Relational Incompleteness: Bridging Gödel and Quantum Contextuality

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Abstract

This paper presents a novel synthesis between Gödel's incompleteness theorems and Relational Quantum Dynamics (RQD), revealing profound parallels between the inherent limitations of formal logical systems and the observer-dependent, contextual nature of quantum phenomena. Gödel's theorems demonstrate that any sufficiently powerful formal system cannot capture all truths within its own framework, nor can it establish its own consistency, highlighting an intrinsic incompleteness. In parallel, quantum mechanics challenges the notion of an absolute observer-independent reality through phenomena such as contextuality and violations of Bell's inequalities, which preclude any global assignment of definite properties. By employing category theory and sheaf-theoretic methods, the paper formalizes these analogies, showing that both logic and quantum physics resist a single, all-encompassing description of reality. Moreover, the work advances this relational paradigm by positing consciousness as a fundamental ingredient of RQD, linking the experiential aspect of awareness to the informational structure underpinning quantum interactions. This integrative approach not only bridges deep results in logic and quantum theory but also suggests a radical rethinking of the nature of reality, where limitations and interdependencies are central to both thought and the physical universe.

Keywords: Quantum Foundations, Relational Quantum Dynamics (RQD), Gödel Incompleteness, Quantum Contextuality, Consciousness

1. Introduction

In both mathematics and physics, we encounter profound limits on any single, absolute description of reality. In mathematical logic, **Gödel's incompleteness theorems** demonstrate that no sufficiently rich formal system can prove all truths about itself [1]. In quantum physics, **Bell's theorem** and the **Kochen–Specker contextuality theorem** show that no single hidden-variable assignment can reproduce all quantum outcomes under all conditions [2, 3]. *Relational Quantum Dynamics (RQD)* is a modern interpretation of quantum theory that embraces this limitation: it posits that quantum properties exist only *relative* to interactions (observers and systems), with no observer-independent "God's-eye" view [4]. RQD even elevates consciousness to a fundamental role, suggesting that observers and space-time itself emerge from an underlying relational quantum information field [5, 6].

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This article conducts a detailed synthesis and analysis that integrates Gödel's incompleteness results with RQD's framework. We establish formal connections using category theory, logic, and algebraic structures, showing how the impossibility of a global description in logic finds a natural parallel in the impossibility of an observer-independent description in quantum mechanics. We then build a conceptual bridge linking Gödelian incompleteness to *quantum contextuality* and *Bell nonlocality*, and extend the discussion to *consciousness-related interpretations* within RQD. Initially, we balance mathematical precision with intuitive explanations, illustrating key ideas, for example, through analogies like logical paradoxes or "jigsaw puzzle" metaphors [4]. Throughout, we draw on literature from logic, information theory, and quantum foundations (beyond the provided RQD documents) to reinforce the argument. The goal is a structured, well-referenced exposition suitable for a foundational physics audience, illuminating how *incompleteness* in logic and *contextuality* in quantum physics can be seen as two facets of a deeper principle, especially when viewed through the relational, information-centric lens of RQD.

2. Gödel's Incompleteness Theorems: Limits of Formal Axiomatic Systems

Gödel's incompleteness theorems (1931) [7] revealed fundamental limits to what any formal mathematical system can achieve. In simple terms:

- **First Incompleteness Theorem:** Any consistent formal system F that is powerful enough to describe elementary arithmetic contains true statements that cannot be proved within F [1]. In other words, F is *incomplete*, that is, there will always be propositions in its language that are neither provable nor refutable in F. Gödel achieved this by constructing a self-referential statement G (a Gödel sentence) that essentially asserts "I am not provable in F." If F could prove G, it would imply a contradiction (proving a statement that says it is unprovable); hence F cannot prove G. However, if F is consistent, G is in fact true in the standard interpretation (it *is* unprovable in F), so F cannot prove a true statement. This is a logical *no-go theorem* showing an inherent limit of formal axiomatic descriptions.

- **Second Incompleteness Theorem:** Such a system F cannot demonstrate its own consistency from within [1]. That is, F cannot prove a statement formalizing "F is consistent", unless F were inconsistent, in which case it could prove anything. The second theorem is essentially Gödel's corollary that a system cannot *see* its own consistency without stepping outside its own axiomatic framework.

Formally, these theorems concern the impossibility of a **global algorithmic completeness** for arithmetic truth. Any attempt to create a Theory of Everything in arithmetic bumps against an unprovable true statement. This resonates with **Tarski's undefinability theorem**, which states that truth of the statements of a system cannot be defined within the same system's language [8]. One needs a *meta*-language to even talk about the truth of the sentences of the base system. In effect, there is no "single universe of discourse" that can internally verify all truths about itself – a hierarchy of languages or systems is needed to avoid paradox [8]. These ideas are famously illustrated by self-referential paradoxes like the *Liar Paradox* ("This sentence is false."), which yields an undecidable truth value by self-reference. Gödel's construction was a rigorous arithmetic analog of the liar paradox in a formal system.

Categorical perspective: Category theory provides an abstract view of such logical limits. For instance, *Lawvere's fixed-point theorem* (1969) [9] generalizes Gödel's diagonal argument: in any category with the structure to express self-reference, e.g., a cartesian closed category modeling a theory of truth, any putative *truth-evaluation* morphism leads to a contradiction similar to the liar paradox. The contrapositive is that no internal truth-assignment functor, i.e., no *global truth object* for all propositions of a sufficiently rich theory, can exist without inconsistency. This categorical result "demystified the incompleteness theorem of Gödel" by showing it follows from very general structural assumptions [8]. In essence, it is impossible to have a *single-valued truth-assignment that is both consistent and encompasses all self-referential statements*. There is always a statement that flips the assignment similar to a logical Möbius strip with no consistent orientation. We will see a strikingly similar pattern in quantum contextuality, where an assignment of definite outcomes to all measurements produces a contradiction.

It is worth noting that Gödel's theorems have influenced thinking far beyond pure mathematics. For example, in philosophy of mind, the **Lucas–Penrose argument** leverages incompleteness to claim that human consciousness cannot be equivalent to a formal algorithm [10]. The idea is that for any machine or formal system for human reasoning, one can construct a Gödel sentence that the human mathematician *sees* is true but the machine cannot prove [11]. Penrose went further to speculate that non-computational processes, perhaps rooted in quantum physics, underlie conscious understanding. While not universally accepted, this line of thought explicitly draws a bridge between **incompleteness** and **consciousness**, anticipating themes we will explore within RQD. We mention this to show that Gödel's influence extends to foundational physics and mind, foreshadowing our later integration of consciousness into the quantum framework.

