# ANDREW ABERDEIN

I owe it was a dangerous project, And you have proved it by dog-logic. Jonathan Swift

In March 1615 His Majesty King James the VI of Scotland and I of England participated in a debate concerning the use of logic by dogs. The debate was one of several entertainments provided for the King during a visit to the University of Cambridge. At first glance, this event might seem wholly frivolous. Certainly the King's enthusiasm for hunting played a part in the choice of topic. But King James was no idle, anti-intellectual prince. He had assembled a hothouse of Protestant theologians to produce the English Bible that still bears his name, and his own collected works were to appear the following year. Moreover, as we shall see, the arguments rehearsed before King James echo down the history of logic, from antiquity to the twenty-first century.

## The Dog Has His Day

The specific proposition which the Cambridge scholars disputed was "Whether dogs could make syllogisms." This requires a little unpacking. Literally, a *syllogism* is a "combined reasoning": a derivation of one statement, the *conclusion*, from two others, the *premisses*. The *major* premiss, which characteristically makes the more general claim, is that conventionally given first; the other is termed the *minor* premiss. Logicians are particularly concerned with *valid* syllogisms, in which the truth of the premisses guarantees the truth of the conclusion. The theory of the syllogism is only a small fragment of modern logic, but until the nineteenth century, it comprised the principal subject matter of the discipline. Most attention was paid to *categorical* syllogisms, in which each of the statements of which the syllogism is comprised expresses a relationship between a pair of categories, or sets. For example, one might reason as follows:

- All pointers are gun dogs.
- All Weimaraners are pointers.
- Therefore, all Weimaraners are gun dogs.

This is the simplest sort of valid syllogism, sometimes known, in accordance with a mediaeval mnemonic, as *Barbara*. Observe that the conclusion *must* be true, if the premisses are. This is a property of the *form* of the argument: one need not know anything about pointers, gun dogs, or Weimaraners to tell that this argument is valid. Categorical syllogisms were invented by Aristotle (384–322 B.C.), the first logician. Other varieties of syllogism were devised by later logicians, such as Chrysippus (280–204 B.C.), who was ultimately responsible for the principal exhibit laid before King James.

At Cambridge in 1615, the claim that dogs use logic was defended by John Preston (1587–1628) of Queens' College. "He instanced in a hound who hath the major proposition in his mind, namely, *The hare is gone either this way or that way*; smells out the minor with his nose, namely, *She is not gone this way*; and follows the conclusion, *Ergo this way*, with open mouth."<sup>1</sup> The inference which the dog is purported to have followed is *disjunctive syllogism*, which we might abbreviate as "P or Q, not-P, therefore Q." Preston was answered by Matthew Wren (1585–1667) of Pembroke College, for whom dogs were distinguished by the excellence of their noses, not their reasoning: surely the dog determined the quarry's path by scent. While the moderator, Simon Reade of Christ's College, agreed with Wren, the King took Preston's side, gave an example of a reasoning dog from his own experience, and suggested that Reade should "thinke better of his dogs or not so highly of himselfe." Reade successfully mollified his sovereign with the suggestion that dogs that hunt by royal prerogative must be exceptions to laws governing common hounds, and the debate concluded in good spirits.

# The Dog's Tale

Preston's dog story was not a new one. As John Mayor observes, the King and the other disputants could have known it from multiple sources. For example, they might have been reading Edward Topsell's (1572–1625) recently published *Historie* of *Foure-footed Beasts*, wherein we learn that

"Ælianus thinkes that Dogges have reason, & vse logick in their hunting, for they will cast about for the game, as a disputant doth for the truth, as if they should say either the Hare is gone on the left hand, or on the right hand, or straight forward, but not on the left or right hand and therefore straight forward. Whereupon he runneth foorth right after the true and infallible footesteps of the Hare."<sup>2</sup>

Or they may have seen the 1600 play *Summer's Last Will and Testament* by Thomas Nash (1567–1601), in which they would have heard (lines 698–708) that

"Chrisippus holds dogs are Logicians, In that, by study and by canvassing, They can distinguish twixt three several things: As when he cometh where three broad ways meet, And of those three hath stayed at two of them, By which he guesseth that the game went not, Without more pause he runneth on the third; Which, as Chrisippus saith, insinuates As if he reasoned thus within himself: Either he went this, that, or yonder way, But neither that, nor yonder, therefore this."

