

Continuity of higher order commutators generated by maximal Bochner-Riesz operator on Morrey space

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Abstract: In this papers, we use the control method of the maximal fractional integral and obtain the boundedness of higher order commutator generated by maximal Bochner-Riesz operator on Morrey space. Moreover, we get its continuity from Morrey space to Lipschitz space and from Morrey space to BMO space.

Key words: BMO space, higher commutator, Morrey space, Lipschitz space

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1 Introduction And Main Results

Let maximal Bochner-Riesz operator B_δ^* [1] be

$$B_\delta^* f(x) = \sup_{t>0} |B_\delta^t(f)(x)|,$$

where $B_\delta^t(x) = t^{-n} B_\delta(x/t)$ is the kernel of B_δ^t , which satisfies $|\frac{\partial^\beta}{\partial x^\beta} B_\delta(x)| \leq C(1+|x|)^{-(\delta+(n+1)/2)}$ for any $x \in \mathbb{R}^n$ and multi-index $\beta \in \mathbb{Z}_+^n$.

For $0 < \beta < 1$, let the homogeneous Besov-Lipschitz space $\dot{\Lambda}_\beta$ [2] be the space of the functions satisfying

$$\|f\|_{\dot{\Lambda}_\beta} = \sup_{x,y \in \mathbb{R}^n, x \neq y} \frac{|f(x) - f(y)|}{|x - y|^\beta}.$$

Suppose $b \in \dot{\Lambda}_\beta$. For suitable function f , define the high order commutator $B_{\delta,*}^{b,m}$ generated by B_δ^* and b as follows

$$B_{\delta,*}^{b,m} f(x) = \sup_{t>0} \int_{\mathbb{R}^n} B_\delta^t(x-y)[b(x) - b(y)]^m f(y) dy.$$

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For $1 < p < \infty$, $\lambda \geq 0$, the classic Morrey space $M^{p,\lambda}(\mathbb{R}^n)$ ^[3] is the space of function $f \in L^p_{loc}$ such that

$$\|f\|_{M^{p,\lambda}} = \sup_{x \in \mathbb{R}^n, r > 0} \left(\frac{1}{|Q(x,r)|^\lambda} \int_{Q(x,r)} |f(y)|^p dy \right)^{1/p} < \infty.$$

Let $1 \leq p < \infty$. Denote the generalized Morrey space^[4] as

$$M^{p,\varphi} = \{f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{M^{p,\varphi}} < \infty\},$$

where

$$\|f\|_{M^{p,\varphi}} = \sup_{x \in \mathbb{R}^n, r > 0} \left(\frac{1}{\varphi(x,r)} \int_Q |f(y)|^p dy \right)^{1/p},$$

φ is a integer increasing function on $\mathbb{R}^n \times \mathbb{R}^+$ satisfying

$$\varphi(x, 2r) \leq D_\varphi \varphi(x, r),$$

and D_φ is a constant, independent of r .

For $1 \leq l < \infty$ and $\beta > 0$, we call that

$$T_l^{l\beta}(f)(x) = \sup_{x \in Q} \left(\frac{1}{|Q|^{1-l\beta/n}} \int_Q |f(y)|^l dy \right)^{1/l}$$

is the maximal fractional integral^[5], where $l < p < n/\beta$ and $1/q = 1/p - \beta/n$.

Let $\lambda < 1/n$ and $1 < q < \infty$. We define $f \in L^q_{loc}(\mathbb{R}^n)$ is belong to $BMO^{\lambda,q}$ space if f satisfies

$$\|f\|_{BMO^{\lambda,q}} = \sup_{x \in \mathbb{R}^n, r > 0} \left(\frac{1}{|Q(x,r)|^{1+\lambda q}} \int_{Q(x,r)} |f(y) - f_Q|^q dy \right)^{1/q} < \infty.$$

Also, we have

$$\|f\|_{BMO^{\lambda,q}} \approx \sup_{x \in \mathbb{R}^n, r > 0} \left(\frac{1}{|Q(x,r)|^{1+\lambda}} \int_{Q(x,r)} |f(y) - f_Q| dy \right) \approx \|f\|_{\dot{\Lambda}_\beta},$$

where $f_Q = \frac{1}{|Q|} \int_Q f(y) dy$, $\lambda = \beta/n$.

