Hobbes on Natural Philosophy as "True Physics" and

Mixed Mathematics

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[...] physics (I mean true physics), that depends on geometry, is usually numbered among the mixed mathematics. *De homine* 10.5 (Hobbes 1994)

[...] all the sciences would have been mathematical had not their authors asserted more than they were able to prove; indeed, it is because of the temerity and the ignorance of writers on physics and morals that geometry and arithmetic are the only mathematical ones. *Anti-White* (Hobbes 1976, 24; MS 6566A, f. 5 verso)¹

1. Introduction

At several points, Hobbes argues that he has provided a unified system, with connections between geometry and natural philosophy.² Some scholars have taken this unity to result from deductive connections between geometry and natural philosophy.³ In this paper, I offer an alternative account of the relationship of Hobbesian geometry to natural philosophy by arguing that mixed mathematics provided Hobbes with a model for thinking about it. In mixed mathematics, one may borrow causal principles from one science and use them in another science without a deductive relationship. Natural philosophy for Hobbes is mixed because an explanation may combine observations with causal principles from geometry. In Hobbesian natural philosophy, one may appeal to everyday experience or experiments for the demonstration of the 'that' and borrow the 'why' from geometry.

¹ I cite by folio number MS fonds Latin 6566A (Bibliothèque nationale, Paris; critical edition is Hobbes [1973]). I have amended Jones' translation to reflect Hobbes's use of *moralis*.

² For example, in *De corpore* 6.6 Hobbes links what he calls "our simplest conceptions," such as 'place' and 'motion', with generative definitions in geometry and, ultimately, with natural philosophy and morality (OL I.62). I cite Hobbes (2005) as EW and Hobbes (1839–45) as OL, followed by volume and page.

³ Peters 1967; Watkins 1973; Hampton 1986; Shapin & Schaffer 1985. My focus will be the relationship between geometry and natural philosophy, but other accounts of Hobbesian unity are also concerned with the relationship of politics to the other sciences. Those supporting the deductivist interpretation of the relationship between geometry and natural philosophy argue that there are also deductive connections between politics and the other sciences. However, others have seen Hobbes's politics as disjoined from the other sciences (Robertson 1886; Taylor 1938; Warrender 1957; for discussion, see also Sorell 1986, 6). Whether Hobbes's politics is related to the other sciences by a deductive connection or is disjoined is beyond the scope of the present paper.

My argument shows that Hobbesian natural philosophy relies upon suppositions that bodies *plausibly* behave according to these borrowed causal principles from geometry, acknowledging that bodies in the world may not behave this way. For example, Hobbes develops an account of simple circular motion in geometry and supposes that the sun moves the air around it by this motion. We do not know as a matter of fact that the sun causes this sort of motion, but we *suppose* that it does – Hobbes describes this as a "possible cause" – and then we explain various phenomena related to light and heat using it. As part of geometry, the principles about simple circular motion have certainty; we can know that simple circular motion has necessary effects. However, when we borrow causal principles related to simple circular motion within a natural-philosophical explanation we cannot know whether the sun actually operates by simple circular motion. As a result, in natural philosophy we have suppositional knowledge of the following form: *if* the sun causes simple circular motion *then* an effect of that propagated motion will be heat and light.

My argument proceeds in two stages. First, I consider Hobbes's relation to Aristotelian mixed mathematics and to Isaac Barrow's broadening of mixed mathematics in *Mathematical Lectures* (1683). I show that for Hobbes maker's knowledge from geometry provides the 'why' in mixed-mathematical explanations.⁴ Next, I examine two explanations from *De corpore* Part IV: (1) the explanation of sense in *De corpore* 25.1-2; and (2) the explanation of the swelling of

⁴ Hobbes uses "mixed mathematics" (*mathematicas mixtas*) in *De homine* 10.5 and in *Anti White*. For general discussion of "mixed mathematics" see Brown (1991). For discussion of making and causal knowledge in Hobbes's geometry, see Jesseph (1996, 88ff). Some connection has been made between Hobbes and others who see making as essential to scientific knowledge, including Bacon and Vico (Barnouw 1980; Gaukroger 1986). Pérez-Ramos (1989) argues that on Bacon's conception of science making, understood as manipulating nature and producing works, is the ideal of scientific knowledge. However, there are two significant differences between Hobbes and Bacon related to maker's knowledge: first, unlike Hobbes does, Bacon never explicitly appeals to making as the guarantee of scientific knowledge, so at best it is perhaps implicit in Bacon's view (Zagorin 1998, 39); and second, Hobbes explicitly holds that we possess maker's knowledge *only* in geometry and civil philosophy, so he could never countenance, as Bacon does on Pérez-Ramos' account, that we possess maker's knowledge in natural philosophy.

parts of the body when they become warm in *De corpore* 27.3. In both explanations, I show Hobbes borrowing and citing geometrical principles and mixing these principles with appeals to experience.⁵

2. Aristotle, Barrow, and Hobbes on Mixed Mathematics

2.1 Aristotle and Isaac Barrow on Mixed Mathematics

In *Posterior Analytics* I, Aristotle argues that "it is not possible to prove a fact by passing from genus to another, e.g., to prove a geometrical proposition by arithmetic" (75a38-39).⁶ For Aristotle, one cannot "prove by any other science the theorems of a different one, except such as are so related to one another that the one is under the other – e.g. optics to geometry and harmonics to arithmetic" (*APo* I.7, 75b14-17). Aristotle argues later that for sciences such as optics the 'that' will come from one science while the 'why' will come from a science which is "above" it (*APo* I.9, 76a4-13). In optics one may borrow geometrical principles because he studies the objects of optics *qua* line and not *qua* object of sight (*Metaph* M.3 1078a14-16). In treating the objects of optics *qua* line, one treats a natural object as a mathematical object.