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<sup>&</sup>lt;sup>1</sup>Thomas Ball, 1628, *Life of Preston*, quoted in John Mayor, 1898, 'King James I on the reasoning faculty in dogs,' *The Classical Review*, **12**, p. 94. Emphasis original.

<sup>&</sup>lt;sup>2</sup>Edward Topsell, 1607, *The Historie of Foure-footed Beasts*, p. 141, quoted in K. J. Höltgen, 1998, 'Clever dogs and nimble spaniels: On the iconography of logic, invention, and imagination,' *Explorations in Renaissance Culture*, **24**.

But they most likely encountered the anecdote in the longest of the essays of the influential Renaissance philosopher Michel de Montaigne (1533–1592). First published in 1580, this appeared in English translation in 1603. Here we may read that

"Chrysippus, albeit in other things as disdainfull a judge of the condition of beasts as any other Philosopher, considering the earliest movings of the dog, who comming into a path that led three severall wayes in search or quest of his Master, whom he had lost, or in pursuit of some prey that hath escaped him, goeth senting first one way and then another, and having assured himself of two, because he findeth not the tracke of what he hunteth for, without more adoe furiously betakes himselfe to the third; he is enforced to confesse that such a dog must necessarily discourse thus with himselfe, 'I have followed my Masters footing hitherto, hee must of necessity pass by one of these three wayes; it is neither this nor that, then consequently hee is gone this other.' And by this conclusion or discourse assuring him selfe, comming to the third path, hee useth his sense no more, nor sounds it any longer, but by the power of reason suffers himselfe violently to be carried through it. This meere logicall tricke, and this use of divided and conjoyned propositions, and of the sufficient numbring of parts: is it not as good that the dog know it by him selfe, as by *Trapezuntius* his logicke?"<sup>3</sup>

Montaigne's version closely follows the accounts in Sextus Empiricus (*fl.* 190 A.D.), and in Plutarch (46–120 A.D.), the acknowledged inspiration for Montaigne's revival of the literary essay. Other ancient versions have survived in the works of Philo of Alexandria (*fl.* 39 A.D.), Claudius Aelianus (170–235 A.D.), Porphyry (233–301 A.D.), St. Basil (330–379 A.D.), and St. Ambrose (340–397 A.D.).<sup>4</sup> These multiple versions of the tale differ in minor details—whether the dog faces two or three possible exits, whether he is chasing a hare or tracking his master—but coincide at least in crediting the story to Chrysippus. Unfortunately, Chrysippus's own account is lost. Sextus follows Chrysippus's arguments closely, but in order to rebut them, making the reconstruction of Chrysippus a delicate task.

# AN OLD DOG TAUGHT NEW TRICKS

The tale has been retold by many different authors, in much the same way, but to very different ends. Two questions posed during the Cambridge debate may help us understand how these different contexts are related: "Are dogs logical?" and, as King James demanded of Dr Reade, "Should we think better of dogs or not so highly of ourselves?" Since each question has two possible answers, that makes for four positions, each of which has been defended by somebody. Firstly, we could say that dogs are logical, and so we should acknowledge that they are almost as clever as we are. Secondly, we could say that dogs are logical, but that shows that we are no cleverer than dogs. Thirdly, we could say that dogs are illogical, but praise them for accomplishing so much without logic. Fourthly, we could say that dogs

 $<sup>^3</sup>Essays$  II.12, 'An Apology for Raymond Sebond,' John Florio's translation, 1603. Emphasis original. Trapezuntius, otherwise known as George of Trebizond (1395–1484), was the author of a popular logic text.

<sup>&</sup>lt;sup>4</sup>The relevant passages, in Latin or Greek, may be found in Mayor, pp. 94–96.

are illogical, which only goes to show the worthlessness of logic, and the superiority of thinking like a dog.

