# AVOIDING INFINITE REGRESS: POSTERIOR ANALYTICS I 22*

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**Article info**
CDD: 185
*Received: 03.10.2019; Accepted: 04.10.2019*
DOI: http://dx.doi.org/10.1590/0100-6045.2019.V42N4.BZ

**Keywords**
Aristotle
Predication
Infinite Regress
Essence

**Abstract:** This article offers a reconstruction of an argument against infinite regress formulated by Aristotle in *Posterior Analytics* I 22. I argue against the traditional interpretation of the chapter, according to which singular terms and summa genera, in virtue of having restrict logical roles, provide limits for predicative chains,

*This paper is a substantially modified version of Chapter 3 of my PhD dissertation (see Zuppolini 2017). I am deeply thankful to my supervisor, Lucas Angioni, and the examiners, Raphael Zillig, Michail Peramatzis, David Bronstein and Manuel Berrón, for their objections and remarks. I presented a draft of this article at the Workshop on Aristotelian Substances and their Substantiality, at UFRGS. I am grateful to the organisers, Raphael Zillig and Wolfgang Sattler, and the other participants, Mary Louise Gill, Marco Zingano, Wellington de Almeida, Jorge Mittelmann, Fabian Mié, and Paulo Ferreira. I also benefited from comments by David Bronstein and Lucas Angioni on the final draft of this article.

preventing them from proceeding ad infinitum. As I intend to show, this traditional reading is at odds with some important aspects of Aristotle's theory of demonstration. More importantly, it fails to explain how his proof is connected to a defence of the existence of ultimate explanations, a connection that must be the case if I 19–22 is advancing a foundationalist way-out to a sceptical challenge raised in I 3.

## 1. INTRODUCTION

Even though the *Posterior Analytics* (hereafter, *APo*) is concerned with ἐπιστήμη, commonly translated as 'knowledge', its doctrine can hardly be classified as an epistemology *stricto sensu*. The main object of the treatise is ἐπιστήμη ἅπλῶς, as defined in *APo* I 2, 71b9–12, a distinguished kind of knowledge peculiar to expert scientists. Aristotle does not present a systematic account of a broader concept of knowledge, nor is he interested in convincing sceptical readers of the possibility of knowledge in general.[1] Nevertheless, he does recognize the need to face a particular sceptical challenge in *APo* I 3. The challenge attacks the notion of demonstration (ἀπόδειξις), which has been defined as "συλλογισμὸν ἐπιστημονικόν" (71b18), i.e. a deductive argument that produces scientific knowledge, which means that its premises must reveal the causal explanation of the conclusion (*APo* I 2, 71b9–19). Now, suppose that the categorical premises from which a given truth is explained require a causal explanation as well. If so, our scientific understanding of the conclusion would remain inaccurate or incomplete unless the demonstration takes the form of a complex argument in which the premises are

[1] See Burnyeat (1981); Taylor (1990, 116); Ferejohn (1991, 2–3).

themselves properly explained.[2] Let us say that the ordered pair ⟨Π, *c*⟩ is a complete demonstration in this sense, where Π is a set of premises $p_1$, $p_2$, … , $p_n$ and *c* is the conclusion the scientist intends to explain. Now, if each $p_i$ is itself demonstrable, there must be, for each $p_i$, a subset of Π, Φ, such that ⟨Φ, $p_i$⟩ would be the complete demonstration of $p_i$. If $p_i$ is not a member of Φ (the set from which it is demonstrated), every scientific truth would be demonstrated from different and more basic premises, which makes Π infinite and impossible to survey with thought (*APo* I 3, 72b10–11).

In *APo* I 3, Aristotle rejects a potential solution to this challenge, according to which demonstrations would proceed "in a circle and reciprocally" (*APo* I 3*,* 72b17–18). In that case, however, if ⟨Π, *c*⟩ were the complete demonstration of *c* (in the sense we have just defined), *c* itself would be a member of Π, which is unacceptable for Aristotle (see *APo* I 2, 72b32–73a6). The philosopher prefers to deny the common assumption, held both by the sceptics and by the proponents of circular demonstration, that all scientific truths are demonstrable. If ⟨Π, *c*⟩ is the complete demonstration of *c*, the set Π is finite because there is a subset of Π that contains only indemonstrable truths, from which the other premises in Π and, consequently, *c* are demonstrated. In *APo* I 3, Aristotle states this foundationalist solution without presenting an argument in its favour. A proper proof is offered only in *APo* I 19–22.[3] The reason for such a delay is simple. This proof is not intended to provide a way out to the problem of infinite regress as it is usually conceived, i.e. as a typical

[2] See *APo* I 24, 86a14–19. See my See my Zuppolini (forthcoming).

[3] Lear (1980, 15–34) describes this argument as a "compactness proof". For a criticism of Lear's position, see Scanlan (1983). On this, see also Crager (2015, 93–95).

epistemological problem concerning the possibility of knowledge in general. After introducing a concept of scientific knowledge and demonstration in *APo* I 1–3, Aristotle presents us with an abstract *model* of demonstrative science, which adopts a specific ontological framework, a given set of semantic principles, and, of course, a underlying logic: the Syllogistic.[4] What Aristotle actually does in *APo* I 19–22 is to prove that *his* model is protected against the threat of infinite regress, which certainly disappoints any reader with generic sceptical concerns.

The proof follows a relatively clear strategy. First, in *APo* I 19–21, Aristotle argues that a syllogistically structured demonstration involving infinitely many steps would contain an infinite chain of universal affirmative predications. Second, in *APo* I 22, the philosopher argues that such predicative chains are impossible. My aim in this paper is to provide a reconstruction of the first of the three arguments formulated in I 22, the longest and most complex of them. More particularly, I shall argue against what I call the 'Traditional Interpretation' of *APo* I 22, according to which singular terms and *summa genera*, in virtue of having restrict logical roles, provide limits for predicative chains, preventing them from proceeding *ad infinitum*. As I intend to show, the Traditional Interpretation is at odds with some important aspects of Aristotle's theory of demonstration. More importantly, it fails to explain how his proof against infinite regress is connected to a defence of the existence of ultimate explanations, a connection that must be the case if *APo* I 19–22 is advancing a foundationalist way-out to the sceptical challenge raised in *APo* I 3. Section 2 contains a brief characterization of the strategy adopted by Aristotle in his proof. In Section 3, I present the Traditional Interpretation and, in Section 4, formulate a set of objections against it. In

[4] On this, see Ferejohn (1991) and Ferejohn (2013, 65; 81).

Section 5, I offer an alternative construal, which is protected from the criticisms raised against the Traditional Interpretation. Section 6 closes the discussion.

