NSS
University of New Mexico

Neutrosophic Regular Filters and Fuzzy Regular Filters in Pseudo-BCI Algebras

Xiaohong Zhang[1, 2], Yingcang Ma[3], and Florentin Smarandache[4]

[1] Department of Mathematics, Shaanxi University of Science & Technology, Xi'an, 710021, P.R.China. E-mail: zxhonghz@263.net
[2] Department of Mathematics, Shanghai Maritime University, Shanghai, 201306, P.R.China. E-mail: zhangxh@shmtu.edu.cn
[3] School of Science, Xi'an Polytechnic University, Xi'an 710048, P.R.China. E-mail: mayingcang@126.com
[4] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA. E-mail: smarand@unm.edu

Abstract. Neutrosophic set is a new mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Pseudo-BCI algebra is a kind of non-classical logic algebra in close connection with various non-commutative fuzzy logics. Recently, we applied neutrosophic set theory to pseudo-BCI algebras. In this paper, we study neutrosophic filters in pseudo-BCI algebras. The concepts of neutrosophic regular filter, neutrosophic closed filter and fuzzy regular filter in pseudo-BCI algebras are introduced, and some basic properties are discussed. Moreover, the relationships among neutrosophic regular filter, fuzzy filters and anti-grouped neutrosophic filters are presented, and the results are proved: a neutrosophic filter (fuzzy filter) is a neutrosophic regular filter (fuzzy regular filter), if and only if it is both a neutrosophic closed filter (fuzzy closed filter) and an anti-grouped neutrosophic filter (fuzzy anti-grouped filter).

Keywords: Neutrosophic set, Pseudo-BCI algebra, Neutrosophic Filter, Neutrosophic Regular Filter, Fuzzy Regular Filter.

1 Introduction

In 1998, Florentin Smarandache introduced the concept of a neutrosophic set from a philosophical point of view (see [16, 17, 18]). The neutrosophic set is a powerful general formal framework that generalizes the concept of fuzzy set and intuitionistic fuzzy set. In this paper we work with special neutrosophic sets, they are called single valued neutrosophic set (see [21]). The neutrosophic set theory is applied to many scientific fields (see [18, 19, 20]), and also applied to algebraic structures (see [1, 2, 15, 19]), it is similar to the applications of fuzzy set (soft set, rough set) theory in algebraic structures (see [11, 14, and 23]).

In 2008, W. A. Dudek and Y. B. Jun [3] introduced the notion of pseudo-BCI algebra as a generalization of BCI algebra, it is also as a generalization of pseudo-BCK algebra (which is close connection with various non-commutative fuzzy logic formal systems, see [4, 24, 26, 27, 28, and 32]). For non-classical logic algebra systems, the theory of filters (ideals) plays an important role (see [9, 12, 13, 25, and 30]). In [7], the notion of pseudo-BCI filter (ideal) of pseudo-BCI algebras is introduced. In 2009, some special pseudo-BCI filters (ideals) are discussed in [10]. Since then, some articles related filters of pseudo-BCI algebras are published (see [29, 31, 33, and 34]).

Recently, we applied neutrosophic set theory to pseudo -BCI algebras in [35]**.** This paper we further study on the applications of neutrosophic sets to pseudo-BCI algebras. We introduce the new concepts of neutrosophic regular filter, neutrosophic closed filter and fuzzy regular filter in pseudo-BCI algebras, and investigate their basic properties and present relationships among neutrosophic regular filters, anti-grouped neutrosophic filter and fuzzy filters.

Note that, the notion of pseudo-BCI algebra in this paper is a dual of the original definition in [3], so the notion of filter is a dual of (pseudo-BCI) ideal in [7, 10].

2 Some basic concepts and properties

2.1 On neutrosophic sets

Definition 2.1[17, 18, 19] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is, $T_A(x)$: $X\to\ ]^-0, 1^+[$, $I_A(x)$: $X\to\ ]^-0, 1^+[$, and $F_A(x)$: $X\to\ ]^-0, 1^+[$. Thus, there is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Definition 2.2[21] Let X be a space of points (objects) with generic elements in X denoted by x. A simple valued neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. Then, a simple valued neutrosophic set A can be denoted by

$$A=\{\langle x , T_A(x), I_A(x), F_A(x) \rangle \mid x\in X\},$$

where $T_A(x)$, $I_A(x)$, $F_A(x)\in[0, 1]$ for each point x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.3[21] The complement of a simple valued neutrosophic set A is denoted by A^c and is defined as ($\forall x\in X$)

$$T_{A^c}(x) = F_A(x),\ I_{A^c}(x) = 1 - I_A(x),\ F_{A^c}(x) = T_A(x).$$

Then

$$A^c=\{\langle x, F_A(x), 1-I_A(x), T_A(x)\rangle \mid x\in X\}.$$

Definition 2.4[21] A simple valued neutrosophic set A is contained in the other simple valued neutrosophic set B, denote $A\subseteq B$, if and only if $T_A(x)\le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x)\ge F_B(x)$ for any x in X.

Definition 2.5[21] Two simple valued neutrosophic sets A and B are equal, written as $A = B$, if and only if $A\subseteq B$ and $B\subseteq A$.

For convenience, "simple valued neutrosophic set" is abbreviated to "neutrosophic set" later.