**Option 1: Dogs as Clever as Men.** On the first position, the dog story is accepted at face value. This is the perspective of Aelianus, and his mediaeval followers, such as the anonymous author of the twelfth-century *Roxburghe Bestiary*. Here the story culminates with the moral: "And so, by rejecting error, Dog finds the truth."<sup>5</sup> Topsell follows suit, as does the Enlightenment polemicist Jean-Baptiste de Boyer, Marquis d'Argens (1704–1771), for whom "this behaviour of the dog is an evident proof that his mind is capable of the three operations of logic, and I do not see why a Shock Dog and a Mastiff may not carry their reasoning as far as a Regent of Philosophy in the College of the Four Nations."<sup>6</sup>

This interpretation of the story helped to establish a curious career for the dog as emblem of logic. Beginning with the *Hortus deliciarum (Pleasure Garden)* of Herrad of Landsberg (d. 1195), numerous emblem books, iconologies and systems of visual mnemonics employ dogs to symbolize logic (Höltgen). Perhaps the most successful example is the woodcut representing logic in the 1503 liberal arts textbook *Margarita Philosophica (Pearl of Wisdom)* by Gregor Reisch (1467–1525). Here, amongst many other mnemonic images, Dame Logic hunts the hare Problem, armed with the sword Syllogism, and accompanied by the dogs Truth, which is hot on Problem's heels, and Falsity, which is sniffing a tree stump—unlabeled. This image was plagiarized by Thomas Murner (1475–1537) as the frontispiece to his 1508 Logica memorativa vel Chartiludium logice (Logic Remembered, or the Logical Card Game). This work, which sought to popularize the teaching of logic by means of elaborately illustrated playing cards, is sometimes credited with inadvertently inspiring tarot card reading.

**Option 2:** Men as Stupid as Dogs. For those in the second camp, the dog's logical acuity serves the very different purpose of undermining what these skeptical authors perceive as an exaggerated reverence for human reason. This take on the story is Sextus's contribution. His broader goal was to suggest that the trust we have in our senses is misplaced: many species of animals have superior, or at least different, sense organs, and so ours is merely one way of perceiving the world. Sextus unleashes his dog in anticipation of the response that human perceptions of the world must be the correct ones, because of our superior intellects. If dogs can perform syllogisms, he suggests, we might not be so superior, and so we must concede that ours is merely one way of seeing the world. Porphyry and Montaigne set their dogs to similar skeptical work.

**Option 3: Dogs Clever Although Illogical.** Proponents of the third and fourth positions seek to debunk the dog story. How do they do this? Wren's suggestion that the dog merely followed its nose is the most obvious strategy. As Samuel Taylor Coleridge (1772–1834) asks, "Why does this story excite either wonder or incredulity? ... the Breeze brought his Master's scent down the fourth Road to the

<sup>&</sup>lt;sup>5</sup>T. H. White, 1954, *The Bestiary: A Book of Beasts*, London, p. 64.

<sup>&</sup>lt;sup>6</sup>Quoted in Luciano Floridi, 1997, 'Scepticism and animal rationality: The fortune of Chrysippus' dog in the history of Western thought,' *Archiv für Geschichte der Philosophie*, **79**. The three operations of logic are generally taken as apprehension, judgment and reasoning; a shock dog is somewhat like a Poodle; a regent is a professor; and the College of the Four Nations was a pre-Revolutionary college of the University of Paris.

Dog's nose, and ... therefore he did not put it down to the Road, as in the two former instances."<sup>7</sup> Paradoxically, it is in the earliest surviving versions, those of Philo and Aelianus, that we find the most direct counter-argument. For Philo's dog the trail runs cold at a deep shaft beside two tracks, which the dog inspects, and then "with no further scenting it jumped into the shaft to track down hastily," whereas Aelianus's "dog came upon a ditch and was puzzled as to whether it had better follow to the left or to the right. And when it had weighed the matter sufficiently, it leapt straight across."<sup>8</sup> In both cases an obstacle has been interposed, seemingly to require the dog to employ disjunctive syllogism, rather than its nose. However, this underestimates the extraordinary canine sense of smell: contrary to popular myth, crossing a stream does nothing to put a scent hound off one's track.<sup>9</sup>

Moreover, Wren's sensible suggestion misses the point. Dismissing the specific story as implausible does not answer the real questions about the intellectual capabilities of dogs which the story raises. Notice that in almost every version of the story the dog is credited with an internal reasoning process, whereby he summarizes his predicament in statements which may serve as the premisses for his argument. For critics of canine cognition this is where the problem starts: do dogs really "say things to themselves" as the story apparently requires? If it is granted that they do, even non-linguistically, they have come close enough to human cognition that whether they also perform syllogisms seems a minor detail. Hence philosophers who deny the dog access to logic proceed by denying that he can form premisses.