## 2. Aristotle's Strategy

Let us begin with a brief characterization of the structure of the proof presented in *APo* I 19–22. Aristotle begins by questioning whether predicative chains can be extended *ad infinitum*. These chains can be of two kinds, the first of which is specified as follows:

T1 Then let *C* be such that it itself no longer holds of anything else and *B* holds of it primitively (i.e. there is nothing else between them). Again, let *E* hold of *F* in the same way, and *F* of *B*. Now must this come to a stop, or is it possible for it to go on *ad infinitum*? [*APo* I 19, 81b30–33; transl. by Barnes 1993]

Later on, in 81b39–40, the philosopher refers to this series as going 'upwards' (ἐπὶ τὸ ἄνω)—and hence I shall call it 'U-series': it begins with a 'fixed' subject and the predicate of each categorical sentence occurs as subject in the next predication:

U-series: $M^1aS$, $M^2aM^1$, $M^3aM^2$, ... , such that $\forall n(M^{n+1}aM^n)$[5]

Aristotle goes on to describe the other kind of predicative chain:

[5] Here and throughout the present paper 'AaB' stands for 'A belongs to all B'.

T2 Again, if nothing is predicated of *A* in itself and *A* holds of *H* primitively and of nothing prior in between, and *H* holds of *G* and this of *B*, must *this* come to a stop, or is it possible for this to go on *ad infinitum*? [*APo* I 19, 81b33–37; transl. by Barnes 1993]

This second series goes the opposite direction: it starts from a given predicate and the subject of each predication becomes the predicate in the next sentence. In 82a1–2, Aristotle refers to this sequence of predications as going 'downwards' (ἐπὶ τὸ κάτω)—thus, I shall call it 'D-series':

D-series: $PaM^1$, $M^1 a\, M^2$, $M^2 a M^3$, ... , such that $\forall n(M^n a M^{n+1})$

As it becomes clear as the discussion proceeds, Aristotle is not interested in knowing whether there can be an infinite set of true sentences with the logical structure of a U- or a D-series.[6] Actually, his point is whether or not U- and D-series can occur *in Aristotelian demonstrations*, i.e. syllogistically structured proofs putting together essence-based explanations and following a specific set of semantic rules (*APo* I 22, 83a18–21).[7] If so, T1 and T2 can be read as raising the following two questions, respectively:

[6] See "ἔστι γὰρ εἰπεῖν ἀληθῶς" in *APo* I 22, 83a1–3. If the semantical rules Aristotle formulates in 83a1–23 are not followed, nothing prevents a predicative chain of true propositions from advancing *ad infinitum* (unless additional and unstated assumptions are made).

[7] It is not without reason that *APo* I 19 begins with a brief discussion about syllogistic reasoning and an anticipation of the semantic concerns explored in *APo* I 22, 83a1–23 (see *APo* I 19, 81b10–29).

Q1 If $\langle \Pi, c \rangle$ is an Aristotelian demonstration, can there be a U-series $\Phi$ such that $\Phi \subseteq \Pi$?

Q2 If $\langle \Pi, c \rangle$ is an Aristotelian demonstration, can there be a D-series $\Psi$ such that $\Psi \subseteq \Pi$?

It is not clear, at first sight, how these questions are related to the problem of infinite regress. Nevertheless, Aristotle moves on, quite abruptly, to a third question, apparently connected to the sceptical challenge raised in *APo* I 3:

T3 Again, is it possible for the terms in between to be infinite if the extremes are determined? I mean e.g. if the *A* holds of *C*, and *B* is a middle term for them, and for *B* and *A* there are different middle terms, and for these others, is it possible or impossible for *these* to go on *ad infinitum*? This is the same as to inquire whether demonstrations can proceed *ad infinitum* and whether there can be demonstrations of everything, or whether terms are bounded by one another [*APo* I 19, 82a2–8; transl. by Barnes 1993].

Aristotle describes the risk of infinite regress as the possibility of all scientific truths being demonstrable. Asking whether it is possible to demonstrate everything is the same as inquiring whether we need infinitely many middle terms in other to explain the connection between two 'extreme' terms (i.e. the bottom major and minor terms, subject and predicate of the conclusion). Thus, the question raised in T3 can be reformulated as follows:

Q3 If $\langle \Pi, c \rangle$ is an Aristotelian demonstration, can $\langle \Pi, c \rangle$ contain infinitely many inferential steps?

Showing how Q3 is connected to Q1 and Q2 is a task Aristotle takes on in the following chapters, *APo* I 20–21. In fact, it can be proved that, if at least one of the branches of a demonstration—structured in any combination of syllogistic moods—contains infinitely many steps, an infinite chain of universal affirmative sentences will be generated, either a U-series or a D-series. It is a matter of dispute whether Aristotle actually established this proof-theoretic result—a dispute that, given our present concerns, we can set aside.[8] What seems undisputable, given how the whole proof is structured, is that Aristotle is relying on the following fact: if the answer to Q3 is affirmative for a given demonstration $\langle \Pi, c \rangle$, then, for the same $\langle \Pi, c \rangle$, an affirmative answer must be given to either Q1 or Q2. Therefore, a proof showing that the answers to Q1 and Q2 must be negative also establishes a negative answer to Q3. It is precisely a proof of this kind that Aristotle tries to formulate in *APo* I 22.

## 3. THE TRADITIONAL INTERPRETATION

In his attempt to justify negative answers to Q1 and Q2, Aristotle seems to endorse, in *APo* I 22, some theses about the relation between language and reality that look familiar to the reader of the *Categories*. In fact, the doctrine of ontological categories is mentioned in more than one passage in the chapter (*APo* I 22, 83a21–23; 83b1–17). One

[8] Lear (1980, 25–30) claims that, although a sound proof can be offered, Aristotle's argument is invalid. Smith (1982; 1986) argues that *APo* I 19–22 relies on a proto-syllogistic that contains only the universal moods *Barbara*, *Celarent*, *Camestres*, and *Cesare*. For a brilliant discussion of this issue, see Crager (2015, 100–124), who convincingly argues that Aristotle came up with a sound strategy of his own, which applies to any combination of syllogistic moods.

could ask why Aristotle would rely on *metaphysical* claims if he intends to deny the occurrence of infinite sequences of predicative sentences, a purpose that is better described as having a *linguistic* character. Here it might be helpful to make a distinction between 'linguistic' and 'metaphysical' predications.[9] A 'linguistic' predication, as it is usually called, is a sentence of the form 'S is P' (or equivalent), where 'S' and 'P' are called 'subject' and 'predicate' in virtue of their grammatical roles. A 'metaphysical' predication, on the other hand, is a relation between entities that determines the truth-value of linguistic predications. The subject of a metaphysical predication S (and not 'S') is not a term, but an entity to which a given attribute P (and not 'P') belongs. In the following, I shall refer to subjects and predicates of linguistic predications as 'linguistic subjects' and 'linguistic predicates', while the subjects and predicates of metaphysical predications will be called 'metaphysical subjects' and 'metaphysical predicates'.

