Toward a New Analysis of Conditional Probability

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Introduction

This article is mainly composed of two discussions. First, we introduce *event-expressions,* individual constants of a new type referring to events, not things. We learn this from Davidson's famous formulation of event-sentences (§§1-3). The first half or more of this article is occupied with this discussion.

The second half is devoted to the creation of a new analysis of probability, especially conditional probability. As seen in the author's other work (Kaneko 2022), probability theory can be reconstructed with predicate logic in the name of inductive logic. This article includes its development and its application to the notion of conditional probability (§§4-6).

§1.[1] Events Expressed by $\exists xFx$

1.1. Davidson's Theory of Events

It is widely known that Donald Davidson (1917-2003)[2] gave the logical formulation of $\exists xFx$ to the action-sentence (Davidson 1967a, p.118), which we call the *event- sentence* hereafter.

(1)[3] We recognize that there is no singular term [i.e. no proper name (or individual constant) nor definite description] referring to a mosquito in "[4]There is a mosquito in here" when we realize that the truth of this sentence is not impugned if there are two mosquitos in the room. […] On the present analysis, ordinary sentences about event[s] like "Doris capsized the canoe yesterda[y]" are related to particular events in just the same way "There is a mosquito in here" is related to particular mosquitos. It is no less true that Doris capsized the canoe yesterday if she capsized it a dozen times than if she capsized it once […] (Davidson 1969, p.167; see also Kaneko 2022, p.168 ⑮)

"There is a mosquito in here" is normally formulated as $\exists x((x\ is\ a\ mosquito) \wedge (x\ is\ in\ here))$, and called a *particular sentence* (cf. Kaneko 2021, sec,95), so Davidson here claimed that it refers to a "*particular*" mosquito.

"Doris capsized the canoe yesterday" has the same structure, according to Davidson:

(2) $\exists x(Capsized\ (Dorris, the\ canoe, x) \wedge On\ (x, yesterday)$[5]

This is read as "*There is an event x such that x is capsizing of the canoe by Dorris and x occurred yesterday*" (cf. Davidson 1967a, p.118). Through this translation, we may say that the event sentence like "Doris capsized the canoe yesterday" has the same structure as "There is a mosquito in here."

1.2. Henkin Witnessing Axiom

Most researchers regarded this analysis of Davidson's as *existential*, because "there is" and ∃ are customarily regarded as such. But the author[6] has come to think that this perception of ∃, or more precisely ∃*x*, is unnecessary. The author owes it to Leon Hankin (1921-2006), who famously introduced the so-called *Henkin witnessing axiom*, which we dub *HK* hereafter (Henkin 1949; see also Barwise et al. 2011, p.547; Kaneko 2021, p.100):

(3) $$\frac{\exists x \Phi x}{\Phi \alpha}\text{HK}$$

"*α*", a meta-variable in a meta-language, is paraphrased into "c_Φ" in an object-language, and defined as the individual constant true of *Φa*. Semantically:

(4) To secure *HK*, the interpretation of *α* in *HK*, or "c_Φ", that is, the object γ_j to which $I_i(c_\Phi)$ is assigned must satisfy[7] *Φa*.

In terms of syntax, *HK*, which is surely a syntactical axiom, simply means $\{\exists x\Phi x\}\vdash\Phi c_\Phi$. And naturally, $\{\Phi c_\Phi\}\vdash\exists x\Phi x$, by a simple, direct application of the existential generalization (cf. Kaneko 2021, sec.109). Therefore, as long as we admit *HK*, through Herbrand's deduction theorem (cf. Kaneko 2021, pt.III ch.3), we gain:

(5) $\vdash \exists x\Phi x \longleftrightarrow \Phi c_\Phi$

This simply means that "c_Φ" can take the place of "∃*x*" plus "*x*" in *Φ*.

1.3. Indefinite Pronouns

The result of (5) convinced the author, who has long been wondering if Frege's idea "∃ *is a second-order function*" is wrong (Kaneko 2023a, sec.10.3), that ∃, or more precisely, ∃x is an indefinite pronoun like "something" or "somebody" (Kaneko 2021, sec.102).

Russel (1905, passim) partially noticed it, but finally agreed with Frege (Moore, 1936, p.90; Cartwright 1960, p.632; Whitehead 1910, p.17). The author's belief in ∃ as an indefinite pronoun is now presented only modestly by a few logicians, it seems.