In this paper, x_0 is the center of cube Q with the side length r . Q_k is the cube with the same center as Q and side length 2^k times the side length of Q .

Theorem 1.1 Suppose $\lambda \geq 0$, $\delta > (n-1)/2$, $0 < m\beta < n$, $0 < \beta < 1$, $1 < p < n/(m\beta)$, $1/p - m\beta/n = 1/q$ and $b \in \dot{\Lambda}_\beta$. We have

$$\|B_{\delta,*}^{b,m}(f)\|_{M^{q,\lambda}} \leq C \|f\|_{M^{p,\lambda(1-\beta m p/n)}}.$$

Corollary 1.2 Let $\varphi_1^{1/p_1}(r) = \varphi_2^{1/p_2}(r)$. Under conditions of theorem 1.1, we get

$$\|B_{\delta,*}^{b,m}(f)\|_{M^{p_2,\varphi_2}} \leq C \|f\|_{M^{p,\varphi_1}}.$$

Theorem 1.3 Let $\delta > (n-1)/2$, $b \in \dot{\Lambda}_\beta$, $0 < \beta < 1$, $0 < m\beta < n$, $1/p - m\beta/n = 1/q$, $1 < p < n/(m\beta)$, $\zeta = \min\{1, \delta - (n-1)/2\}$, and $-1/q < \lambda < \zeta$. We have

$$\|B_{\delta,*}^{b,m}\|_{BMO^{q,\lambda}} \leq C\|f\|_{M^{p,1+p(\lambda-m\beta/n)}}.$$

Theorem 1.4 Suppose $\delta > (n-1)/2$, $1/p - m\beta/n = 1/q$ and $b \in \dot{\Lambda}_\beta$, $0 < \beta < 1$, $0 < m\beta < n$, $1 < p < n/(m\beta)$. Let $\zeta = \min\{1, \delta - (n-1)/2\}$ and $1 - (mp\beta)/n < \lambda < 1 + (\zeta - m\beta)p/n$. We get

$$\|B_{\delta,*}^{b,m}\|_{\dot{\Lambda}_{m\beta+(\lambda-1)n/p}} \leq C\|f\|_{M^{p,\lambda}}.$$

2 $[M^{p,\lambda(1-\beta mp/n)}(\mathbb{R}^n), M^{q,\lambda}(\mathbb{R}^n)]$ -Type Continuity.

Lemma 2.1 Let $\delta > (n-1)/2$, $b \in \dot{\Lambda}_\beta$, $0 < m\beta < n$, $1 < p < n/(m\beta)$ and $1/p - m\beta/n = 1/q$, Then there is a constant $C > 0$, independent of f , such that

$$\|T_{\delta,*}^{b,m} f\|_q \leq C\|f\|_p.$$

Proof of lemma 2.1 For $x \in \mathbb{R}^n$ and $\varepsilon > 0$ with $0 < m\beta - \varepsilon < m\beta + \varepsilon < n$, we choose a $\xi > 0$ such that

$$\xi^{2\varepsilon} = M_{m\beta-\varepsilon} f(x) / M_{m\beta+\varepsilon} f(x).$$

Write

$$\begin{aligned} T_{\delta,t}^{b,m} f(x) &= \int_{|x-y|<\xi} B_\delta^t(x-y)[b(x)-b(y)]^m f(y) dy \\ &+ \int_{|x-y|\geq\xi} B_\delta^t(x-y)[b(x)-b(y)]^m f(y) dy = D_1 + D_2. \end{aligned}$$

By the inequality^[4] $|B_\delta^t(x-y)| \leq |x-y|^{-n}$ and $b \in \dot{\Lambda}_\beta$, we have

$$\begin{aligned} |D_1| &\leq C \sum_{j=0}^{\infty} \int_{2^{-j-1}\xi \leq |x-y| < 2^{-j}\xi} \frac{|f(y)|}{|2^{-j}\xi|^{n-m\beta}} dy \\ &\leq C \sum_{j=0}^{\infty} \frac{(2^{-j}\xi)^\varepsilon}{|2^{-j}\xi|^{n-m\beta+\varepsilon}} \int_{|x-y| < 2^{-j}\xi} |f(y)| dy \\ &\leq C \sum_{j=0}^{\infty} 2^{-j\varepsilon} \xi^\varepsilon M_{m\beta-\varepsilon} f(x) \leq C \xi^\varepsilon M_{m\beta-\varepsilon} f(x). \end{aligned}$$