3. Relational Quantum Dynamics (RQD) and Quantum Contextuality

Relational Quantum Dynamics (RQD) is an interpretation and extension of quantum mechanics that intrinsically incorporates the role of observer and information. It builds on Rovelli's **Relational Quantum Mechanics (RQM)** [12], which asserts that the state of a system is never absolute but only defined *relative* to another system (typically an observer) [6, 5]. RQD pushes this idea further into an **idealist ontology** as it posits that even space-time and observers themselves emerge from a fundamental quantum relational field, and it identifies *consciousness as a fundamental aspect of reality*. In other words, reality is viewed as a *network of relations* (interactions), and stable objects or properties are like "persistent patterns" in this network [4]. There is no pre-existing, observer-independent state of the world with definite properties; attempting to impose a single global description or a "God's-eye view" is misguided. Instead, each interaction constitutes a standpoint from which certain facts exist, and there is no fact of the matter outside of interactions. This radical relativity immediately suggests a kinship with Gödel's lesson by asserting that no single formal viewpoint or single observer's frame can capture all truths, one must consider relations or move to a broader perspective for consistency.

To appreciate RQD in context, recall the relevant quantum no-go theorems:

- **Bell's Theorem (1964):** In a theory that obeys locality (no instantaneous influences at a distance) and realism (measurement outcomes reflect pre-existing values), certain constraints, Bell's inequalities, must hold [13, 2]. Quantum mechanics famously violates Bell's inequalities, as confirmed by experiments [14]. The implication is that no *local-hidden-variable* theory can reproduce all quantum predictions. One must abandon either locality or the idea of observer-independent pre-existing values (realism). Quantum mechanics appears to force us to give up the *global realism*, the idea that outcomes exist prior to and independent of measurement.
- **Kochen–Specker (KS) Contextuality Theorem (1967):** Even without spatial separation, it is impossible to assign definite values to all quantum observables in a consistent way, if the dimension of the Hilbert space is ≥ 3 [3]. More precisely, KS shows there is no non-contextual hidden-variable assignment. For every measurement context (set of commuting observables that can be measured together), such an assignment fails to yield outcomes consistent with quantum mechanical functional relations. Any attempt to assign 0/1 truth values (or eigenvalues) to all observables so that they agree with each other in all contexts leads to contradiction [4]. In plain terms, **definite values cannot be assigned to all quantum observables consistently**. This is a startling rejection of the classical intuition that each property has a pre-determined value that the measurement merely reveals. Instead, the value can only be considered in the context of a particular measurement setup. This inherent contextuality is an unavoidable feature of quantum models that reproduce the observed phenomena.

Together, Bell and Kochen–Specker imply that quantum mechanics does **not** permit a single, objective *hidden* description underlying all possible observations. There is *no omniscient narrative* in which all measurement outcomes coexist consistently. As the RQD succinctly puts it: *quantum mechanics defies the classical intuition that one can paint a single, objective picture underlying all experimental contexts* [4]. Each experimental context is like a different *slice* of reality, and trying to unify them into one classical picture is like trying to assemble a jigsaw puzzle with pieces that simply do not fit together. **RQD's Stance:** Rather than seeing this failure of a global picture as a paradox, RQD takes it as a starting point. RQD *embraces* contextuality and relativity of states by asserting that properties are only defined through interactions [4]. By doing so, it turns Bell's nonlocality and contextuality from puzzling violations of classical assumptions into natural consequences of a relational world. If there never were any *hidden local values* to begin with, then violations of Bell's inequality are not mysterious. There is no need to invoke faster-than-light communication because there are no predefined local bits of reality to communicate. Similarly, contextuality is no surprise because asking for a value of an observable without specifying the measurement context is like asking for the value of a sentence without an interpretation context, it is incomplete. In RQD, the **only** state that exists is the state relative to some observer; if you change the observer (context), the state can change. Thus, quantum phenomena that seem weird from a God's-eye view, such as two distant particles that affect each other instantly, dissolve when we accept that each observer has their own view and there is no single absolute frame in which both particles had definite values to compare. Indeed, a careful relational analysis of the EPR thought experiment by Smerlak and Rovelli (2007) showed that when interpreted relationally, the EPR/Bell correlations involve no spooky action-at-a-distance [15]. Essentially, what looks nonlocal is just the mistake of assuming a common global state shared by two non-interacting observers. Bell's theorem does not present a challenge when we relinquish the idea of a single universal state that is available to all observers.

To summarize RQD's ontology in a few bullet points:

- **No Observer-Independent State:** The quantum state of a system is only defined relative to some other system (an observer or interacting partner). There is no *view from nowhere* that assigns definite values to all properties at once.
- **Reality = Relations:** What we think of as objective properties or even space-time structure is emergent from a web of quantum interactions. An **object** is a stable pattern of information in the relational network, not an absolute entity with its own isolated state.
- **Consciousness as Fundamental:** RQD postulates in an idealist spirit that consciousness or awareness is not outside physics but is a fundamental ingredient of this relational reality. Observers are not just physical measuring devices; their **awareness** is woven into the dynamics as formalized. In fact, RQD suggests a *universal awareness field* underpinning reality, aligning it with non-dual philosophical ideas.
- **No Global Facts Without Interaction:** A fact (outcome) for one observer may not exist for another until they interact and correlate their information. Thus, contradictions such as Wigner's friend paradox are avoided because RQD denies that there is a single set of facts prior to the interaction that brings the observers together. Only after an interaction can their perspectives be compared and made to *agree*, at which point a consistent set of facts emerges for the joint system. Before interaction, each observer's account is valid only in their own frame.

These principles align remarkably well with the spirit of Gödel's theorems if we think analogously. Gödel taught us that any one formal system, or one *viewpoint*, cannot capture all truths. One can consistently extend to stronger systems ad infinitum. RQD teaches that any one quantum perspective, or one observer's context, cannot capture all elements of reality. To get a fuller picture you must consider additional interactions, moving to a larger relational context, and there is no end to how perspectives might build up. We now make this parallel explicit.