At the very beginning of *APo* I 22, Aristotle discusses essence-specifying sentences, in which there is a definitional connection between the (linguistic) subject and the (linguistic) predicate (see *APo* I 22, 83b5–8). Since the definition and the essence of things can be known, he argues, there cannot be infinite series of predications of this kind:

T4 For items predicated in what something is, the case is plain: if it is possible to define anything, or if what it is to be something can be known, and if you cannot survey infinitely many items, then the items predicated in what something is must be finite [*APo* I 22, 82b37–83a1; transl. by Barnes 1993].

[9] See Bogen & McGuire (1985, 1–2); Code (1985); Lewis (1985; 1991); Crager (2015, 129–132); Zuppolini (2018, 121–122). See also Owen Goldin's contribution to the present volume.

After these brief words, Aristotle examines sentences in which the linguistic predicate is not mentioned in the definition of its linguistic subject; the corresponding metaphysical predicate, in turn, is not part of the essence of the respective metaphysical subject:[10]

T5 You can say truly that the white thing is walking, and that that large thing is a log, and again that the log is large and the man is walking. When you speak in these two ways you make different sorts of statement. When I assert that the white thing is a log, I say that something which is incidentally white is a log, and not that the white thing is the underlying subject for the log. For it is not the case that, being white or just what is some particular white, it came to be a log—hence it is not a log except incidentally. But when I say that the log is white, I do not say that something different is white and that that is incidentally a log, as when I say that the musical thing is white (I am then saying that the man, who is incidentally musical, is white). Rather, the log is the underlying subject which came to be white not in virtue of being something different from what is a log or a particular log [*APo* I 22, 83a1–14; transl. by Barnes 1993, with changes].

Here, Aristotle draws a distinction between—as it is usually phrased since ancient commentators—'natural' and 'unnatural' predications.[11] A natural (linguistic) predication like 'the log is white' and 'the man is walking' is such that the linguistic subject denotes the metaphysical subject of the

[10] For a similar division of the chapter, see Philoponus (235.10–236.23); Barnes (1993, 175); Angioni (2007, 108–109).

[11] See Philoponus (235.17–23); cf. Barnes (1993, 114).

associated metaphysical predication appropriately. In the sentence 'the log is white', the linguistic subject '(the) log' is appropriate because the corresponding metaphysical subject is white 'without being something different from just what is a log or a particular log' (οὐχ ἕτερόν τι ὂν ἢ ὅπερ ξύλον ἢ ξύλον τί, *APo* I 22, 83a13–14). Unnatural (linguistic) predications such as 'the white thing is walking' and 'that large thing is a log' have as their linguistic subjects terms that fail to refer to the corresponding metaphysical subjects in this way. When we say, for instance, that "the musical thing is white", what we actually mean is that "the man, who is incidentally musical, is white" (83a10–12). Unnatural predications do not affirm "one thing of one thing" (83b17): the connection between the linguistic subject and the linguistic predicate do not mirror the connection between the actual metaphysical subject and one distinguished metaphysical predicate, signifying instead a complex state of affairs in which two attributes happen to be in the same underlying (and not properly specified) subject. For Aristotle, scientific discourse should avoid predications of this kind (see *APo* I 22, 83a20–21).

Terms that capture *just what* a given metaphysical subject *is* (ὅπερ ἔστι) are said to "signify substance" (οὐσίαν σημαίνει, *APo* I 22, 83a24–25). This is congenial to the well-known thesis, defended in the *Categories*, that substances are the metaphysical subjects *par excellence*, which makes them the primary realities on which everything else ontologically depends. Thus, it seems natural to understand T5 as claiming that scientific propositions are supposed to describe and explain the world as it is, which involves referring to the basic metaphysical subjects as what they are, namely, substances. In fact, the whole discussion at the beginning of *APo* I 22 seems reminiscent of the *Categories*. The distinction between essential and non-essential predications is explored, with a peculiar vocabulary, in *Cat.* 2, 1a20–b9. A predicate P is *said of a subject* S (καθ᾽ ὑποκειμένου λέγεσθαι) when P belongs essentially to S (e.g. *man* is said of *Socrates*), while P is *in the subject* S (ἐν ὑποκειμένῳ εἶναι) when P belongs non-essentially or 'accidentally' to S (e.g. *whiteness* is in *Socrates*).[12] Aristotle seems committed to two claims concerning these predicative connections: (i) if P is *said of the subject* S, then S and P belong in the same category; (ii) if P is *in the subject* S, S and P belong in different categories and S is a substance.[13] Someone could take these claims as the motivation for the semantic rule prescribed in T5. That is to say, T5 would be arguing that, since S must be a substance if P is a non-essential attribute of S, the corresponding (linguistic) predication must have as (linguistic) subject a term that denotes an entity in the category of substance.[14]

Given all that, it is not completely implausible to think that Aristotle found in the *Categories* a metaphysical framework that could justify negative answers to Q1 and Q2. According to a common construal of this framework, all entities in a porphyrian tree can occur as subject or predicate in (metaphysical) predications with two exceptions: (i) particular substances, such as Socrates or Secretariat (which can only occur as subjects), and (ii) *summa genera*, such as substance or quality (which can occur only as predicates).[15] Thus, several interpreters claim that, in *APo* I 22, Aristotle argues that terms naming particular substances and *summa*

[12] As has been noted, Aristotle's vocabulary is misleading, since 'said-of' connections are metaphysical (and not linguistic) predications (see "τῶν ὄντων" in 1a20). On this, see Ackrill (1963, 75).

[13] See Furth (1988, 14); Ferejohn (1991, 82).

[14] As I shall argue in Sections 4 and 5, this is an incorrect interpretation of T5.

[15] This is true if we exclude transcategorical attributes, such as *being* and *one*.

