hue) to imprecise qualities, would be unable to capture the similarity relations between imprecise mental qualities belonging to different quality-spaces, and (as we will discuss in the next section) would be unable to accommodate views that allow the precision of one dimension (such as hue) to vary independently of the precision of other dimensions (such as brightness). To capture precision, we need more than just tweaks to existing models; instead, we need structural changes to the whole quality-space model framework.

In light of these problems, some might wonder whether the standard framework was always meant to be an idealization. What if the standard framework was never intended to capture all mental qualities? However, there is a paucity of literature in both philosophy and cognitive science on how to model imprecise qualities, and the isomorphism claims frequently advanced in discussions of quality-spaces are evidence that the standard framework has been presumed to be representationally adequate. Nevertheless, the principal aim of this paper is to build on existing research on modeling mental qualities. Whether or not the regional framework has been implicitly assumed, it has certainly not been explicitly developed, and its implications for the structure of experience have not been widely appreciated.

§ 2 | The Regional Framework

The *regional framework* models mental qualities using *regions*, or sets of points, rather than just individual points. In what follows, I develop the formal structure of the regional framework, address some of the technical challenges, and explain why the framework is theoretically fruitful and philosophically significant.

---

[22] A further motivation concerns the idea that more precise mental qualities have more specific contents (in that they eliminate more possibilities). This aspect of precision is not captured by the standard framework, since every mental quality simply corresponds to a single point. By contrast, it will be obvious how the regional framework does better.

The Basic Formal Structure

Let us begin by identifying the formal structure of the standard framework so that we can see the contrast with the regional framework. Any standard model requires a way of representing individual qualities and a way of representing degrees of similarity between qualities. In light of this, we can think of standard models as comprised of a pair of elements: a set S of points (representing individual qualities) and a distance metric d (where greater distances[23] represent lower degrees of phenomenal similarity).[24]

There are three main desiderata when constructing a model in the standard framework. First, the points in the model should stand in one-to-one correspondence with the qualities in the target quality-space. Second, points that are more distant in the model should represent qualities that are less phenomenally similar to each other. Third, points should have distance zero just in case the qualities represented by those points are phenomenally identical. If these constraints are satisfied, then the structural properties of the model mirror the structural properties of the quality-space. Since standard models cannot capture precision structure, none of these desiderata can be fully satisfied. The challenge in what follows is to show that the analogous desiderata can be satisfied using the regional framework.

The regional framework represents experiences using *regions*, or sets of points, rather than individual points. The size of a region inversely corresponds to the degree of precision of the quality represented by that region. More specifically: mental quality x_A is more precise than x_B just in case region A (corresponding to x_A) is smaller than region B (corresponding to x_B). There is a question of whether we can develop a richer characterization of the relationship between degrees of precision and region sizes, such as whether differences or ratios between region sizes corre-

---

[23] I assume that a metric is needed to capture relations of phenomenal similarity, though see Gert [2017] for an opposing view. Note that this assumption makes the task of developing the regional framework harder, since it is unobvious how to develop a metric over regions that corresponds to phenomenal similarity. For discussion of how metric structure can be extracted from ordinal data, see Beals et al [1968].

[24] Note that the set S of points and metric d suffice to determine the dimensionality of the space. See Hurevicz & Wallman [1948] on measures of dimensionality.

spond to differences or ratios between degrees of precision. The answer to this question will depend on one's view about the structure of precision itself: in particular, on whether precision has ordinal, interval, or ratio structure.[25] I will be neutral on this issue in this paper: the regional framework is compatible with any of these views about the structure of precision.

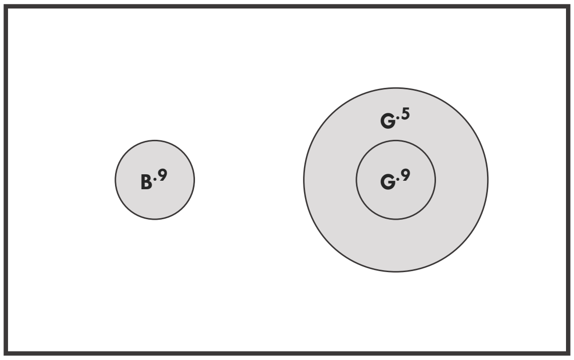
We saw above that standard models can be specified with just a set $\mathcal{S}$ of points and a distance metric d. The regional framework requires adding more structure. To specify a regional model, we need not only the set $\mathcal{S}$ of points and the point-distance metric d, but also a set $\mathcal{R}$ of regions (meaning a subset of the powerset of $\mathcal{S}$), a measure μ on $\mathcal{S}$, and two metrics on regions which will be formally defined later. Over the course of this section, I will explain each of these elements in detail.[26]

In the rest of the paper, I will denote regions using small-caps letters (ex: region A), perceptible qualities using lowercase letters (ex: perceptible quality a), mental qualities using lowercase x with a subscript for the corresponding region (ex: mental quality x_{A}), functions using lowercase script letters (ex: the measure μ) and elements of the regional framework using uppercase script letters (ex: the set $\mathcal{S}$ of points). I will also illustrate regions using diagrams like the one below, where precision values are denoted using numbers from 0–1 in superscript, with higher numbers denoting higher degrees of precision:[27]

[25] See Stevens [1946] for discussion of these different measurement scales.

[26] Strictly speaking, specifying a regional model requires specifying only $\mathcal{S}$, $\mathcal{R}$, d, and μ. This is because the metrics on regions ($qual$ and $prec$) can be recovered from d and μ. However, it will be better for expository purposes to discuss all of the elements of the regional framework in the same way.

[27] The box represents a space, the bounded shapes represent regions, and the regions are denoted using the linguistic convention described above. Note that the interpretation of the dimensions in the figures is not important.



$B^{.9}$ represents a relatively precise blue quality.

$G^{.9}$ represents a relatively precise green quality.

$G^{.5}$ represents a relatively imprecise green quality.

FIGURE 1: A pictorial representation of some regions in a regional model.

The rest of this section proceeds as follows: First, I discuss the spaces of regional models, focusing mostly on the set $\mathcal{S}$ of points and the point-distance metric d. Second, I discuss the structure of imprecise qualities, focusing mostly on the set $\mathcal{R}$ of regions. Third, I discuss similarity with respect to qualitative character versus similarity with respect to precision, focusing mostly on the new metrics on regions, $qual$ and $prec$. Along the way, I explain how the formal framework interacts with a variety of philosophical issues concerning the quality-space model framework.

The Structure of the Space

The space of any given quality-space model is determined by the set $\mathcal{S}$ of points and the point-distance metric d. Since these are the elements that characterize standard models, most of the theoretical issues concerning these elements have already been addressed in prior work. However, there is one new question concerning these elements that arises with the regional framework: namely, whether the spaces in regional models should be discrete or continuous. In a discrete space there is a finite number of points between any two distinct points; in a continuous space there is a continuum of points between any two distinct points. A discrete model would enable us to simply "export" all the points of the set $\mathcal{S}$ of a standard model into a regional model.[28] In contrast, a continuous model would require a new set $\mathcal{S}$ that has infinitely many points.