There has been some debate regarding the status of mathematical objects for Aristotle, given this account of mixed mathematics. Whereas Lear understands them as fictional objects (1982), Lennox views them as resulting from "taking a delimited cognitive stance toward an object" (1986, 37). In other words, one considers an object in a certain way. As I will discuss below, this is how Hobbes describes mathematical objects.

⁵ Additional instances of Hobbes borrowing principles from geometry in natural philosophy beyond those I discuss include the following: *De corpore* 26.6 (OL I.349), 26.8 (OL I.353), and 26.10 (OL I.357). Each of these explanations borrows geometrical principles related to circular motion from *De corpore* 21 (they cite 21.10, 21.11, and 21.4, respectively). Hobbes similarly borrows geometrical principles from *De corpore* 22.6 and *De corpore* 24.2 in optics in *De homine* 2.2 (OL II.8) (Adams 2014b, 39-40).

⁶ See also *Physics* II.2 and *Metaphysics* M.1-3 (esp. 1078a14-17). For discussion, see McKirahan (1978), Lennox (1986), Wallace (1991), Hankinson (2005).

Hobbes's contemporary Isaac Barrow appeals to and revises Aristotle's account of mixed mathematics in his *Mathematical Lectures* (1685). It is worthwhile to compare Barrow's view to Hobbes's because of their similar outlook in mathematics, especially since both held, against John Wallis, that geometry had priority over arithmetic (Jesseph 1993). In Lecture II, Barrow criticizes Aristotle and Plato for having distinguished pure from mixed mathematics by assuming that there are two kinds of things: intelligible things, the subject of pure mathematics, and sensible things, the subject of mixed mathematics (Mahoney 1990, 185). Barrow argues that "there exists in fact no other quantity different from that which is called magnitude, or continuous quantity, and, further, it alone is rightly to be counted the object of mathematics..." (Barrow 1685, 39; trans. Mahoney 1990, 186). Since "magnitude is the common affection of all physical things," there is "no part of natural science which is not able to claim for itself the title of 'Mathematical'" (Barrow 1685, 40).

Some have taken Barrow's criticisms of the pure/mixed distinction as a rejection of mixed mathematics.⁷ However, one might instead view Barrow's criticisms as a broadening of the purview of mixed mathematics (Malet 1997, 280ff). Indeed, Barrow continues in *Mathematical Lectures* to describe what will be the *new* mixed mathematical disciplines, if his account is correct. In a way that will resonate with Hobbes's comments from *De homine* 10.5, discussed below, Barrow articulates the properly understood relationship between geometry and physics as follows: "…to return to Physics, I say there is no Part of this which does not imply Quantity, or to which geometrical Theorems may not be applied, and consequently which is not

⁷ Mahoney (1990, 186). Similarly, Jesseph argues that Hobbes rejects the distinction between pure and mixed mathematics since Hobbes understands "body as the fundamental object of mathematics" (1999, 74-76). Nevertheless, Hobbes himself describes pure mathematics as that which treats quantities in the abstract (*in abstracto*), which is how he articulates the project of *De corpore* Part III, and takes "true physics" to be part of mixed mathematics (discussed below).

some Way dependent on Geometry" (Barrow 1734, 22; Barrow 1685, 41). As support for broadening mixed mathematics beyond the normally included disciplines, such as optics or harmonics, Barrow favorably mentions Aristotle's claim in *APo* (79a13-16) that "the physician chooses the cause from Geometry" when explaining why circular wounds heal more slowly (Barrow 1685, 40).

Seeing Barrow as broadening the purview of mixed mathematics will connect Barrow to Hobbes, but there are important differences from Aristotle for both. For example, Barrow and Hobbes include motion in geometry (Mancuso 1996, 94ff), something which for Aristotle must be kept separate from mathematics (*Phys* II.2, 193b.35). The incorporation of motion into geometry makes kinds of motion themselves the *subject* of geometry, as we find, for example, in Lecture II of Barrow's *Geometrical Lectures* (1860) and in Hobbes's discussions of motion in *De corpore* Part III, which part is entitled "Proportions of Motions and Magnitudes" (e.g., fermentation as a kind of circular motion considered in geometry is discussed below). As I will argue, Hobbesian mathematical principles also depart from the Aristotelian model, and from Barrow's model, by being grounded in maker's knowledge.⁸

2.2 Hobbes on Mixed Mathematics

In discussing scientific knowledge, Hobbes appeals to the distinction between a demonstration of the 'that' and a demonstration of the 'why':⁹

We are said to know [*scire*] some effect when we know what its causes are, in what subject they are, in what subject they introduce the effect and how they do it.

⁸ Recent work has focused on Aristotelian aspects of Hobbes's natural philosophy, in particular Leijenhorst (2002). See also Leijenhorst (1996).

⁹ Some connection has been made in the literature between Zabarella and in Hobbes, but there are significant differences between the two (see Hattab 2014; Dear 1998, 150-153).

Therefore, this is the knowledge [*scientia*] τοῦ διότι or of causes. All other knowledge [*cognitio*], which is called τοῦ ὅτι, is either sense experience or imagination remaining in sense experience or memory (*De corpore* 6.1; Hobbes 1981, 287-289).

Hobbes connects scientific knowledge with causal knowledge, distinguishing it from knowledge (*cognitio*) of the 'that'.¹⁰ For Hobbes, causal knowledge is available only to *makers* – we make figures in geometry so this accords a special epistemic status to the geometrical principles we borrow in mixed mathematical explanations (more on this below).