This goes back to Aristotle. Unlike his teacher Plato (427–348 B.C.), who distinguished only between the perceptions derived from our senses and the beliefs founded upon them, Aristotle introduced a third sort of mental content, that of appearances, which mediates between the two.<sup>10</sup> Hence Aristotle could deny, as Plato could not, that animals were rational. Seemingly rational behavior by animals could now be explained away as the product of some process at the appearance level. Animals, for Aristotle, do not have beliefs, but they do have appearances upon which rudimentary internal reasoning processes can operate. The crucial difference is that animals are not aware of their own reasoning process. For an appearance to qualify as a belief one must be convinced of it, that is persuade oneself that it is true ('Rationality,' p. 317). Hence, not just reasoning, but self-aware reasoning is required, if one is to have beliefs, and thereby qualify as rational.

Philo makes use of this strategy in his critique of the dog story. The dog has an appearance of no scent from two of the trails, and by an instinctive natural capacity proceeds in the direction of his quarry. We might interpret his behavior in accordance with the disjunctive syllogism, but that no more means that he is explicitly employing it than it would if we made similar observations of a human being eliminating false leads. People look for things all the time; they seldom think of syllogisms while doing so. Hence Philo is in the third position outlined

<sup>&</sup>lt;sup>7</sup>*The Friend*, No. 5, 1809. Emphasis original.

<sup>&</sup>lt;sup>8</sup>Philo: *De Animalibus adversus Alexandrum*, 45; Aelianus: *De Natura Animalium*, VI.59, both quoted by Floridi. Philo's original Greek survives only in an Armenian translation, which Floridi has read in French, and Mayor quotes in Latin. Mayor's version of Philo resembles Aelianus's account.

 $<sup>^9 \</sup>mathrm{See,}$  for example, MythBusters, Season 4, Episode 5: 'Dog Myths.' Original air date: 14 March 2007.

<sup>&</sup>lt;sup>10</sup>See Richard Sorabji, 1996, 'Rationality'. In Michael Frede and Gisela Striker, eds, *Rationality* in Greek Thought, Oxford, pp. 311–334.

above: the dog is not logical, but does not need to be. St. Thomas Aquinas (1225–1274) also seems to interpret the story this way. On his version, "the dog, chasing the stag, reached a crossroads, and having smelled the first and the second path, missing any information, ran with all certainty in the third direction, without further investigation, as if he had used a syllogism."<sup>11</sup> Indeed, Chrysippus himself may have held this view: Sextus only has him say that "the dog in effect reasons."<sup>12</sup> Hence the whole point of his story could have been not to suggest that dogs are really logical, but to illustrate the way that their natural intellect of appearances might be mistaken for reason ('Rationality,' p. 316).

Option 4: Dogs Clever Because Illogical. Finally, for those who retell the story from the fourth position, this natural intellect seems rather more attractive and useful than the much vaunted logic which the dog lacks. This suggestion is found in Plutarch, and reinforced by the church fathers SS. Basil and Ambrose. The former saint observes approvingly that "Whereas the wise of our world may spend a life-time of laborious meditation on the combinations of syllogisms, dogs manage to clear up such problems naturally."<sup>13</sup> This perspective was especially influential among the empiricist philosophers of the early modern period, who trusted in experience and mistrusted what they saw as a bankrupt tradition of logical obfuscation perpetrated by mediaeval obscurantists. Thus Pierre Gassendi (1592–1655) asks "what olfactory sense is logic endowed with that it sniffs out and runs to ground the hidden nature of things?" This trend culminates with David Hume (1711–1776), for whom "the whole conduct of life depends" not on logic, but on "experimental reasoning ... which we possess in common with beasts"<sup>14</sup> This "is nothing but a species of instinct or mechanical power, that acts in us unknown to ourselves; and in its chief operations, is not directed by any such relations or comparisons of ideas, as are the proper objects of our intellectual faculties."