Though discrete models are finite, continuous models are arguably better because of their flexibility. A continuous model can capture qualities at arbitrary levels of precision, including even qualities with greater precision than the most precise qualities for human experiences. Furthermore, it is mathematically simpler to specify certain kinds of formal constraints on regions in continuous spaces, which is an advantage that will be relevant in the next subsection. For these reasons, I will assume for the rest of the paper that the models under consideration are continuous (though most of the discussion will apply also to discrete models). Note that while continuous spaces have infinitely many points, they may still be bounded, in that all points lie within a fixed distance from each other. Consider how the interval of real numbers from 0 to 1 is continuous but bounded by the limit points 0 and 1.

In a continuous model, individual points are probably best thought of as idealizations: they are the maximally specific values of the dimensions of a quality-space, even if it turns out that no mental qualities actually correspond to regions comprised of a single point. This may raise the worry that regional models have more structure than is strictly necessary. However, scientific models often idealize, especially when doing so leads to simpler formalisms. For example, suppose that all physical objects are composed of particles, that there is a finite set of elementary particles, and that all elementary particles have discrete mass values. Then there are some mass values that no physical objects could have, since no combination of elementary particles would generate that mass value. Nevertheless, it may still be useful to represent mass using real numbers (which have continuous structures). By the same lights, it may be useful to model mental qualities using continuous spaces even if it turns out that points in the spaces are idealizations.

Permissible Regions

Any regional model must specify the set $\mathcal{R}$ of *regions*, which are subsets of the set $\mathcal{S}$ of points. As we will see, this new element generates new philosophical

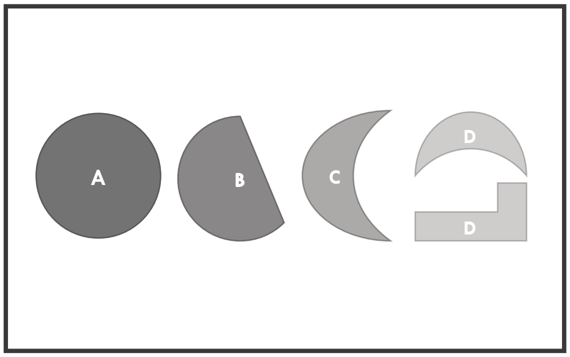
[28] I assume that standard models must have a finite number of points, since that is needed to satisfy the desideratum of one-to-one correspondence between points and qualities (at least if we assume that there are finitely many mental qualities for any given quality-space).

questions about the space and structure of imprecise qualities that are difficult to even formulate without the appropriate theoretical resources.

Why is there a need for $\mathcal{R}$ at all? Some might think that we could simply take imprecise qualities to be represented by the subsets of $\mathcal{S}$, avoiding the need to posit a whole new set $\mathcal{R}$. However, $\mathcal{R}$ is a crucial element in the regional framework, for it allows us to distinguish different theories of the structure and space of imprecise qualities. As we will see, it may not be the case that every subset of $\mathcal{S}$ corresponds to a possible mental quality. Consequently, we need a way of distinguishing regions, which can be any subset of $\mathcal{S}$, from *permissible regions*, which are the subsets of $\mathcal{S}$ that are members of $\mathcal{R}$ and that are to be interpreted as representing possible mental qualities. To put it another way, different specifications of $\mathcal{R}$ correspond to different theories of the structure and space of imprecise qualities.

Since there are as many ways of specifying $\mathcal{R}$ as there are sets of subsets of $\mathcal{S}$, it will be useful to focus on formal constraints on $\mathcal{R}$ that permit different kinds of permissible regions. One class of constraints concerns *sizes*: what are the minimal and maximal sizes of permissible regions? Questions about size constraints correspond to questions about degrees of precision. For example, we might wonder whether there could be maximally imprecise qualities whose regions cover entire quality-spaces (such as an experience representing something as merely colored) or whether there could be super-precise qualities whose regions are arbitrarily small (such as an experience representing something as red$_{345.71}$). For the most part I will set aside questions about region sizes, though we will later address how to empirically investigate the region sizes for particular mental qualities.

The more interesting class of constraints on $\mathcal{R}$ concerns *shapes*: which kinds of shapes can permissible regions take? Questions about shape constraints correspond to questions about the structure of precision. These questions are more difficult to formulate linguistically, and it will take a bit of space to explain what different shape constraints look like and how they correspond to different philosophical theories. As examples, we will focus on three different formal constraints on region shapes: CONNECTIVITY, CONVEXITY, and UNIFORMITY. Each of these constraints may be thought of as a distinct hypothesis about the space and structure of imprecise qualities. To get a feel for which kinds of shapes these formal constraints permit or exclude, consider the four sample regions in the diagram below:



CONNECTIVITY permits A, B, C
CONVEXITY permits A, B
UNIFORMITY permits A

FIGURE 2: Formal constraints on permissible regions.

The most permissive of the three constraints is CONNECTIVITY, according to which all permissible regions are connected.[29] A region is *connected* just in case it has no discontinuities, meaning any two points in the region can be connected via a continuous path of adjacent points that all lie within the region. CONNECTIVITY excludes region D from FIGURE 2 but permits regions A, B, and C. More generally, the principle rules out mental qualities that represent arbitrary collections of perceptible qualities, such as a mental quality that represents something as either crimson or aquamarine (but nothing else). However, CONNECTIVITY still permits regions that are rather strangely shaped, such as region C.

This brings us to CONVEXITY, according to which all permissible regions are convex.[30] A region is *convex* just in case for every pair of points within the region, every point on the straight line-segment that joins the pair of points is also within the region. In other words, any region must contain all points within the straight lines connecting its boundaries. CONVEXITY excludes regions C and D but permits regions A and B. This means that CONVEXITY is more restrictive than CONNECTIVITY but still allows irregularly shaped regions, such as region B.

[29] FORMAL DEFINITION: if $\mathrm{A} \in \mathcal{R}$, then A is not the union of two disjoint open sets of $\mathcal{S}$, where A is open just in case $\forall a \in \mathrm{A}, \exists \epsilon > 0$ where the ball $\mathrm{B}(s, \epsilon) = \{x \in \mathcal{S} \mid d(x, s) < \epsilon\} \in \mathcal{S}$.

[30] FORMAL DEFINITION: if $\mathrm{A} \in \mathcal{R}$, then $\forall a, c \in \mathrm{S}$ such that $d(a, c) > 0$, A contains all points $b \in \mathcal{S}$ such that $d(a, b) + d(b, c) = d(a, c)$.

The last constraint within our examples is UNIFORMITY, according to which all permissible regions are balls.[31] A region is a *ball* just in case it includes all and only the set of points that are within a certain distance from a center.[32] UNIFORMITY excludes regions B, C, and D, permitting only region A, meaning that the principle excludes all the irregular regions permitted by the previous constraints. But UNIFORMITY also excludes the possibility of mental qualities that differ in their degree of imprecision across different dimensions, such as a color experience that is precise in hue but imprecise in brightness. This might make some worry that UNIFORMITY is too restrictive.[33]

The preceding discussion merely scratches the surface. My present aim is not to evaluate which of these formal constraints is most plausible, but instead to show how the regional framework provides us with tools that enable more rigorous and systematic theorizing about the structure of experience. And in doing so, the regional framework enables us to formulate interesting hypotheses about the space and structure of mental qualities that are hard to express without the appropriate framework. Consider how difficult it would be to demarcate these hypotheses or even formulate the relevant questions using only natural language.