Hobbes's emphasis upon causal knowledge does not make the 'that' unimportant. On the contrary, Hobbes admits that knowledge 'that' in the case of both natural history and political history is "[...] very useful (no, indeed necessary) for philosophy [...]" (OL I.9; Hobbes 1981, 189). Similarly, in *De homine* 11.10 he claims that "[...] histories are particularly useful, for they supply the experiences/experiments [*experimenta*] on which the sciences of the causes rest" (OL II.100). Natural philosophers must know the 'that' from natural or political history, or from sense experience.

Hobbes's geometry that provides the 'why' in natural philosophy is a physical geometry. For Hobbes, definitions of simple geometrical figures must instruct how to make those figures (OL I.63). The definition for 'line' is one of the more intuitive¹¹ examples of Hobbesian

¹⁰ Hobbes argues in *Examinatio et Emendatio* that the term 'demonstration' should be reserved only for the 'why' (OL 4.38; for discussion, see Jesseph 1999, 204-205).

¹¹ Even if the maker's-knowledge view in Hobbes is intuitive, this does not mean it is without problems. For example, how does one differentiate 'line' from 'straight line' using only Hobbesian resources? Also, what is the source of our knowledge of "point" and how does that relate to maker's knowledge? Hobbes never attempts to resolve such difficulties associated with grounding mathematics in generative definitions. Furthermore, there is some tension between Hobbes's explicit appeal in earlier works, such as *Six Lessons* (1656) and *De homine* (1658), to *making* as what gives us scientific knowledge in geometry (discussed in the present paper) and his appeal in his later works, such as *Examinatio et Emendatio* (1660) and *De Principiis et Ratiocinatione Geometrarum* (1666), to the "natural light" in mathematical contexts (for examples of the latter, see OL IV.95, OL IV.395, OL IV.446; see also a reference in *Principia et Problemata...* [1674; OL V.213], discussed in Jesseph [1999, 187]). In these latter

geometry as maker's knowledge: "a line is made from the motion of a point" (Hobbes 1981, 297; OL I.63). One knows the nature of the line by moving a point, and the motion of that point creates a line. We make more complex geometrical figures by using the line: a surface is made by the motion of a line, and a solid is made by the motion of a surface (OL I.63). These generative definitions in geometry give efficient causes and describe the mechanical way in which a body is moved to make a new body.¹² Hobbes assumes that a maker in geometry moves directly from *knowledge of the causes* of a figure that are acquired in construction to the *properties of the figure* that was constructed (Gauthier 1997, 512).

Hobbes's contention with Euclid's principles is that Euclid "maketh not" and so his principles "ought not to be numbered among the principles of geometry" (EW VII.184; see also EW VII.202). Hobbes's criticism is that without providing a definition that specifies the mechanical procedure for constructing a figure one cannot have causal knowledge about that kind of figure. However, *De corpore* Part III moves beyond simple geometrical definitions for objects like lines and surfaces, treating topics as far-ranging as endeavor and refraction. Thus before discussing the geometrical principles that Hobbes borrows for the two explanations to be discussed below, I discuss the connection between Part III and the simple geometry of points and lines.

De corpore Part III concerns geometry because it treats *both* motion and magnitude, which for Hobbes are "the most common accidents of bodies" (OL I.75). At the end of Part III, he advises that "…we have considered motion and magnitude in themselves and in the abstract"

texts, Hobbes countenances our ability to grasp a definition by the natural light, something that runs contrary to *making* what establishes geometrical principles. My focus is on Hobbes's claims about the epistemic standing of geometry as it relates to natural philosophy, around the mid-1650s when he is working on and publishes *De corpore*, and not on the coherence of those views with his later appeals to the natural light.

¹² See Jesseph (1999, 203-204) on Hobbes's appeal to efficient causes, not formal causes, as a solution to the question of how mathematical demonstration provides causal knowledge.

(OL I.314). Part IV treats phenomena of nature and concerns the "motion and magnitude of the bodies of the world, or which themselves exist in reality" (OL I.314).

This distinction between (1) the features of bodies in the world and (2) the abstract features of bodies, such as motion and magnitude, is essential for understanding Hobbesian geometry and mixed mathematics. Hobbes distinguishes between pure and mixed mathematics, but he does so in a way that does not fall prey to Barrow's criticisms of that distinction already discussed; sciences in which we discover abstract (causal) principles are pure, and sciences in which we borrow these principles for explanations are mixed, as in *De homine* 10.5 (1658):

[...] since one cannot proceed in reasoning about natural things that are brought about by motion from the effects to the causes without a knowledge of those things that follow from that kind of *motion*; and since one cannot proceed to the consequences of motions without a knowledge of *quantity*, which is geometry; nothing can be demonstrated by physics without something also being demonstrated *a priori*. Therefore physics (I mean true physics) [*vera physica*], that depends on geometry, is usually numbered among the mixed mathematics [*mathematicas mixtas*]. [...] Therefore those mathematics are pure which (like geometry and arithmetic) revolve around quantities in the abstract [*in abstracto*] so that work [in them] requires no knowledge of the subject; those mathematics are mixed, in truth, which in their reasoning some quality of the subject is also considered, as is the case with astronomy, music, physics, and the parts of physics that can vary on account of the variety of species and the parts of the universe (Hobbes 1994, 42; OL II.93).¹³

¹³ I have modified Gert's (Hobbes 1994) translation.

In the section immediately preceding this quotation, Hobbes connects maker's knowledge and geometry. He claims that we possess maker's knowledge in geometry because "we ourselves draw the lines." However, since "the causes of natural things are not in our power" we can demonstrate only what their causes may be.