Definition 2.6[21] The union of two neutrosophic sets A and B is a neutrosophic set C, written as $C=A\cup B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x)=\max(T_A(x), T_B(x)),\ I_C(x)=\max(I_A(x), I_B(x)),$$
$$F_C(x)=\min(F_A(x), F_B(x)),\ \forall x\in X.$$

Definition 2.7[21] The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as $C=A\cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x)= \min(T_A(x), T_B(x)),\ I_C(x)=\min(I_A(x), I_B(x)),$$
$$F_C(x)=\max(F_A(x), F_B(x)),\ \forall x\in X.$$

Definition 2.8[20] Let A be a neutrosophic set in X and $\alpha, \beta, \gamma\in[0, 1]$ with $0\le\alpha+\beta+\gamma \le 3$ and (α, β, γ)-level set of A denoted by $A^{(\alpha, \beta, \gamma)}$ is defined as:

$$A^{(\alpha, \beta, \gamma)}=\{ x\in X \mid T_A(x)\ge\alpha, I_A(x)\ge\beta, F_A(x)\le\gamma\}.$$

2.2 On pseudo-BCI algebras

Definition 2.9[3] A pseudo-BCI algebra is a structure $(X; \le, \to, \rightsquigarrow, 1)$, where "$\le$" is a binary relation on X, "$\to$" and "$\rightsquigarrow$" are binary operations on X and "1" is an element of X, verifying the axioms: for all $x, y, z\in X$,

(1) $y\to z\le(z\to x)\rightsquigarrow(y\to x)$, $y\rightsquigarrow z\le(z\rightsquigarrow x)\to(y\rightsquigarrow x)$;

(2) $x\le(x\to y)\rightsquigarrow y$, $x\le(x\rightsquigarrow y)\to y$;

(3) $x\le x$;

(4) $x\le y, y\le x \Rightarrow x=y$;

(5) $x\le y \Leftrightarrow x\to y =1 \Leftrightarrow x\rightsquigarrow y =1$.

If $(X; \le, \to, \rightsquigarrow, 1)$ is a pseudo-BCI algebra satisfying $x\to y = x\rightsquigarrow y$ for all $x, y\in X$, then $(X; \to, 1)$ is a BCI-algebra.

Proposition 2.1[3, 7, 10] Let $(X; \le, \to, \rightsquigarrow, 1)$ be a pseudo-BCI algebra, then X satisfy the following properties ($\forall x, y, z\in X$):

(1) $1\le x \Rightarrow x=1$;

(2) $x\le y \Rightarrow y\to z\le x\to z$, $y\rightsquigarrow z\le x\rightsquigarrow z$;

(3) $x\le y, y\le z \Rightarrow x\le z$;

(4) $x\rightsquigarrow(y\to z)=y\to(x\rightsquigarrow z)$;

(5) $x\le y\to z \Leftrightarrow y\le x\rightsquigarrow z$;

(6) $x\to y\le(z\to x)\to(z\to y)$, $x\rightsquigarrow y\le(z\rightsquigarrow x)\rightsquigarrow(z\rightsquigarrow y)$;

(7) $x\le y \Rightarrow z\to x\le z\to y$, $z\rightsquigarrow x\le z\rightsquigarrow y$;

(8) $1\to x=x$, $1\rightsquigarrow x=x$;

(9) $((y\to x)\rightsquigarrow x)\to x=y\to x$, $((y\rightsquigarrow x)\to x)\rightsquigarrow x=y\rightsquigarrow x$;

(10) $x\to y\le(y\to x)\rightsquigarrow 1$, $x\rightsquigarrow y \le(y\rightsquigarrow x)\to 1$;

(11) $(x\to y)\to 1=(x\to 1)\rightsquigarrow(y\rightsquigarrow 1)$,
$(x\rightsquigarrow y)\rightsquigarrow 1=(x\rightsquigarrow 1)\to(y\to 1)$;

(12) $x\to 1=x\rightsquigarrow 1$.

Definition 2.10[7] A nonempty subset F of pseudo-BCI algebra X is called a pseudo-BCI filter (briefly, filter) of X if it satisfies:

(F1) $1\in F$;

(F2) $x\in F, x\to y\in F \Rightarrow y\in F$;

(F3) $x\in F, x\rightsquigarrow y\in F \Rightarrow y\in F$.

Definition 2.11[29] A pseudo-BCI algebra X is said to be anti-grouped pseudo-BCI algebra if it satisfies the following identity:

(G1) $\forall x, y, z\in X$, $(x\to y)\to(x\to z)= y\to z$,

(G2) $\forall x, y, z\in X$, $(x\rightsquigarrow y)\rightsquigarrow(x\rightsquigarrow z)= y\rightsquigarrow z$.

Proposition 2.2 [29] A pseudo-BCI algebra X is an anti-grouped pseudo-BCI algebra if and only if it satisfies:

$\forall x\in X$, $(x\to 1)\to 1=x$ or $(x\rightsquigarrow 1)\rightsquigarrow 1=x$.

Definition 2.12[29] A filter F of a pseudo-BCI algebra X is called an anti-grouped filter of X if it satisfies

(GF) $\forall x\in X$, $(x\to 1)\to 1\in F$ or $(x\rightsquigarrow 1)\rightsquigarrow 1\in F\Rightarrow x\in F$.

Definition 2.13[29] A filter F of a pseudo-BCI algebra X is called a closed filter of X if it satisfies

(CF) $\forall x\in X$, $x\to 1\in F$.

Definition 2.14[34] A filter F of pseudo-BCI algebra X is said to be regular if it satisfies:

(RF1) $\forall x, y\in X$, $y\in F$ and $x\to y\in F \Rightarrow x\in F$.

(RF2) $\forall x, y\in X$, $y\in F$ and $x\rightsquigarrow y\in F \Rightarrow x\in F$.