*genera* could play the role of limits in predicative chains, preventing them from advancing *ad infinitum*. For instance, a sequence of the form $\{PaM^0, M^0a\ M^1, M^1aM^2, ... \}$ would be interrupted by a sentence ‘$M^iaS$’ in which ‘S’ names a particular substance, which would preclude the occurrence of a D-series. A sequence $\{M^0aS, M^1aM^0, M^2aM^1, ... \}$, in turn, would end up, at some point, with a sentence ‘$PaM^i$’ in which ‘P’ signifies a *summum genus* (substance, quality, quantity etc.), which would prevent the predicative chain from becoming a U-series. I shall refer to this reading as the ‘Traditional Interpretation’, given that a significant number of authors have analysed the argument in *APo* I 22 along these lines.[16]

We must also note that the Traditional Interpretation seems in accordance with what Aristotle says in *Prior Analytics* (hereafter, *APr*) I 27:

T6 Now of all the things there are, some are such that they cannot be predicated truly and universally of anything else (for instance, Cleon or Calllias, that is, what is individual and perceptible), but other things may be predicated of them (for each of these is both a man and an animal). Some things are themselves predicated of others, but nothing else is prior and predicated of them. And some things are both predicated themselves of others and others of them, as man is predicated of Callias and animal of man [*APr* I 27, 43a25–32; transl. by Striker 2009].

In T6, Aristotle divides entities into three groups: (i) those that are not predicated of anything else, but of which some things can be predicated; (ii) those that are predicated

[16] Philoponus (233.26–29, 244.28–31, 247.17–22; 250.20–251.7); Aquinas (*in An. Post*. lib. 1, l.34, n.5); Demos (1944, 257–259); Ross (1949, 578–579); Hamlyn (1961, 119–120); Loux (1991, 42).

of other things but of which nothing can be predicated; and (iii) those that can be both predicated of other things and other things of them. The first class is exemplified by individual and perceptible substances (43a32–36), while the third class covers intermediary items such as *man*, which are predicated of things like *Callias* and of which things like *animal* are predicated. Aristotle does not give an example of the second class, but just announces that he will argue "later" (πάλιν ἐροῦμεν) that "one also comes to a halt if one goes upwards" (43a36–37). The occurrence of the phrase "ἐπὶ τὸ ἄνω" here does suggest that T6 refers to *APo* I 19–22 and it is easy to see why anyone advocating the Traditional Interpretation would like to take the *summa genera* as examples of the second class listed in T6.[17]

## 4. OBJECTIONS TO THE TRADITIONAL INTERPRETATION

Let me now raise some difficulties for the Traditional Interpretation. (1) We know Aristotle endorses the thesis that particular substances are the ultimate metaphysical subjects. However, how does this fact prevent the occurrence of a D-series, i.e. an infinite chain of linguistic predications? One might claim that if particular substances are the bottom limits of metaphysical predications, their names and descriptions also interrupt all descending sequences of linguistic predications. Well, Q1 and Q2 concern the occurrence of U-series and D-series *in Aristotelian demonstrations*. If so, Aristotle would be committed to the view that, in his model of demonstrative science, language and reality are perfectly isomorphic, i.e. there would

[17] For a detailed discussion of T6 and the occurrence of singular terms and *summa genera* in the syllogistic, see Almeida (2013).

be a one-to-one correspondence between the chains of metaphysical predications reality is made of, one the one hand, and the chains of linguistic predications that make up scientific demonstrations, on the other. However, the philosopher is hostile to the presence of singular terms in demonstrative sciences. Scientific discourse should not refer to particular subjects individually, since demonstrations concern their common attributes, which belong to them universally, as members of one kind or another. The knowledge that the triangle has the sum its internal angles equal to two right angles (hereafter, 2R) is in some way prior to the knowledge that a given figure in a semi-circle (*APo* I 1, 71a19–21) or the isosceles (*APo* I 4, 73a28–34) have the same property. After all, 2R belongs to them *as triangles*.[18] Particular objects x, y, z (…) have the demonstrable properties they have *qua* members of a universal kind K, i.e. *in virtue of* being Ks.[19] In pursuing a demonstration, the scientist must identify the relevant kind on whose nature the occurrence of the demonstrable attribute is grounded. For that reason, these explanatorily relevant kind-terms will be the (linguistic) subjects in demonstrative sciences, not names and descriptions of particular substances, which are irrelevant for scientific purposes.[20]

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[18] See also *APo* I 5, 74a16–b4; I 24, 85b4–15; b23–27; 85b38–86a3; II 17, 99a30–b7; 73b25; *APo* I 24, 85b5–7 *APo* I 9, 76a4–9.

[19] See Lennox (1987, 91); Kosman (1973, 375); Angioni (2014, 97–98; 2016, 97–100); Zuppolini (2014, 15–17; 2018, 130–132).

[20] I am not saying that in the *APo* Aristotle abandons his view that particular substances somehow ground the existence of universal substances and non-substantial beings. My point is just that, if we make the argument in *APo* I 22 dependent on the occurrence of singular terms in demonstrations, that argument would be in conflict with Aristotle's contention that scientific propositions should not refer to the particulars in its domain individually, but as members of a kind in virtue of which they have certain

(2) A similar objection concerns terms signifying *summa genera*, which are supposed to prevent U-series from occurring in demonstrations, according to the Traditional Interpretation. Being a substance, a quality or a quantity does not seem to be the kind of feature Aristotle would take as a demonstrable attribute, which is peculiar to a given subject-kind and belongs to it in virtue of this kind being precisely what it is—as the triangle has 2R in virtue of being a plane three-sided rectilinear figure etc. This could hardly be the case of highly abstract predicates such as the *summa genera*. Although they are relevant to metaphysical speculation, it is hard to see how a scientist could tell a clear-cut causal story explaining why they are connected to a specific subject and not another—as a geometer, for instance, is able to explain why 2R belongs to the triangle and no other figure. If there is a causal story to be told, it is certainly not the kind of explanation departmental sciences are supposed to set out.