Hobbes provides similar reasons for why geometry is demonstrable two years earlier in *Six Lessons* (1656):

Geometry therefore is demonstrable for the lines and figures from which we reason are drawn and described by ourselves and civil philosophy is demonstrable because we make the commonwealth ourselves. But because of natural bodies we know not the construction but seek it from the effects there lies no demonstration of what the causes be we seek for but only of what they may be (EW VII.184).

So in these two texts from the 1650s, Hobbes holds that making something gives one direct access to the causes of the thing made. Only God has access to the causes of natural things, but humans have access to the causes of things that they make – geometrical figures and commonwealths.

The connection between geometry and maker's knowledge influences how we should understand Hobbes's claims about "true physics" in the extended quotation above from *De homine* 10.5. To reason from the effects to causes in natural philosophy, one must know already what the causes may be. If we understand *a priori* to be "from the causes" for Hobbes, then we are able demonstrate "from the causes" when prior to a natural-philosophical investigation we already possess geometrical causal principles.¹⁴

¹⁴ My focus is upon the relationship between natural philosophy and geometry, but Hobbes also uses principles from 'first philosophy' in natural philosophy. Jesseph (2006, 138-139) sees two parts to natural philosophy: first, *a priori* principles of 'first philosophy' (what Jesseph calls the "persistence principle" from *De corpore* 8.19 and the "action by contact" principle of *De corpore* 9.17); and second, the use of these principles within natural philosophy by hypothesis (e.g., OL I.339; OL I.354; OL I.417). Hobbes uses other principles from first philosophy, in addition to these two, e.g., a principle related to the division of bodies and places used in *De corpore* 25.6 (OL I.321) and another principle related to the necessity of an effect following from a necessary cause used in *De corpore* 25.13 when explaining deliberation (OL I.333). Thus a complete account of Hobbes's claim that "something also being demonstrated *a priori*" will include both first philosophy and geometry (discussed more below).

Even though 'body' is the fundamental object of mathematics, we find in this quotation from *De homine* 10.5 that Hobbes nevertheless divides mathematics into pure and mixed (cf. fn. 7). Pure mathematics treats quantities in the abstract, but natural philosophy also considers the qualities that "vary on account of the variety of species and the parts of the universe" (Hobbes 1994, 42; OL II.93). So for Hobbes "true physics" "depends on geometry" and, since it also must consider qualities unique to certain kinds of bodies, it is mixed mathematics – physical suppositions or experience are mixed with geometrical principles.

Hobbes similarly distinguishes between pure and mixed mathematics in *Anti-White*. He claims that only arithmetic and geometry are presently mathematical because no one has written anything in morals or physics that is not "open to question." If previous writers had not been so ignorant, morals and physics would also be mathematical (see epigraph above; Hobbes 1976, 24; MS 6566A, f. 5 *verso*). Hobbes argues that, in addition to geometry and arithmetic, mixed mathematics should also "be counted among mathematical" sciences (MS 6566A, f. 6). Among mixed mathematics, Hobbes includes astronomy, mechanics, optics, and music and leaves the door open for "others yet untouched" (MS 6566A, f. 6). Hobbes's idea that mixed mathematics must be broadened in this way, even to disciplines not yet existing, resonates with Barrow's view that even disciplines such as medicine, politics, and zoology should be seen as dependent upon geometry and thus as mixed mathematics (Barrow 1734, 21-22).

In *Anti-White*, Hobbes argues that mixed mathematics should be counted as part of mathematics since they consider "quantity and number, not [merely] abstractly [*non abstracte*], but with regard to the motion of the stars, or the motion of heavy [bodies], or with regard to the action of shining [bodies], and of those which produce sound [...]" (MS 6566A, f. 6). So in

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addition to considering quantity and number, which pure mathematics does, mixed mathematics treats the unique qualities bodies possess.

Hobbes's claim that in pure mathematics we treat a body in the abstract differs from seeing abstraction as grounding mathematics. Hobbes, like Aristotle, is licensed in borrowing the 'why' from geometry because one treats a natural object like a mathematical object; he uses the term "considers" to refer to this activity. For example, Hobbes's difficulty with Euclid's definition of 'point' as "a breadthless length" is that "there is no such thing as a broad length" (EW VII.202).¹⁵ Instead, Hobbes argues that a line is "a body whose length is *considered* without its breadth" (EW VII.202; emphasis added). Similarly, we "consider" bodies as points, like when we call the earth a point when discussing its annual revolution (OL I.98-99). Thus discussions about lines in geometry refer to bodies considered without breadth as mathematical objects, though as bodies they actually do have breadth. This way of characterizing Hobbesian mathematical objects has affinity with Lennox's understanding of Aristotelian abstraction as "taking a delimited cognitive stance toward an object" (Lennox 1986, 37). In other words, for Aristotel one considers an object in a certain way, and the same can be said for Hobbes.

However, whereas Aristotle arrives at mathematical principles by abstracting away physical features, Hobbes arrives at mathematical principles by an analysis of complex conceptions received in experience down to the "simplest conceptions" (OL I.62) and then by synthetically constructing geometrical figures and providing definitions for them using those simplest conceptions. For example, we are able to formulate the definition of 'line' when we understand the simplest conceptions of 'motion' and 'point' and use those conceptions in our definition (Adams 2014b). Another example is the complex conception 'square', which we

¹⁵ Hobbes's criticisms of Euclid's account of mathematical definitions apply to Wallis as well (OL IV.41-42).

analyze into the following conceptions: "plane, bounded by a certain number of lines equal to one another, and right angles" (Hobbes 1981, 293, OL I.61). We continue to analyze these conceptions until we terminate in our simplest conceptions. Then in synthesis we put 'square' back together, making first lines from the motions of points and making a surface from the motion of a line, so that when we have ended the synthesis, which was begun from the termination points of the analysis, we know the "cause of the square" (Hobbes 1981, 293, OL I.61).¹⁶ Since simplest conceptions like 'motion' are "manifest *per se*," the definition of 'motion' requires no demonstration; the definition of 'motion' will simply be an "explication" of what is contained in the simplest conception 'motion' (OL I.62). Since simplest conceptions and their definitions are manifest *per se* they are vouchsafed. However, the definitions we create with simplest conceptions, such as the definition for 'line', are vouchsafed because we make the figures signified by the terms of our definitions and thus gain maker's knowledge.