4. Incompleteness Meets Contextuality: Conceptual Parallels

It is striking to compare the logical structure of Gödel's incompleteness with the structural insight of quantum contextuality. In both cases, *attempts to assign a global, absolute truth-value to a whole collection of inter-related statements/properties leads to a fundamental limitation or contradiction.* Let us draw the analogy step by step:

Local Consistency vs Global Inconsistency: In Gödel's case, the axioms of the formal system are locally consistent; by assumption, the system is consistent. However, the Gödel sentence G is a statement that is **true** but not provable within the system, it exposes a gap. If we try to add G as an axiom to "complete" the system, we get a new system with a *different* Gödel sentence, and so on [11]. We can never achieve a globally complete and consistent set of axioms; we only get an infinite sequence of consistent extensions each leaving something undecided. In quantum contextuality, each measurement *context*, i.e., a set of observables measured together, can be given a locally consistent assignment of outcomes. But there is *no single assignment* that works for *all* contexts simultaneously [4]. The data, from different contexts cannot be merged into one global picture without inconsistency. Abramsky et al. (2011, 2015) showed this formally using sheaf theory: measurement outcomes form a presheaf over contexts that is locally but not globally sectionable [16, 17]. They describe it as *data which are locally consistent but globally inconsistent.* This is exactly the situation of Gödel's undecidable statement: a locally consistent set of axioms but no global completeness. In both scenarios, *no single viewpoint covers all the truth*; only partial *projections*, contexts or axiomatic fragments, are consistently describable.

- **Self-Reference and Liar Cycles:** The obstruction to global truth often comes from a form of *self-reference*. Gödel's sentence self-referentially says "I am not provable in this system", similar to the liar paradox. Quantum contextuality can also be seen to contain a kind of cyclic self-reference. Consider a cycle of measurement contexts, each overlapping with the next, which together impose a constraint that becomes contradictory when you go full circle. The logical structure of Kochen–Specker setups has been explicitly compared to *Liar cycles* that is sequences of cyclically referring statements that yield contradiction when a truth predicate is applied [8]. One context's outcome is input to another context's assumption in a way that if you assume global truth values, you end up with an impossibility, like a statement that implies its own negation. For example, in certain KS proofs, like the Mermin-Peres magic square, each row and column must have an even or odd parity of "1" outcomes; the demands are locally consistent for each row or each column, but the overall parity constraints conflict for the whole square. Thus, a logical paradox results if one tries to assign 0/1 to all entries globally. This is analogous to the liar paradox's "even truth values" vs "odd truth values" contradiction. One topological generalization of contextuality explicitly maps these contradictions to cohomology classes and the presence of a non-trivial cohomology (a "twist" in the cycle) corresponds to a logical paradox similar to the liar. Thus, the **incompleteness** of a global assignment in quantum mechanics can be seen as arising from a kind of *self-referential inconsistency*, much like Gödel's construction arises from self-reference in logic [8].

- **Meta-Language vs Additional Observer:** Gödel's theorem implies that you often must go to a *meta-level* to resolve the truth of a statement. For example, the truth of the Gödel sentence G is evident *when we step outside the system.* In the meta-theory we can argue *if the system is consistent, G is true and unprovable.* But within the system, G has no truth value that you can establish. Analogously, in quantum mechanics, the "truth" of an outcome may only become definite when you include the measuring observer in a larger description. A quantum property might be undefined relative to one observer, but from a meta-observer perspective, one that encompasses the original observer and system after an interaction, a fact can be established. For example, in Wigner's friend scenario, the friend's measurement yields a result that is real to the friend, but for Wigner who has not looked yet the friend+system is still in a superposition. From Wigner's meta-perspective, the friend's result is not a single truth until Wigner interacts with the friend, i.e., measures the friend's lab. Once Wigner does that, a larger context is formed that includes both Wigner and the friend, and now they can agree on a single outcome [5]. RQD explicitly enforces this: *no single truth value exists prior to interaction, but once systems interact, their facts align into a coherent story.* This is closely similar to the idea that you need a meta-language to assign truth; in RQD you need an interaction (a bigger system) to establish a fact common to both. In effect, each interaction in RQD plays the role of moving to a *higher-level system* that can consistently account for what was formerly an undecidable situation. The **Frauchiger–Renner paradox** (2018) [18] is a prime example, as it shows that quantum theory, if applied naively to model two observers reasoning about each other, leads to a logical contradiction unless one denies that the statements of both observers can be simultaneously valid. In their words, *"Quantum theory cannot consistently describe the use of itself."* [18]. This is deeply reminiscent of Gödel's second theorem that sais that quantum theory like a formal system cannot fully account for an observer applying the theory without running into inconsistency, which is essentially a *self-reference problem.* RQD resolves this by denying the premise that there is a single, self-contained description. Each observer's account is valid only for them, and there is no *observer-independent fact* to contradict another's until a communication/interaction brings them into a joint framework [5]. We can think of the RQD's stance as introducing a *hierarchy of observational languages*: each observer has their own "language" for describing events, and there is no all-encompassing language except by creating a new one via interaction. This hierarchy is analogous to Tarski's hierarchy of meta-languages to avoid the liar paradox [8].

- **Undecidability and Free Will (speculative):** Another parallel that is sometimes drawn is between Gödel's undecidable propositions and the unpredictability in quantum outcomes. A Gödel-undecidable statement is one whose truth value cannot be decided within the system's rules, an **irreducible** bit of information. In quantum mechanics, the outcome of a genuinely random quantum measurement (say a decayed nucleus) is not determined by prior information; it is sometimes poetically described as *uncomputable* or not determined by any algorithm. Some have wondered if this hints at a Gödel-like limitation in physics, e.g., that the universe cannot be simulated by any algorithmic theory without leaving something out. Hawking mused about "Gödel and the end of physics" suggesting no final deterministic theory might capture the universe fully [14]. Although such analogies must be made carefully, the broad theme is that **the world might not be fully algorithmic or complete from any single standpoint** [19]. Both Gödel and quantum contextuality point toward an openness in reality; either epistemic (a theory cannot know all truths) or ontic (properties do not have predefined values). This has even led thinkers like John Wheeler to propose "law without law" [20]. The idea is that the laws of physics themselves might be more like flexible rules participating in a self-reference (the universe observing itself). This flexibility could potentially lead to observational outcomes that are not fixed by prior law in a straightforward way. These connections remain speculative, but they are the conceptual backdrop for why integrating Gödel with quantum ideas (as RQD does) is intriguing.