(6) Variables are a kind of auxiliary symbol. In some ways[,] they behave like individual constants, since they can appear in the list of arguments immediately following [i.e. put after] a predicate or function symbol. But in other way[,] they are very different from individual constants. In particular, their semantic function [i.e. role] is not to refer to objects. Rather, they are placeholders that indicate relationships between quantifiers and the argument positions of various predicates. (Barwise et al. 2011, pp.230-231)

In addition to this passage, reading the discussion about *multiple quantification* (Iida 1987, pp.35f.) also known as *mixed quantification* (Barwise et al. 2011, pp.302) is enough to convince the author that $\exists x$, and $\forall x$, are no more than *indefinite pronouns*, that is, *something/body* and *everything/body* respectively (Kaneko 2021, pt.IV ch.3).

§2. Events Expressed by Witnessing Constants

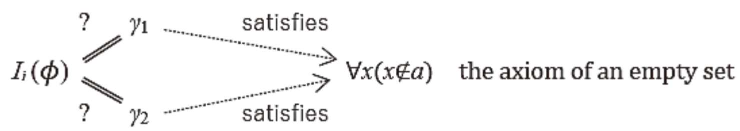
2.1. Indefiniteness

The consequence of *HK*, i.e. (5), convinced the author that $\exists x$ is an indefinite pronoun. Let us think over its semantical aspect more.

The most famous instance of c_Φ in *HK* is ϕ in set theory (Kaneko 2021, app.2). ϕ is introduced as an individual constant *syntactically* first, and then, through the axiom of extensionality (cf. Kaneko 2021, p.194 ⑫), ϕ gains a *semantical* status as an individual constant. Let us see into this.

At the time of introducing ϕ through the axiom of "an" empty set (cf. Kaneko 2021, p.98 (168 ②)) plus *HK*, ϕ has no definite interpretation (cf. Kaneko 2021, p.192):

(7)

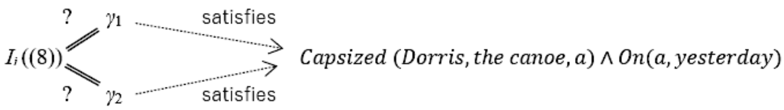


This picture of ϕ is exactly the same as that of the event Davidson talked about (cf. (1)). He said the event like "Doris's capsizing the canoe yesterday" is *particular*, which means it is *indefinite* like ϕ in (7).

Let us dig into that picture. First, we introduce an event through the preceding sentence (2). And then, though *HK*, we gain the following individual constant:

(8) $c_{\exists x(Capsized\ (Dorris, the\ canoe, x) \wedge On(x, yesterday))}$

Logicians may call this a *witnessing constant*, too (cf. Kaneko 2021, p.100). Like (7), constant (8) refers to an object indefinitely:

(9)


Davidson had (or should have had) this picture when he stated (1) above. But again, he was also aware that not every event follows that picture:

> (10) Some actions [and events] are difficult or unusual to perform more than once in a short or specified time, and this may provide a specious reason in some cases for holding that action sentences [and event sentences] refer to unique actions. [We t] hus [deal] with[8] "Jones got married last Saturday", "Doris wrote a cheque at noon", "Mary kissed an admirer at the stroke of midnight". [⋯] A special case arises when we characterize actions [and events] in ways that logically entrail that at most one action so characterized exixt[s. ⋯] "Brutus killed Caesar" is then arguably equivalent (by way of Russel's theory of descriptions) to "The killing Caesar by Brutus occurred". (Davidson 1969, p.168)

Davidson's lack of knowledge of logic blurs the point of the discussion; we would like to clarify it in the sequel.

2.2. Uniqueness

Deep knowledge of logic tells us that the indefiniteness of the interpretation, if the uniqueness is proved, will dissolve. The *uniqueness* here means the following (cf. Kaneko 2021, p.192):

(11) $\forall x \forall y\ (\Phi x \wedge \Phi y\ \rightarrow\ x = y)$

In the case of ϕ, it is proved through the axiom of extensionality, as said earlier (§2.1). But in the case of the event, we directly recognize its uniqueness, for example:

(12) $\exists x(Performed\ (I, x) \wedge At\ (x, there))$

This is read as "I did something there" (cf. Kaneko 2022, p.169). It is a kind of reflection on my past act, and in a relevant context, I remember the act as unique, so that automatically the following holds:

(13) $\forall x \forall y\ ((Performed\ (I, x) \land At\ (x, there)) \land (Performed\ (I, y) \land At\ (y, there))\ \rightarrow\ x = y)$

This secures the uniqueness of the witnessing constant coming from (12), which should be introduced by *HK* in advance:

(14) $c_{\exists x(Performed\ (I,x)\ \land\ At\ (x,there))}$

With the help of (13), we may say (14) refers to the event in question uniquely, recognizing it epistemologically, as it were.