Qualitative Similarity

In the standard framework, it is straightforward to define a point-distance metric d that represents the similarity relations between different mental qualities. By contrast, identifying the right similarity metric in the regional framework is much more challenging. For the regional framework, we need a metric that takes as input regions of arbitrary size and shape yet still produces outputs that systematically correspond to degrees of phenomenal similarity. More specifically, let x_A, x_B, and x_C be mental qualities and let A, B, and C be the corresponding regions: if a metric

[31] FORMAL DEFINITION: if $A \in \mathcal{R}$, then $\exists s \in \mathcal{S}$ and $\epsilon > 0$ where $A = \{x \in \mathcal{S} \mid d(x, s) < \epsilon\}$.

[32] In a continuous n-dimensional space, balls are usually bounded by n-dimensional spheres, though note that balls near the boundary points of a space may have non-spherical shapes.

[33] There are other natural constraints in between UNIFORMITY and CONVEXITY, such as the constraints that all permissible regions are (1) regular polygons or (2) ellipsoids.

m is to serve as the similarity metric for the regional framework, it ought to satisfy the following constraints:[34]

(1) if x_A is phenomenally identical to x_B, then $m(A, B) = 0$.

(2) if x_A is more phenomenally similar to x_B than to x_C, then $m(A, B) > m(A, C)$.

Before addressing candidates for metrics, let me first mention the *measure*, which will be used (alongside the point-distance metric d) to construct our candidates for metrics on regions. The measure μ takes as input a subset of $\mathcal{S}$ and outputs a size value.[35] Since the size of a region represents its degree of imprecision, μ can be thought of as telling us the degree of imprecision of the mental quality represented by a region. In discrete models, it is natural to simply take the size of a region to be the number of points in the region. However, such a measure does not work well in continuous models, since regions in continuous models will typically have infinitely many points. Instead, we need the standard mathematical measure: the Lebesgue measure, which is a generalization of the notions of length, area, volume, and so forth. The mathematical details of the Lebesgue measure are not particularly philosophically relevant. What is important is that the Lebesgue measure has the properties we would intuitively want a measure to have: in particular, it produces intuitive size values in continuous spaces of arbitrary dimensionality.[36] With μ on the table, we are in position to examine candidates for metrics on regions.

A first pass is to turn to the default way of determining distance between regions: the least distance function, which takes the distance between regions A and

---

[34] Actually, I will later argue that these constraints ought to be relativized to different kinds of phenomenal similarity. But this point does not matter for the moment.

[35] Why is the measure on the set $\mathcal{S}$ of points rather than the set $\mathcal{R}$ of regions? Since a measure takes as input a subset of a set, a measure on $\mathcal{R}$ would output the sizes of sets of regions (rather than the sizes of regions). Consequently, determining the size of a region in $\mathcal{R}$ requires measuring the corresponding subset of $\mathcal{S}$.

[36] See Tao [2011] for a detailed overview of the Lebesgue measure.

B to be the lowest distance value between any pair of points between A and B.[37] However, suppose that A and B are distinct but overlap (meaning that A and B contain some but not all of the same points). Since A and B are distinct, they represent distinct qualities, such as phenomenal red and phenomenal reddish-orange. But since A and B overlap, there is a point in A that has distance zero to a point in B. As a consequence, the least distance function has the result that the distance from A to B is zero. Since A and B represent distinct qualities, and since distance zero represents phenomenal identity, we have the wrong result.

A more promising candidate is the average distance function, avg, which takes the distance between regions A and B to be the average distance from points in A to points in B. More specifically, $avg(\text{A}, \text{B})$ takes a point in A, determines the average distance between that point and all the points in B, and repeats the procedure for every point in A, and then averages the averaged distance values.[38] Since the average distance function is sensitive to all the points in A and B, it is an improvement over the least distance function. Yet avg is also inadequate. Consider the average distance from any region A to itself. So long as A contains more than one point, there will be some pair of points a and b in A where $d(a, b) > 0$. As a consequence, the average distance from a region to itself must also be non-zero. But non-zero distance values represent phenomenally distinct qualities. This means we get the absurd result that all mental qualities that are not maximally precise are not maximally similar to themselves. The source of the problem is that avg does not differentiate between points that are shared between regions versus points that belong to only one region. When regions are disjoint, avg delivers intuitive results. But when regions overlap, avg runs into problems. We need a new metric that is sensitive to this difference.

Now we can turn to my proposal, which I call the 'qualitative similarity metric', or '$qual$'. Though $qual$ might appear somewhat complex at first glance, it is

---

[37] FORMAL DEFINITION: Let $inf(\text{A})$ denote the infimum of set A. Then the least distance from A to B = $inf\{d(a, b) \mid a \in \text{A}, b \in \text{B}\}$.

[38] FORMAL DEFINITION: The formal definition of avg differs in discrete versus continuous spaces. For discrete spaces, $avg(\text{A}, \text{B}) = \sum_{a\in\text{A}} \sum_{b\in\text{B}} d(a, b) \div \mu(\text{A})\mu(\text{B})$. But since summations over divergent series are undefined, in continuous spaces summation must be replaced with integration, where $avg(\text{A}, \text{B}) = \int_{\text{A}}(\int_{\text{B}} d(a, b)\mathrm{d}\mu(b))\mathrm{d}\mu(a) \div \mu(\text{A})\mu(\text{B})$. For a more comprehensive discussion of these average distance functions, see Fujita [2013].

simple and intuitive after getting a feel for how it works. The metric is motivated by a simple observation: any case involving overlapping regions can be treated as a pair of cases involving disjoint regions. To see how it works, let us begin with an example illustrated by the diagram below:

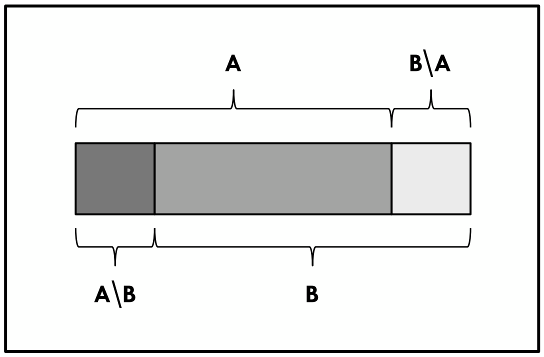


FIGURE 3: A partially overlaps with B.

Suppose we wish to determine the similarity between the mental qualities represented by A and B. The *qual* metric works by taking the average distance from A to the subregion of B that does not overlap with A, the average distance from B to the subregion of A that does not overlap with B, and then taking the weighted average of the two average distances (with the weighting in proportion to the relative sizes of A and B). In other words, we first find avg(A, B\A), then find avg(B, A\B), and then determine the weighted average of the two results. The *qual* metric is illustrated pictorially in the diagram below:

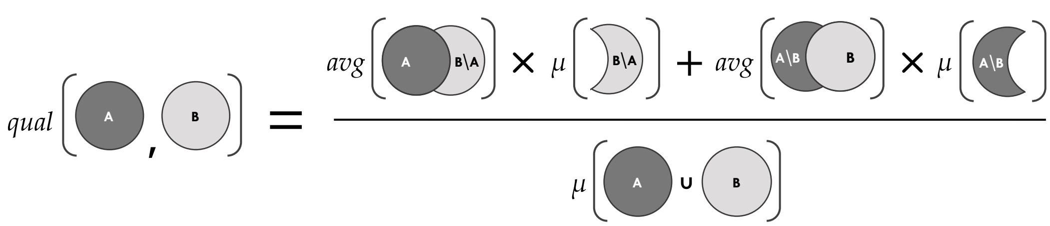


FIGURE 4: The qualitative similarity metric.

