T. FRANZÉN. *Gödel's Theorem. An Incomplete Guide to its Use and Abuse.* Wellesley, Mass.: A K Peters, 2005. x + 172pp. $24.95. ISBN 1-56881-238-8

Reviewed by

R. ZACH, *Department of Philosophy, University of Calgary, Calgary T2N 1N4, Canada*

On the heels of Franzén's fine technical exposition of Gödel's incompleteness theorems and related topics (Franzén 2004) comes this survey of the incompleteness theorems aimed at a general audience. *Gödel's Theorem. An Incomplete Guide to its Use and Abuse* is an extended and self-contained exposition of the incompleteness theorems and a discussion of what informal consequences can, and in particular cannot, be drawn from them. The book is divided into seven chapters. A brief introduction outlines the aims and contents of the book, a lengthy second chapter introduces the incompleteness theorems and outlines their proofs in non-technical terms, and chapter 3 discusses computability and its connections with the incompleteness theorems. Chapter 7 deals with the completeness theorem, and chapter 8 outlines and criticizes Chaitin's work on information-theoretic complexity and its relationship to incompleteness. An appendix fills in some of the technical details. The remaining three chapters (4−6) are devoted to dispelling confusions about incompleteness. Chapter 4, "Incompleteness Everywhere", dispenses with some basic misconceptions, examples range from atrocious yet all-too-common claims made in Internet discussions ("Gödel's theorems show that the Bible is either inconsistent or incomplete.") to published remarks by the likes of Freeman Dyson and Stephen Hawking. As one might expect, the corrections here are often basic (e.g., pointing out that the Bible is not a formal system of arithmetic), but just as often they are quite subtle. The (purported) implications of Gödel's theorems for the character of mathematical knowledge and for the nature of the mind (the anti-mechanist arguments of Lucas and Penrose) receive extended treatment in chapters 5 ("Skepticism and confidence") and 6 ("Gödel, minds, and computers"), respectively.

A book like Franzén's has been necessary for some time. To the chagrin of logicians and logically-minded philosophers, Gödel's theorems and their consequences have certainly been misunderstood and misrepresented too often. *Gödel's Theorem* is a valuable antidote to this, and its readers will gain a better understanding of the theorems and will be able to avoid—and spot—such misunderstandings. But the book will also be useful to those of us who are often confronted with misconceptions of Gödel's theorems in the popular press and in, e.g., the minds of our students. Although anyone who's taken an intermediate logic course will be able to point out many of the basic mistakes commonly made in interpreting the incompleteness theorems, such as that they apply directly only to formal systems in which a sufficient amount of arithmetic can be represented, many other mistakes are much more subtle, e.g., taking the fact that any consistent theory T to which Gödel's theorem applies can be consistently extended to incompatible theories (by adding Con_T and $\neg\mathrm{Con}_T$) as indicating a "postmodern condition" in mathematics in which many different systems of mathematics exist side-by-side (a conclusion discussed in section 2.8). So in addition to being a good source of accurate information about incompleteness to a general audience, the book is also a useful manual for professional logicians, philosophers, and mathematicians which conveniently surveys the various mistakes often encountered and how to correct them.

Franzén's book is accessible, well written, and often funny. Its faults, such as they are, are minor. In some cases I have found the informal presentation of formal results a little too terse, and perhaps an interested student or lay person will find it difficult to follow steps in the exposition which are second nature to trained logicians. It could also be made more useful by adding more references. For instance, I would have liked to know the sources of Dyson's and Hawking's comments discussed in section 4.4, and it would be a service especially to readers interested in the philosophical aspects of the theorems to include pointers to the relevant literature on the philosophy of mathematics, of science, and of mind more generally. (Hawking's lecture can be found on the World Wide Web; see the references.) Perhaps Franzén will make such pointers available on a companion web page or in a second edition (some errata are listed at http://www.sm.luth.se/~torkel/eget/tic.html). Although Franzén's criticisms are all very much to the point, sometimes it seems as if he gives too short shrift to quite respectable philosophical arguments. Both Dyson and Hawking, in quotations in section 4.4, for instance, claim that Gödel's theorems show that we can never have a complete physical theory (since this theory will contain a lot of mathematics, including the requisite amount of arithmetic for the theorems to apply). Of course, Franzén is perfectly correct in pointing out that the incompleteness established by Gödel's theorems concerns (in the first instance) the arithmetical part, and that they say nothing about the completeness of the description of physical reality such theories may contain. But there are important philosophical questions here, specifically, what should count as a physical statement, what statements we would expect to be proved by a physical theory, and what it means to say that a physical theory is “complete” (a question Franzén himself raises). Although Dyson and Hawking are too quick in their applications of Gödel's theorems, one *can* usefully apply Gödel's theorems once the questions to which they are applied are made more precise; an example is Shapiro's (1983) criticism of Field's nominalist program (1980) on the basis of a point which is similar to Hawking's. Just as it isn't (and shouldn't be) Franzén's aim to present the technical results in all their formal detail in such a book, it also isn't to survey and assess all the philosophical uses of the incompleteness results in all their detail. Nevertheless, some readers might be left with the impression that the only legitimate uses of Gödel's theorems are mathematical, and that most, if not all philosophical uses are spurious. There are of course legitimate and important uses of Gödel's theorems in philosophy (e.g., in relation to consistency proofs and Hilbert's program, which is discussed in chapter 5, and to the limits of formal systems generally). A comprehensive survey of these would of course require another book. On the other hand, the the book, in combination with suitable additional readings from the philosophical literature, would make an excellent text for a seminar on the philosophical implications of Gödel's results which might be suitable even for students without an extensive background in logic.

References

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