Proposition 2.3 [34] Let X be a pseudo-BCI algebra, F a filter of X. Then F is regular if and only if F is anti-grouped and closed.

Definition 2.15[31, 33] A fuzzy set A in pseudo-BCI algebra X is called fuzzy filter of X if it satisfies:

(FF1) $\forall x\in X, \mu_A(x)\leq\mu_A(1)$;

(FF2) $\forall x, y\in X, \min\{\mu_A(x), \mu_A(x\to y)\}\leq\mu_A(y)$;

(FF3) $\forall x, y\in X, \min\{\mu_A(x), \mu_A(x\rightsquigarrow y)\}\leq\mu_A(y)$.

Definition 2.16[31] A fuzzy set A: $X \to[0, 1]$ is called a fuzzy closed filter of pseudo-BCI algebra X if it is a fuzzy filter of X such that:

(FCF) $\mu_A(x\to 1) \geq \mu_A(x), \forall x\in X$.

Definition 2.17[31] A fuzzy set A in pseudo-BCI algebra X is called fuzzy anti-grouped filter of X if it satisfies:

(1) $\forall x\in X, \mu_A(x)\leq\mu_A(1)$;

(2) $\forall x, y, z\in X, \min\{\mu_A(y), \mu_A((x\to y)\to(x\to z))\}\leq\mu_A(z)$;

(3) $\forall x, y, z\in X, \min\{\mu_A(y), \mu_A((x\rightsquigarrow y)\rightsquigarrow(x\rightsquigarrow z))\}\leq\mu_A(z)$.

Proposition 2.4[31] Let A be a fuzzy filter of pseudo-BCI algebra X. Then A is a fuzzy anti-grouped filter of X if and only if it satisfies:

$$\forall x\in X, \mu_A(x)\geq\mu_A((x\to 1)\to 1), \mu_A(x)\geq\mu_A((x\rightsquigarrow 1)\rightsquigarrow 1).$$

Definition 2.18[35] A neutrosophic set A in pseudo-BCI algebra X is called a neutrosophic filter in X if it satisfies: $\forall x, y\in X$,

(NSF1) $T_A(x)\leq T_A(1)$, $I_A(x)\leq I_A(1)$ and $F_A(x)\geq F_A(1)$;

(NSF2) $\min\{T_A(x), T_A(x\to y)\}\leq T_A(y)$, $\min\{I_A(x), I_A(x\to y)\}\leq I_A(y)$ and $\max\{F_A(x), F_A(x\to y)\}\geq F_A(y)$;

(NSF3) $\min\{T_A(x), T_A(x\rightsquigarrow y)\}\leq T_A(y)$, $\min\{I_A(x), I_A(x\rightsquigarrow y)\}\leq I_A(y)$ and $\max\{F_A(x), F_A(x\rightsquigarrow y)\}\geq F_A(y)$.

Proposition 2.5[35] Let A be a neutrosophic filter in pseudo-BCI algebra X, then $\forall x, y\in X$,

(NSF4) $x\leq y \Rightarrow T_A(x)\leq T_A(y)$, $I_A(x)\leq I_A(y)$ and $F_A(x)\geq F_A(y)$.

Definition 2.19[35] A neutrosophic set A in pseudo-BCI algebra X is called anti-grouped neutrosophic filter in X if it satisfies: $\forall x, y, z\in X$,

(1) $T_A(x)\leq T_A(1)$, $I_A(x)\leq I_A(1)$ and $F_A(x)\geq F_A(1)$;

(2) $\min\{T_A(y), T_A((x\to y)\to(x\to z))\} \leq T_A(z)$, $\min\{I_A(y), I_A((x\to y)\to(x\to z))\} \leq I_A(z)$ and $\max\{F_A(x), F_A((x\to y)\to(x\to z))\} \geq F_A(z)$;

(3) $\min\{T_A(y), T_A((x\rightsquigarrow y)\rightsquigarrow(x\rightsquigarrow z))\} \leq T_A(z)$, $\min\{I_A(y), I_A((x\rightsquigarrow y)\rightsquigarrow(x\rightsquigarrow z))\} \leq I_A(z)$ and $\max\{F_A(x), F_A((x\rightsquigarrow y)\rightsquigarrow(x\rightsquigarrow z))\} \geq F_A(z)$.

Proposition 2.6[35] Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is a neutrosophic filter in X if and only if A satisfies:

(i) T_A is a fuzzy filter of X;

(ii) I_A is a fuzzy filter of X;

(iii) $1-F_A$ is a fuzzy filter of X, where $(1-F_A)(x) = 1-F_A(x)$, $\forall x\in X$.

Proposition 2.7[35] Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is an anti-grouped neutrosophic filter in X if and only if A satisfies:

(i) T_A is a fuzzy anti-grouped filter of X;

(ii) I_A is a fuzzy anti-grouped filter of X;

(iii) $1-F_A$ is a fuzzy anti-grouped filter of X, where $(1-F_A)(x)=1-F_A(x)$, $\forall x\in X$.

3 Neutrosophic regular filters and neutrosophic closed filters

Definition 3.1 A neutrosophic set A in pseudo-BCI algebra X is called a neutrosophic regular filter in X if it is a neutrosophic filter in X such that: $\forall x, y\in X$,

(NSRF1) $\min\{T_A(y), T_A(x\to y)\}\leq T_A(x)$, $\min\{I_A(y), I_A(x\to y)\}\leq I_A(x)$ and $\max\{F_A(y), F_A(x\to y)\}\geq F_A(x)$;

(NSRF2) $\min\{T_A(y), T_A(x\rightsquigarrow y)\}\leq T_A(x)$, $\min\{I_A(y), I_A(x\rightsquigarrow y)\}\leq I_A(x)$ and $\max\{F_A(y), F_A(x\rightsquigarrow y)\}\geq F_A(x)$.