And the metric is expressed formally in the following equation:[39]

$$qual(\text{A, B}) = \frac{avg(\text{A, B}\setminus\text{A}) \times \mu(\text{B}\setminus\text{A}) + avg(\text{B, A}\setminus\text{B}) \times \mu(\text{A}\setminus\text{B})}{\mu(\text{A}\cup\text{B})}$$

To verify that $qual$ produces the right results, we can check the general behavior of the metric across cases of identity, disjointness, containment, and overlap between regions: IDENTITY: $qual(\text{A, B}) = 0$. This means that whenever two qualities are represented by the same region, those qualities are represented as maximally similar. DISJOINTNESS: $qual(\text{A, B}) = avg(\text{A, B})$. This means that $qual$ collapses to avg if A and B do not overlap—and as observed above, avg produces intuitively correct verdicts in such cases. CONTAINMENT: $qual(\text{A, B}) > 0$. This means that qualities that differ in precision are never represented as maximally similar. Moreover—presuming A contains B—increasing the difference in size between A and B also increases their distance, meaning that $qual$ predicts that similarity decreases as the difference in precision increases. OVERLAP: $qual(\text{A, B}) > 0$. In fact, the more A and B overlap, the lower the distance between them, with the limit of the distance approaching zero as degree of overlap tends towards identity. More generally, $qual$ entails that regions with points that are more distant are themselves more distant.

These results are evidence that $qual$ satisfies the desiderata on a metric outlined earlier. This is a significant finding: neither the least distance metric nor the average distance metric produced results that plausibly correspond to degrees of similarity, and other standard candidates for metrics on regions fare poorly as well.[40] A bonus is that $qual$ works even in spaces whose dimensions lack ordinal structure.

---

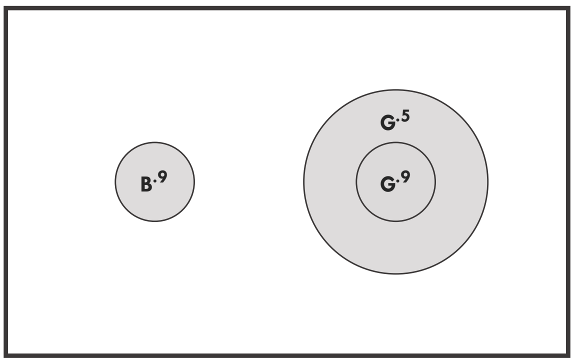
[39] Why are the numerators $\mu(\text{B}\setminus\text{A})$ and $\mu(\text{A}\setminus\text{B})$ rather than simply $\mu(\text{B})$ and $\mu(\text{A})$? The reason is that this enables $qual$ to treat overlapping parts of regions differently from disjoint parts of regions, avoiding the issues we encountered with avg. If the numerators were $\mu(\text{B})$ and $\mu(\text{A})$, then the function would not entail that $qual(\text{A, A}) = 0$. For more detailed discussion of the formal properties of $qual$, see Fujita [2013].

[40] For example, the Hausdorff metric is another standard metric, but its output depends only on certain local maxima and minima points, rather than all points in the relevant regions. See Rockellar & Wets [2005] for discussion of this metric.

This point is important, for any framework for modeling mental qualities must be able to accommodate dimensions such as hue or pitch.

Precision Similarity

Nevertheless, there remains an aspect of phenomenal similarity that *qual* does not capture. Consider again the diagram from the start of this section:



$B^{.9}$ represents a relatively precise blue quality.

$G^{.9}$ represents a relatively precise green quality.

$G^{.5}$ represents a relatively imprecise green quality.

FIGURE 5: $qual(B^{.9}, G^{.9}) = qual(B^{.9}, G^{.5})$.

Remember that when regions are disjoint, *qual* works the same way as *avg*. Because of this, $qual(B^{.9}, G^{.9}) = qual(B^{.9}, G^{.5})$, meaning that *qual* takes the precise blue quality to be as similar to the precise green quality as to the imprecise green quality. But $B^{.9}$ is more similar to $G^{.9}$ than it is to $G^{.5}$. After all, $B^{.9}$ and $G^{.9}$ are similar with respect to precision, whereas $B^{.9}$ and $G^{.5}$ are not. Therefore, *qual* is not directly sensitive to the relative sizes of regions. How should we proceed in light of this result?

My view is that this result is a feature rather than a flaw, for we can now formally distinguish two different dimensions of phenomenal similarity. On the one hand, two mental qualities might be similar with respect to *qualitative character*, or the aspects of phenomenal character that correspond to phenomenal properties such as hue, loudness, and painfulness and that correspond to the dimensions of quality-space models. On the other hand, two mental qualities might be similar with respect to *precision*, a structural feature of experience that does not correspond to any particular dimension. It is easy to get an intuitive grip on the difference between these kinds of phenomenal similarity. Consider, for example, the difference in kind of similarity when comparing a precise phenomenal red quality to a precise phenomenal orange quality versus when comparing a precise phenomenal red quality to an imprecise phenomenal red quality. Our analysis of $qual$ indicates that it is the right metric for qualitative similarity. But it must be supplemented with another metric, which I will call '$prec$', that measures precision similarity.

Recall that degree of imprecision is represented by size of region. In light of this, a core desideratum for $prec$ is that the difference in size between regions A and B should be greater than that between A and C just in case the precision similarity between mental qualities x_A and x_B is less than that between x_A and x_C. This criterion leads to two natural options for defining $prec$: (1) in terms of absolute difference (i.e., the absolute value of the difference in size between A and B), or (2) in terms of absolute ratio (i.e., the ratio in size between A and B, where the numerator is the size of the smaller region). Though both options satisfy the desideratum mentioned above, I think the absolute ratio measure is somewhat more plausible. If we were to adopt the absolute difference measure, then we would get the result that very precise mental qualities can differ only marginally in precision similarity (since the absolute difference in sizes between small regions will never be very large). But on the contrary, it seems plausible that there can be as much precision dissimilarity between precise experiences as between imprecise experiences.

If $prec$ is a matter of absolute ratio, then it is easy to define (it is also easy to define $prec$ if we instead adopt the absolute difference option). Let $min(\text{A, B})$ be the size of the smaller region between A and B and $max(\text{A, B})$ be the size of the larger region between A and B. Then we can formally define $prec$ as follows:

$$prec(\text{A, B}) = \frac{min(\text{A, B})}{max(\text{A, B})}$$

And the diagram for *prec* is straightforward:

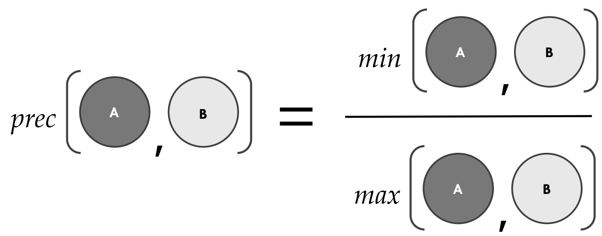


FIGURE 6: The precision similarity metric.

