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Definite Descriptions: What Frege Got Right and Russell Didn't¹

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It seems that the present king of France has really gotten Gottlob Frege's account² of definite descriptions into trouble. Now, the present king of France could not have done it all himself—he does not exist. Rather, the trouble for Frege arises because descriptions such as 'the present king of France' are proper parts of the English language and can occur in well-formed English sentences.

By the law of excluded middle, if two sentences are of the form A and $\sim A$ and we assign a truth value to either one, the other is bound to have the opposite truth value. According to Frege's formulation of sense and nominatum, the following two sentences are such opposites: 'The present king of France is bald', and 'The present king of France is not bald'. Consequently, if one of them is true, the other must be false and vice versa.³ Seeing that neither of the two seems any more false or true than the other, Frege did not want to allow such a truth value assignment. But then he had to mercilessly expel from the realm of "logic talk" sentences that contain descriptions such as 'the present king of France'. Henceforth, I will call descriptions that correspond to no actual entity 'empty'.⁴

But some lines of intuition suggest that sentences containing empty descriptions can be evaluated with respect to their truth (and thus can be incorporated into "logic talk"). Here is one such example, embedded in a hopefully somewhat natural-sounding conversation:

1. "The present king of France is not bald. In fact, I got a strand of his hair from a friend who just yesterday saw it fall out and picked it up."
2. "But surely you're wrong. It can't be that you possess a strand of hair from the present king of France. He doesn't exist at all."

What happens here is that 2's statement, 'It can't be that you possess a strand of hair from the present king of France', seems to be decisively true.⁵ But Frege's account prevents 2's statement from having a truth value, as it contains an empty description, 'the present king of France'.

Bertrand Russell's account may then be viewed as a welcome proposal that successfully addresses intuitions about empty descriptions. According to Russell,⁶ there are things beyond mere predications of, say, baldness or nonbaldness, going on in a sentence containing a definite description; and those are on the one hand the assertion of existence and on the other, the assertion of uniqueness. As we shall see in section 2, these added assertions provide a wide range of ways in which sentences containing definite descriptions can be false.

Frege also treats issues of existence and uniqueness. His treatment of those two components does not, however, enter into matters of truth evaluation. Instead, uniqueness is incorporated into what a sense is, while existence is incorporated into what it is for a sense to have a nominatum. In

other words, Frege's "logic talk" operates at a level distinct from the one at which uniqueness and existence operate. In order to get to discussions of truth and falsity, the definite descriptions involved have already had to pass the test of being senses that nominate something, that is, tests of uniqueness and existence. By contrast, the three components—uniqueness, existence and predication—operate. And Russell suggests that truth evaluations take place at this level, so that all three components contribute to such evaluations.

In this essay, I take a stand that is distinct from that of Frege or Russell about the levels at which the three components operate. I will argue that Frege got the arrangement of the three right by stipulating two distinct levels of operation and by claiming that the uniqueness component should operate at a level prior to the one at which truth evaluations take place. Russell, on the other hand, got the level of operation for the existence component right by placing it on the level at which truth evaluations occur. My thesis then is that in sentences containing definite descriptions, there are two levels of operation; at one is uniqueness and at the other are predication and existence. And it is only at the latter level that truth evaluations come into play. In section 1, I will present Frege's view of the three components along with formal truth evaluations and logical implications. The same points about Russell's view will be covered in section 2. In section 3, I will compare and evaluate how the two views treat uniqueness. In doing so, I will give examples which show that Frege's view captures intuitions about definite descriptions which do not uniquely refer better than Russell's does.

For ease of exposition, I will introduce some new terms in addition to the previously defined term 'empty'. 'Overflowing' is a semantic property that a definite description has if and only if the description fails to refer uniquely to an entity. An example of an overflowing description is 'the capital of Holland', as there exist two capitals of Holland, namely Amsterdam and The Hague. As it is questionable whether 'the capital of Holland' makes a genuine reference to either of the cities, I will call them 'referables' instead of 'referents' of the description. Referables then are the entities that fit an overflowing description. In the case of overflowing descriptions, I further distinguish between homogeneous and nonhomogeneous predications about those descriptions. Homogeneous predications are such that they yield the same truth value for all of the referables. An example of such a predication is 'The capital of Holland is pretty', if both Amsterdam and The Hague are pretty. Nonhomogeneous predications are such that they yield True for some but not all of the referables. For instance, 'The capital of Holland is large' is nonhomogeneous when Amsterdam is large but the Hague is not.