Similarly, we get

$$\begin{aligned}
|D_2| &\leq C \sum_{j=1}^{\infty} \int_{2^{j-1}\xi \leq |x-y| < 2^j\xi} \frac{|2^j\xi|^{m\beta}}{|2^{j-1}\xi|^n} |f(y)| dy \\
&\leq C \sum_{j=1}^{\infty} \frac{(2^j\xi)^{-\varepsilon}}{|2^j\xi|^{n-m\beta-\varepsilon}} \int_{|x-y| < 2^j\xi} |f(y)| dy \\
&\leq C \sum_{j=0}^{\infty} 2^{-j\varepsilon} \xi^{-\varepsilon} M_{m\beta+\varepsilon} f(x) \leq C \xi^{-\varepsilon} M_{m\beta+\varepsilon} f(x).
\end{aligned}$$

Thus, by the above selection of ξ we get

$$|T_{\delta,t}^{b,m} f(x)| \leq C(\xi^\varepsilon M_{m\beta-\varepsilon} f(x) + \xi^{-\varepsilon} M_{m\beta+\varepsilon} f(x)) = C[M_{m\beta+\varepsilon} f(x)]^{1/2} [M_{m\beta-\varepsilon} f(x)]^{1/2}.$$

Noting $1 < p < n/(m\beta)$, there is an $\varepsilon > 0$, such that $1 < p < n/(m\beta + \varepsilon)$. Let $1/q_1 = 1/p - (m\beta - \varepsilon)/n$, $1/q_2 = 1/p - (m\beta + \varepsilon)/n$, $l = 2q_1/q$, $l' = 2q_2/q$, then $q_1, q_2 > 0, l' > 1$ and $1/l + 1/l' = 1$. Thus, by inequality (1), we have

$$\begin{aligned}
\|T_{\delta,t}^{b,m} f\|_q^q &\leq C \int_{\mathbb{R}^n} |M_{m\beta-\varepsilon} f(x)|^{q/2} |M_{m\beta+\varepsilon} f(x)|^{q/2} dx \\
&\leq C \left(\int_{\mathbb{R}^n} |M_{m\beta-\varepsilon} f(x)|^{q l/2} dx \right)^{1/l} \left(\int_{\mathbb{R}^n} |M_{m\beta+\varepsilon} f(x)|^{q l'/2} dx \right)^{1/l'} \\
&\leq C \left(\int_{\mathbb{R}^n} |M_{m\beta-\varepsilon} f(x)|^{q_1} dx \right)^{q/2q_1} \left(\int_{\mathbb{R}^n} |M_{m\beta+\varepsilon} f(x)|^{q_2} dx \right)^{q/2q_2} \\
&\leq C \left(\int_{\mathbb{R}^n} |f(x)|^p dx \right)^{q/2p} \left(\int_{\mathbb{R}^n} |f(x)|^p dx \right)^{q/2p} = C \|f\|_p^q.
\end{aligned}$$

Lemma 2.2^[6] Let $T_l^{l\beta}$ be the maximal fractional operator, $\lambda \geq 0$, $1 \leq l < p < n/\beta$ and $1/p - \beta/n = 1/q$, we have

$$\|T_l^{l\beta}(f)\|_{M^{q,\lambda}} \leq C \|f\|_{M^{p,\lambda(1-\beta p/n)}}.$$

Proof of Theorem 1.1 Let $f = f\chi_{2Q} + f\chi_{(2Q)^c} = f_1 + f_2$. Write

$$\begin{aligned}
&\frac{1}{|Q|} \int_Q |B_{\delta,t}^{b,m} f(y) - B_{\delta,t}^{b,m} f_2(x_0)| dy \\
&\leq C \frac{1}{|Q|} \int_Q |B_{\delta,t}^{b,m} f_1(y)| dy + C \frac{1}{|Q|} \int_Q |B_{\delta,t}^{b,m} f_2(y) - B_{\delta,t}^{b,m} f_2(x_0)| dy \\
&= E_1 + E_2.
\end{aligned}$$