With *prec*, we have a solution to the problem from earlier. The situation involved a precise blue experience $\text{B}^{.9}$, a precise green experience $\text{G}^{.9}$, and an imprecise green experience $\text{G}^{.5}$. We noted that *qual* does not capture the precision similarity between the precise blue experience and the precise green experience. But *prec* is designed to capture this second dimension of similarity (without encroaching on the qualitative similarity captured by *qual*).[41] The result is expressed formally below:

$$qual(\text{B}^{.9}, \text{G}^{.9}) = qual(\text{B}^{.9}, \text{G}^{.5})$$
$$prec(\text{B}^{.9}, \text{G}^{.9}) > prec(\text{B}^{.9}, \text{G}^{.5})$$

I mentioned previously that the regional framework assumes only that precision has ordinal structure, whereby we can talk of one precision value being greater than another. The fact that *prec* appeals to ratios may lead some to wonder whether the framework is tacitly assuming that precision furthermore has ratio structure, whereby we can talk of ratios between precision values. This is a natural worry, but it is off the mark: even those who think that precision has only ordinal structure can agree that *prec* produces the right verdicts on precision similarity. On the current analysis, mental quality x_{A} is more precise than x_{B} just in case region A is smaller than B, and the precision similarity between x_{A} and x_{B} is greater than that between x_{A} and x_{C} just in case the absolute ratio of A to B is greater than that of A to C. The appeal to ratios occurs only on the formal side of these biconditionals: there is no talk of ratios between precision values. In other words, while *prec* appeals to the ratio structure of region sizes to determine degrees of precision similarity, it does

[41] Strictly speaking, *prec* is a pseudometric (rather than a metric) since any metric m must satisfy the condition that $\mu(\text{A}, \text{B}) = 0$ just in case $\text{A} = \text{B}$.

not ascribe ratio structure to precision itself. As an analogy, consider how even though regions in the regional framework are built out of points, imprecise experiences need not themselves be thought of as being built out of precise experiences.

The separation of $qual$ and $prec$ may lead some to wonder whether there is an overall phenomenal similarity metric that captures phenomenal similarity simpliciter. Speaking for myself, I am skeptical that there is an objective fact of the matter about how to compare the kinds of phenomenal similarity tracked by $qual$ and $prec$. However, the regional framework itself is designed to accommodate a wide range of views. For those who think there are such objective facts, we could always develop a more general metric that captures both qualitative and precision similarity. For example, a simple method would be to sum the outputs of $qual$ and $prec$, with a weighting to scale their values relative to each other. However, even if there is an objectively correct metric for overall phenomenal similarity, it remains plausible that $qual$ and $prec$ track two natural kinds of phenomenal similarity.

At this point, some might wonder how it even makes sense to develop a formal framework for modeling mental qualities before empirically investigating those mental qualities. It is true that in order to determine how particular mental qualities map to particular regions within a particular model, we must empirically investigate the similarity relations between those mental qualities. But in order to construct a model using that empirical data, we need a general framework for mapping collections of data to formal structures. In other words, empirical investigation of precision requires having a theoretical framework for interpreting those empirical results, and developing that theoretical framework requires the kinds of arguments I have made in this paper.

§ 3 | Applications

In this final section, I explain how the regional framework sheds light on the connections between precision and discriminatory grain, how empirical methods can be used to construct regional models for particular domains of mental qualities, and how the regional framework can be extended to capture probabilistic interpretations of precision. These discussions will be brief, but they will still illustrate some of the power and potential of the regional framework.

Discriminatory Grain

In the quality-space literature, it is standardly taken for granted that we can investigate mental qualities via their functional roles. Consider how judgments of similarity or distinctness between perceptible qualities are taken to be evidence of phenomenal similarity or phenomenal distinctness between the corresponding mental qualities. Given this, we might ask whether there is a distinctive functional role associated with precision. This section explains and explores the following conjecture: precision correlates with discriminatory grain.[42]

As a first pass, think of *discriminatory grain* as the number of discriminations a subject can make over a set of physical objects using a particular perceptual capacity.[43] If Aya can make 36 discriminations over 50 color chips while Beto can make only 9 discriminations over the same 50 color chips, then Aya's color discrimination capacities are more fine-grained than Beto's. Though this initial gloss elicits the intuitive connection between precision and discriminatory grain, it also raises some questions. Suppose a and b are perceptible qualities, that x_A and x_B are the corresponding mental qualities, and that A and B are the regions for x_A and x_B. How should we think about discriminability when A and B overlap?

Let us say that a is *strongly discriminable* from b just in case A is disjoint from B, and that a is *weakly discriminable* from b just in case A partially overlaps with B. If a and b are strongly discriminable, then the subject can be sure (taking their experience at face value) that a is distinct from b. If a and b are only weakly discriminable, then the subject cannot be sure (solely on the basis of their experience) whether a and b are distinct. Nevertheless, weak discriminability still entails that the way a looks is distinct from the way b looks (since A and B are distinct regions and distinct regions represent distinct qualities). This distinction between strong and weak discriminability is useful for understanding the sense in which perceptual *in*discriminability is (or is not) transitive. Let us say that a and b are *indiscriminable* (in either

[42] See Hellie [2005] and Pelling [2008] for similar philosophical analyses of discriminability and imprecise experiences. This section may be thought of as building on this prior work by providing a formal treatment of the issues using the resources of the regional framework.
[43] Other factors that may influence discriminatory grain include environment and noise. For simplicity, I will assume that measures of discriminatory grain are always relativized to optimal environments and factor out (or hold constant) noise.

sense) just in case they are not discriminable. Then strong indiscriminability is non-transitive while weak indiscriminability is transitive.

We can now use the regional framework to formulate a more rigorous measure of discriminatory grain: the *discriminatory grain* of a perceptual capacity corresponds to the size of the maximally large set of disjoint permissible regions associated with that perceptual capacity. Putting it another way, discriminatory grain is a maximization of strong discriminations conditional on a minimization (i.e., zero) of weak discriminations. In the example from earlier, Aya's color discriminatory capacities involve precise color experiences corresponding to relatively small regions, meaning that there will be a relatively large maximal set of disjoint permissible regions associated with Aya's color perception capacities. By contrast, Beto's color discriminatory capacities involve imprecise color experiences corresponding to relatively large regions, meaning that there will be a relatively small maximal set of disjoint permissible regions associated with Beto's color perception capacities. To picture this, imagine taking the set of regions associated with a perceptual capacity and using those regions to "fill up" the corresponding space with as many regions as possible, as illustrated below:

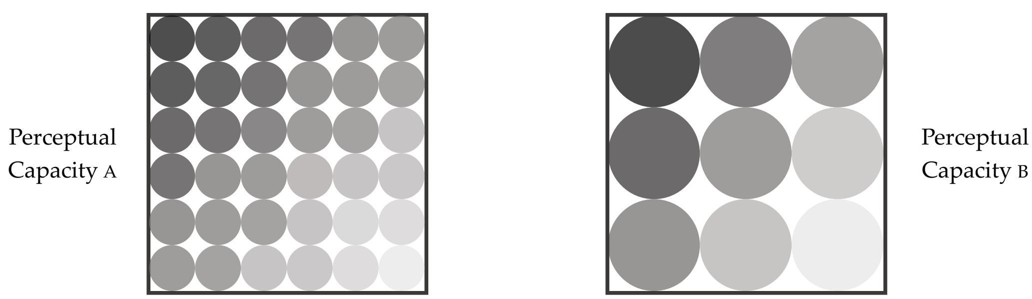


FIGURE 7: A has greater discriminatory grain than B.