Section 1

On Frege's view about sense and nominatum, sense by its very nature corresponds to a definite nominatum (187). Thus 'the capital of Holland' does not express a sense because it is overflowing: it may nominate The Hague or Amsterdam equally well. Therefore, in order to express a sense of, say, The Hague, further information is required to pick out The Hague uniquely. Something like 'the smaller capital of Holland' will do, for it uniquely picks out The Hague, thus qualifying for expressing a sense.⁷

Let us suppose that in the case of the capital of Lilliput, just as in the case of most capitals, the above ambiguity does not arise.⁸ Then 'the capital of Lilliput' is not overflowing; it passes—

even if only vacuously—on uniqueness and thus expresses a sense of the city that is the capital of Lilliput. This city, however, does not in fact exist. As a consequence, ‘the capital of Lilliput’ lacks a nominatum and is, in other words, an empty description.

But according to Frege, evaluations of truth are about the nominata of sentences, as sentences are what nominate the True or the False (190). Meanwhile, the principle of compositionality ensures that simple sentences, among others, are extensional: if we replace a component by another with the same nominatum, the nominatum of the whole sentence remains unaltered (190). This is brought about by the workings of the principle: the nominata of the parts of a sentence are combined in a systematic way to yield the nominatum of the sentence. Thinking in terms of function-argument application, in the sentence ‘John sneezes’, ‘John’ and ‘sneezes’ are proper parts of the sentence and ‘John’ nominates John, the person, whereas ‘sneezes’ nominates a characteristic function which assigns to each entity (in this case, John) the truth value True just in case the entity is a sneezer and assigns False otherwise. (For a more elaborate account of this, see Dowty chapter 2, especially 42-43.)

According to the evaluation suggested above, the sentence ‘The capital of Lilliput is pretty’, may not have a nominatum for the following reason: the nominatum of ‘is pretty’ is a function that assigns truth values to entities. But in the present case, there is no entity to which any function can assign anything as there is no entity nominated by ‘the capital of Lilliput’. Hence, the sentence ‘The capital of Lilliput is pretty’ cannot nominate the True or the False.

A consequence of this, which Frege was content to admit, is that sentences containing empty descriptions have only senses but not nominata (187, 189). On this view, sentences like ‘The capital of Holland is pretty’ as well as ‘The capital of Lilliput is pretty’ cannot be evaluated with respect to their truth because they fail on one or the other of the two criteria that need to be fulfilled in order for truth values to be compositionally obtained:

1. ‘The capital of ----’ has to express a sense; that is, there should be at most one entity designated by the expression. (This is the criterion for uniqueness.)
2. A sense, in turn, has to nominate something. (this is the criterion for existence.)

The question of truth arises only after the requirements in both (1) and (2) have been met. Thus if we are asked to evaluate a sentence ‘The capital of Q is pretty’ with respect to its truth, we expect that it has already been established that ‘the capital of Q’ nominates exactly one city in order for our endeavor to succeed. Then our only concern is about predication: whether or not the function ‘is pretty’ yields True or False for the capital of Q. Concisely put, the motto of the Fregean would be "No sense means no truth value; no nominatum also means no truth value. In order to engage in ‘logic talk,’ you’ve got to have both."

Section 2

It was precisely this limitation that inspired Russell to reformulate Frege’s account of descriptions. The chief achievement of Russell’s alternative lies in its allowing that truth and falsity may be the business of sentences like ‘The capital of Lilliput is pretty’ and ‘The capital of

Holland is pretty'. Russell obtained this result by making uniqueness and existence figure in "logic talk" rather than making them prerequisites for "logic talk." According to Russell (213), the following three claims are asserted in the sentence 'The capital of Q is pretty':

1. There is at least one city that is the capital of Q (the existence component);
2. There is at most one city that is the capital of Q (the uniqueness component);
3. Whatever city is the capital of Q, is pretty (the predication component).