Definition 3.2 A neutrosophic set A in pseudo-BCI algebra X is called a neutrosophic closed filter in X if it is a neutrosophic filter in X such that: $\forall x\in X$,

(NSCF) $T_A(x\to 1)\geq T_A(x)$, $I_A(x\to 1)\geq I_A(x)$, $F_A(x\to 1)\leq F_A(x)$.

Proposition 3.1 Let A be a neutrosophic regular filter in pseudo-BCI algebra X. Then A is closed.

Proof: Suppose $x\in X$. By Definition 2.9 (2) and Proposition 2.1 (12) we have

$$x \leq (x\to 1)\rightsquigarrow 1= (x\to 1)\to 1.$$

From this and Proposition 2.5 we get

$$T_A(x)\leq T_A((x\to 1)\to 1),\ I_A(x)\leq I_A((x\to 1)\to 1),$$
$$F_A(x)\geq F_A((x\to 1)\to 1).$$

Moreover, by Definition 2.18 (NSF1) and Definition 3.1 (NSRF1)

$$T_A((x\to 1)\to 1)=\min\{T_A(1), T_A((x\to 1)\to 1)\}\leq T_A(x\to 1),$$
$$I_A((x\to 1)\to 1)=\min\{I_A(1), I_A((x\to 1)\to 1)\}\leq I_A(x\to 1),$$
$$F_A((x\to 1)\to 1)=\max\{F_A(1), F_A((x\to 1)\to 1)\}\geq F_A(x\to 1).$$

Thus,

$$T_A(x)\leq T_A((x\to 1)\to 1)\leq T_A(x\to 1),$$
$$I_A(x)\leq I_A((x\to 1)\to 1)\leq I_A(x\to 1),$$
$$F_A(x)\geq T_A((x\to 1)\to 1)\geq T_A(x\to 1).$$

By Definition 3.2 we know that A is closed.

By Proposition 2.4 and Proposition 2.7 we can get the following proposition.

Proposition 3.2 Let A be a neutrosophic filter of pseudo-BCI algebra X. Then A is an anti-grouped neutrosophic filter of X if and only if it satisfies: $\forall x\in X$,

$$T_A(x)\geq T_A((x\to 1)\to 1),\ T_A(x)\geq T_A((x\rightsquigarrow 1)\rightsquigarrow 1);$$
$$I_A(x)\geq I_A((x\to 1)\to 1),\ I_A(x)\geq I_A((x\rightsquigarrow 1)\rightsquigarrow 1);$$
$$F_A(x)\leq F_A((x\to 1)\to 1),\ F_A(x)\leq F_A((x\rightsquigarrow 1)\rightsquigarrow 1).$$

Proposition 3.3 Let A be a neutrosophic regular filter in pseudo-BCI algebra X. Then A is anti-grouped.

Proof: Suppose $x\in X$. By Definition 2.9 and Proposition 2.1 we have

$$x\to((x\to 1)\to 1)= x\to((x\to 1)\rightsquigarrow 1)=1.$$

From this we get

$$T_A(x\to((x\to 1)\to 1))=T_A(1),\ I_A(x\to((x\to 1)\to 1))=I_A(1),$$

$$F_A(x\to((x\to1)\to1))=F_A(1).$$

Thus, applying Definition 3.1 (NSRF1) we get

$$T_A(x)\geq\min\{T_A((x\to1)\to1),\ T_A(x\to((x\to1)\to1))\}$$
$$=\min\{T_A((x\to1)\to1),\ T_A(1)\}=T_A((x\to1)\to1),$$
$$I_A(x)\geq\min\{I_A((x\to1)\to1),\ I_A(x\to((x\to1)\to1))\}$$
$$=\min\{I_A((x\to1)\to1),\ I_A(1)\}=I_A((x\to1)\to1),$$
$$F_A(x)\leq\max\{F_A((x\to1)\to1),\ F_A(x\to((x\to1)\to1))\}$$
$$=\max\{F_A((x\to1)\to1),\ F_A(1)\}=F_A((x\to1)\to1).$$

Similarly, we can prove that

$$T_A(x)\geq T_A((x\rightsquigarrow1)\rightsquigarrow1), I_A(x)\geq I_A((x\rightsquigarrow1)\rightsquigarrow1),$$
$$F_A(x)\leq F_A((x\rightsquigarrow1)\rightsquigarrow1).$$

By Proposition 3.2 we know that A is anti-grouped.

Proposition 3.2 Assume that A is both an anti-grouped neutrosophic filter and a neutrosophic closed filter in pseudo-BCI algebra X. Then A satisfies: $\forall x\in X$,

$$T_A(x)=T_A(x\to1),\ I_A(x)=I_A(x\to1),\ F_A(x)=F_A(x\to1).$$

Proof: For any $x\in X$, by Definition 3.2 we have

$$T_A(x\to1)\geq T_A(x),\ I_A(x\to1)\geq I_A(x),\ F_A(x\to1)\leq F_A(x).$$

Moreover, $\forall x\in X$, by Definition 2.19 and Definition 3.2,

$$T_A(x)\geq\min\{T_A((x\to1)\to(x\to x)),\ T_A(1)\}$$
$$=\min\{T_A((x\to1)\to1),\ T_A(1)\}$$
$$=T_A((x\to1)\to1)\geq T_A(x\to1),$$
$$I_A(x)\geq\min\{I_A((x\to1)\to(x\to x)),\ I_A(1)\}$$
$$=\min\{I_A((x\to1)\to1),\ I_A(1)\}$$
$$=I_A((x\to1)\to1)\geq I_A(x\to1),$$
$$F_A(x)\leq\max\{F_A((x\to1)\to(x\to x)),\ F_A(1)\}$$
$$=\max\{F_A((x\to1)\to1),\ F_A(1)\}$$
$$=F_A((x\to1)\to1)\leq F_A(x\to1).$$