These connections between precision and discriminability can be applied to the puzzle of the phenomenal sorites.[44] The puzzle concerns cases where a subject cannot discriminate (on the basis of their perceptual experiences) *a* from *b* or *b* from *c* yet can discriminate *a* from *c*. A natural hypothesis is that phenomenal sorites cases

[44] See Fara [2001], Hellie [2005], and Pelling [2008] on phenomenal sorites cases.

involve situations where there is partial overlap between A and B, partial overlap between B and C, and non-maximal overlap between A and C. Equivalently, these are situations where a and b are not strongly discriminable, b and c are not strongly discriminable, and a and c are at least weakly discriminable. In fact, the regional framework not only enables us to identify the general conditions for when a phenomenal sorites might occur, but also provides the basis for developing a probabilistic model of those conditions. To do that, we need to first define degree of overlap:

$$overlap(\mathrm{A}, \mathrm{B}) = \frac{\mu(\mathrm{A} \cap \mathrm{B})}{½(\mu(\mathrm{A}) + \mu(\mathrm{B}))}$$

This definition of $overlap$ yields a value of 1 just in case A and B are identical and a value of 0 just in case A and B are disjoint. With $overlap$, we can sharpen the conjecture that precision is connected to discriminatory grain by developing principles connecting degree of overlap between regions to the likelihood of a phenomenal sorites case occurring with the experiences represented by those regions. In particular, let $p(\mathrm{A}, \mathrm{B}, \mathrm{C})$ be a function that is intended to capture the probability that the subject judges (on the basis of experiences corresponding to regions A, B, and C) that $a = b$ and $b = c$ but $a \neq c$. Now consider the following principles, which are plausible constraints for developing the function p:

(1) the greater $overlap(\mathrm{A}, \mathrm{B})$ and $overlap(\mathrm{B}, \mathrm{C})$, the greater $p(\mathrm{A}, \mathrm{B}, \mathrm{C})$.
(2) if either $overlap(\mathrm{A}, \mathrm{B}) = 0$ or $overlap(\mathrm{B}, \mathrm{C}) = 0$, then $p(\mathrm{A}, \mathrm{B}, \mathrm{C}) = 0$.
(3) the greater $overlap(\mathrm{A}, \mathrm{C})$, the smaller $p(\mathrm{A}, \mathrm{B}, \mathrm{C})$.
(4) if $overlap(\mathrm{A}, \mathrm{C}) = 1$, then $p(\mathrm{A}, \mathrm{B}, \mathrm{C}) = 0$.
(5) $0 \leq p(\mathrm{A}, \mathrm{B}, \mathrm{C}) \leq 1$.

These constraints are all satisfied by the formula below, which uses degree of overlap to model the probability that a set of pairwise discriminatory judgments (of a, b, and c) based off of mental qualities represented by regions A, B, and C yields a phenomenal sorites case:

$$p(\mathrm{A}, \mathrm{B}, \mathrm{C}) \simeq overlap(\mathrm{A}, \mathrm{B}) \times overlap(\mathrm{B}, \mathrm{C}) \times (1 - overlap(\mathrm{A}, \mathrm{C}))$$

The symbol '$\simeq$' is intentionally ambiguous, for there is a question of whether we must also accommodate other factors that influence perceptual judgments, such as epistemic norms, basing abilities, psychophysical noise, and so forth. If we can abstract away from such factors to isolate the relationship between precision and discriminability, then the formula may capture a linear relationship. If such abstractions are unfeasible, or if the relationship between precision and discriminability is non-linear even after abstraction, then the formula may capture only a monotonic relationship. But even in the latter case, the regional framework gives us powerful formal tools for understanding the phenomenal sorites (and more generally, the relationship between precision and discriminability).

A noteworthy result is that even the mental qualities captured by standard models are best represented by regions, since phenomenal sorites cases occur even for the maximally fine discriminatory capacities of normal humans. This illuminates the fact that the regional framework is a general framework for modeling all mental qualities, rather than a specialized tool for dealing with a particular kind of mental quality. Putting it another way: the regional framework serves as the successor (rather than merely a supplement) to the standard framework.

Empirical Methodology

What kind of methodology is required to construct a regional model for a particular domain of mental qualities? Suppose we start with a standard model that captures a set of precise mental qualities. The initial step is to convert that standard model into a regional model representing those same qualities. This requires mapping points in the standard model to regions in the regional model such that (1) every point in the former is mapped to a distinct region in the latter, (2) similarity relations are preserved, and (3) the boundaries of the space remain the same. The basic procedure for this conversion is relatively straightforward, though there is a question of how to determine when two regions overlap. Let us set that issue aside for the moment—we will return to it soon.

After converting the standard model to a regional model, the challenge is to identify mental qualities that are more imprecise and to map them onto regions, with the constraints that distance values (outputted by $qual$ and $prec$) correspond to degrees of qualitative and precision similarity (between the relevant mental qualities). In other words, for any particular mental quality, we need methods for identifying its corresponding region's location, size, and shape. The easiest of these tasks is location, for the methodology used to determine locations of points in standard models generalizes to determining locations of regions in regional models. Because of this, I will focus on region sizes and region shapes.

To determine the size of a region for a particular mental quality, we need to partition perceptual capacities into subclasses, where subclasses are individuated by fineness of grain of perceptual discrimination abilities. For example, since color discrimination is more coarse-grained outside of the center of the visual field, color perception might be divided into subclasses corresponding to angular distance from the center of the visual field. More precisely, these subclasses can be individuated by distances between just-noticeably different physical stimuli, where two physical stimuli that are just-noticeably different for a more fine-grained perceptual capacity subclass will be indiscriminable for a more coarse-grained perceptual capacity subclass. Then, given our conjecture connecting precision to discriminatory grain, each set of just-noticeably different stimuli identified by each perceptual capacity subclass will correspond to a set of mental qualities at a different degree of precision. In other words, this method enables us to identify different classes of mental qualities corresponding to regions of different sizes.[45]

The remaining challenge is to devise a method for identifying region shapes. In the previous section, we discussed the idea that there are different patterns of discrimination for cases involving overlapping versus disjoint regions. In particular, disjoint regions should never lead to judgments that two objects are the same, whereas overlapping regions should sometimes lead to such judgments, with the frequency of such judgments monotonically increasing with the degree of overlap between regions. These asymmetries in discrimination patterns provide a way of triangulating the shape of a region.

The most straightforward procedure for identifying region shapes would be to induce experiences that involve both an imprecise quality and a precise quality. This would require collecting data from subjects making perceptual judgments about objects that are perceived using different perceptual capacity subclasses. For

---

[45] There are numerous studies in psychophysics that implement this kind of procedure. See, for example, Strasburger [2011] on peripheral vision or Bruns et al [2014] on touch.

example, this might require subjects to make perceptual judgments about a color chip that they see via foveal vision versus color chips that they see via peripheral vision. If the subject judges the chips to be the same color, then that is evidence that the region representing the mental quality associated with the fine-grained perceptual capacity subclass at least partially overlaps with the region representing the mental quality associated with the coarse-grained perceptual capacity subclass. By collecting enough data to identify which regions representing precise qualities overlap with the region representing the imprecise quality, this method would enable us to approximate the region shapes for imprecise mental qualities.