(3) Another obvious difficulty for the Traditional Interpretation is the status of mathematical sciences in the face of T5. If the terms allowed to occur as (linguistic) subject in natural (accidental) predications must "signify substance" (οὐσίαν σημαίνειν), it is tempting to postulate that, when 'PaS' express an accidental relation between S and P (in which case P is *in* S, as opposed to being *said of* S), the linguistic subject 'S' must be a substance-term such as 'man', 'horse' or 'animal'—a substance-term being a term that denotes exclusively entities in the category of substance. However, for Aristotle, the metaphysical subjects studied by mathematical sciences (number, point, figure, triangle etc.) are not substances, and the corresponding linguistic subjects ('number', 'point', 'figure', 'triangle' etc.) are not substance-terms in this sense. As a result, non-definitional

demonstrable attributes. I am grateful to David Bronstein for pressing me on this issue.

mathematical statements would be all unnatural predications. Since mathematical sciences are clearly demonstrative—and arguably the paradigms of demonstrative science—, it seems highly implausible that Aristotle would exclude them from his proof against infinite regress. Therefore, we should try to avoid reading T5 as determining that only substance-terms such as 'man' or 'horse' can be subjects of non-definitional propositions.[21]

(4) Apart from all this, the greatest weakness of the Traditional Interpretation, I believe, is the fragile connection it establishes between Q1/Q2 and Q3. Even if 'S' and 'P' signify a particular substance and a *summum genus* respectively, a series of the form $\{PaM^i, \ldots, M^jaS\}$ could be infinite 'by division', i.e. infinitely many intermediate terms could be inserted between 'S' and 'P'.[22] If so, a negative answer to Q3 would not be secured. In fact, the logical properties of terms denoting individuals and ontological categories do not seem relevant to establishing a foundationalist solution to the problem of infinite regress. Alternatively, the connection between Q1/Q2 and Q3 can be better illustrated as follows. Let us explore the scenario discussed in T1 and suppose the following sequence of inferences in *Barbara*, in which the minor premises are immediate, while the major premises are always demonstrated from prior premises:

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[21] For similar objections, see Crager (2015, 133) and David Bronstein's contribution to this volume.

[22] See Demos (1944, 257) who advocates a version of the Traditional Interpretation, but takes T3 as considering the possibility of a predicative series being infinite by division.

$$\underline{PaM^{n+1} \quad \mathbf{M^{n+1}aM^{n}}}$$
$$PaM^{n}$$
(...)
$$\underline{PaM^{3} \quad \mathbf{M^{3}aM^{2}}}$$
$$\underline{PaM^{2} \quad \mathbf{M^{2}aM^{1}}}$$
$$\underline{PaM^{1} \quad \mathbf{M^{1}aS}}$$
$$PaS$$

In this case, it is easy to see how Q1 would be related to Q3. Asking whether a U-series is generated from 'S' is the same as asking whether there is a middle term '$M^{i}$' which is 'immediately' or 'indemonstrably' connected to 'P'. In this case, Q3 is equivalent to the following question (cf. Lear 1980, 22):

Q3.1 Is it the case that $\forall i \exists j((PaM^{j}, M^{j}aM^{i} \vdash PaM^{i})$ & $M^{j}$ explains why $PaM^{i})$?

Following the situation described in T2, suppose now a sequence of syllogisms in *Barbara* in which the minor premises are demonstrable, while the major premises are immediate:

$$\underline{\mathbf{M^{n}aM^{n+1}} \quad M^{n+1}aS}$$
$$M^{n}aS$$
(...)
$$\underline{\mathbf{M^{2}aM^{3}} \quad M^{3}aS}$$
$$\underline{\mathbf{M^{1}aM^{2}} \quad M^{2}aS}$$
$$\underline{\mathbf{PaM^{1}} \quad M^{1}aS}$$
$$PaS$$

Again, in this scenario, the connection between Q2 and Q3 is pretty straightforward. Asking whether this demonstration involves infinitely many steps is the same as asking whether a D-series could emerge from the major term P, which is equivalent to asking whether there is no middle term $M^{i}$ to which the bottom minor term S is 'immediately' connected:

Q3.2 Is it the case that $\forall i \exists j((M^i a M^j, M^j a S \vdash M^i a S)$ & $M^j$ explains why $M^i a S)$?

As we can see, 'P' signifying a *summum genus* is irrelevant to establishing a negative answer to Q3.1, in the same way as 'S' naming a particular substance does not secure a negative answer to Q3.2. If the aim of demonstrative arguments is to provide causal explanations of their respective conclusions, we expect Aristotle's defence of his foundationalism to be related to the recognition of *ultimate explanations*. Therefore, it would be preferable to pursue an interpretation of *APo* I 22 that explains the sense in which denying U- and D-series in demonstrations is a way of affirming the existence of indemonstrable premises in science.

## 5. An Alternative Interpretation

Before I present my interpretation of *APo* I 22, I must recognize that the text is extremely difficult and the structure of the argument is far from clear. Identifying the premises involved is not an easy task, since Aristotle does not combine them in a clear step-by-step reasoning. It is not without reason that some of the comments on it are notably discouraging.[23] We, as interpreters, are left with one possible strategy: investigate whether the text contains premises that, once identified and articulated, can be used to establish that U- and D-series cannot occur in Aristotelian demonstrations. If we are lucky, the resulting interpretation

[23] Barnes (1993, 181) concludes his commentary on *APo* I 22 with the following words: "None of the arguments is successful; and they cannot be reformulated in such a way as to furnish proof of Aristotle's contention".

will not be liable to the same objections raised against the Traditional Interpretation.

Let me begin by arguing that T5 does not need to be read as stating that only substance-terms such as 'man' or 'animal' can be subjects of natural predications. From now on, I shall use the term 'E-sentence' to refer to statements of the form 'PaS' in which 'P' signifies a (metaphysical) predicate P that is essential to S. The term 'A-sentence', in turn, will be used when P is an 'accidental' or non-essential attribute of S.[24] In fact, nothing in T5 suggests that only typical substance-terms can be (linguistic) subjects in A-sentences. All the passage implies is that an A-sentence 'PaS' is a natural predication iff. 'S' refers to the objects it denotes $s_1, s_2, \ldots, s_n$ by signifying *just what* $s_1, s_2, \ldots, s_n$ *are* (ὅπερ ἐστιν).[25] If we use a more contemporary vocabulary, we would say that 'S' is a sortal or individuative term, distinguished by what Quine (1960, 90–95) calls 'divided reference.' By specifying what the denoted objects essentially are, such terms give us a criterion for counting these objects as discrete entities, which makes them suitable for quantificational expressions such as 'every S' or 'some S'. It is true that, according to Aristotle, terms that behave in this way 'signify substance' (οὐσίαν σημαίνειν), as we have seen. However, the semantical feature of 'signifying substance' is not defined as 'denoting exclusively entities in the category of substance'. Rather, to 'signify substance' is to signify an attribute that is *essentially* predicated of everything

[24] Since we are discussing predicative sentences in demonstrations, the relevant 'accidental' attributes here will be, of course, demonstrable attributes, also known as *per se* accidents—attributes that belong to a subject *per se* but not as a part of its essence (see *Met.* V 30, 1025a30–34). For reasons of exposition, however, I shall follow Aristotle in T5 and use non-demonstrable attributes as examples, e.g. white predicated of human beings.