That is,

$$T_A(x)\geq T_A(x\to1),\ I_A(x)\geq I_A(x\to1),\ F_A(x)\leq F_A(x\to1).$$

Therefore,

$$\forall x\in X,\ T_A(x)=T_A(x\to1),\ I_A(x)=I_A(x\to1),\ F_A(x)=F_A(x\to1).$$

Theorem 3.1 Let A be a neutrosophic filter in pseudo-BCI algebra X. Then the following conditions are equivalent:

(i) A is both an anti-grouped neutrosophic filter and a neutrosophic closed filter in X;

(ii) A satisfies: $\forall x\in X$,

$$T_A(x)=T_A(x\to1),\ I_A(x)=I_A(x\to1),\ F_A(x)=F_A(x\to1).$$

(iii) A is a neutrosophic regular filter in X.

Proof: (i) $\Rightarrow$ (ii) See Proposition 3.2.

(iii) $\Rightarrow$ (i) See Proposition 3.1 and Proposition 3.3.

(ii) $\Rightarrow$ (iii) Suppose that A satisfies: $\forall x\in X$,

$$T_A(x)=T_A(x\to1),\ I_A(x)=I_A(x\to1),\ F_A(x)=F_A(x\to1).$$

For any $x, y\in X$, using Proposition 2.1 (6) we have

$$y\to1\leq(x\to y)\to(x\to1).$$

From this, applying Propostion 2.5,

$$T_A(y\to1)\leq T_A((x\to y)\to(x\to1)),$$
$$I_A(y\to1)\leq I_A((x\to y)\to(x\to1)),$$
$$F_A(y\to1)\geq F_A((x\to y)\to(x\to1)).$$

From these, by Definition 2.18 we get

$$\min\{T_A(y\to1),\ T_A(x\to y)\}$$
$$\leq\min\{T_A((x\to y)\to(x\to1)),\ T_A(x\to y)\}=T_A(x\to1),$$
$$\min\{I_A(y\to1),\ I_A(x\to y)\}$$
$$\leq\min\{I_A((x\to y)\to(x\to1)),\ I_A(x\to y)\}=I_A(x\to1),$$
$$\max\{F_A(y\to1),\ F_A(x\to y)\}$$
$$\geq\max\{F_A((x\to y)\to(x\to1)),\ F_A(x\to y)\}=F_A(x\to1).$$

Moreover, by condition (ii),

$$T_A(y\to1)=T_A(y),\ T_A(x\to1)=T_A(x);$$
$$I_A(y\to1)=I_A(y),\ I_A(x\to1)=I_A(x);$$
$$F_A(y\to1)=F_A(y),\ F_A(x\to1)=F_A(x).$$

Therefore,

$$\min\{T_A(y),\ T_A(x\to y)\}\leq T_A(x),$$
$$\min\{I_A(y),\ I_A(x\to y)\}\leq I_A(x),$$
$$\max\{F_A(y),\ F_A(x\to y)\}\geq F_A(x).$$

Similarly, we can get

$$\min\{T_A(y),\ T_A(x\rightsquigarrow y)\}\leq T_A(x),$$
$$\min\{I_A(y),\ I_A(x\rightsquigarrow y)\}\leq I_A(x),$$
$$\max\{F_A(y),\ F_A(x\rightsquigarrow y)\}\geq F_A(x).$$

By Definition 3.1 we know that A is a neutrosophic regular filter in X.

4 Fuzzy regular filters and neutrosophic filters

Definition 4.1 A fuzzy filter A in pseudo-BCI algebra X is called to be regular if it satisfies:

(FRF1) $\forall x, y\in X$, $\min\{\mu_A(y),\ \mu_A(x\to y)\}\leq\mu_A(x)$;

(FRF2) $\forall x, y\in X$, $\min\{\mu_A(y),\ \mu_A(x\rightsquigarrow y)\}\leq\mu_A(x)$.

Lemma 4.1[9, 33] Let X be a pseudo-BCI algebra. Then a fuzzy set μ: $X\to[0, 1]$ is a fuzzy filter of X if and only if the level set $\mu_t=\{x\in X \mid \mu(x)\geq t\}$ is filter of X for all $t\in Im(\mu)$.

Theorem 4.1 Let X be a pseudo-BCI algebra. Then a fuzzy set μ: $X\to[0, 1]$ is a fuzzy regular filter of X if and only if the level set $\mu_t=\{x\in X \mid \mu(x)\geq t\}$ is regular filter of X for all $t\in Im(\mu)$.