2.3. Definite Description

The author has already presented this insight before (Kaneko 2022, p.168 ⑰). It may affect Russel's *theory of description* as Davidson mentioned it (cf. (10)).

(15) $\iota x(Performed\ (I, x) \land At\ (x, there))$

Compare this so-called *definite description* with constant (14) above. We have secured the latter's interpretation, i.e. $I_i(c_{\exists x(Performed\ (I,x) \land At(x,there))})$, being unique through direct, epistemological recognition of (13) above. Thus, constant (14) itself needs no change; description (15) is unnecessary. This is true of ϕ as well. (14) and ϕ instantiate the nature of what Davidson and Russel wrongly conceived of as and called a definite description.

§3. Singular Sentences Regarded as Event-Sentences

3.1. Individual Constants Regarded as Event-Expressions

We have gained *event-expressions*, as it were, in the form of individual constants; (12) plus *HK* provides (14), first, and then, through the recognition of (13), we convince ourselves of (14) referring to the event uniquely. What does this imply? It is that we are always eligible to think of an individual constant as an *event-expression*.

For example, when we see a singular sentence "*Fc*", its translation is normally "Mike is a man", etc., that is, we normally interpret "*c*" as a thing (or a *person*), but now, we are also eligible to translate "*Fc*" into "What I did there is parking a car", etc. (cf. Kaneko 2022, sec.121, sec.134-136), that is, we can regard "*c*" as *a description of event* "what I did there". Let us dig into this new dimension of logic.

3.2. Time Built in the Constant

One of the headaches in logic is whether or not we should ascribe *time* to "and," namely a *conjunction*[9]. For example (cf. Kaneko 2021, sec.17; Kaneko 2019, sec.44),

(16) Mike got up and brushed his teeth.

Here "and" is translated into a conjunction, but if so, on account of the commutative law, the following logically holds:

(17) Mike brushed his teeth and got up.

This is bizarre, which makes us at a loss how to handle time amid sentences connected by a conjunction.

But the event-sentence we have learned would save us from this deadlock. Translate (16) into this:

(18) $\exists x(GetUp\ (Mike, x) \wedge At\ (x, 7{:}\,00\ a.m.on\ 20/6/2023)) \wedge$
$\exists x(Brushed\ (Mike, Mike's\ teeth, x) \wedge At\ (x, 7{:}\,05\ a.m.on\ 20/6/2023))$

Let "*Fa*" be the translation of "$GetUp\ (Mike, a) \wedge At\ (a, 7{:}\,00\ a.m.on\ 20/6/2023)$" and "*Gb*" be that of "$Brushed\ (Mike, Mike's\ teeth, b) \wedge At\ (b, 7{:}\,05\ a.m.on\ 20/6/2023)$". Then, from (18) plus *HK*, we gain:

(19) $Fc_F \wedge Gc_G$

This is *the first option* to handle the problem of (17) mentioned above.

In contrast with this, there should be another option. We see it as well. First, we adopt the following two event-sentences, instead of (18) above.

(20) $\exists x(Performed\ (Mike, x) \wedge At\ (x, 7{:}\,00\ a.m.on\ 20/6/2023))$

(21) $\exists x(Performed\ (Mike, x) \wedge At\ (x, 7{:}\,05\ a.m.on\ 20/6/2023))$

From these, by applying HK, we gain the following two witnessing constants:

(22) $c_{\exists x(Performed\ (Mike,x) \wedge At\ (x,7:00\ a.m.on\ 20/6/2023))}$

(23) $c_{\exists x(Performed\ (Mike,x) \wedge At\ (x,7:05\ a.m.on\ 20/6/2023))}$

Using these individual constants, we can translate (16) into this:

(24) $GettingUp\ (c_{\exists x(Performed\ (Mike,x)\wedge At\ (x,7{:}00\ a.m.on\ 20/6/2023))})$
$\wedge\ TeethBrushing\ (c_{\exists x(Performed\ (Mike,x)\wedge At\ (x,7{:}05\ a.m.on\ 20/6/2023))})$

This is *the second option* to handle the problem of (17) mentioned above. Both the first option and the second option work well, yet our emphasis has been put on the thought that we can have individual constants of events, or event-expressions. The second option endorses this thought, while the first option did not, so we choose the second option hereafter.