[25] See Angioni (2006, 114–117); Angioni (2007).

of which it is predicated (*APo* I 22, 83a24–32). A term such as 'white' signifies an attribute (whiteness) that is predicated non-essentially of some of the things to which it belongs—for instance, a white man or a white horse (although we might say that a particular shade of white is essentially white).[26] In contrast, a term like 'man' signifies an attribute that belongs essentially to any object to which it belongs.[27]

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[26] There is no textual evidence in *APo* I 22 indicating that if 'S is P' is an E-sentence, then S must be essential to everything of which it is predicated; and, as I will try to show, Aristotle's argument does not depend on that. If so, either (1) the distinction between natural and unnatural predications applies exclusively to A-sentences or (2) S being essential to everything to which it belongs is not a necessary condition for an E-sentence 'S is P' being natural. For a defence of option (2), see Philoponus (235.30–236.8). I intend my reconstruction of the argument to be neutral between (1) and (2). What is common to (1) and (2) is that, even if (e.g.) whiteness is not essential to everything to which it belongs (for instance, whiteness is a non-essential attribute of Socrates), 'white is a colour' should not be taken as unnatural, either because it is natural (option 2) or because the distinction natural/unnatural does not apply to E-sentences (option 1). For a different view, see Malink (2013, 163–165)—especially his claim that all natural predications (including E-sentences) must have as linguistic subjects what he calls 'essence-terms'.

[27] In his contribution to this volume, Bronstein raises an interesting objection to the view that all subjects in natural predications must signify an attribute that is essential to everything to which it belongs. Take for instance the major premise in the syllogism about thunder in II 8: 'thunder belongs to extinction of fire'. Of course, 'extinction of fire' does not signify substance (i.e. extinction of fire is not essential to everything to which it belongs), and therefore this predication would be unnatural despite being a premise in a demonstration. I do not think that this statement express a typical accidental predication, in which the subject is the substratum in which the attribute inheres—and this, I believe, must the case even for *per se*$_2$ predications (as defined in *APo* I 4,

This feature is common to the subject-terms in mathematical sciences. Even though triangles are not substances, the term 'triangle' signifies an attribute that is essential to all triangles, which cannot exist as the objects they are independently of being triangles.[28]

If demonstrations must not involve unnatural predications, T5 imposes two constraints on scientific discourse: (i) terms that 'signify substance' cannot be predicates in A-sentences; (ii) only terms that 'signify substance' can be subjects in A-sentences. As I will try to show, these two constraints correspond to two premises in Aristotle's argument. Assuming that 'AaB' is a scientific predication iff. 'AaB' is true & 'AaB' is not an unnatural predication, we have:

P1 If 'AaB' is an A-sentence & 'BaC' is a scientific predication, 'BaC' is an E-sentence.

73a37–b3), *pace* Bronstein (see *Met.* V 18, 1022a29–31; Zuppolini 2018). If so, even in my view, 'extinction of fire' does not need to signify substance (which means extinction of fire does not need to be essential to everything to which it belongs). Rather, the major premise in the thunder syllogism—and similar cases, such as eclipse (II 8), or leaf-shedding (II 17)—is a converted definition, where the *definiendum* is predicated of the causal part of its *definiens*, not an A-sentence. Anyway, I cannot see how Aristotle's argument would work without P1 (see below). I am grateful to Bronstein for calling my attention to this problem.

[28] We could say that, in the expression 'οὐσίαν σημαίνειν', 'οὐσίαν' refers not to the ontological category comprising the concrete individuals of the common sense (like Socrates and Secretariat), but rather to the relational notion of 'substance of', i.e. 'the substance of' Socrates understood as the factor that grounds the being of Socrates, in the sense of making Socrates what Socrates is. For similar views, see Ross (1949, 577); Angioni (2006, 124–126); Peramatzis (2010, 162–163).

P2 If 'AaB' is an A-sentence & 'CaA' is a scientific predication, 'CaA' is an E-sentence.

A third premise can be identified if we take a closer look at *APo* I 22, 83a24–30, the passage in which Aristotle introduces the expression 'signify substance.'

T7 Again, items that signify substance signify, of what they are predicated of, just what that item is or just what a particular sort of it is. Items that do not signify substance, but are said of some other underlying subject which is neither just what that item is nor just what a particular sort of it is, are incidental. E.g. white of a man: a man is neither just what is white nor just what is some particular white—rather, presumably, animal: a man is just what an animal is [*APo* I 22, 83a24–30; transl. by Barnes 1993, with changes].

Here Aristotle discusses a feature of terms that 'signify substance': if 'P' signifies substance and P belongs to S, then P belongs *essentially* to S, which means that whatever belongs *essentially* to P belongs *essentially* to S as well. T7 contains a sort of test to determine whether a word signifies substance or not: the transitivity of essential predication, a principle Aristotle also defends elsewhere (see *Cat.* 3, 1b10–16). A term such as 'white' does not 'signify substance' insofar as it fails to specify what (e.g.) a white man essentially is (ὅπερ ἔστι). If 'white' signified substance, a white man, for instance, would be *just what* white *is* (ὅπερ λευκὸν ἐστιν), i.e. a colour of a certain kind. Since this is not the case, 'white' does not signify substance. 'Animal', however, signifies substance insofar as every animal, man included, is *just what* animal *is* (ὅπερ ζῷον ἐστιν), i.e. a sentient being etc.[29] If this is correct, more than just specifying a feature of terms that 'signify substance', T7 gives us another premise in Aristotle's argument:

P3 If 'AaB' is an E-sentence & 'BaC' is an E-sentence, then 'AaC' is an E-sentence.

A fourth (and crucial) premise in the argument can be extracted from T4 and three other passages in I 22 (83b8; 13–15; 26–27), where the same view is restated. T4 contains a simple reasoning: (i) essences are knowable; (ii) if a subject had infinitely many essential predicates, its essence would be

[29] Some interpreters seem to read 83a24–25 as if τὰ οὐσίαν σημαίνοντα (83a24) refers to "essential" or "substantial predicates", while οὗ in καθ᾽ οὗ κατηγορεῖται refers back to ἐκεῖνο or ἐκεῖνό τι, which stand for the corresponding subject. See Ross (1949, 577–578); Barnes (1993, 177); Peramatzis (2010, 163). In this reading, Aristotle would be making a trivial statement, namely, that predicates signifying substance are essential attributes of their subjects. But T7 seems to be saying something more. First, the Greek allows the antecedent of οὗ to be omitted from the text, καθ᾽ οὗ κατηγορεῖται meaning "the thing of which [items signifying substance] are predicated". Second, in 83a28–30, λευκὸν and ζῷον are being compared as predicates of ἄνθρωπος, and ἐκεῖνο in the general formula of 83a24–25 is replaced by λευκὸν and ζῷον, not ἄνθρωπος. If so, in 83a24–25, ἐκεῖνο should be taken as referring back to the predicates (τὰ οὐσίαν σημαίνοντα), not to the subject, and ὅπερ ἐκεῖνο stands for the essence *of the predicate*—I do not think the shift from plural (τὰ οὐσίαν σημαίνοντα) to singular (ἐκεῖνο) is problematic, given that the plural is clearly distributive. For a detailed defence of this reading, see Angioni (2006, 124–125). See also Philoponus (240.14–24). Additionally, in this interpretation, T7 gives us another premise in the argument, as I am about to show.