Proof: Assume that μ is fuzzy regular filter of X. By Lemma 4.1, for any $t\in Im(\mu)$, we have

$$\mu_t=\{x\in X \mid \mu(x)\geq t\}\text{ is filter of }X.$$

If $y\in\mu_t$ and $x\to y\in\mu_t$, then

$$\mu(y)\geq t,\ \mu(x\to y)\geq t.$$

From this and Definition 4.1 (FRF1) we get

$$\mu_A(x)\geq\min\{\mu_A(y),\ \mu_A(x\to y)\}\geq t.$$

This means that $x\in\mu_t$. Similarly, we can prove that

$$y\in\mu_t\text{ and }x\rightsquigarrow y\in\mu_t\Rightarrow x\in\mu_t.$$

By Definition 2.14 we know that μ_t is regular filter of X

Conversely, assume that the level set $\mu_t=\{x\in X \mid \mu(x)\geq t\}$ is regular filter of X for all $t\in Im(\mu)$. By Lemma 4.1 we know that μ: $X\to[0, 1]$ is a fuzzy filter of X. Let $x, y\in X$, denote $t_0=\min\{\mu_A(y),\ \mu_A(x\to y)\}$, then $t_0\in Im(\mu)$ and

$$\mu(y)\geq t_0,\ \mu(x\to y)\geq t_0.$$

This means that $y\in\mu_{t_0}$ and $x\to y\in\mu_{t_0}$. Since μ_{t_0} is regular filter of X, by Definition 2.14 we have $x\in\mu_{t_0}$, that is

$\mu(x)\geq t_0=\min\{\mu_A(y), \mu_A(x\to y)\}$.

It follows that Definition 4.1 (FRF1) holds. Similarly, we can prove that $\forall x, y\in X$, $\min\{\mu_A(y), \mu_A(x\rightsquigarrow y)\}\leq\mu_A(x)$. Therefore, μ: $X\to[0, 1]$ is a fuzzy regular filter of X.

Similar to Theorem 4.1 we can get the following proposition (the proofs are omitted).

Proposition 4.1 Let X be a pseudo-BCI algebra. Then a fuzzy set μ: $X\to[0, 1]$ is a fuzzy closed filter of X if and only if the level set $\mu_t=\{ x\in X \mid \mu(x)\geq t\}$ is closed filter of X for all $t\in Im(\mu)$.

By Theorem 6 in [31] we have

Theorem 4.2 Let μ be a fuzzy filter of pseudo-BCI algebra X. Then the following conditions are equivalent:

(i) μ is fuzzy closed anti-grouped filter of X;

(ii) $\forall x\in X$, $\mu_A(x\to 1)=\mu_A(x)$.

(iii) μ is a fuzzy regular filter of X.

Theorem 4.3 Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is a neutrosophic closed filter in X if and only if A satisfies:

(i) T_A is a fuzzy closed filter of X;

(ii) I_A is a fuzzy closed filter of X;

(iii) $1-F_A$ is a fuzzy closed filter of X, where $(1-F_A)(x)=1-F_A(x)$, $\forall x\in X$.

Proof: Assume that A is a neutrosophic closed filter in X. By Definition 3.2 we have ($\forall x\in X$)

$T_A(x\to 1)\geq T_A(x)$, $I_A(x\to 1)\geq I_A(x)$, $F_A(x\to 1)\leq F_A(x)$.

Thus,

$(1-F_A)(x\to 1)=1-F_A(x\to 1)\geq 1-F_A(x)=(1-F_A)(x)$.

Therefore, using Definition 2.16, we get that T_A, I_A and $1-F_A$ are fuzzy closed filters of X.

Conversely, assume that T_A, I_A and $1-F_A$ are fuzzy closed filters of X. Then, by Definition 2.16,

$T_A(x\to 1)\geq T_A(x)$, $I_A(x\to 1)\geq I_A(x)$,

$(1-F_A)(x\to 1)\geq(1-F_A)(x)$.

Thus,

$F_A(x\to 1)=1-(1-F_A)(x\to 1)\leq 1-(1-F_A)(x)=F_A(x)$.

Hence, applying Definition 3.2 we get that A is a neutrosophic closed filter A in X.

By Theorem 4.2, Theorem 4.3, Theorem 3.1 and Proposition 2.7 we can get the following results.

Theorem 4.4 Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is a neutrosophic regular filter in X if and only if A satisfies:

(i) T_A is a fuzzy regular filter of X;

(ii) I_A is a fuzzy regular filter of X;

(iii) $1-F_A$ is a fuzzy regular filter of X, where $(1-F_A)(x)=1-F_A(x)$, $\forall x\in X$.

Theorem 4.5 Let X be a pseudo-BCI algebra, A be a neutrosophic set in X such that $T_A(x)\geq\alpha_0$, $I_A(x)\geq\beta_0$ and $F_A(x)\leq\gamma_0$, $\forall x\in X$, where $\alpha_0\in Im(T_A)$, $\beta_0\in Im(I_A)$ and $\gamma_0\in Im(F_A)$. Then A is a neutrosophic closed filter in X if and only if (α, β, γ)-level set $A^{(\alpha, \beta, \gamma)}$ is closed filter of X for all $\alpha\in Im(T_A)$, $\beta\in Im(I_A)$ and $\gamma\in Im(F_A)$.

Proof: Assume that A is neutrosophic closed filter in X. By Theorem 4.3 and Proposition 4.1, for any $\alpha\in Im(T_A)$, $\beta\in Im(I_A)$ and $\gamma\in Im(F_A)$, we have

$(T_A)_\alpha=\{x\in X \mid T_A(x)\geq\alpha\}$, $(I_A)_\beta=\{x\in X \mid I_A(x)\geq\beta\}$ and $(1-F_A)_{1-\gamma}=\{x\in X \mid (1-F_A)(x)\geq 1-\gamma\}=\{x\in X \mid F_A(x)\leq\gamma\}$ are closed filters of X.