We choose l satisfying $1 \leq l < p < n/(m\beta)$. Take a s such that $1/s = 1/l - m\beta/n$. By the lemma 2.1 and Hölder inequality, we have

$$\begin{aligned}
E_1 &\leq C \frac{1}{|Q|} \int_Q |T_{\delta,t}^{b,m} f_1(y)|^s dy)^{1/s} \leq C \frac{1}{|Q|^{1/s}} \left(\int_Q |f(y)|^l dy \right)^{1/l} \\
&\leq C \frac{1}{|Q|^{1-m\beta l/n}} \int_Q |f(y)|^l dy)^{1/l} \leq C T_l^{m\beta l} f(x).
\end{aligned}$$

We turn to E_2 . Noticing $z \in (2Q)^c, y \in Q$ and $|y - z| \sim |x_0 - z|$, we have

case 1 $t < r$

$$\begin{aligned} |B_{\delta,t}^{b,m} f_2(y) - B_{\delta,t}^{b,m} f_2(x_0)| &\leq C \sum_{k=1}^{\infty} \left| \int_{Q_{k+1} \setminus Q_k} B_{\delta}^t(y-z)[b(y) - b(z)]^m f(z) dz \right| \\ &\leq C \sum_{k=1}^{\infty} |2^k r|^{m\beta} t^{-n} \int_{Q_{k+1} \setminus Q_k} \left(1 + \frac{|x-z|}{t}\right)^{-(\delta + \frac{n+1}{2})} |f(z)| dz \\ &\leq C \sum_{k=1}^{\infty} 2^{-k[\delta - (n-1)/2]} \left(\frac{1}{|Q_{k+1}|^{1-m\beta l/n}} \int_{Q_{k+1}} |f(z)|^l dz\right)^{1/l} \leq CT_l^{m,\beta l} f(x). \end{aligned}$$

case 2 $t \geq r$

(1) We consider $\delta < (n+1)/2$. By the mean value theorem, we obtain

$$\begin{aligned} |T_{\delta,t}^{b,m} f_2(y) - T_{\delta,t}^{b,m} f_2(x_0)| &\leq C \sum_{k=1}^{\infty} |2^k r|^{m\beta} \left| \int_{Q_{k+1} \setminus Q_k} B_{\delta}^t(y-z) f(z) dz - \int_{Q_{k+1} \setminus Q_k} B_{\delta}^t(x_0-z) f(z) dz \right| \\ &\leq C \sum_{k=1}^{\infty} |2^k r|^{m\beta} t^{-n-1} \int_{Q_{k+1} \setminus Q_k} \left(1 + \frac{|x_0-z|}{t}\right)^{-(\delta + \frac{n+1}{2})} |y-x_0| |f(z)| dz \\ &\leq C \sum_{k=1}^{\infty} \left(\frac{r}{t}\right)^{\frac{n+1}{2} - \delta} 2^{-k[\delta - (n-1)/2]} \left(\frac{1}{|Q_{k+1}|^{1-m\beta l/n}} \int_{Q_{k+1}} |f(z)|^l dz\right)^{1/l} \leq CT_l^{m,\beta l} f(x). \end{aligned}$$

(2) When $\delta \geq (n+1)/2$, we also obtain

$$\begin{aligned} |T_{\delta,t}^{b,m} f_2(y) - T_{\delta,t}^{b,m} f_2(x_0)| &\leq C \sum_{k=1}^{\infty} |2^k r|^{m\beta} t^{-n-1} r \int_{Q_{k+1} \setminus Q_k} \left(1 + \frac{|x_0-z|}{t}\right)^{-(\delta + \frac{n+1}{2})} |f(z)| dz \\ &\leq C \sum_{k=1}^{\infty} 2^{-k} T_l^{m,\beta l} f(x) \leq CT_l^{m,\beta l} f(x). \end{aligned}$$

Then, by the lemma 2.2, we gain

$$\begin{aligned} \|B_{\delta,t}^{b,m}(f)\|_{M^{q,\lambda}} &\leq C \|M(B_{\delta,t}^{b,m})(f)\|_{M^{q,\lambda}} \leq C \|(B_{\delta,t}^A)^{\sharp}(f)(x)\|_{M^{q,\lambda}} \\ &\leq C \|T_l^{m,\beta l}(f)\|_{M^{q,\lambda}} \leq C \|f\|_{M^{p,\lambda(1-m\beta p/n)}}. \end{aligned}$$

Taking the supreme of the left side about E_1 and E_2 for any $t > 0$, this completes the proof of theorem 1.1.