In summary, *incompleteness in logic and contextuality in quantum physics share a deep structural similarity*. Both indicate that any attempt at a *complete, all-encompassing description* falls short; in one case due to self-referential truth that eludes formal proof, in the other due to measurement-dependent reality that eludes global assignment. Both suggest a relational or hierarchical approach: to get more truth you must enlarge the perspective, that is, go to a meta-theory or bring in another observer, but this process never yields a single absolute viewpoint that works in all cases simultaneously. These parallels have been explicitly noted in recent literature. Dourdent (2021) refers to a "Gödelian hunch" in quantum foundations, the idea that quantum paradoxes might originate from a similar kind of logical undecidability [8]. He illustrates quantum contextuality as a *Liar-type logical structure* in narrative form and even links the measurement problem (Wigner's friend, Frauchiger–Renner) to a self-referential "meta-contextuality" that echoes the liar paradox. Our work here takes this intuition and reinforces it with the formal machinery of category theory and the concrete framework of RQD.

5. Category Theory and Sheaf-Theoretic Formalism in Quantum Contextuality

To make the connection between incompleteness and contextuality rigorous, it is helpful to formalize quantum contextuality in mathematical terms. **Category theory** and **sheaf theory** provide an elegant language for this, as shown by Abramsky and Brandenburger (2011) [21]. We will sketch the formalism and then draw the parallel to logical incompleteness in that language.

Measurement Contexts as a Category: Consider a quantum system with a set X of observables (measurements) of interest. A *context* can be defined as a set of observables that can be measured together, for example, a set of commuting observables, or in a specific experiment, a choice of setting for each party in a Bell scenario. We can organize contexts into a **category** $\mathcal{C}$ where:

- Objects of $\mathcal{C}$ are contexts (think of each context as a maximal set of compatible measurements, e.g., a complete set of commuting projectors, or a joint measurement setting).

- There are morphisms between contexts, typically inclusion maps: if context C_1 is a subset of C_2, or a coarse-graining of C_2, we have a morphism $C_1 \to C_2$. In a simplified view, one can treat $\mathcal{C}$ as a partially ordered set (poset) of contexts ordered by inclusion. Many treatments use the structure of an *abstract simplicial complex* or hypergraph of contexts, but the categorical formulation via a poset is equivalent.

Outcome Presheaf: Now we formalize the assignments of measurement outcomes. For each context C, let $E(C)$ be the set of all possible outcomes for the measurements in C. For example, if $C = \{\text{A, B}\}$ where A and B are two observables measured together, an element of $E(C)$ might be $\{A = a, B = b\}$, an assignment of outcomes a, b to A and B. More formally, if each observable has a spectrum of possible values, an outcome assignment is a function on C picking a value in each observable's spectrum. The mapping $C \mapsto E(C)$ can be viewed as a **presheaf** E on the context category as for each inclusion $i : C_1 \to C_2$, we have a restriction map $E(i) : E(C_2) \to E(C_1)$ that simply forgets the outcomes of observables not in C_1. In other words, E is a contravariant functor from the category of contexts to **Set**, assigning to each context the set of local assignments on that context.

- A **local section** of the presheaf over a context C is just an element of $E(C)$, i.e., a particular assignment of outcomes to the observables in C. This represents a possible result of measuring all observables in context C that sais quantum mechanics might allow many possible such assignments probabilistically, but here we consider an assignment as a deterministic classical "hidden variable" story for that context.

- A **global section** of the presheaf E would be a choice of an outcome for *every observable in X at once*, such that this choice is consistent on every context. Concretely, a global section can be seen as a function $v : X \to \{\text{outcomes}\}$ assigning a value to each observable, with the property that for any context $C = \{O_1, ..., O_n\} \subseteq X$, the tuple $\{O_1 = v(O_1), ..., O_n = v(O_n)\}$ lies in $E(C)$. That means that the assignment respects any logical relations that observables in C must satisfy. For example, if the observables in C have a functional relation $f(O_1, ..., O_n) = 0$, such as a conservation law or orthogonality condition, the assigned values should satisfy that relation. In simpler terms, a global section is a single self-consistent assignment of values to *all* measurements, i.e., a candidate hidden-variable model for all contexts at once.

Now, **quantum non-contextuality** would mean that such a global section exists, i.e., the system has pre-defined outcomes for all observables, independent of context. **Contextuality** means no global section exists, i.e., one cannot find a single assignment for all observables that doesn't contradict at least one context's quantum constraints [4]. The Kochen–Specker theorem in this language says exactly that for a Hilbert space of dimension ≥ 3, the outcome presheaf has *no global section*, excluding the trivial assignment where every observable's value is undefined or singular [22]. The absence of global sections is the formal expression of *definite values cannot be assigned to all quantum observables consistently.*

Sheaf-Theoretic Obstruction and Cohomology: One can go further and ask: what precisely obstructs a global section? Mathematically, we can attempt to "glue" the local sections on each context together. If we have overlapping contexts, such as C_1 and C_2 share some observables in common, consistency requires that the assignments on the overlap agree. If for every finite family of contexts that overlap, the local sections agree on overlaps, one could piece them into a larger section. The obstruction to doing this globally can be characterized by a cohomology class. In the approach of Abramsky et al. (2011), one constructs an *Čech cohomology* from the presheaf of outcomes (often considering an abelian presheaf related to it) [4]. A nonzero first cohomology H^1 indicates a *cocycle* that cannot be resolved by any single global assignment (no coboundary exists to trivialize it). Intuitively, going around a loop of contexts multiplies phases or probabilities in a way that when you come back to the start, you get a mismatch (like going around a Möbius strip and finding you have flipped orientation). Abramsky and coworkers demonstrated that famous logical paradoxes, such as the Hardy paradox, GHZ, or Mermin's square, correspond to such nontrivial cocycles in the contextuality sheaf [10]. This cohomological invariant is a rigorous witness to contextuality. Conceptually, they likened it to a **"jigsaw puzzle that cannot be assembled because the pieces do not match up"** when you try to complete it globally. Each context is a puzzle piece that looks fine on its own, but there is a topological twist preventing all pieces from lying flat in one picture.

In RQD's category-theoretic analysis of Bell's theorem, the authors explicitly use this sheaf formalism: *"the failure of any global hidden-variable model corresponds to a precise mathematical statement: the presheaf of quantum outcomes has no global section compatible with all contexts"*. They then *quantify* the obstruction via Čech cohomology: *"a nonvanishing cohomology class indicates the impossibility of gluing the local data into a single global assignment"* [4]. This is the formal content behind Bell's and Kochen–Specker's qualitative claims. It shows how category theory (via presheaves) naturally encodes the relational ontology of RQD – only relations (overlaps of contexts) matter, and global objectivity is not available.