Thus $(T_A)_\alpha\cap(I_A)_\beta\cap(1-F_A)_{1-\gamma}$ is a closed filters of X. Moreover, by Definition 2.8, it is easy to verify that (α, β, γ)-level set $A^{(\alpha, \beta, \gamma)}=(T_A)_\alpha\cap(I_A)_\beta\cap(1-F_A)_{1-\gamma}$. Therefore, $A^{(\alpha, \beta, \gamma)}$ is closed filter of X for all $\alpha\in Im(T_A)$, $\beta\in Im(I_A)$ and $\gamma\in Im(F_A)$.

Conversely, assume that $A^{(\alpha, \beta, \gamma)}$ is closed filter of X for all $\alpha\in Im(T_A)$, $\beta\in Im(I_A)$ and $\gamma\in Im(F_A)$. Since $T_A(x)\geq\alpha_0$, $I_A(x)\geq\beta_0$ and $F_A(x)\leq\gamma_0$, $\forall x\in X$, then

$$(T_A)_\alpha=\{x\in X \mid T_A(x)\geq\alpha\}=(T_A)_\alpha\cap X\cap X$$
$$=(T_A)_\alpha\cap(I_A)_{\beta_0}\cap(1-F_A)_{1-\gamma_0}=A^{(\alpha,\beta_0,\gamma_0)};$$
$$(I_A)_\beta=\{x\in X \mid I_A(x)\geq\beta\}=X\cap(I_A)_\beta\cap X$$
$$=(T_A)_{\alpha_0}\cap(I_A)_\beta\cap(1-F_A)_{1-\gamma_0}=A^{(\alpha_0,\beta,\gamma_0)};$$
$$(1-F_A)_{1-\gamma}=\{x\in X \mid (1-F_A)(x)\geq 1-\gamma\}$$
$$=X\cap X\cap\{x\in X \mid F_A(x)\leq\gamma\}$$
$$=(T_A)_{\alpha_0}\cap(I_A)_{\beta_0}\cap\{x\in X \mid F_A(x)\leq\gamma\}=A^{(\alpha_0,\beta_0,\gamma)}.$$

Thus,

$(T_A)_\alpha=\{x\in X \mid T_A(x)\geq\alpha\}$, $(I_A)_\beta=\{x\in X \mid I_A(x)\geq\beta\}$ and $(1-F_A)_{1-\gamma}=\{x\in X \mid (1-F_A)(x)\geq 1-\gamma\}=\{x\in X \mid F_A(x)\leq\gamma\}$ are closed filters of X.

From this, applying Proposition 4.1, we know that T_A, I_A and $1-F_A$ are fuzzy closed filters of X. By Theorem 4.3 we get that A is neutrosophic closed filter in X.

Similarly, we can get

Lemma 4.2 Let X be a pseudo-BCI algebra, A be a neutrosophic set in X such that $T_A(x)\geq\alpha_0$, $I_A(x)\geq\beta_0$ and $F_A(x)\leq\gamma_0$, $\forall x\in X$, where $\alpha_0\in Im(T_A)$, $\beta_0\in Im(I_A)$ and $\gamma_0\in Im(F_A)$. Then A is a (anti-grouped) neutrosophic filter in X if and only if (α, β, γ)-level set $A^{(\alpha, \beta, \gamma)}$ is (anti-grouped) filter of X for all $\alpha\in Im(T_A)$, $\beta\in Im(I_A)$ and $\gamma\in Im(F_A)$.

Combining Theorem 4.5, Lemma 4.2 and Theorem 3.1 we can get the following theorem.

Theorem 4.6 Let X be a pseudo-BCI algebra, A be a neutrosophic set in X such that $T_A(x)\geq\alpha_0$, $I_A(x)\geq\beta_0$ and $F_A(x)\leq\gamma_0$, $\forall x\in X$, where $\alpha_0\in Im(T_A)$, $\beta_0\in Im(I_A)$ and $\gamma_0\in Im(F_A)$. Then A is a neutrosophic regular filter in X if and only if (α, β, γ)-level set $A^{(\alpha, \beta, \gamma)}$ is regular filter of X for all $\alpha\in Im(T_A)$, $\beta\in Im(I_A)$ and $\gamma\in Im(F_A)$.

Conclusion

The neutrosophic set theory is applied to many scientific fields, and also applied to algebraic structures. This paper applied neutrosophic set theory to pseudo-BCI algebras, and some new notions of neutrosophic regular filter, neutrosophic closed filter and fuzzy regular filter in pseudo-BCI algebras are introduced. In addition to studying the basic properties of these new concepts, this paper also considered the relationships between them, and obtained some necessary and sufficient conditions.

Acknowledgment

This work was supported by National Natural Science Foundation of China (Grant No. 61573240, 61473239).

References

[1] A. A. A. Agboola, B. Davvaz, and F. Smarandache, Neutrosophic quadruple algebraic hyperstructures, Annals of Fuzzy Mathematics and Informatics, 14 (1) (2017), 29–42.

[2] R. A. Borzooei, H. Farahani, and M. Moniri, Neutrosophic deductive filters on BL-algebras, Journal of Intelligent & Fuzzy Systems, 26 (2014), 2993–3004.

[3] W. A. Dudek, and Y. B. Jun, Pseudo-BCI algebras, East Asian Mathematical Journal, 24 (2) (2008), 187–190.

[4] G. Georgescu and A. Iorgulescu, Pseudo-BCK algebras: an extension of BCK algebras, in: Combinatorics, Computability and Logic. Springer Ser. Discrete Math. Theor. Comput. Sci., 2001, 97–114.

[5] P. F. He, X. L. Xin and Y. W. Yang, On state residuated lattices, Soft Computing, 19 (8) (2015), 2083–2094.