Proof of Corollary 1.2

$$\begin{aligned} & \left(\frac{1}{\varphi_2(r)} \int_Q |B_{\delta,t}^{b,m} f(x)|^q dx \right)^q = \frac{|Q|^{\lambda/q}}{\varphi_2^{1/q}(r)} \left(\frac{1}{|Q|^\lambda} \int_Q |B_{\delta,t}^{b,m} f(x)|^q dx \right)^{1/q} \\ & \leq \frac{|Q|^{\lambda/q} \varphi_1(r)^{1/p}}{\varphi_2(r)^{1/q} |Q|^{\lambda/p - \lambda m \beta/n}} \left(\frac{1}{\varphi_1(r)} \int_Q |f(x)|^p dx \right)^{1/p} = C \left(\frac{1}{\varphi_1(r)} \int_Q |f(x)|^p dx \right)^{1/p}. \end{aligned}$$

3 $[M^{p,1+p(\lambda-m\beta/n)}(\mathbb{R}^n), BMO^{\lambda,q}(\mathbb{R}^n)]$ -Type Continuity .

Proof of Theorem 1.3 Write $f = f\chi_{2Q} + f\chi_{(2Q)^c} = f_1 + f_2$, then

$$\begin{aligned} & \left(\frac{1}{|Q|^{1+q\lambda}} \int_Q |B_{\delta,t}^{b,m} f(x) - B_{\delta,t}^{b,m} f_2(x_0)|^q dx \right)^{1/q} \leq C \left(\frac{1}{|Q|^{1+q\lambda}} \int_Q |B_{\delta,t}^{b,m} f_1(x)|^q dx \right)^{1/q} \\ & + C \left(\frac{1}{|Q|^{1+q\lambda}} \int_Q |B_{\delta,t}^{b,m} f_2(x) - B_{\delta,t}^{b,m} f_2(x_0)|^q dx \right)^{1/q} = F_1 + F_2. \end{aligned}$$

By the lemma 2.1, we have

$$F_1 \leq C \frac{1}{|Q|^{1/q+\lambda}} \left(\int_Q |f_1(x)|^p dx \right)^{1/p} \leq C \|f\|_{M^{p,1+p(\lambda-m\beta/n)}}.$$

Now, we consider F_2 . Noticing $x \in Q$, we have $y \in (2Q)^c$.

case 1 $t < r$. We then obtain

$$\begin{aligned} & |B_{\delta,t}^{b,m} f_2(x) - B_{\delta,t}^{b,m} f_2(x_0)| \\ & \leq C \sum_{k=1}^{\infty} |2^{k+1}Q|^{1+\lambda t^{\delta-(n-1)/2}} |2^k r|^{-[\delta+(n+1)/2]} \left(\frac{1}{|Q_{k+1}|^{1+p(\lambda-m\beta/n)}} \int_{Q_{k+1}} |f(y)|^p dy \right)^{1/p} \\ & \leq C \left(\frac{t}{r} \right)^{\delta-(n-1)/2} \sum_{k=1}^{\infty} 2^{-k[\delta-(n-1)/2-n\lambda]} r^{n\lambda} \|f\|_{M^{p,1+p(\lambda-m\beta/n)}} \leq C r^{n\lambda} \|f\|_{M^{p,1+p(\lambda-m\beta/n)}}. \end{aligned}$$

case 2 $t \geq r$.

(1) When $\delta < (n+1)/2$, by the mean value theorem, we have

$$\begin{aligned} & |T_{\delta,t}^{b,m} f_2(x) - T_{\delta,t}^{b,m} f_2(x_0)| \\ & \leq C \sum_{j=2}^{\infty} |2^{k_0+j}|^{m\beta t^{-n-1}} \int_{B_{k_0+j+1} \setminus B_{k_0+j}} |x-x_0| \left(1 + \frac{|x-y|}{t}\right)^{-[\delta+(n+1)/2]} |f(y)| dy \\ & \leq C \sum_{k=1}^{\infty} 2^{-k[\delta-(n-1)/2-n\lambda]} r^{n\lambda} \|f\|_{M^{p,1+p(\lambda-m\beta/n)}} \leq C r^{n\lambda} \|f\|_{M^{p,1+p(\lambda-m\beta/n)}}. \end{aligned}$$

(2) When $\delta \geq (n+1)/2$, we similarly gain

$$\begin{aligned} & |T_{\delta,t}^{b,m} f_2(x) - T_{\delta,t}^{b,m} f_2(x_0)| \leq C \sum_{k=1}^{\infty} 2^{-k(1-n\lambda)} r^{n\lambda} \|f\|_{M^{p,1+p(\lambda-m\beta/n)}} \\ & \leq C r^{n\lambda} \|f\|_{M^{p,1+p(\lambda-m\beta/n)}}. \end{aligned}$$

So, we get estimation for F_2 which is

$$F_2 \leq C \|f\|_{M^{p,1+p(\lambda-m\beta/n)}}.$$

These yields the desired result.