Now we apply the commutative law to (24), a result of the second option, to gain:

(25) $TeethBrushing\ (c_{\exists x(Performed\ (Mike,x)\wedge At\ (x,7{:}05\ a.m.on\ 20/6/2023))})$
$\wedge\ GettingUp\ (c_{\exists x(Performed\ (Mike,x)\wedge At\ (x,7{:}00\ a.m.on\ 20/6/2023))})$

This is read as "What Mike did at 7:05 a.m. is brushing his teeth and what he did at 7:00 a.m. is getting up." There is nothing bizarre; this is how we can solve the problem of time.

To put it another way, or to review the kernel of our argument heretofore, the time attributed to a conjunction, ∧, is the time at which the sentence is *asserted*, while the problem of time we found in (16) to (17) relates to the time built in each event-sentence (if we take the first option) or the time built in the event-expression (if we take the second option).

These discussions may relate to *the problem of the law of contradiction* on one hand (cf. Kaneko 2019, sec.44-48), and on the other hand, may relate to *the stochastic process or the random process* (cf. Uchii 1974, pp.42f.) put forward by Bruno de Finetti (1906-1985), which we shall touch on later (§6.1).

§4. New Horizon of Probability

4.1. From Things to Events

Individual constants we use in logic relate not only to *things* but also to *events*. This is the insight we have gained so far. And it allows us to express a *temporal order* even in conjunctions. Let us see into the impact of this.

One crucial application of it is to *probability.* Probability theory necessarily needs event-expressions, as we handled it under the name of the second option above (§3.2). See the following sentence:

(26) I drew a white ball out of a bag (where there are one red ball and three white balls).

This is a typical statement in math classes at school. Teachers might translate (26) into "W_1", not merely "W", to emphasize that it is the first draw[10]. In terms of logic, we wonder how this subscript "1" should be handled syntactically. It refers to something individual for sure, so it allows us to use an individual constant.

(27) Wc_1

The individual constant "c_1" is a replacement for the subscript "1" in "W_1". We ask, then, whether it refers to a *thing*. True, "c_1" could be such a definite description as "*the ball I took on the first draw*." It could refer to a thing in this sense. However, another situation would never allow of this approach. For example,

(28) The coin showed its head on my first throw.

Similarly to (27), we can formulate this, as follows:

(29) Hc_1

Does this, "c_1", refer to a *thing*? Maybe, it is such a description as "the side the coin showed on my first throw", but such a description is hard to be thought referring to a *thing*, because "the side" is practically hard to be regarded as a thing. Presumably, in engaging on probability, we refer to events, not things, as a matter of course[11]. We explore this possibility further.

4.2. Predicates Newly Formulated

Statements in probability theory relate to event-expressions, presumably. If so, we will certainly be able to translate Wc_1 and Hc_1 above in the same manner as we analyzed (16) above. Let us try:

(30) $\exists x(Drew\ (I, the\ ball, x) \wedge TheFirst\ (x))$

(31) $\exists x(Threw\ (I, the\ coin, x) \wedge TheFirst\ (x))$

These correspond to the pair of logical formulations, (20) and (21). By applying *HK* (§1.2) to these, we gain the following two individual constants:

(32) $c_{\exists x(Drew\ (I, the\ ball, x) \wedge TheFirst\ (x))}$

(33) $c_{\exists x(Threw\ (I, the\ coin, x) \wedge TheFirst\ (x))}$

(32) is the detailed expression of "c_1" in (27) above. (33) is the detailed expression of "c_1" in (29) above. Each refers to an event, not a thing.