[6] P. F. He, B. Zhao and X. L. Xin, States and internal states on semihoops, Soft Computing, 21 (11) (2017), 2941–2957.

[7] Y. B. Jun, H. S. Kim and J. Neggers, On pseudo-BCI ideals of pseudo-BCI algebras, Matematicki Vesnik, 58 (1-2) (2006), 39–46.

[8] H. S. Kim, Y. H. Kim, On BE-algebras, Sci. Math. Japon., 66(1) (2007), 113–116.

[9] M. Kondo and W.A. Dudek, On the transfer principle in fuzzy theory, Mathware & Soft Computing, 12 (2005), 41–55.

[10]K. J. Lee and C. H. Park, Some ideals of pseudo BCI-algebras, Journal of Applied Mathematics and Informatics, 27 (1-2) (2009), 217–231.

[11]L. Z. Liu, Generalized intuitionistic fuzzy filters on residuated lattices, Journal of Intelligent & Fuzzy Systems, 28 (2015), 1545–1552

[12]Z. M. Ma, B. Q. Hu, Characterizations and new subclasses of I-filters in residuated lattices, Fuzzy Sets and Systems, 247 (2014), 92–107.

[13]Z. M. Ma, W. Yang, Z. Q. Liu, Several types of filters related to the Stonean axiom in residuated lattices, Journal of Intelligent & Fuzzy Systems, 32 (1) (2017), 681–690.

[14]B. L. Meng, On filters in BE-algebras, Sci. Math. Japon., 71 (2010), 201–207.

[15]A. Rezaei, A. B. Saeid, and F. Smarandache, Neutrosophic filters in BE-algebras, Ratio Mathematica, 29 (2015), 65–79.

[16]F. Smarandache, Neutrosophy, Neutrosophic Probability, Set, and Logic, Amer. Res. Press, Rehoboth, USA, 1998.

[17]F. Smarandache, Neutrosophy and Neutrosophic Logic, Information Sciences First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA, 2002.

[18]F. Smarandache, Neutrosophic set–a generialization of the intuituionistics fuzzy sets, International Journal of Pure and Applied Mathematics, 24 (3) (2005), 287–297.

[19]F. Smarandache, Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications, Pons Publishing House, Brussels, 2017

[20]C. A. C. Sweety, I. Arockiarani, Rough sets in neutrosophic approximation space, Annals of Fuzzy Mathematics and Informatics, 13 (4) (2017), 449–463.

[21]H. Wang, F. Smarandache, Y. Q. Zhang, et al, Single valued neutrosophic sets, Multispace & Multistructure. Neutrosophic Transdisciplinarity, 4 (2010), 410–413.

[22]J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, Applied Mathematical Modelling, 38 (2014), 1170–1175.

[23]J. M. Zhan, Q. Liu and Hee Sik Kim, Rough fuzzy (fuzzy rough) strong h-ideals of hemirings, Italian Journal of Pure and Applied Mathematics, 34(2015), 483–496.

[24]X. H. Zhang, Y. Q. Wang, and W. A. Dudek, T-ideals in BZ-algebras and T-type BZ-algebras, Indian Journal Pure and Applied Mathematics, 34(2003), 1559-1570.

[25]X. H. Zhang and W. H. Li, On pseudo-BL algebras and BCC-algebra, Soft Computing, 10 (2006), 941–952.

[26]X. H. Zhang, Fuzzy Logics and Algebraic Analysis, Science Press, Beijing, 2008.

[27]X. H. Zhang and W. A. Dudek, BIK+-logic and non-commutative fuzzy logics, Fuzzy Systems and Mathematics, 23 (4) (2009), 8–20.

[28]X. H. Zhang, BCC-algebras and residuated partially-ordered groupoid, Mathematica Slovaca, 63 (3) (2013), 397–410.

[29]X. H. Zhang and Y. B. Jun, Anti-grouped pseudo-BCI algebras and anti-grouped pseudo-BCI filters, Fuzzy Systems and Mathematics, 28 (2) (2014), 21–33.

[30]X. H. Zhang, H. J. Zhou and X. Y. Mao, IMTL(MV)-filters and fuzzy IMTL(MV)-filters of residuated lattices, Journal of Intelligent & Fuzzy Systems, 26 (2) (2014), 589–596.

[31]X. H. Zhang, Fuzzy commutative filters and fuzzy closed filters in pseudo-BCI algebras, Journal of Computational Information Systems, 10 (9) (2014), 3577–3584.

[32]X. H. Zhang, Fuzzy 1-type and 2-type positive implicative filters of pseudo-BCK algebras, Journal of Intelligent & Fuzzy Systems, 28 (5) (2015), 2309–2317.

[33]X. H. Zhang, Fuzzy anti-grouped filters and fuzzy normal filters in pseudo-BCI algebras, Journal of Intelligent and Fuzzy Systems, 33 (2017), 1767–1774.

[34]X. H. Zhang and Choonkil Park, On regular filters and well filters of pseudo-BCI algebras, Proceedings of the 13th International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD 2017), IEEE, 2017.

[35]X. H. Zhang, Y. T. Wu, and X. H. Zhai, Neutrosophic filters in pseudo-BCI algebras, submitted, 2017.

[36]Topal, S. and Smaradache, F. A Lattice-Theoretic Look: A Negated Approach to Adjectival (Intersective, Neutrosophic and Private) Phrases. The 2017 IEEE International Conference on INnovations in Intelligent SysTems and Applications (INISTA 2017); (accepted for publication).

Received: July 3, 2017. Accepted: July 19, 2017.