Since these three claims are strung together in a conjunction within the scope of an existential quantifier, using the conventional truth value assignment for conjunction, we can generate the truth table below:

TRUTH TABLE 1

	1. <i>existence</i>	2. <i>uniqueness</i>	3. <i>predication</i>	4. <i>sentence</i>
<i>a.</i>	T	T	T	T
<i>b.</i>	T	T	F	F
<i>c.</i>	T	F	T	F
<i>d.</i>	T	F	F	F
<i>e.</i>	F	T	T	F
<i>f.</i>	F	T	F	F
<i>g.</i>	F	F	T	F
<i>h.</i>	F	F	F	F

Following are a few examples based on the supposition that all cities except for London are pretty (we will take the Kripkean Pierre's word on this one):²

(*a*) is about a sentence such as 'The capital of Sweden is pretty'.

(*b*) is about a sentence such as 'The capital of England is pretty'.

(*c*) is about a sentence such as 'The capital of Holland is pretty'.

(*e*) is about a sentence such as 'The capital of Lilliput is pretty' (by using our previous assumption that 'the capital of Lilliput' is not overflowing).

Notice that except for (*a*) all other instances are false, And once we consider the truth and falsity of the three components, it becomes apparent that (*b*), (*c*) and (*e*), though all false, are false on account of the falsity of different components. Further, there are a total of eight distinct ways in which the truth values of the components may combine and only one of those combinations yields a true sentence. Thus Russell's formulation provides us with the expressive power to

construct false sentences in seven distinct ways. Accordingly, we may imagine situations in which the following sentences are uttered: "It can't be that the capital of England is pretty because that city isn't pretty at all!" or "It can't be that the capital of Lilliput is pretty because there is no such city!"--both of which are true because they negate false sentences.

Let us pause for a moment and consider another such utterance: "It can't be that the capital of Holland is not pretty!" Arguably, this is true because its non-negated version, 'The capital of Holland is not pretty', is false because it yields False for both the uniqueness and predication conjuncts. The latter is false because both Amsterdam and The Hague are pretty. Meanwhile, in the case of (c), the predication component was true because both The Hague and Amsterdam are pretty. But how do we evaluate the predication component of a sentence like 'The capital of Holland is large' where Amsterdam is large and The Hague is not? In deciding what truth value to assign to such nonhomogeneous cases the more plausible of two approaches seems to be the following:

A. the predication component is true just in case it is true for all the referables (this means making a universal commitment).

Indeed, this is exactly what the predication component of Russell's account stipulates (see 3 above). The alternative would be that

B. the predication component is true just in case it is true of some of the referables (this means making an existential commitment).

When there is a unique referent, the truth conditions in (B) are the same as in (A). Though our intuitions may not be sufficiently clear to take us either way, we need not take a firm stand on choosing one or the other of these alternatives, as the subsequent discussion can be carried out for both cases. (And unless we introduce quantifiers other than the existential and the universal, these two choices exhaust our options.) Following Russell, I will adopt (A) for the remainder of this section.

In each of the eight cases (a) through (h) in which the uniqueness component is false (that is, where the description is overflowing), there are several ways in which the predication's being true of the referables may vary. In fact, in the case of the Amsterdam-The Hague ambiguity, there are 11 different ways in which the sentence 'The capital of Holland is pretty' could be false (lines b through h₃):

TRUTH TABLE 2

	1. <i>existence</i>	2. <i>uniqueness</i>	3. <i>predication</i> (n_1, n_2) ¹⁰	4. <i>sentence</i>
<i>a.</i>	T	T	T	T
<i>b.</i>	T	T	F	F

<i>c.</i>	T	F	T (T, T)	F
<i>d₁.</i>	T	F	F (T, F)	F
<i>d₂.</i>	T	F	F (F, T)	F
<i>d₃.</i>	T	F	F (F, F)	F
<i>e.</i>	F	T	T	F
<i>f.</i>	F	T	F	F
<i>g.</i>	F	F	T (T, T)	F
<i>h₁.</i>	F	F	F (T, F)	F
<i>h₂.</i>	F	F	F (F, T)	F
<i>h₃.</i>	F	F	F (F, F)	F

Generally speaking, for an overflowing description with n referables, there are $3+2 \cdot 2^n$ distinct ways in which a sentence involving that description could be false. As seen in the truth table above, this equation yields 11 when $n=2$. In the above equation, 3 is the number of cases in which there is a unique referent and the sentence is false (lines *b*, *e* and *f*). 2^2 is the number of combinations of two truth values in case the overflowing description involves a twofold ambiguity of referables. In one of these cases, the predication component is true (e.g., in line *c*), while in the remaining cases, it is false (e.g., in lines *d₁* through *d₃*). We have the same number of variations in lines (*g*) and (*h₁*) through (*h₃*), and thus 4 is multiplied by 2. Using the same equation, it turns out that there are 259 ways a sentence like 'The dwarf sneezed' could be false in the context of Snow White's story.