We newly introduce *predicates* for these constants, further. Those individual constants refer to events; events cannot be white nor heads as a matter of course, so we must revise predicates for them:

(34) $Wx \longleftrightarrow_{\text{def.}}$ *x leads to an appearance of a white ball*
(35) $Hx \longleftrightarrow_{\text{def.}}$ *x leads to an appearance of the head of the coin*

Constants (32) and (33) above occupy "*x*" here. We do not read any causal relations in "*leads to*" in these predicates, (34) and (35)[12], though thinking of these predicates as causal is possible by applying an additional predicate like "*x is performed at a 60 degrees angle to the surface of the table*" to the constant of (33), for example, which we shall never detail in the present text[13].

4.3. Occurrences Newly Formulated

Next, we formulate *occurrences*, such as "*h*" in "*P*(*h*)", in other words, those propositions to which probabilities are practically assigned (Kaneko 2022, sec.5).

(36) $Wc_{\exists x(Drew\ (I,the\ ball,x)\wedge TheFirst\ (x))}$
(37) $Hc_{\exists x(Threw\ (I,the\ coin,x)\wedge TheFirst\ (x))}$

These are our formulations of occurrences. Thus, the expression "$P\left(Wc_{\exists x(Drew\ (I,the\ ball,x)\wedge TheFirst\ (x))}\right)$" is adopted, for example. Note that "*W*" here is read in accordance with (34), not with (27), and similarly, "*H*" here is read in accordance with (35), not with (29).

Rudolf Carnap (1891-1970), who invented *inductive logic*, did not pay attention to this type of expression; it was because his research was devoted to statements about *things*, such as "All swans are white". But probability theory as we handle it relates to events, such as drawing a ball, throwing a coin, etc. Exactly here, the necessity to create new logic of probability emerges.

§5. Tree Diagram

5.1. Why Logic?

Readers may have been wondering why we must apply *symbolic logic* to the discussions on probability, because *set theory* will do normally, as we saw in math classes at school (Chart Institute 2011, pp.332f.; Matsuzaka 1990, pp.739f.; Kaneko 2022, sec.147).

The most critical reason for this is the fact of logic underlying sets. For example, $A \cap B =_{\text{def.}} \{x \mid x \in A \wedge x \in B\}$, which means the conjunction underlies the intersection, *but not vice versa*.

We bring this aspect of logic more into discussions on probability governed by set theory. For this purpose, we stick to a *tree diagram*, a basic approach to probability (cf. Kaneko 2022, p.182). Since we do not know what a tree diagram is in essence, we initially give logical formulations to it (§§5.3-6.1). It will open up a new horizon of probability. With it, we move on to an analysis of conditional probability, which is the original aim of this paper, to tell the truth (§§6.2-6.4).

5.2. Direct Answer

In the preceding section (§4.3), we have already given a logical formulation to statement (26) above, a statement typical of math classes at school. We develop it further to think about conditional probability as well as the tree diagram. Consider the following *situation*[14], first:

(38) There are one red ball and three white balls in the bag. You draw a ball, not putting it back to the bag.

Again, consider the following *solved problem*[15]:

(39) Given that, in situation (38), you have already drawn a white ball, what is the probability of your drawing a red ball next?

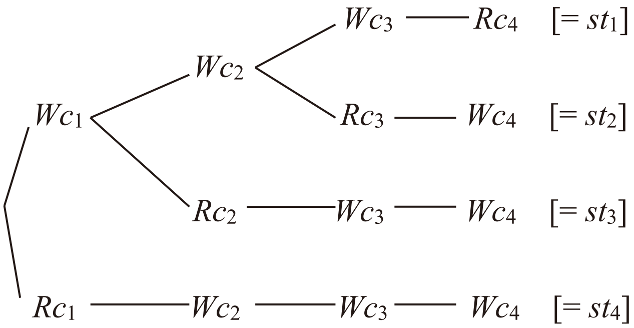
This is a question in conditional probability, but we can answer $\frac{1}{3}$ to it *directly* by reviewing the preceding situation of (38).

We address this under the name of a *direct answer*. A direct answer is so *intuitive* that we need not insert any processes of calculation, say, a tree diagram. So it appears. But that is not the case, even though in math classes at school, teachers avoid applying a tree diagram, which they initially prefer mentioning, when it comes to the solved problem of conditional probability. In the sequel, we pursue, instead of them, a logical character of conditional probability by reference to a tree diagram.

5.3. Tree Diagram

A tree diagram to describe the situation of (38) is somehow drawn in the following way.

(40)



Notations in the square brackets are anticipating the succeeding discussions on *state-descriptions* (§6.1).