From Regions to Fields

According to what we can call the *probabilistic view*, precision is associated with probabilistic representation.[46] I will be neutral on whether the probabilistic view is correct, but I will briefly mention how the regional framework can be naturally extended to accommodate it.

Let me start by explaining the basic structure underlying the probabilistic view. Suppose that a subject perceives an object via a color experience, and let $p(a)$ be the probability (according to the content of that color quality) that the object has color a. On the probabilistic view, it is possible that $p(\text{blue}) = .2$, $p(\text{teal}) = .6$, and $p(\text{green}) = .2$, even when blue, teal, and green correspond to regions of the same size. Since these probabilities can vary even when region size is held fixed, the regional framework does not have the formal structure needed to model this kind of situation. The limitation is due to the fact that regions are "flat," in that they do not assign different weights to different points. Speaking more pictorially, the probabilistic view takes imprecision to not only have range (which is captured by regions) but also depth (which corresponds to the probabilities associated with any given point).

Nevertheless, the probabilistic view can be accommodated by moving to a *field framework*. A *field* on a space of points is an assignment of values to every point in the space.[47] Equivalently, a field is a function from points to values. A region may

[46] See Morrison [2016] and Munton [2016] for argument in favor of the probabilistic view. See Denison [2017] and Nanay [forthcoming] for arguments against.

[47] This notion of 'field' should be distinguished from the algebraic notion, where a *field* is an algebraic structure that permits addition, subtraction, multiplication, and division.

be thought of as a special case of a field, where the region assigns to each point either (say) $\top$ (if the point is inside the region) or $\bot$ (if the point is outside the region). But fields can assign a broader range of values (where a natural constraint for the probabilistic view is that the integral of the field must equal 1). Under the field framework, the precision structure of a mental quality would correspond to the structure of the field for that quality. For example, the color quality mentioned above would be represented by a field that assigns higher values in the teal region of the quality-space than in the blue and green regions.[48]

As with the regional framework, there are novel challenges in identifying which formal constraints on fields are most plausible, in developing the right similarity metrics on fields, and in constructing particular field models. Solving these challenges is beyond the scope of this paper, but it is obvious how the regional framework provides a basis for developing these more sophisticated formal tools. Thus, if the probabilistic view is correct, the evolution of the quality-space model framework progresses from points to regions to fields.[49]

Conclusion

A regional model can be specified via a tuple: $<\mathcal{S}, d, \mathcal{R}, \mu, qual, prec>$, where $\mathcal{S}$ is a set of points, d is the point-distance metric, $\mathcal{R}$ is a set of regions (representing possible mental qualities), μ is a measure on $\mathcal{S}$ (representing degrees of precision), $qual$ is a metric on regions (representing qualitative similarity), and $prec$ is a metric on regions (representing precision similarity). The structure of the regional framework makes it more powerful and flexible than the standard framework, and enables us to formulate novel hypotheses about the space and structure of mental qual-

[48] I suggest fields rather than probability distributions because the relevant probability distributions would range over perceptible qualities rather than mental qualities. Given this, I think it is better to think of mental qualities themselves as modeled by fields, even if their contents can be modeled by probability distributions over perceptible qualities.

[49] A more speculative suggestion: the field framework might also be used to model *phenomenal vivacity* (consider the phenomenal contrast between perceptual and imaginative experiences), where more vivid mental qualities would correspond to fields with greater integrals.

ities, formally differentiate two distinct dimensions of phenomenal similarity, generate a probabilistic model of the phenomenal sorites, and deploy a new theoretical tool in the empirical investigation of consciousness.

A striking consequence is that the structure of the mental qualities of conscious experiences is fundamentally different from the structure of the perceptible qualities of external objects. Whereas both mental qualities and perceptible qualities have similarity structure, only mental qualities have precision structure. Because of this, the isomorphism thesis discussed at the beginning of this paper is false. The standard framework remains adequate for modeling perceptible qualities. But capturing the precision structure of mental qualities requires implementing structural changes to the entire quality-space model framework.

The fact that the isomorphism thesis is false may strike some as puzzling. It may be tempting to think that perceptible qualities *must* have precision structure, since perceptible precision may simply be defined as whatever property of perceptible qualities corresponds to phenomenal precision. The problem with this line of reasoning is that the perceptible correlate of phenomenal precision is determinability: imprecise mental qualities correspond to more determinable perceptible qualities. However, mental qualities *also* have determinability structure, and (as we saw in §1) determinability and precision are mutually dissociable. Therefore, the fact that there is a perceptible correlate of precision does not vindicate the isomorphism thesis. And as far as I can see, there are no other credible candidates for anything playing the role of perceptible precision. Because of this, mental qualities have strictly more structure than perceptible qualities. This illustrates, once again, the importance of precision for understanding the structure of conscious experiences.

For those who will undertake future investigations of mental qualities, there is good reason to adopt the regional framework from the outset. By doing so, one attains increased power and flexibility with little added cost. Though I focused earlier on how a standard model can be transformed into a regional model, there is no methodological advantage to starting with a standard model and subsequently converting it. In fact, the very procedures deployed to construct standard models can likewise be deployed to construct regional models. And as we saw earlier, even the mental qualities captured by standard models are often better captured by regions, since even those mental qualities give rise to phenomenal sorites cases.

A goal of this paper has been to exhibit the prospects for formal phenomenology, or the application of formal tools to the study of conscious experiences. By formally modeling conscious experiences, we not only sharpen our understanding of how conscious experiences are structured, but also progress our understanding of the mind without needing to resolve long-standing theoretical disputes about the mind-body problem, the nature of perception, or the physical correlates of consciousness. In my view, this kind of project is one of the most promising ways of moving consciousness research towards a systematic science.

References

Anton-Erxleben, K., Herrmann, K. & Carrasco, M. (2013). Independent effects of Ayaptation and attention on perceived speed. Psychological Science, 24 (2), 150-159. 10.1177/0956797612449178

Beals, Richard ; Krantz, David H. & Tversky, Amos (1968). Foundations of multidimensional scaling. *Psychological Review* 75 (2):127-142.

Block, Ned (2012). The Grain of Vision and the Grain of Attention. *Thought: A Journal of Philosophy* 1 (3):170-184.

Block, Ned (2015). The Puzzle of Perceptual Precision. Open Mind.

Bruns, P., Camargo, C. J., Campanella, H., Esteve, J., Dinse, H. R., & Röder, B. (2014). Tactile acuity charts: a reliable measure of spatial acuity. PloS one, 9(2), e87384. doi:10.1371/journal.pone.0087384

Byrne, Alex (2011). Sensory qualities, sensible qualities, sensational qualities. In Brian McLaughlin, Ansgar Beckermann & Sven Walter (eds.), *The Oxford Handbook of Philosophy of Mind*. Oxford: Oxford University Press.

Churchland, Paul (2005). Chimerical colors: Some phenomenological predictions from cognitive neuroscience. Philosophical Psychology 18 (5):527-560

Churchland, Paul (2007). On the reality (and diversity) of objective colors: How color ‐ qualia space is a map of reflectance ‐ profile space. Philosophy of Science 74 (2):119-149.

Clark, Austen (2000). *A Theory of Sentience*. Oxford University Press.