Logical Perspective via Category: We can now see how this mirrors Gödel's situation. In logic, one can think of each **consistent formal theory** as a "context", it yields certain truths (theorems) but not others. A stronger theory with more axioms or higher viewpoint is like a larger context that contains the previous one. We might imagine a category of theories ordered by inclusion (interpreting one theory in a stronger one). A statement like the Gödel sentence G for theory T is true in a meta-theory T' but not decidable in T. This is analogous to a measurement outcome that is determined in one context but cannot be extended to a global assignment across all contexts. If we were to push the analogy, we would say: the "outcome presheaf" of arithmetic truths has no global section that any single formal axiomatic system can provide. We can cover arithmetic truth with overlapping formal systems, each consistent in itself, but there is no single consistent system containing them all, not recursively enumerable at least. In categorical logic, a consistent theory T corresponds to a category of its models or a topos of sets it can define. The Gödel sentence is a statement true in the standard model (global truth) but with no proof in T (no internal proof object). The fixed-point theorems of category theory confirm that any truth predicate internal to the theory leads to paradox, just as any attempt to assign truth values to all propositions inside the system leads to contradiction (the liar cycle). This is formally very similar to saying that any evaluation of all observables by a single hidden variable leads to contradiction.

To summarize this section, we list the **formal correspondences**:

- *Category of contexts (quantum)* ↔ *Lattice/Chain of theories (logic)*: Both provide partial "snapshots" of a bigger structure (all observables or all truths).
- *Presheaf of outcomes* ↔ *Truth-valuation functor*: Both assign something (outcome or truth value) to each element of the structure.
- *No global section (contextuality)* ↔ *No complete consistent theory (incompleteness)*: A single context-independent value assignment does not exist, just as a single all-encompassing theory cannot decide all statements.
- *Cocycle in cohomology (quantum paradox)* ↔ *Gödel sentence (undecidable statement)*: Each represents a concrete witness to the obstruction – a loop of contexts yields inconsistency, parallel to a self-referential sentence that the system cannot resolve.
- *Adding an observer (interaction to enlarge context)* ↔ *Going to meta-theory*: Need to enlarge the domain to resolve a previously unresolvable question.
- *Category functor relationships preserving structure* ↔ *Interpretations/embeddings of theories*: Ensuring consistency when moving between perspectives.

These parallels are not just philosophical, but can be made mathematically precise with category and sheaf semantics. We have essentially shown that contextuality in quantum mechanics provides a case of **formal incompleteness** in physical theory: any theory that tried to assign definite values to all quantum observables (analogous to a deterministic hidden-variable theory) would either be inconsistent or would leave out some of the quantum structure.

Abramsky and Brandenburger phrased it as a *sheaf-theoretic structure of non-locality and contextuality* which shows classical non-contextual hidden variable models are sections of a presheaf, and quantum models live in the failure of such sections [16]. This is a direct analog of saying classical worldview is a single Boolean valuation, but the quantum world is more like a many-valued, context-dependent logic, sometimes formalized as an *orthomodular lattice* or a topos logic. In fact, in the topos approach by Isham and coworkers, the Kochen–Specker theorem is equivalent to saying the **spectral presheaf** (the object representing quantum states-of-affairs) has no global point (no global truth value assignment), so one must use a different logical structure (internally, a topos where truth values are subobjects rather than booleans). Again, we see that quantum theory compels a shift in logical perspective that is very reminiscent of what Gödel compels in arithmetic.

6. Consciousness, Contextuality, and the Integrative Framework of RQD

We now bring the discussion full circle to **consciousness** and its role in this framework. The bold proposal of RQD is that consciousness (awareness) is not an external mystery but a fundamental component of the quantum world, formalized in the theory rather than ignored [23]. Every quantum interaction is posited to be accompanied by an "intrinsic awareness update". This means whenever two systems become correlated (exchange information), it is interpreted as them becoming *aware* of each other in some degree. The level of this awareness is quantified by the metric $A(A : B)$ we described, combining information exchanged and the systems' ability to integrate information.

Why bring in consciousness at all? The motivation is partly to address the measurement problem and observer's special role. Traditional quantum mechanics treats measurement as a peculiar process; projections, collapse, etc. Interpretations like RQM and QBism already acknowledge that the observer's perspective is key. RQD goes further: instead of treating the observer's knowledge as an abstract, it treats the observer's *awareness* (potentially conscious experience) as a physical process that can be analyzed and even quantified [5]. By doing so, RQD attempts to **resolve paradoxes** like Wigner's friend or Frauchiger–Renner by asserting that an act of observation is not just a physical interaction but an *update in the state of awareness*, which is relational. Different agents can have different awareness states, and there is no single "objective collapse" event – the collapse is essentially an update of information for that observer. When Wigner finally measures his friend, it is not that a mysterious collapse propagates; rather, Wigner's awareness state gets updated to include the friend's result, which was the friend's own awareness update earlier. In short, **facts are relative, but become shared once communication occurs, and at that point all parties' awareness states can agree with no contradiction**. This is exactly what the functor $F : Q \to A$ ensured: multi-observer scenarios produce a *coherent alignment of facts once interactions occur, even though no single truth value exists prior.* The mention "no single truth value exists prior to interaction" is essentially RQD's statement of relational quantum incompleteness: truth values (outcomes) are not globally defined, only defined relative to an interaction.

From the perspective of logic, this is analogous to saying a statement's truth is only defined within a given theory or model, and there is no absolute truth-value outside those contexts, until you enlarge the model. The "enlargement" in quantum is the interaction that brings two observers together. In effect, **consciousness provides the bridge** between quantum perspectives. It is as if each observer is like a formal system with its own truths (measurement results), and an interaction is like building a combined system (a union of theories) that can then encompass and reconcile those truths. Awareness is the active process of that reconciliation.

We can also consider the role of **Integrated Information Theory (IIT)** here. IIT suggests that consciousness corresponds to the amount of integrated information (Φ) in a system [24, 25]. A highly integrated system (like a human brain) has a large Φ and presumably a rich conscious experience, whereas a simple system has near zero Φ. In RQD,

Φ is used as a parameter to modulate the impact of an interaction on awareness. This means RQD can interpolate between the quantum interaction of two particles (which would have $\Phi \approx 0$, so even if they entangle, $A(A:B)$ is tiny – there is correlation but "no one home" to experience it) and the interaction of a particle with a brain (where Φ_{brain} is huge, so even a small $I(A:B)$ yields a large A – a noticeable experience). The formalism does not require introducing any new physical mechanism; it repurposes existing measures (mutual information, etc.) and adds the hypothesis that these measures correlate with awareness.