Section 3

Thus Russell's generous theory stands in sharp contrast with Frege's since the latter denies the possibility of there being even one case in which a sentence containing an empty or an overflowing description has a truth value. And according to Frege, even if both the uniqueness and the existence criteria are in place, there is only one way a sentence may be false: when the predication is false for the argument of the sentence.¹¹

As we have seen in the conversation about the present king of France's strand of hair (above), Russell's theory is at an advantage relative to Frege's in that it allows for truth values in the case of empty descriptions such as 'the present king of France'. And intuition tells us that there exist such cases. My view, however, is that Russell's generosity was excessive; while he was right to admit sentences containing empty descriptions into the realm of "logic talk," I will suggest that he should not have let overflowing descriptions slip in as well.¹²

The central question to ask is the following: Does the issue of evaluation with respect to truth arise in the case of sentences containing overflowing descriptions? In other words, can we have and would we want to have "logic talk" about sentences with definite descriptions which do not pass the test of uniqueness? In arguing against uniqueness being part of "logic talk," I will suggest a quantification-based treatment for definite descriptions and will show how first, this treatment necessitates that uniqueness be a prerequisite for truth evaluation rather than part of it, and second, that this view of definite descriptions matches the way such expressions are commonly understood—indeed much better than Russell's view does.

Let us consider the following two sentences:

1. The capitals of Holland are pretty.
2. A capital of Holland is pretty.

Clearly, (1) makes a universal commitment in that it predicates 'is pretty' of all capitals of Holland (see footnote 5). We can translate this into the following symbolic logic expression:

1'. $\forall x(x \text{ is a capital of Holland} \rightarrow x \text{ is pretty})$

By contrast, (2) is patently existential:

2'. $\exists x(x \text{ is a capital of Holland} \wedge x \text{ is pretty})$

'The capital of Holland' seems to bring about quite some confusion: should we evaluate a sentence containing it in accordance with (1') or with (2')? (As before, taking a standard system of logic, the existential and universal quantifiers are the only ones we have.) In particular, how should we regard the sentence: 'The capital of Holland is large' (again, assuming Amsterdam is large and The Hague is not)? If we choose (1'), the sentence is false, as not all Dutch capitals are large. If we choose (2'), the sentence is true, as one of the Dutch capitals (namely Amsterdam) is large. Additionally, by the principle of excluded middle, 'It is not the case that the capital of Holland is large' is true according to (1') since The Hague is not large, and false according to (2') because it is not the case that Amsterdam is nonlarge. It would be interesting to have such intuitions or any intuitions about specific truth value assignments in this case; I have not managed to have either.

This may then suggest that a sentence of the form

3. The capital of Q is pretty

does not on its own make a universal or an existential commitment. It is in fact quantifierless. It does, however get quantified by default in the case where 'the capital of Q' picks out a unique actual city, since then the readings (1') and (2') always yield identical truth value assignments:

3'. $\forall x(x \text{ is a capital of Q} \rightarrow x \text{ is pretty})$

3'. $\exists x(x \text{ is a capital of Q} \wedge x \text{ is pretty})$

And since those two options are the only ones we have, no ambiguity arises as to what the truth value of the sentence is in a quantified form. In such a case, we may use either quantifier and thus evaluate the sentence with respect to its truth. Meanwhile, in the case of sentences such as 'the capital of Holland is pretty', the quantifier ambiguity never gets resolved and thus the sentence may not take on a form that would let it be part of "logic talk."