Empiricism, which privileges experience over reason, is invariably contrasted with rationalism, which reverses these priorities. However, the deprecation of logic by empiricist philosophers coincided with a deprecation of dogs, or rather of the intellectual capabilities of animals in general, by their rationalist rivals. The chief rationalist René Descartes (1596–1650) is well-known for his contention that non-human animals are unthinking automata. He left no explicit comment on Chrysippus's dog; the closest he comes is a remark in a letter that, "as for what concerns the capacity of understanding and thinking that Montaigne and some others attribute to brutes, I cannot be of the same opinion"<sup>15</sup> Under this dual assault the dog began to lose his status as patron beast of logic. For Thomas Hobbes (1588–1679), writing his *Leviathan* in 1651, the imagination works "as a spaniel ranges the field, till he finds the scent." Although the dog still represents a thought process, it is no longer a logical inference, but rather a crude search algorithm. By 1711, the date of its earliest citation in the OED, "dog('s) logic" is a term of abuse: "bad, spurious,

 $<sup>^{11}</sup>Summa\ Theologica,$ Prima Secundae, Quaest. XIII, Art. II.3, quoted by Floridi. My emphasis.

<sup>&</sup>lt;sup>12</sup>Pyrrhon. Hypotyp., A 69, in Benson Mates, 1997, The Skeptic Way: Sextus Empiricus's Outlines of Pyrrhonism, Oxford, p. 98. My emphasis.

 $<sup>^{13}</sup>Homelia$  n. IX.4, cited by Floridi.

<sup>&</sup>lt;sup>14</sup>David Hume, 1748, An Enquiry concerning Human Understanding, section 9, p. 108.

 $<sup>^{15}\</sup>mathrm{Letter}$  to William Cavendish, Marquess (later Duke) of Newcastle, 23 November 1646, quoted by Floridi.

bastard, mongrel" logic, according to the lexicographers.<sup>16</sup> The once celebrated logical dog was now in full retreat, with his tale between his legs.

## The Dog Returns to his Syllogism

In recent decades the dog has made an unexpected reappearance in works of logical scholarship. To understand why, let us look more closely at what we know of his first appearance. Sextus tells us that

"According to Chrysippus, who was certainly no friend of nonrational animals, the dog even shares in the celebrated dialectic. In fact, the author says that the dog uses repeated applications of the fifth indemonstrable argument-schema when, arriving at a juncture of three paths, after sniffing at the two down which the quarry did not go, he rushes off on the third without stopping to sniff. For, says this ancient authority, the dog in effect reasons as follows: the animal either went this way or that way or the other; he did not go this way and he did not go that; therefore, he went the other." (*Pyrrhon. Hypotyp.*, A 69)

The aside in the first sentence, subsequently echoed by Montaigne, flags up what Sextus takes to be self-contradiction in Chrysippus: denying that animals can be rational and then attributing a syllogism to a dog. As we saw above, this may be a misreading: Chrysippus might have shared Aristotle's view that primitive inferring could take place without rationality. Sextus uses "dialectic" to refer to the process of logical argument, and the "fifth indemonstrable argument-schema" is what Chrysippus called the disjunctive syllogism. It is indemonstrable because he takes it as axiomatic: something just so obviously correct that no further justification is required, or, indeed, possible. This view is not universally endorsed.

Indeed, some twentieth-century logicians have sought to banish disjunctive syllogism. Their concern is that its presence in a system of logic validates what they call *irrelevant* inferences. These are arguments held to be valid by mainstream logicians, but in which the conclusion is seemingly unrelated to the premisses. Consider the following case. Suppose your neighbor owns a small yappy dog, which you have heard from a usually reliable source to be a Pomeranian. You might harmlessly conclude that the dog was either a Pomeranian or a Rottweiler. This might seem an odd conclusion to draw, but it is a secure one: if one of the components of an "or"-statement is true then the "or"-statement must be true, even if the other component is obviously false. But if you were now to learn that the dog was not really a Pomeranian (perhaps it was a Shock Dog), you might apply disjunctive syllogism as follows: the dog is either a Pomeranian or a Rottweiler; it is not a Pomeranian; so it must be a Rottweiler. Something has definitely gone wrong. Proponents of *relevance logic* would place the blame with disjunctive syllogism.

The substantial technical complications to devising a working system of relevance logic need not detain us. But the debate has tossed a bone to Chrysippus's dog.

<sup>&</sup>lt;sup>16</sup>The OED cites *The Examiner*, No. 50, attributing it, wrongly, to Jonathan Swift (1677–1745). Swift did use the phrase, to comparable purpose, in 1723: see the epigraph at the beginning of this chapter, taken from 'Upon the horrid plot discovered by Harlequin, the Bishop of Rochester's French dog,' in R. Maynard Leonard, ed., 1893, *The Dog in British Poetry*, p. 229. The OED traces "dog-rimes" back to 1611, and the usage of "dog" within plant names, "frequently denoting an inferior or worthless sort," to 1548.