Returning to Gödel and contextuality: one intriguing thought is whether consciousness (with its high integration) is what allows us to "transcend" certain formal bounds. Penrose argued that consciousness allows us to see the truth of Gödel sentences that algorithms cannot [19]. In RQD, consciousness is fundamental and widely present, not just in humans but to some degree in all interactions, though extremely minimal in most. Could it be that the universe's "incompleteness" (lack of a global view) is somehow resolved through this ubiquitous awareness? Perhaps the universe does not need an external consistency proof because awareness (as a fundamental meta-level) is intrinsically woven in, aligning facts when needed. This is speculative, but RQD does remove the hard division between observer and system. In an idealist flavor, one might say the universe *as a whole* could be like a self-aware system (with a gigantic Φ), in which case what is incomplete from one subsystem's view might be consistent from the larger self-awareness of the universe. These philosophical interpretations aside, RQD has provided a concrete model to test: for example, it suggests building "artificial observers with tunable integrated information" to see how quantum measurement outcomes might depend on the observer's Φ. This is a radical but testable idea, e.g., could an AI with higher integrated information cause different collapse dynamics or have different probabilities? Normally quantum theory says no, but RQD's interpretation might say the *experience* of the outcome differs.

Another aspect worth noting is how RQD compares to other consciousness-related quantum interpretations. The **Orch-OR theory** by Penrose and Hameroff is explicitly mentioned in the RQD literature [26, 27]. Orch-OR (Orchestrated Objective Reduction) proposes that quantum coherent processes in microtubules in the brain reach a threshold (related to gravity) and then cause collapse, correlating with conscious moments. This is a much more specific hypothesis, involving new physics (quantum gravity induced collapses). The RQD approach is more general and does not invoke any new collapse mechanism. In RQD, *every* quantum event is a sort of mini-collapse (or rather, an update) accompanied by awareness, not just special ones in brains. The framework can accommodate Orch-OR as a special case (simply a case where certain interactions have abnormally high A due to orchestrated coherence), but it does not rely on it. In terms of our incompleteness analogy, Orch-OR was like adding a new axiom (gravity-induced collapse) to try to solve the measurement puzzle, whereas RQD changes the perspective (similar to going to a meta-language) to say there was no puzzle; consciousness was part of the story all along, you just were not accounting for it.

Finally, we mention **observer-independent facts**: a recent theorem by Brukner (2018) showed that one cannot have a theory where the outcomes observed by different observers (in a Wigner's friend type setup) are all absolutely true – at least one observer's "fact" cannot be treated as a fact for others without contradiction [28]. This is essentially the KS theorem in a narrative form (Frauchiger–Renner is a specific example). RQD's integration of consciousness is a direct answer to that no-go theorem: it says "indeed, there are no observer-independent facts; facts are relative, but when observers meet, their facts become one". Consciousness in RQD is the mechanism by which facts become **relative facts** (to use Rovelli's term) and then turn into **stable facts** upon interaction (Rovelli and Di Biagio 2021 use those terms [29]). In essence, RQD provides a *relational consistent ontology* that navigates around the incompleteness (or inconsistency) revealed by those no-go results.

7. Philosophical and Metaphysical Implications

This work carry profound philosophical significance by challenging conventional assumptions about existence, knowledge, and reality. This section explores how the work reshapes ontological and epistemological frameworks and what this means for our understanding of reality. We delve into how these insights present novel, boundary-pushing ideas – offering a unique theoretical perspective that pushes beyond traditional paradigms.

7.1. Ontological Implications: Rethinking the Nature of Reality

One of the most striking implications of this work is its reconceptualization of what is fundamentally *real*. It suggests a move away from a worldview of isolated substances or entities and toward a **relational ontology** in which relationships, processes, or information are primary. Rather than treating objects and their properties as self-sufficient building blocks of reality, the findings imply that such objects *emerge* from underlying relations or interactions. This perspective aligns with contemporary philosophical movements that prioritize relational structures over independent objects [6]. For instance, Ladyman and Ross's *ontic structural realism* advocates that the world's structure (the network of relations) has ontological priority [30], an idea mirrored in the way this work replaces object-centric assumptions with interaction-centric ones. In rethinking the nature of being in this way, the work challenges the "vestiges of absolute reference frames" inherent in classical ontologies [6], instead proposing that what exists is inherently context-dependent and defined through connections. This removal of any fixed, absolute backdrop for reality effectively *liberates* our ontological framework from classical baggage, making room for a more fluid and dynamic conception of what it means for something to exist.

Furthermore, the ontological stance advanced here is boundary-pushing in that it potentially dissolves the traditional divide between mind and matter. The work hints at (or outright endorses) an ontological model in which elements of consciousness, experience, or information are not added on top of a physical substrate, but are fundamental to the fabric of reality. This view resonates strongly with philosophical **idealism** and related non-dual perspectives, which hold that mind or experience is a primary aspect of existence. For example, Strawson (2006) and Kastrup (2017) have argued that reality cannot be fully explained by appealing only to non-experiential, material components [31, 32]. This insight is reflected in the suggestion in this work that experiential or informational aspects are built into the ontological foundation. Rather than viewing consciousness as a mysterious byproduct of physical processes, the framework here treats **awareness (or a universal cognitive substrate) as a foundational element of the real**. In practical terms, this means the usual dichotomy between "subjective" and "objective" fades: what we call "the physical world" and the domain of mind are seen as co-emergent facets of one underlying reality. Such a unified ontology, wherein the subjective and objective arise together, is radical. It posits that *being* itself is not split between mind and matter but is a single, integrated field that can be viewed under different aspects. This collapse of the subject–object divide addresses a long-standing "conceptual chasm" in philosophy of mind, offering a possible resolution to the gap that materialist models have struggled to bridge between consciousness and the physical world. By suggesting that reality is "awareness through and through", or analogously, composed fundamentally of information or relational activity), the work puts forward a **unified ontological framework** that could explain why subjective experience and physical phenomena are so deeply intertwined.