Coates, Paul & Coleman, Sam (eds.) (2015). *Phenomenal Qualities: Sense, Perception, and Consciousness*. Oxford University Press UK.

Crane, Tim and French, Craig, "The Problem of Perception", The Stanford Encyclopedia of Philosophy (Spring 2017 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2017/entries/perception-problem/>.

Cutter, Brian (2019). Indeterminate perception and colour relationism. *Analysis* 79 (1):25-34.

Debats, N. B., Kingma, I., Beek, P. J., & Smeets, J. B. (2012). Moving the weber fraction: the perceptual precision for moment of inertia increases with exploration force. *PloS one*, *7*(9), e42941. doi:10.1371/journal.pone.0042941

Denison, Rachel N. (2017). Precision, Not Confidence, Describes the Uncertainty of Perceptual Experience: Comment on John Morrison's "Perceptual Confidence". *Analytic Philosophy* 58 (1):58-70.

Dretske, Fred (1981). Knowledge and the Flow of Information. MIT Press.

Gescheider, G. A. (1997). Psychophysics: The fundamentals (3rd ed.). Mahwah, NJ, US: Lawrence Erlbaum Associates Publishers.

Fara, Delia Graff (2001). Phenomenal continua and the sorites. *Mind* 110 (440):905-935.

Fazekas, Peter & Overgaard, Morten (2018). A Multi-Factor Account of Degrees of Awareness. *Cognitive Science* 42 (6):1833-1859.

Fechner, G., 1860, *Elements of Psychophysics*, H.E. Adler (trans.), New York: Holt, Reinhart & Winston, 1966.

Fink, Sascha Benjamin (2015). Phenomenal precision and Some Possible Pitfalls – A Commentary on Ned Block. Open MIND.

Fujita, O. Japan J. Indust. Appl. Math. (2013) 30: 1. https://doi.org/10.1007/s13160-012-0089-6

Gärdenfors, Peter (2014). The Geometry of Meaning: Semantics Based on Conceptual Spaces. MIT Press.

Gert, Joshua (2017) Quality spaces: Mental and physical, Philosophical Psychology, 30:5, 525-544, DOI: 10.1080/09515089.2017.1295303

Gescheider, G. A. (1997). Psychophysics: The fundamentals (3rd ed.). Mahwah, NJ, US: Lawrence Erlbaum Associates Publishers.

Goodman, Nelson (1951). *The Structure of Appearance*. Harvard University Press.

Hansen, Thorsten, Pracejus, Lars, & Gegenfurtner, Karl R. (2009). Color perception in the intermediate periphery of the visual field. *Journal of Vision* 2009;9(4):26. doi: https://doi.org/10.1167/9.4.26.

Hellie, Benj (2005). Noise and perceptual indiscriminability. Mind 114 (455):481-508.

Hilbert, David R. & Byrne, Alex (2008). Basic sensible qualities and the structure of appearance. Philosophical Issues 18 (1):385-405.

Horst, Steven (2005). Phenomenology and psychophysics. Phenomenology and the Cognitive Sciences 4 (1):1-21.

W. Hurevicz, G. Wallman, (1948) Dimension theory, Princeton University Press.

Intriligator, J., and P. Cavanagh. ''The Spatial Resolution of Visual Attention.'' Cognitive Psychology 43 (2001): 171–216.

Kind, Amy (2017). Imaginative Vividness. *Journal of the American Philosophical Association* 3 (1):32-50.

Kleiner, Johannes. 2020. Mathematical Models of Consciousness. Forthcoming in Entropy. arXiv preprint arXiv:1907.03223.

Klincewicz M. (2011) Quality Space Model of Temporal Perception. In: Vatakis A., Esposito A., Giagkou M., Cummins F., Papadelis G. (eds) Multidisciplinary Aspects of Time and Time Perception. Lecture Notes in Computer Science, vol 6789. Springer, Berlin, Heidelberg

Kostic, Daniel (2012). The vagueness constraint and the quality space for pain. *Philosophical Psychology* 25 (6):929-939.

Logvinenko, A. D. (2015). The geometric structure of color. *Journal of Vision, 15*(1). Article ID 16.

Luce, R. D., & Krumhansl, C. L. (1988). Measurement, scaling, and psychophysics. In R. C. Atkinson, R. J. Herrnstein, G. Lindzey, & R. D. Luce (Eds.), *Stevens' handbook of experimental psychology: Perception and motivation; Learning and cognition* (pp. 3-74). Oxford, England: John Wiley & Sons.

Morrison, John (2016). Perceptual Confidence. Analytic Philosophy 57 (1):15-48.

Munton, Jessie (2016). Visual Confidences and Direct Perceptual Justification. *Philosophical Topics* 44 (2):301-326.

Murray, David J. (1993). A perspective for viewing the history of psychophysics. Behavioral and Brain Sciences 16 (1):115.

Nanay, Bence (forthcoming). Perceiving Indeterminately. *Thought: A Journal of Philosophy*.

Pelling, C. (2008). Exactness, Inexactness, and the Non-Transitivity of Perceptual Indiscriminability. Synthese, 164(2), 289–312. JSTOR.

Renero, Adriana (2014). Consciousness and Mental Qualities for Auditory Sensations. *Journal of Consciousness Studies* 21 (9-10):179-204.

Rockafellar, R. Tyrrell; Wets, Roger J-B (2005). *Variational Analysis*. Springer-Verlag. p. 117. ISBN 3-540-62772-3.

Rosenthal, David (2010), "How to Think about Mental Qualities," Philosophical Issues: Philosophy of Mind, 20 (October): 368-393.

Rosenthal, David (2015), “Quality Spaces and Sensory Modalities,” in The Nature of Phenomenal Qualities: Sense, Perception, and Consciousness, ed. Paul Coates and Sam Coleman, Oxford: Oxford University Press, pp. 33-65.

Rosenthal, D. M. (2016). Quality spaces, relocation, and grain. In J. R. O’Shea (Ed.), Wilfrid Sellars and his legacy. Oxford: Oxford University Press.

Sellars, W. (1963). Science, perception and reality. London: Routledge.

Stazicker, James (2011). Attention, Visual Consciousness and Indeterminability. *Mind and Language* 26 (2):156-184.

Stevens, S.S. (1946). On the Theory of Scales of Measurement. *Science* 103, 677-680.

Strasburger, Hans; Rentschler, Ingo; Jüttner, Martin (2011). "Peripheral vision and pattern recognition: A review". *Journal of Vision*. 11 (5): 13.

Tao, Terence (2011). *An Introduction to Measure Theory Vol. 126*. Amer Mathematical Society.

Thomas, Nigel J.T., "Mental Imagery", The Stanford Encyclopedia of Philosophy (Spring 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2018/entries/mental-imagery/>.

Tononi, Giulio (2007). The information integration theory of consciousness. In Max Velmans & Susan Schneider (eds.), *The Blackwell Companion to Consciousness*. Blackwell. pp. 287--299.

Williamson, Timothy (1994). *Vagueness*. Routledge.

Wilson, Jessica M. (2013). A Determinable-Based Account of Metaphysical Indeterminability. *Inquiry: An Interdisciplinary Journal of Philosophy* 56 (4):359-385.

Young, Benjamin D., Keller, Andreas, and Rosenthal, David (2014), ‘Quality Space Theory in Olfaction’, in Frontiers in Consciousness Research.