In the same way that a sentence like (3) may get quantified, a definite description such as 'the capital of Sweden' can be disambiguated based on the following two readings:

4. a capital of Sweden and

4'. the capitals of Sweden (this is equivalent to 'all capitals of Sweden')

The fact that both (4) and (4') pick out Stockholm means that the uniqueness criterion has been fulfilled. Yet again, 'the capital of Holland' makes different commitments about The Hague and Amsterdam when read as 'a capital of Holland' rather than 'the capitals of Holland'. The fact that this ambiguity may not be resolved constitutes a failure of uniqueness.

The above treatment can be extended to evaluate empty descriptions as well. The reason why we may care to perform such an evaluation is one that Cresswell motivates as follows:

I say that Mr. Pickwick [from the Pickwick Papers] . . . [does] exist all right, but not in the actual world, only in another possible world. . . . Possible worlds are things we can talk about or imagine, suppose, believe in or wish for. . . . The actual world is the only world which is actual, yet there are many other worlds which might have been actual. (3-5)

Then in the case of a definite description, whether it be 'the present king of France' or the 'capital of Lilliput', its properties that are independent of existence should be nonvacuously evaluable by looking at appropriate possible worlds.

Uniqueness is one such existence-independent property. The nonvacuous nature of testing empty definite descriptions for uniqueness comes from the following intuition: while it seems that the capital of Lilliput would most likely turn out to be a unique city if it were to exist, the description 'the dwarf' in the context of Snow White's story would be overflowing if the story were actual, thus failing on uniqueness. It lies outside the scope of this essay to explore the nature of such a counterfactual evaluation, but it is well to appeal to already existing theories of counterfactuals (e.g., Lewis and Stalnaker). These can be adapted to testing uniqueness for definite descriptions, whether they be empty or not. For example, uniqueness can be evaluated for 'the capital of Lilliput' by the following steps:

- i. Find a possible world p that is most like the actual one except for the fact that in p there exists a capital of Lilliput. Clearly, this hinges on an appropriate respect of similarity to be established between the actual world and possible worlds, on the basis of which we find p .
- ii. Check whether 'a capital of Lilliput' and 'the capitals of Lilliput' pick out the same entities in p .
- iii. If so, the description passes the test of uniqueness and can enter "logic talk."
Otherwise, it fails to do either.

In the case of the above 'the dwarf', the possible world that is most like the actual one except that in it some dwarf of Snow White's exists, will contain more than one such dwarf.¹³ As a result, the answer to (ii) will be 'no', and any sentence containing this description will thus fail to enter "logic talk" by (ii) and (iii). Steps (i) and (ii) can be expressed in a more traditional counterfactual conditional form:

$\forall x(x \text{ is a capital of Lilliput} \rightarrow \forall y(y \text{ is a capital of Lilliput} \rightarrow y = x))$

More generally, for a definite description 'the ϕ ' to pass the test of uniqueness means making the following counterfactual true:

$\forall x(\phi x \rightarrow \forall y(\phi y \rightarrow y = x))$

This account then corresponds to Frege's view in that passing the test of uniqueness is the only way a definite description may, by default, be quantified and thus enter "logic talk." When a definite description fails to do this, no quantifier may be affixed to the sentence containing the description and as a result, the sentence will not be evaluable with respect to its truth. At the same time, the test of uniqueness arises only in the case of definite descriptions and hence all others will automatically enter "logic talk" to be evaluated based on the existence and predication components only.

By contrast, on Russell's account, in truth table (2), lines (c), (d₁) through (d₃), (g), and (h₁) through (h₃) yield truth evaluations for predications of all sorts applied to 'the capital of Holland'. As in the example 'The capital of Amsterdam is large', intuitions fail at least in cases in which the predication is nonhomogenous with respect to the referables (that is, a property is predicated of some but not all the referables).

Further if we carry out Russell's schema for the case of the overflowing description 'the dwarf' in Snow White's story, we will obtain a gigantic truth table that will provide a specially computed truth value for, say, 'The dwarf sneezed' in each of the situations where a certain dwarf sneezed and the others did not, or another one was the single sneezer, or two dwarfs sneezed and five did not, or three dwarfs sneezed and four did not, and so forth. (Naturally, since all cases will be false for the existence component, all lines of the table that may in fact obtain will yield False.) But if I were to judge whether those sentences were true or false, I would be too perplexed to come up with an answer for any one instance, let alone distinguish the ways in which each answer would be computed. As I doubt my lack of intuitions in this respect is in any way unique, I find it more appropriate to turn to an alternative truth table that consists of only such lines that reflect everyday, intuitively plausible truth assignments.