unknowable (which means this subject could not be defined); therefore, (iii) one single (metaphysical) subject cannot have an infinite number of essential (metaphysical) predicates. One could object that the inference (i),(ii)/(iii) is valid, but philosophically vicious, since it uses an epistemological assumption ((i) = essences are knowable) to justify a claim about how reality is metaphysically structured ((iii) = one subject cannot have infinitely many essential attributes), while a good philosophical justification would proceed the other way around. However, since this is clearly the inference made in T4, this criticism has to be taken as an objection to Aristotle, not to interpretations of Aristotle's text. For better or worse, the final premise needed in the argument is precisely the linguistic counterpart of (iii):

P4 There cannot be infinitely many (true) E-sentences '$A^1$aB', '$A^2$aB', … , '$A^n$aB.'

Now we are in a position to show how P1-P4 can be used to prove that the answers to Q1 and Q2 must be negative. Let us first analyse the following predicative chain, going 'upwards' from a subject S:

$$\Phi = \{M^1aS, M^2aM^1, M^3aM^2, \ldots\}$$

The first predication of the chain, '$M^1$aS', is either an E-sentence or an A-sentence, i.e. either $M^1$ is part of the essence of S or not. I shall now analyse each one of these two alternatives.

Suppose first that '$M^1$aS' is an E-sentence (Hypothesis 1; H1, for short). Now, we have to ask whether the other predications in Φ are also E-sentences (H1.1) or whether an A-sentence comes up at some point (H1.2). Let us follow H1.1 and assume that all the other predications in Φ are E-sentences as well. If '$M^1$aS' and '$M^2$a$M^1$' are E-sentences, '$M^2$aS' (which follows in *Barbara* from '$M^1$aS' and '$M^2$a$M^1$) is also an E-sentence (given P3). For the same reason, if '$M^2aS$' and '$M^3aM^2$' are E-sentences, '$M^3aS$' is also an E-sentence, the same being true for every $M^i$ in $\Phi$. Thus, if the series $\Phi$ were infinite, there would be infinitely many E-sentences, each of which attributes a different essential predicate to S.

$$\Phi = \{M^1aS, M^2aM^1, M^3aM^2, \ldots\}$$

$M^2aS$

$M^3aS$

$(\ldots)$

According to P4, this result is impossible, which means $\Phi$ could not be infinite under H1.1. Therefore, $\Phi$ must contain at least one A-sentence. Let us then explore H1.2 and assume that the first A-sentence in $\Phi$ is '$M^2aM^1$'. In that case, the next sentence '$M^3aM^2$' would have to be an E-sentence, since a predicate in an A-sentence (such as '$M^2$' in '$M^2aM^1$') does not 'signify substance' and therefore cannot be itself the subject of another A-sentence (given P2). Now, if '$M^2$' does not signify *the substance of* the objects in the domain, but one of their *accidents*, neither could the essential predicates of '$M^2$' (like '$M^3$') signify their substance.[30] If '$M^3aM^2$' is an E-sentence, '$M^4aM^3$' would also be an E-sentence, since '$M^3$',

[30] Otherwise $M^3$ would be essential not only to $M^2$, but also to items of which $M^2$ is accidentally predicated (like $M^1$ and S), since there is nothing to which terms signifying substance belong accidentally. For an illustration of this step, replace 'S' by 'man', '$M^1$' by 'animal', '$M^2$' by 'white', and '$M^3$' by 'color'.

as an essential predicate of '$M^2$', does not signify substance either, and hence cannot be subject in an A-sentence (again, P2). Thus, if Φ were infinite, there would be infinitely many E-sentences starting from '$M^3aM^2$', from which infinitely many E-sentences with '$M^2$' as subject would follow (given P3). Since this result is impossible (given P4), we must conclude again that Φ is finite under H1.2. Since H1.1 and H1.2 are exhaustive alternatives, Φ cannot be infinite under H1, i.e. under the assumption that '$M^1aS$' is an E-sentence.

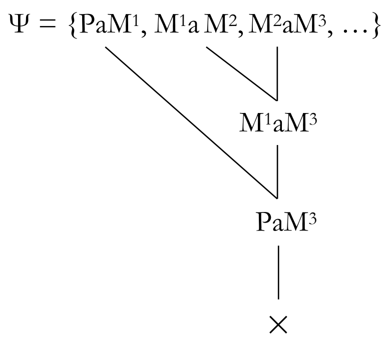
Alternatively, let us assume that '$M^1aS$' is an A-sentence (H2). Here, we can follow the same reasoning we applied to H1.2. If '$M^1aS$' is an A-sentence, the next sentence in the series, '$M^2aM^1$', would have to be an E-sentence (given P2). In that case, however, the next proposition '$M^3aM^2$' and all the other predications in Φ will be E-sentences as well, since none of the $M^i$s would signify substance. If so, there would be infinitely many E-sentences with '$M^1$' as the subject-term (given P3), which cannot be the case (given P4). Since H1 and H2 are exhaustive (the first predication in Φ is either an E-sentence or an A-sentence), it follows that U-series cannot occur in syllogistic demonstrations, which means that the answer to Q1 is negative.

I shall now address Q2, which concerns predicative chains going 'downwards' from a predicate P:

$$\Psi = \{PaM^1, M^1a\,M^2, M^2aM^3, \ldots\}$$

The first predication of the series, '$PaM^1$', will be either an E-sentence or an A-sentence. Again, I shall explore each one of these two options.