3. De corpore Part IV as Mixed-Mathematical "True Physics"

3.1 Explaining Sense in De corpore 25.1-2

De corpore Part IV follows the second path of philosophy, beginning from "known effects or phenomena," but since the actual causes of natural things are unavailable to us Part IV shows how "they can be generated" (OL I. 315-316). These comments reflect different levels of certainty for geometry and natural philosophy. Maker's knowledge found in geometrical construction is the paradigm of scientific knowledge for Hobbes because when we make a figure by drawing it we know its actual causes. Natural philosophy lies somewhere between (1) the certainty of geometry and (2) the limited prudence characterizing those who rely solely on

¹⁶ Adams (2014a, 416-417) discusses this example in detail.

memory and associations. Borrowing maker's knowledge from geometry transfers some of the certainty had there to a natural-philosophical explanation, but only *suppositional certainty*: we know that if such a motion *were* present in actual bodies then certain effects would necessarily follow from that motion.¹⁷

Explaining sensation is necessary because Hobbesian natural philosophy starts from the appearances of nature. So Hobbes first discusses "appearing itself," which he calls the "most admirable" of all appearances (OL I.316). Since appearances are the starting points by which all other things are known, he argues that sense is the principle by which all other principles are known: "all knowledge may be said to be derived from [sense]" (OL I.316). Succinctly, we know only through phantasms, but the only way that we become aware of and inspect phantasms is by sense. Thus, an inquiry into phantasms must begin with sense.

Sense is a stopping point against any potential regress which might require a faculty beyond it to be supposed, such as a faculty of the intellect. We are aware of sense not by some other faculty, but by *sense itself* since anyone who has sensed remembers that he has sensed: "For sensing oneself [as] having sensed is to remember" (OL I.317). Hobbes explains sensation in *De corpore* 25 with three separate definitions of sense in a series of refinements, with each refinement due to borrowing a causal principle from either geometry or first philosophy.¹⁸ As will be discussed below, the first definition understands sense as "nothing other than motion of

¹⁷ Jesseph (2006, 139) makes a similar point regarding applying principles from first philosophy to natural philosophy.

¹⁸ The relationship between geometry and natural philosophy is my focus, but in the explanation of sensation that I discuss below Hobbes borrows principles from geometry and first philosophy; the second refinement of the definition borrows the principle of action by contact from *De corpore* 9.7. Jesseph (2006) has treated the relationship between Hobbes's first philosophy and natural philosophy. As already discussed, Hobbes understands geometry to be maker's knowledge – it has special epistemic standing because we know actual causes. However, we do not have maker's knowledge of the principles of first philosophy; they are conceptual truths that are true by stipulation (see Jesseph 2006, 130). This distinction agrees with how Hobbes differentiates between first philosophy as consisting in "those things which are closest to the most universal definitions" and geometry as "those things which can be demonstrated by simple motion" (OL I.77).

some of the parts inside the sentient, which moved parts are parts of the organs by which we sense" (OL I.317). I will show that this definition mixes an appeal to experience – something that can be "observed" – with a borrowed principle related to mutation and motion from first philosophy. Two further refinements to this initial definition are developed by borrowing additional principles, one from first philosophy and the other from geometry. I have summarized the steps Hobbes takes to reach the final definition as follows:

Hobbes's Explanation of Sense in De corpore 25

- 1. All sense is mutation of the sentient body (known from experience; the 'that').
- 2. All mutation is motion or endeavour, and endeavour is also motion (borrowed principle from first philosophy; *De corpore* 9.9).
- 3. Therefore, all sense is motion in the sentient body. (from 1 and 2)
- 4. The motion of any body A occurs only by means of some other body B which is contiguous to A and presses upon A (borrowed principle from first philosophy; *De corpore* 9.7).
- 5. Thus, all sense occurs by contiguous bodies pressing upon the sentient body, i.e., the organs of sense, which motion continues in the sentient body (from 3 and 4).

*Remaining explanandum: why do we perceive bodies as outside of us if sense can be explained solely in terms of contiguous bodies pressing against one another?

- 6. All resistance is endeavour contrary to another endeavor, i.e., reaction (borrowed principle from geometry, *De corpore* 15.2; the 'why').
- 7. Supposition: the body of the sentient has an internal endeavour outward which resists inward motion from the objects of sense.
- 8. Thus, (if the sentient body behaves according to supposition in 7) the resistance of the internal parts of the sentient's body against the inward motion from the object causes our perception of bodies as outside of us.

Hobbes appeals to everyday experience - the fact that the appearances of things

continually change - to demonstrate the 'that':

... it is proper in the first place to observe that our phantasms are not always the

same, but new ones are constantly being created and old ones are disappearing,

just as the organs of sense are turned now to one, now to another object.

Therefore, they are produced and pass away, from which it is understood that they are some mutation of the sentient body (OL I.317).

Observing the phantasms that arise in experience, it is evident that with each new body we encounter a new phantasm arises and disappears once we turn elsewhere. Since phantasms are continually produced and pass away, there must be mutation in the sentient body.