The new problem which the dog now poses is that, although his behavior still seems rational, the principle of logic by which it has been explained for over two thousand years is, according to the relevance logician, not a principle of logic after all. How is this to be resolved? The answer lies in exposing a fundamental ambiguity in the understanding of "or."<sup>17</sup>

Logicians typically interpret "or"-statements as statements about the truth of the components. Hence "The hare is gone either this way or that way" is understood as saying that one of the following statements is true: "The hare is gone this way"; "The hare is gone that way." (Whether it says that *exactly* one of them is true, or that *at least* one of them is true, is an important debate in its own right, but of no significance here, since it is obvious that only one of them can be true: the hare has not gone both ways.) This interpretation is called the *extensional* or *truth functional* sense of "or," since it makes the truth of an "or"-statement a function of the truth (or falsity) of its components.

However, there is another way of reading "or"-statements: as disguised "if"statements. Logicians call this the *intensional* sense of "or." On this reading an "or"-statement says of each of its components that, if that component is false, then (one of) the other(s) must be true. Hence "The hare is gone either this way or that way" becomes "If the hare is not gone this way then it is gone that way, and if not that way then this." This interpretation captures the assumption credited to the dog at least as well as the more familiar statement. However, the dog's syllogism would no longer be disjunctive syllogism, "P or Q, not-P, therefore Q," but rather "If not-P then Q, not-P, therefore Q." This syllogism is accepted as valid by virtually all logicians, and certainly by relevance logicians, whose grasp of the dog's logical acumen is thus vindicated. But note that this trick cannot be performed on every instance of "or." Your belief that your neighbor's dog was a Pomeranian would not support the belief that if it wasn't a Pomeranian then it was a Rottweiler. You could still believe of the dog that it was either a Pomeranian or a Rottweiler, but but only in the extensional sense of "or," which, for the relevance logician would not support the application of disjunctive syllogism. Hence the troublesome inference that a small yappy dog might somehow be concealing an extra hundred pounds of muscle is safely neutered.

Of course, you might have an independent reason for believing the intensional form of this "or"-statement: that, if your neighbor's dog is not a Pomeranian, then it is a Rottweiler. Perhaps you have seen him and his girlfriend walking one of each, and you couldn't be sure whose dog was which. Or perhaps you hear that he has recently acquired a dog, and he has said that he could own no other breeds. But if, in one of these rather contrived situations, you were to learn that he doesn't have a Pomeranian, then your inference that he must have a Rottweiler would be entirely sound. We should also note that the distinction between the two senses of "or" will collapse if an inappropriate sense of "if" is used. Somewhat counter-intuitively, mainstream logicians define "if P then Q" as equivalent to "not-P or Q": on this account of "if" there is no difference between extensional and intensional "or," at

<sup>&</sup>lt;sup>17</sup>What follows is inspired by Stephen Read, 2004, 'In defence of the dog: Response to Restall,' in S. Rahman & al, eds, Logic, Epistemology, and the Unity of Science, Kluwer, pp. 175–180. See also Jay Garfield, 1990, 'The dog: Relevance and rationality,' in J. M. Dunn and A. Gupta, eds, Truth or Consequences: Essays in Honor of Nuel D. Belnap, Jr., Kluwer, pp. 97–109. The dog's relevance logic début was in Alan Ross Anderson and Nuel D. Belnap, Jr., 1975, Entailment: The Logic of Relevance and Necessity, Princeton, vol. I, §25.1.

least as defined above. A central motivation of relevance logic is its more attractive treatment of "if": not the least of its attractions is that it sustains this distinction.

As the investigations in this chapter suggest, a simple tale can be of great philosophical value. In his earlier incarnations the dog helped us see what it means to attribute logical reasoning, not just to dogs, but to ourselves. In his most recent role the dog has clarified a significant debate between two competing conceptions of logic. Once again the faithful creature has proved himself the logician's best friend.

Humanities and Communication, Florida Institute of Technology, 150 West University Blvd, Melbourne, Florida 32901-6975, U.S.A.

 $E\text{-}mail \ address: \texttt{aberdein@fit.edu}$