4 $[M^{p,\lambda}(\mathbb{R}^n), \dot{\Lambda}_{[m\beta+n/p(\lambda-1)]/n}(\mathbb{R}^n)]$ -Type Continuity .

Proof of Theorem 1.4 We write

$$\begin{aligned} \int_Q |B_{\delta,t}^{b,m} f(x) - T_{\delta,t}^{b,m} f_2(x_0)| dx &\leq C \int_Q |T_{\delta,t}^{b,m} f_1(x)| dx \\ &+ \int_Q |T_{\delta,t}^{b,m} f_2(x) - T_{\delta,t}^{b,m} f_2(x_0)| dx = G_1 + G_2. \end{aligned}$$

By the Hölder inequality and the theorem 1.1 ,we get

$$G_1 \leq C \left(\int_Q |T_{\delta,t}^{b,m} f_1(x)|^q dx \right)^{\frac{1}{q}} |Q|^{1-1/q} \leq C |Q|^{1+[m\beta+(\lambda-1)n/p]/n} \|f\|_{M^{p,\lambda}}.$$

Now, we estimate G_2 . Noting $x \in Q$, $y \in (2Q)^c$, we know $|x - y| \sim |x_0 - y|$.

case 2 $t < r$. We get

$$\begin{aligned} &|T_{\delta,t}^{b,m} f_2(x) - T_{\delta,t}^{b,m} f_2(x_0)| \\ &\leq C \sum_{k=1}^{\infty} 2^{-k[\delta-(n-1)/2-m\beta+(1-\lambda)n/p]} |Q|^{m\beta/n-1/p+\lambda/p} \|f\|_{M^{p,\lambda}} \\ &\leq C |Q|^{m\beta/n-1/p+\lambda/p} \|f\|_{M^{p,\lambda}}. \end{aligned}$$

case 2 $t \geq r$.

(1) When $\delta < (n+1)/2$, we have

$$\begin{aligned} &|T_{\delta,t}^{b,m} f_2(x) - T_{\delta,t}^{b,m} f_2(x_0)| \\ &\leq C \left(\frac{r}{t}\right)^{(n+1)/2-\delta} \sum_{k=1}^{\infty} 2^{-k[\delta-(n-1)/2-m\beta+(1-\lambda)n/p]} |Q|^{m\beta/n-1/p+\lambda/p} \|f\|_{M^{p,\lambda}} \\ &\leq C |Q|^{m\beta/n-1/p+\lambda/p} \|f\|_{M^{p,\lambda}}. \end{aligned}$$

(2) When $\delta \geq (n+1)/2$, we have

$$\begin{aligned} &|T_{\delta,t}^{b,m} f_2(x) - T_{\delta,t}^{b,m} f_2(x_0)| \\ &\leq C \sum_{k=1}^{\infty} 2^{-k[1-m\beta+(1-\lambda)n/p]} |Q|^{m\beta/n-1/p+\lambda/p} \|f\|_{M^{p,\lambda}} \\ &\leq C |Q|^{m\beta/n-1/p+\lambda/p} \|f\|_{M^{p,\lambda}}. \end{aligned}$$

We gain

$$G_2 \leq C|Q|^{1+[m\beta+(\lambda-1)n/p]/n} \|f\|_{M^{p,\lambda}}.$$

Together with G_1 and G_2 , we have

$$\frac{1}{|Q|^{1+[m\beta+(\lambda-1)n/p]/n}} \int_Q |B_{\delta,t}^{b,m} f(x) - T_{\delta,t}^{b,m} f_2(x_0)| dx \leq C \|f\|_{M^{p,\lambda}}.$$

Taking the supreme of the left side for any $t > 0$, we get the ideal result. This finishes the proof of theorem 1.4.

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