Hobbes next considers what follows from knowing that phantasms are mutation. He borrows a principle from first philosophy (see fn. 18):

But that all mutation is something having been moved or endeavoured, (which endeavor [*conatus*] is also motion) in the internal parts of the thing changed has been shown (cap. 9., art. 9.) from this: that while the smallest parts of some body stay the same having been mutually positioned, nothing new happens to those parts, (unless perhaps it may be possible that every part may be moved at the same time), except that it both be and appear to be the same, which at first it was and appeared to be (OL I.317).¹⁹

Adding this borrowed principle from *De corpore* 9.9 to the earlier demonstration of the 'that', Hobbes formulates the first definition: "...sensation in the sentient can be nothing other than motion of some of the parts inside the sentient, which moved parts are parts of the organs by which we sense" (OL I.317). The principle that "all mutation is something having been moved or endeavoured" implies that motion must be responsible for the mutation involved in sensation. Hobbes notes with this first definition that we have the "subject of sense," which is the organs of

¹⁹ The Molesworth edition and the 1655 edition (cf. Hobbes 1655, 224) incorrectly record this citation as to *De corpore* 8.9. Schumann (Hobbes 1999, 268) corrects the citation to *De corpore* 9.9.

sense in which phantasms are created. We have also discovered "part of its nature": it is "some internal motion in the sentient" (OL I.317-318).

Hobbes borrows a second principle from *De corpore* 9 in the first refinement of the definition of sensation in *De corpore* 25. He wants to show that the motion that causes sensation in the sentient must originate from the internal motions of the parts of the object of sense and be carried to the subject of sense. He does this by borrowing from first philosophy again: "...it has been shown (cap. 9., art 7.) that motion cannot be generated except by [a body] moved and contiguous. From which the immediate cause of sensation is understood to be in this, that it both touches and presses the first organ of sense" (OL I.318). He formulates the second definition: "...sense is some internal motion in the sentient, generated by some motion of the internal parts of the object, and propagated through media to the inmost parts of the organ" (OL I.318).

Thus far we have discussed steps 1-5 of Hobbes's definition. As already mentioned, principles in first philosophy have their justification in being conceptual truths (fn. 18). As conceptual truths, Hobbes's claims in step 3 and step 5 are merely the application of the definition of 'mutation' and the principle of action by contact to what has been demonstrated from experience, i.e., that sensation is a mutation of the sentient body. With the last refinement, however, we find Hobbes doing something different. He borrows a geometrical principle related to 'resistance' and must *suppose* that sentient bodies behave according to that causal principle.

Hobbes has "almost defined what sense may be" (OL I.318). It remains to explain why we perceive objects of sensation as outside of us rather than inside of us. Given his account of phantasms as caused by *internal* motion in the sentient, he has not yet explained why we do not perceive those objects as *inside of us*. Since Hobbes explains sensation by appeal only to the cause of motion, this problem is present for Hobbes in a way that it is not for others. Consider

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the case of vision, though this problem applies to all of the senses equally for Hobbes. Hobbes's explanation of vision employs neither an image on the back of the retina nor the natural triangulation similar to "surveyors" (*mensorium*) like in Kepler's account in *Ad Vitellionem Paralipomena* (chapter III, Propositio IX; 1604, 63). Furthermore, Hobbes's explanation does not assume that our ability to know an object's location occurs "as if by natural geometry" like Descartes in *Dioptrique* (1637/2001, 104). Instead, Hobbes endeavors to explain all aspects of vision by motion alone. The motion from the object of sense that is transmitted by a medium and the motion from the reaction of a sentient's body are the only *explanantia* that are mechanically intelligible for Hobbes.

To be consistent with this constraint, Hobbes posits that we perceive phantasms as caused by outside bodies because of outward motion from the resistance of our body against the inward motion. Hobbes borrows a causal principle for 'resistance' from *De corpore* 15.2 (Part III): "Likewise, it has been shown (cap. 15., art. 2.) that all resistance is the endeavour contrary to [another] endeavor, that is, reaction" (OL I.318). In sensation, this reaction occurs because "the natural internal motion of the organ itself" (OL I.318) resists the motion from the object of sensation. Since the endeavour moves outward due to this resistance, the phantasm "always appears as something situated outside of the organ" of sense (OL I.318). Rather than being a conceptual truth from first philosophy, like the earlier borrowed principles, this principle of resistance from Part III is geometrical because it, like the concept of 'endeavour' (introduced in *De corpore* 15.2 and used in the definition of 'resistance'), is a kind of simple motion which can be treated according to proportions. For example, it is intelligible to compare the proportion of the velocities of two endeavours on Hobbes's account (OL I.178). This borrowed causal principle allows Hobbes to formulate the final definition of sensation: "[...] a phantasm made by means of a reaction from an endeavour to [the] outside, which is generated by an internal endeavour from the object, and there remains for some time" (OL I.319). Since we are explaining the behavior of natural bodies, we cannot know with certainty that the body of the sentient *actually* reacts against the inward motion in this way. However, we can use the borrowed principle to form a supposition: *if* the body of the sentient resists the inward motion with an outward-directed endeavour, *then* the motion would continue until it left the body. Hobbes describes the final definition of sense as "from the explication of its causes and its order of generation" (OL I.318-319). My goal has been to show that this explication of the causes within natural philosophy is possible only by borrowing a causal principle from geometry.

3.2 Explaining the Swelling of the Parts of the Body when Warm in De corpore 27.3

Prior to explaining light, heat, and color in *De corpore* 27, Hobbes introduces several suppositions. First, he supposes that no matter how small some bodies may be we will "suppose" only that their size is not smaller than what the phenomena themselves require (OL I.364). Second, regarding the motion of the bodies under consideration, he supposes only what is needed for the "explication of [their] natural causes" (OL I.364). Finally, he supposes that "in the parts of pure ether" there is no motion except what is transferred "by the bodies floating in it" and that these parts of the ether are not liquid (OL I.364).