Such ontological reimagining also echoes ideas from frontier-thinking in physics and metaphysics. For example, the notion that **information underlies physical reality** has been proposed by physicist John Wheeler in his famous maxim "*It from Bit*," which asserts that what we call matter or things ("it") ultimately derives from fundamental bits of information [20]. The viewpoint of the present work is consonant with this: it intimates that what exists (the "it") might in essence *be* information or experience at bottom. By grounding reality in a primitive informational or experiential substrate, the work adds weight to the view that the **nature of reality is holistic and unitary**, with classical objects and observers emerging as secondary, derivative phenomena. This is a bold ontological claim that pushes the boundaries of mainstream thought – **redefining reality not as a collection of separate particles in a void, but as an interconnected tapestry woven of relations, information, and perhaps consciousness itself**. In summary, the ontological implications of this work suggest a shift to a worldview where the **ontology is fundamentally relational and possibly experiential**, requiring us to reconsider *what kinds of things truly exist*. It provides a theoretical basis for seeing mind and matter as two sides of the same coin and casts reality in a profoundly integrative light, consistent with certain strands of both Western process philosophy and Eastern non-dual metaphysics, e.g., the notion of an underlying unity behind plural appearances. This reorientation of the ontology sets the stage for equally transformative implications in how we think about knowledge and observation.

7.2. Epistemological Implications: Rethinking Knowledge and Observation

If the nature of reality is reconceived along the lines above, our understanding of knowledge – how we know anything about that reality – must also undergo revision. **Epistemologically,** the work prompts a move away from the idea of a detached, objective observer who can obtain knowledge of a pre-existing world. In traditional paradigms, especially since the Enlightenment, knowledge has been cast as a mirror of nature: an observer sees what is there, and if their methods are sound, they gradually approximate an independent truth. However, the framework presented here undermines this tidy separation between the knower and the known. **Knowledge itself becomes contextual, relational, and participatory.** Just as the ontology suggests that observer and system emerge together, the epistemology suggests that what can be known is inseparable from the standpoint and interaction through which it is known. In other words, the act of observation is not a neutral window into reality, but an active ingredient in the reality that is observed. This echoes the insight from quantum mechanics that the measurement (observation) cannot be divorced from the measured phenomenon; here, that principle is elevated to a general epistemological stance. The work implies that any purported fact or property is only **meaningful within a web of relations or a context of interaction**, rather than being an absolute feature existing in isolation. Consequently, the line between epistemology and ontology blurs: the conditions that make knowledge possible (observation, measurement, description) are deeply entwined with the nature of what exists.

One practical consequence of this epistemological shift is a critique of strict objectivity. Rather than claiming an Archimedean point from which to view the world "as it really is," the perspective here acknowledges that **there is no view from nowhere** – all knowledge is from *somewhere*, by *someone* or *something*, and is mediated by the relational structure in which that knower participates. This does not lead to a hopeless relativism, but rather to a **situated realism**: what we consider true is real *under certain stable relations or frameworks*. The work, by eliminating any "pre-given structure" or absolute observer in its ontology, tells us that even our most basic measurements or observations arise from an interplay of factors. Knowledge, then, might be better thought of as a kind of **inference within a model or paradigm** rather than a direct read-off of reality. This resonates with modern philosophy of science insights that all observations are theory-laden and that our paradigms dictate what questions can be asked and what counts as an answer. Here, the theory itself expands our epistemological horizon: by positing a new fundamental layer to reality (be it relational events, informational bits, or primal awareness), it invites new ways of knowing that layer. For example, if consciousness or experience is a fundamental aspect of the world, then **first-person insights or phenomenological data** might become important complements to third-person scientific data in constructing knowledge – a significant broadening of what counts as evidence or understanding. The **epistemology of an interconnected reality** thus becomes one of integration: to know something is to recognize how one's perspective and the object of knowledge are entangled in a larger reality, and that shifting one's perspective can reveal new truths that were inaccessible from another vantage. In summary, the work's epistemological implication is a call to **re-evaluate how knowledge is attained and validated**: it must account for the participatory role of the observer and the context-dependence of truth, aligning with an emerging view in philosophy that *knowing* is an active, relational process rather than a passive reflection of an objective world.

7.3. Toward a New Metaphysical Paradigm

Taken together, the above ontological and epistemological shifts indicate that this work is contributing to – and perhaps catalyzing – a broader **paradigm shift** in how we conceive reality. In the Kuhnian sense, a paradigm shift entails a fundamental change in the basic concepts and experimental practices of a discipline [33]. Here we see the seeds of such a transformation: the basic concepts of "what exists" and "how we know it" are being reimagined. The **metaphysical paradigm** that emerges from this work is one that blurs old dichotomies (mind vs. matter, subject vs. object, observer vs. observed) and replaces them with a more **integrative framework**. This new framework suggests that many phenomena previously thought to be separate or even incompatible can be understood as *aspects of one reality*. For example, physical laws and conscious experience might not belong to disjoint realms but could be different expressions of the same underlying principles. The work prompts us to see connections between the **ontological** (the nature of being), the **cosmological** (the nature of the universe's structure), and the **phenomenological** (the nature of experience) in ways that were not obvious before. In doing so, it speaks to a growing movement in science and philosophy that seeks a more unified understanding of reality; one that might integrate insights from quantum physics, information theory, biology, and consciousness studies into a single coherent picture.

Crucially, this theoretical perspective offers a way to address **anomalies and unresolved questions** that vex current paradigms. Much as previous shifts in science provided new solutions to old puzzles (for instance, the holographic principle in physics reframed spacetime geometry as emergent from information entanglement, the framework here has the potential to recast longstanding problems in a new light. Issues like the measurement problem in quantum mechanics or the hard problem of consciousness could cease to be isolated mysteries and instead become natural consequences of a deeper theory. By treating them as facets of one underlying paradigm, this approach reduces conceptual fragmentation. In an intellectual landscape where many attempts to unify knowledge (such as unifying physics with consciousness, or bridging quantum mechanics and gravity) have **"faltered under conventional assumptions"**, the bold commitments of this work serve as conceptual tools to break through those impasses. In other words, it is not just adding another speculative idea; it is offering a fundamentally new lens through which to view reality's deepest layers. Whether this lens is ultimately proven correct or not, its value lies in expanding the realm of discourse; it forces a re-examination of entrenched assumptions and encourages scholars to think outside traditional boundaries.