Diagram (40) exhausts all the *possibilities* in situation (38). Note that "Wc_1", "Rc_1", etc. are read in accordance with what we have learned so far; that is, "Wc_1" is read as "Your first draw leads to an appearance of a white ball", and so on; the individual constant "c_1" is read like (32) above, while the predicate "W" is read like (34) above.

The trial is made only *twice,* but the diagram must exhaust all the *four* possibilities. This is the kernel of this diagram.

§6. New Approach to Conditional Probability

6.1. First Step

By reference to (40), we pursue a logical character of conditional probability, which teachers in match classes at school are apt to give a direct answer (§5.2).

Its logical character initially comes into sight when we find the following set of *state-descriptions* behind (40).

(41)

$$\left.\begin{array}{l} Wc_1 \wedge Wc_2 \wedge Wc_3 \wedge Rc_4 \longleftrightarrow_{\text{def.}} st_1 \\ Wc_1 \wedge Wc_2 \wedge Rc_3 \wedge Wc_4 \longleftrightarrow_{\text{def.}} st_2 \\ Wc_1 \wedge Rc_2 \wedge Wc_3 \wedge Wc_4 \longleftrightarrow_{\text{def.}} st_3 \\ Rc_1 \wedge Wc_2 \wedge Wc_3 \wedge Wc_4 \longleftrightarrow_{\text{def.}} st_4 \end{array}\right] \cdots str_1$$

Carnap's idea of the state-description is not fully explained here; readers unfamiliar with it are kindly asked to read Kaneko (2022, p.113), for example. "st_1" etc. represents a state-description in (41), and "str_1" refers to a *structure-description*.

Only one structure-description appears in (41), which means all the state-descriptions are *isomorph* (Kaneko 2022, p.125); this implies that no notion of *statistics* emerges at this point, which finally enables us to find the following *correspondences*:

(42) (i) $Wc_1 - Wc_2 - Wc_3 - Rc_4$ [a branch of the tree diagram]

(ii) $Wc_1 \wedge Wc_2 \wedge Wc_3 \wedge Rc_4$ [a state-description]

(iii) $\langle Wc_1, Wc_2, Wc_3, Rc_4 \rangle$ [an ordered 4-tuples]

These three are the same as long as we deal with situation (38). The last one is close to *the stochastic process* or *the random process* as de Finetti called it (§3.2).

By attributing a *temporal order* to the individual constant of Wc_1, for example, we get able to express the order even in a *conjunction*[16] where the *commutative law* holds; $Wc_1 \wedge Wc_2$ is equivalent to $Wc_2 \wedge Wc_1$, but this never spoils the temporal order Wc_1 keeps; that is, Wc_1 occurred first, and then, Wc_2 occurred, as discussed in §3.2 above. This is most critical[17].

6.2. Second Step

We can express a temporal order even in conjunctions. This enables us to see a logical structure of the tree diagram as we saw it in math classes at school most frequently. Diagram (40) is a good example, where structure-descriptions as Carnap formulated them appear (cf. (41)). On this basis, we can define conditional probability so logically as to be applied to Q (39) above:

(43) $$P(Rc_2|Wc_1) = \frac{P(Wc_1 \wedge Rc_2)}{P(Wc_1)}$$

This is the formulation of conditional probability asked about in Q (39) above. Note that $Wc_1 \wedge Rc_2$ in the numerator is possible only in our course of arguments. The notations of set theory predominant in math classes at school (Chart Institute 2011, pp.352-361; Matsuzaka 1990, pp.762-772) never take the place of this.

6.3. Third Step

By reference to the correspondences asserted in (42), we can equate each branch of the tree diagram in (40) with an *elementary occurrence* as Pierre-Simon Laplace (1749-1827) called it (cf. Kaneko 2022, p.137). On account of equipossibility of elementary occurrences, the following holds:

(44) $$P\,(\textit{an elementary occurrence}) = \frac{1}{\textit{the number of elementary occurrences}}$$

Moreover,

As long as we stay in the equation of (42), this formulation of Laplace's is the same as the probability assignment formulated by Carnap.:

(45) $P(st_i) = \frac{1}{the\ number\ of\ state{-}descriptions\ (isomorphic\ to\ st_i)}$

This hold as long as $\frac{1}{the\ number\ of\ structure{-}descriptions}$ = 1 in Carnap's original formulation (cf. Kaneko 2022, p.129).