To explain the cause of the light (lux) of the sun, Hobbes introduces an additional supposition: that the sun "by its simple circular motion" moves the parts of the ether that are near

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it (OL I.364).²⁰ Circular motion propagates through the medium, reaching the organ of sense and the heart of the perceiving human. Referring back to his explanation of sensation in *De corpore* 25, Hobbes states that the endeavour outward is "called light [*lumen*] or the phantasm of a lucid [*lucidi*] [body]" (OL I.365). These considerations provide the possible cause of the light of the sun (*lucis solaris*; OL I.365).

Thus far, Hobbes seems to have asserted mysteriously that the endeavour moving outward from a perceiver's body is the cause of the light (*lux*) of the sun. Hobbes is, of course, using the vocabulary relevant to the distinction between *lux* and *lumen* when he provides this possible cause of the *lux* of the sun.²¹ However, Hobbes uses this vocabulary to differentiate between two motions that he posits as causing the perception of light: (1) the motion from the luminous body and (2) the resistance against that motion by the sentient body. The *lux* of the sun is the simple circular motion propagated through media, and the *lumen* is created because of the outward reaction of perceivers' bodies to the *lux*.

Hobbes's account in *Elements of Law* helps clarify this point. Instead of the simple circular motion that we find in *De corpore*, in *Elements of Law* Hobbes posits that the motion produced by fire and other lucid bodies is "dilation, and contraction of it self alternately, commonly called scintillation or glowing" (Hobbes 1650, 13). Apart from this difference in the type of motion between *Elements of Law* and *De corpore*, though, his account of the perception of light is largely the same:²²

²⁰ Galileo's *Dialogo* may be a source of Hobbes's use of simple circular motion (Brandt 1927, 330ff; Mintz 1952). Simple circular motion is also a topic Barrow treats in his *Geometrical Lectures* (see Lecture II; 1670).

²¹ Lindberg (1978, 356) discusses the entrenchment of the *lux/lumen* distinction through the Latin translation of Avicenna's *De Anima*. For Avicenna *lux* referred to the light from luminous bodies, such as the sun, and *lumen* referred to the effect of *lux* upon the medium and the non-luminous bodies which it lit.

²² The terms *lux* and *lumen* are not present, of course, since *Elements of Law* was composed in English.

Now the interiour coat of the Eye is nothing else but a piece of the *Optick* nerve; and therefore the motion [from the lucid body] is still continued thereby into the *Brain*, and by *resistance* or reaction of the Brain, is also a *rebound* into the Optick nerve again; which we *not conceiving* as motion or rebound from within, do think it *without*, and call it *light* [...] (Hobbes 1650, 14).

Thus, *lux* from the sun is nothing other than simple circular motion coming from the lucid body (or *scintillation* in *Elements of Law*). This motion propagates through a medium, continues to the eye, and then rebounds outward after meeting resistance, creating *lumen*, which causes the phantasm of light.

Following this explanation of the possible cause of the *lux* of the sun, Hobbes focuses on the felt heat that accompanies the light of the sun (OL I.365). This explanation occurs in three steps, two of which are appeals to experience. The first appeal to experience is used to differentiate the Hobbes's intended explanandum from another, and the second is the demonstration of the 'that'. In the third step, he borrows a principle related to a type of circular motion he calls 'fermentation' from *De corpore* 21.5. Like Barrow does in his *Geometrical Lectures* (see fn. 20), Hobbes treats simple circular motion as a part of geometry. Hobbes understands fermentation as type of circular motion wherein bodies perpetually change place, which is why he establishes the principles related to fermentation in the section of *De corpore* concerned with geometry (Part III).

Hobbes's first appeal to experience shows what may be inferred about lucid bodies from the heat they cause in us. We know by experience what it is to perceive heat in ourselves when we grow warm, but we know what it is "in other things by ratiocination" (OL I.365). He distinguishes between (1) the sensation of heat and (2) what we can know about the things that

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produce heat: "we recognize fire or the sun making warm, but we do not recognize that it may be hot" (OL I.365). Although in the case of heat that is caused in us by other creatures we know that those creatures are themselves hot, like heat caused by a dog laying on one's lap, Hobbes argues that we cannot make the same inference from the heat caused in us by the sun to the properties of the sun itself. The inference Hobbes is opposing would claim something like the following: anytime a creature causes heat in another body, that creature itself is warm; thus, when a body like the sun causes warmth, it must be warm. Hobbes thinks that we can no more assert this than we can say that "fire causes pain, therefore [fire] is in pain" (OL I.365).

Hobbes makes a second appeal to the everyday experience of being warm to demonstrate the 'that':

...when we are growing hot, we learn that the spirits, blood, and whatever is fluid

in our bodies is called forth from the interior parts to exterior as the degree of heat

is more or less, and the skin swells up (OL I.365).

Hobbes focuses upon a feature of the experience of being warm – the pores sweat and the skin swells. This appeal to experience demonstrates that this sweating and swelling occurs when a body is heated.