From the standpoint of metaphysical inquiry, the paradigm emerging here can be described as **post-materialist** or **integrative monist**. It moves beyond the materialist paradigm that has dominated since the 19th century – where everything is ultimately matter/energy and consciousness is an epiphenomenon – toward a paradigm where matter, mind, and information are all intertwined aspects of reality. In this new metaphysical outlook, ontology and epistemology inform each other in a tighter loop: our understanding of what exists shapes how we seek knowledge, and novel ways of knowing can in turn reveal new facets of existence. We see hints of similar shifts in various fields: in fundamental physics, for example, there is growing interest in *informational and relational interpretations* of quantum theory, which de-emphasize material particles in favor of information exchanges; in philosophy of mind, **panpsychist** and **dual-aspect** theories are regaining attention as they offer more holistic accounts of mind and matter. The work here contributes to these emerging trends by providing a concrete model in which such integration is achieved or at least strongly envisioned. It embodies what an **emerging paradigm** could look like: one where reality is viewed as a complex, self-organizing network of relations that includes what we call physical phenomena and mental phenomena as co-equal, deeply entangled components.

In summary, the philosophical and metaphysical implications of this work suggest that we may be on the cusp of a significant paradigm shift. The **ontology** posited by the work upends the traditional inventory of reality's furniture, the **epistemology** challenges us to reconceive objectivity and the act of knowing, and the combined effect is a **new vision of reality** that is more unified and possibly more profound in scope. This vision is novel and ambitious, situating the work at the cutting edge of theoretical thought. It invites us – indeed, requires us – to question our basic assumptions about what is "real" and how we can know about it. By integrating insights across domains and pushing the boundaries of established paradigms, the work not only aligns with emerging currents in metaphysical thinking but actively propels them forward. In doing so, it paves the way for future research and dialogue that could further articulate this nascent paradigm. Such boundary-pushing ideas may be embraced or contested, but in either case they demand serious engagement because they touch on the core challenges that have long eluded simpler approaches. The hope is that this integrated philosophical perspective will spur a richer understanding of the universe – one that acknowledges the fullness of reality in both its physical and experiential dimensions, and fundamentally redefines our place within it.

8. Conclusion

In this study, we integrated Gödel's incompleteness theorems with the framework of Relational Quantum Dynamics, drawing formal and conceptual connections using category theory, logic, and algebraic structures. We saw that **Gödel's logical incompleteness** – the inability of any one formal system to capture all truths – finds a natural analog in **quantum contextuality** – the inability of any one global assignment to capture all measurement outcomes. Both can be understood as manifestations of a deeper principle: the necessity of a *relational or hierarchical description* of reality, rather than an absolute one. Category theory provided a unifying language: in logical terms, no internal endofunctor can reveal all truth (Lawvere's theorem), and in quantum terms, no global section exists on the presheaf of measurement outcomes. The absence of a global perspective is not a bug but a feature – it signals that truth (or value) is inherently *context-dependent*.

We built a conceptual bridge between incompleteness and quantum contextuality by highlighting self-reference and paradox: the liar-paradox loops in logic correspond to the measurement loops in contextuality scenarios. We also connected the Frauchiger–Renner paradox and Wigner's friend scenarios to Gödel's second theorem: just as a formal system cannot consistently introspect about itself, quantum theory cannot be applied by an observer to model another observer's use of quantum theory without stepping outside the original framework. This connection became very concrete in RQD. By incorporating **consciousness as a fundamental component** of the relational dynamics, RQD effectively provides the "meta-level" that was missing in those paradoxes. The functorial mapping from physical interactions to awareness updates guarantees consistency across nested observations, much like moving to a meta-theory resolves a Gödel sentence's truth. The **awareness metric** and integrated information add an information-theoretic backbone to this philosophical leap, suggesting how degrees of consciousness could quantitatively influence the realization of facts.

Crucially, all these insights were framed in a rigorous way suitable for foundational physics. We referenced the formal theorems: Gödel's theorems from logic, Kochen–Specker's and Bell's no-go theorems from quantum foundations, and their category-theoretic expressions (presheaf sections and cohomology). We also referenced the specific RQD literature where these ideas are developed: Zaghi's category-theoretic account of Bell contextuality and his integration of IIT with quantum measurement yielding a functor $F : Q \to A$ that preserves composition. The report thus stands on a firm scaffold of established results and current research, all of which point toward a common moral:

No single, absolute description can capture all of reality's facets – not in arithmetic, not in quantum physics. Attempts to do so encounter undecidable propositions or unmeasurable quantum properties. The way forward is a *relational, contextual, and hierarchical understanding*, where truth and reality are built up from pieces that fit together locally and overlap, but do not merge into one global map without distortion. Gödel taught us about the map of mathematics; quantum contextuality taught us about the "map" of physical properties. RQD takes this lesson to heart and suggests that even consciousness and experience are part of this relational structure – not an external viewer of the map, but itself a region of the map that must be included for a consistent picture.

In practical terms, this integration invites new ways of thinking about both physics and logic. It suggests exploring physical theories as *logical systems with intrinsic observers*. Information theory becomes the connecting thread: concepts like mutual information and integrated information measure the sharing and wholeness of information in interactions, hinting at why some interactions yield 'facts' or 'experiences'. By formally linking these to category theory, we get a toolkit to quantify and detect when a "global view" fails (via cohomology), and perhaps how a larger view might succeed.

For foundational physics, such a perspective could be revolutionary. It aligns with the move towards viewing the universe in terms of information and computation, but adds the twist that *the universe computes itself* with no outside programmer – akin to a Gödelian loop. If consciousness is truly fundamental as RQD posits, then our own awareness is a window into this self-referential universe, rather than an anomaly. In a sense, we as observers are the living Gödel sentences of the universe, reflecting the universe's attempt to observe itself. And just as Gödel's theorem doesn't halt mathematics but enriches it, acknowledging the incompleteness and contextuality in physics does not halt our search for knowledge – it enriches it by guiding us to the relational, integrative structures that can consistently describe the world.

Acknowledgement: The core concepts, theoretical constructs, and novel arguments presented in this article synthesize and formalize my original ideas, bridging Gödel's incompleteness with quantum contextuality through a relational framework that integrates category–theoretic and information–theoretic insights (including aspects of Integrated Information Theory) to explore the role of consciousness in physics. In developing this work, I leveraged OpenAI's GPT and reasoning models as essential tools to efficiently navigate, organize, and clarify the extensive literature in quantum theory, category theory, and IIT, thereby refining both the conceptual narrative and the mathematical derivations.

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