As a result, the following holds:

(46) *P* (*an elementary occurrence, i.e. each branch of the tree diagram*)
= *P* (*a state-description*)
= $\frac{1}{4}$

What underlies this is (40) to (42). Note that each branch of the tree diagram is regarded as an elementary occurrence.

6.4. Solution

(46) means each branch in (40) as well as each state-description in (41) has the probability of $\frac{1}{4}$. Associated with this, the numerator of (43) and the denominator of (44) are calculated in the following way.

$Wc_1 \land Rc_2$ in the numerator of the right side of (43) is *realized* in st_3 alone, so its probability is $\frac{1}{4}$.

Wc_1 in the denominator of the right side of (43) is *realized* in st_1, st_2, and st_3. So its probability is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$. We reach this result of calculation by reference to Laplace's well-known *premier principe* (cf. Kaneko 2022, p.137).

(47) $P\,(h) = \frac{the\ number\ of\ elementary\ occurrences\ where\ h\ is\ realized}{the\ number\ of\ elementary\ occurrences}$

Lastly, by applying the definition of conditional probability logically formulated above, i.e. (43), we gain the solution $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$. This is how we reach the solution $\frac{1}{3}$ logically, not directly nor intuitively (§5.2).

§7 Concluding Remarks

Regrettably enough, we must stop here to present our discussions in a compact manner. Various interesting topics emerge form here, but our arguments have already exceeded one article's capacity. So we would rather summarize them for later reviews:

(48) Event-expressions (§§1-3)

The first half of this paper is devoted to establishing event-expressions. It follows from it that event-expressions can take the place of customary interpretations of individual constants as things. This enables us to express a temporal order even in conjunctions where the commutative law holds (§3.2).

(49) Application to Probability Theory (§§4-6)
We have applied event-expressions to the solved problem of conditional probability. Through the formulation of diagram (40), we reached a logical solution of the solved problem, which includes a new analysis of conditional probability.

The impact of these is still blurred. So the author is now planning to present it in other pieces of writing (Kaneko 2023b; Kaneko 2023c).

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1 This article is composed of sections numbered with §, and subsections counted as "1.1", for example.

2 The years of life and death are attached to the initial appearance of the key thinker.

3 Citations and important statements are consecutively numbered.

4 We prefer using double quotation marks to single quotation marks consistently in this article without notification.

5 Kashiwabata, in a part of his book (1997, p.8), translated the expression specifying the time when the event in question occurred, into a predicate like "*This-morning* (*x*)," but it is clearly wrong. Davidson (1967b, p.158) preferred translating it, using a function, into "$t(x)$ = *yesterday*", on the other hand.

6 The author of this article, Yusuke Kaneko, is referred to as "the author."

7 Regarding the notion of satisfaction, see Kaneko 2021, sec.134; it is an object that satisfies a formula.

8 Originally, "Thus with". The author does not know how to handle this expression as of now.

9 The present article sometimes calls a connective "∧" a conjunction (§3.2), for example, and at other times, a whole sentence "$p \wedge q$" a conjunction (§6.1). Expressions of this kind differ by authors (Nolt et al. 2011, p.47; Barwise et al. 2011, p.71; Kaneko 2021, p.9).

10 This idea is found in Chart Institute 2011, pp.325-326, for example; see also Kaneko 2022, p.188. The author is forced to admit the formulations and discussions in the present section (§4.1) being still rough ones. More adequate ones are presented in Kaneko 2023b, sec.6. sec.9.

11 See n.10 above again, however.

12 Nevertheless, predicates (34) and (35) are close to F_2, i.e. the predicate including causation presented in Kaneko 2022, sec.193.

13 Kaneko (2022, sec.135) detailed this. On the other hand, it is claimed that drawing a ball, throwing a coin, etc., are *gambles*, not allowing of causal analyses (Kaneko 2022, sec.83).

14 "Situation" is a term the author devised for considering the case where the probability is asked about (Kaneko 2022, p.2).

15 The author owes this expression "solved problem" to Nolt et al. 2011, and it is used in English math classes customarily.

16 See n.9 above.

17 Uchii (1974, p.42) did not notice the significance of this point when he equated *the stochastic process* or *the random process* as de Finetti called it (§3.2) with a conjunction customarily used in symbolic logic.

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