This truth table will, in accordance with Frege's view, make no mention of truth value assignments concerning uniqueness since that is a criterion to be tested prior to looking up a truth evaluation. It will, however, include Russell's existence component so as to support intuitions above about the present king of France. Following is the table we have:

TRUTH TABLE 3

	1. <i>uniqueness</i>	2. <i>predication</i>	3. <i>sentence</i>
<i>a.</i>	T	T	T
<i>b.</i>	T	F	F
<i>c.</i>	F	T	F
<i>d.</i>	F	F	F

Thus in his treatment of definite descriptions that are overflowing, Frege got right what Russell didn't.

NOTES:

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²Frege's theory discussed here comes from "On Sense and Nominatum." All page references to Frege will be to this paper.

³Bertrand Russell raises the same line of argument in "On Denoting" 202.

⁴The property 'empty' is a semantic one I apply to the syntactic category 'definite description'. An empty definite description is one that does not refer to or nominate anything actual (e.g., 'the present king of France' or 'the unicorn in my garden' are empty definite descriptions). When it is unambiguous to do so, I sometimes write 'empty description', omitting 'definite'.

⁵For another formulation of this, see "On Denoting" 201.

⁶The view of Russell that I contrast with Frege's view comes from "Descriptions." All subsequent page references to Russell will be to this paper.

⁷This point is not explicit in Frege's paper. I do, however, take it to be not just a possible extension but a necessary consequence of Frege's theory on the following grounds: when introducing descriptions starting with 'the', there are two possible continuations using countable nouns:

- a. there may be a plural noun following the definite article, say, 'capitals of Holland', or
- b. there may be a singular one, say, 'capital of Holland'.

In (a), the plural form brings it about that the nominatum is a collection/set containing all capitals of Holland. Meanwhile, the singular form assumes the entity itself to be the nominatum. In cases such as (a) the possibility of violating the uniqueness criterion does not arise. But in cases such as (b), in which entities themselves are nominated, the existence of two nominable entities will violate the uniqueness criterion. Then Frege's requirement that there be a unique nominatum for a sense suggests that (b) does not express sense while descriptions like (a) do. (To see the shift from nominating entities to nominating collections of entities, compare 'The collection of capitals Holland' [a case of singular use] with a and b.)

⁸In section 3, I will discuss how descriptions may pass the test of uniqueness without passing the test of existence. Until then, I suggest we take unique to mean "at most one."

⁹At this point, (g) and (h) cannot arise, as empty descriptions vacuously pass uniqueness. As we will see in section 3, there exists a formulation in which it is possible for descriptions to be empty and overflowing.

¹⁰As stipulated in Russell's component (3), we assume that the predication component of sentences containing overflowing descriptions (in this case, the referables are n_1 and n_2) is computed as a universally quantified sentence, yielding True if the predication is true for all referables and False otherwise.

¹¹Although the examples given so far concern only cases in which the subject of a sentence is an empty or an overflowing description, the arguments in sections 1 and 2 can be carried out for empty or overflowing descriptions in object positions of sentences as well.

¹²A note of clarification before we proceed: in this essay, I concede the point Keith Donellan makes about the attribution/reference distinction (in "Reference and Definite Descriptions"). In accordance with his conclusions, I will take Russell's theory of descriptions, as well as Frege's theory of sense and nominatum, to cover only issues concerning attribution/assertion. Thus the question of what city has been *meant* by a speaker tittering the words 'the capital of Holland' is not relevant here. Instead, we are to examine what has been asserted, regardless of speakers' or listeners' intentions to take a reference to be one thing or another.

¹³It is clear that our choice of p in this case cannot depend on the commonly used notion of similarity between p and the actual world, because based on that choice, a world in which only one dwarf attends to Snow White can be on the whole more similar to ours than one in which seven dwarfs do. The notion of similarity that would be more appropriate for evaluations involving fiction would take into account the fact that in the original story we tell Snow White had seven dwarfs tending her. The delineation of a notion of similarity that would take into account such facts is a complex task which is not undertaken in this essay.

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