Let us assume first that '$PaM^1$' is an E-sentence (H3). Suppose, in addition, that all the other predications in Ψ are also E-sentences (H3.1). Under this supposition, one could think that it is possible to argue as we did in the case of U-series, and use P3 to show that, if Ψ were infinite, there would be infinitely many E-sentences with one of the terms in Ψ as subject (which would be impossible according to P4). However, this line of reasoning does not work for D-series: for any $M^i$ in Ψ there will be *finitely* many sentences from '$M^{i-1}aM^i$' back to '$PaM^1$', as we can see in the following diagram:[31]




Nevertheless, it can be shown that, if Ψ were infinite, there would be infinitely many essential predicates belonging to the same particular subject (one of the particulars in the domain), even if this subject is not mentioned in Ψ. Still under the assumption that Ψ contains only E-sentences (H3.1), let us say that none of the terms in Ψ signifies substance, e.g. 'sensible quality', 'colour', 'white' etc. If Ψ were infinite, it would be possible to derive infinitely many E-sentences with (e.g.) 'seashell white' as subject-term (even if 'seashell white' does not occur in Ψ): '*sensible quality* holds of *seashell white*', '*colour* holds of *seashell white*', '*white* holds of *seashell white*' etc.[32] Similarly, if all terms in Ψ signify substance—e.g. 'living being', 'animal', 'man' etc.—, each of

[31] Barnes (1993, 180) makes a similar point about 84a7–28.

[32] Assuming, of course, that seashell white is a particular among shades of white, i.e. something that cannot be predicated essentially of a more specific shade of white.

them would be predicated of a particular object in the domain (in this case, a particular man). Therefore, infinitely many E-sentences with (e.g.) 'Socrates' as subject-term would follow from Ψ (even if 'Socrates' does not occur in Ψ): '*living being* holds of *Socrates*', '*animal* holds of *Socrates*', '*man* holds of *Socrates*' etc. None of these results is possible (given P4), so Ψ does not proceed *ad infinitum* under H3.1.

Still assuming that '$PaM^1$' is an E-sentence (H3), suppose in addition that an A-sentence comes up at some point (H3.2)—let it be '$M^1aM^2$'. In this case, the next predication '$M^2aM^3$' is necessarily an E-sentence: as the subject of an A-sentence, '$M^2$' signifies substance and therefore can be predicate only in E-sentences (given P1). In fact, the same is true for any '$M^i$' in Ψ provided that $i \geq 2$, since all of them would signify substance. Again, if it were infinite, Ψ would contain infinitely many predicates—e.g. 'living being', 'animal', 'man' etc.—, from which infinitely many E-sentences about any particular subject in the domain could be obtained: '*living being* holds of *Socrates*', '*animal* holds of *Socrates*', '*man* holds of *Socrates*' etc. (again, this holds good even if 'Socrates' does not occur in Ψ). Since H3.1 and H3.2 are exhaustive alternatives, Ψ must be finite under H3.

For similar reasons, the same result follows from the assumption that '$PaM^1$' is an A-sentence (H4). In that case, the next predication '$M^1aM^2$' is necessarily an E-sentence (given P1). In fact, if Ψ begins with an A-sentence, every '$M^i$' in Ψ, starting with '$M^1$', would signify substance. Once again, Ψ would contain infinitely many terms signifying substance, all of which would be predicated of any particular object in the domain—even if none of them is individually mentioned in Ψ. Well, H3 and H4 are exhaustive ('$PaM^1$' is either an A-sentence or an E-sentence), which means that Ψ must terminate at some point. In other words, a D-series cannot occur in Aristotelian demonstrations, and the answer to Q2 is also negative.

## 6. CONCLUSION

Let me now close our discussion by noting that, in his summary of the argument (*APo* I 22, 83b17–31), Aristotle reaffirms P1, P2 and P4, which is further evidence in favour of my reconstruction:

T8 We have supposed that one thing is predicated of one thing, and that items which do not signify what something is are not predicated of themselves. For these are all incidental (though some hold of things in themselves and some in another way), and we say that all of them are predicated of an underlying subject, and that what is incidental is not an underlying subject [*APo* I 22, 83b17–22; transl. by Barnes 1993, with changes].

T9 The incidentals are said of items in the substance of each thing, and these latter are not infinite [*APo* I 22, 83b26–27; transl. by Barnes 1993, with changes].

In T8, Aristotle seems to subscribe to P2 once again. Predicates of a given subject (e.g. Socrates) that do not signify *what* that subject *is* (e.g. white and musical) cannot be predicated of each other (i.e. white cannot be predicated of musical). In other words, an accident cannot be predicated of another accident.[33] In T9, the philosopher restates P1 (καθ᾽ ὧν μὲν γὰρ λέγεται τὰ συμβεβηκότα, ὅσα ἐν τῇ οὐσίᾳ ἑκάστου) and P4 (ταῦτα [= ὅσα ἐν τῇ οὐσίᾳ

[33] Unless, of course, one is part of the essence of the other (as colour is predicated of white). I am also assuming that, given the "γὰρ" in 83b19, the referent of "πάντα" in the same line is the same as the referent of "αὐτὰ" in 83a18. Nevertheless, T8 would be extra (and perhaps dispensable) textual evidence in favour of P2, if one prefers to pursue another reading of these lines.

ἑκάστου] δὲ οὐκ ἄπειρα). Probably assuming P3 (already explored in T7), Aristotle then concludes his argument and denies that U- and D-series can occur in demonstrations (83b24–25; 27–28).

Our reading also has significant advantages in comparison to the Traditional Interpretation. First, the philosopher establishes negative answers to Q1 and Q2 without including singular terms and *summa genera* in scientific discourse (see objections (1) and (2) in Section 4).[34] Second, the linguistic subjects in natural predications are not restricted to substance-terms—i.e. terms signifying exclusively entities in the category of substance—, so the argument against infinite regress also applies to mathematical sciences (see objection (3)). Finally, our reading makes it easier to understand how *APo* I 19–22 can be taken as a defence of Aristotle's foundationalist project (see objection (4)). As we have seen, the argument relies heavily on the claim that no subject has infinitely many essential predicates. Well, essences are the primary causes studied by demonstrative sciences, with the corresponding definitions playing the role of indemonstrable principles. Thus, it is not surprising that the claim that every demonstration contains a finite number of steps is ultimately grounded in the belief that essences are knowable. If so, a more substantial defence of Aristotle's foundationalism depends on specifying how exactly the scientist gets to know essences and definitions, and this is precisely the gap *APo* II attempts to fill in. Therefore, to a certain degree, Aristotle succeeds in protecting *his model* of demonstrative science (with its

[34] In T6, Aristotle is concerned with the restricted roles that certain terms assume in syllogistic inferences. Even if 43a36–37 refer prospectively to *APo* I 22 (as providing reasons for rejecting U-series), nothing in *APr* I 27 indicates that Aristotle's argument in *APo* I 22 relies on the occurrence of singular terms and *summa genera* in demonstrations.

peculiar doctrine of predication and underlying logic) from the threat of infinite regress. Whether or not the more sceptical reader is satisfied is a different matter, especially if she doubts that essences can be known. Aristotle does not seem to bother.

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