Next Hobbes provides the 'why' for the skin's swelling and the possible "cause of the heat of the sun" (OL I.365). Hobbes's stated explanandum is the cause of the heat of the sun, but given his reservations on the inference that can be made by 'heat of the sun' the explanandum is the cause of the *sensation of heat* from the light of the sun. To provide this cause, Hobbes borrows a principle related to fermentation from *De corpore* 21.5.²³

 $^{^{23}}$ Hobbes claims that in chapter 21 he has explained how the air is moved by the "simple circular motion of the sun." However, as Schumann notes (Hobbes 1999, 304 fn. 2), this likely refers to an earlier version of *De corpore* since there is no discussion of the simple circular motion of the sun in 21.5. Although this claim is absent from extant versions of *De corpore* 21.5, for my purposes it is sufficient that Hobbes does introduce the concept of

Hobbes first supposes that the sun's simple circular motion moves the air around it so that the parts of the air "perpetually change their places with one another" (OL I.366). This motion is propagated from the sun to the air that surrounds humans. Hobbes identifies this perpetual change of place with the process of fermentation, drawing attention to an earlier demonstration in *De corpore* where he explains how water is drawn up into the clouds by the same cause of the circular motion of fermentation.²⁴ Like water that forms clouds when drawn from the ocean, Hobbes explains how "from our bodies the fluid parts from the insides to the outsides may be drawn out by the same fermentation" (OL I.366).

Hobbes is drawing on common knowledge that fermentation causes heat, but he is describing this common notion in terms of a particular kind of simple motion. Fermentation is type of circular motion that involves the perpetual change of place by the parts of air that results in the joining together of homogeneous parts and the production of heat (*De corpore* 21.5, OL I.263-265). His interest in this type of motion is to show how as a type of simple circular motion, we can use fermentation to explain how air draws water into the clouds and human sweating.

fermentation in that article, and that fermentation provides the causal principle for the explanation of the possible cause of the heat of the sun. Furthermore, Hobbes's supposition that the sun moves the air around it by simple circular motion occurs throughout De corpore (OL I.351; OL I.358; OL I.364; OL I.367-368; OL I.381). ²⁴ There is a discrepancy between extant versions of *De corpore* related to this reference to cloud formation and fermentation. Both Latin editions of De corpore (1655; 1668) and also the Molesworth English Works and English edition of 1656 have this as a reference to a demonstration contained in *De corpore* 26.8, but such a demonstration is absent from 26.8. However, this reference is missing from the Molesworth Latin Works edition, and Schumann follows OL, claiming that this demonstration Hobbes cites might have been in an earlier version of *De corpore* 21.11 (cf. Hobbes 1999, 305). Schumann may have found evidence for this claim regarding *De corpore* 21.11 (though he does not state this) because of a later reference in *De corpore* 28.14 to the formation of clouds. There the 1655 edition (cf. Hobbes 1655, 276) and the Molesworth Latin Works edition (OL I.391) cite an explanation of the formation of clouds that is also supposed to be in De corpore 21; Schumann (Hobbes 1999, 323) explains the fact that such an explanation is missing in De corpore 21 by supposing that it might have been present in an earlier version of chapter 21. On this citation to chapter 21, the English Works edition (EW I.480) follows the English edition of 1656 (Hobbes 1656, 357), recording this citation as being to De corpore 26 instead. In positing an earlier version of De corpore 21 as the likely location of this explanation, Schumann neglects the possibility that this explanation of the formation of clouds due to fermentation was moved to *De corpore* 28.2, where Hobbes does discuss the formation of clouds (EW I.468-469; OL I.381). Perhaps Hobbes had once included this explanation in De corpore 21 (Part III) as an example of how the cause of fermentation as a type of circular motion could be used as the 'why' in natural philosophy, but later he moved so that it was within the section on natural philosophy (Part IV).

This is a peculiar usage of fermentation for sure, but Hobbes is taking the abstract geometrical concept of fermentation as this type of motion and using it as a cause in explanations of phenomena as diverse as sweating in human bodies and cloud formation over the ocean, both of which are mixed explanations that take into account the "qualities of the subject," as described in *De homine* 10.5. The account of fermentation in *De corpore* 21.5 is geometrical not merely because it is within Part III, but more importantly because it describes a type of simple circular motion irrespective of the unique qualities of particular bodies like human bodies or rain clouds. As a type of simple motion, Hobbes, like Barrow, holds it that it is the work of geometry to explicate it.²⁵

When parts of air that are contiguous to the body of an animal ferment by perpetually changing places with one another, "the parts of the animal contiguous to the medium may endeavour to enter into the spaces of the divided parts" (OL I.366). Therefore, the "most fluid and separable" parts of the animal go out first, and their place is filled by other parts which are able to transpire through the pores of the skin (OL I.366).

What happens to the non-fluid parts of animal bodies that are not able to be separated in this way? Although these are not separated, it is "necessary that thus the whole mass be moved" into the place left by those fluid parts that are being drawn outside of the body "so that all places may be filled" (OL I.366). When the non-fluid parts of the body endeavour this way, the body swells: "…the mass of the body, all striving at the same time in that way, swells" (OL I.366). Hobbes now claims that we have a "possible cause" of the heat from the sun. Thus, when we

 $^{^{25}}$ A complete discussion of Hobbes's view that 'simple circular motion' is part of geometry is beyond the scope of the present paper. Hobbes begins his discussion of simple circular motion, of which fermentation is a type, in *De corpore* 21. There, from the construction of a figure, he begins with a demonstration that any body that moves by simple circular motion "carries" straight lines with it such that each straight line so carried is always parallel to itself (OL I.258-60).

become warm and begin to sweat it is not because the air around us is hot. Rather, this occurs because the parts of the air around us are continually changing place (fermenting), causing the liquid part of our bodies to leave and the other parts to swell.

This may seem like a strange explanation; for it might appear that these fluids simply exit the body because *qua* fluids they do so more easily than other parts of the body. However, the reason why these fluids exit the body is found in the account of fermentation in *De corpore* 21.5 (OL I.263-265). These fluids exit the body because they are being separated from the non-fluidic parts of the body and, through fermentation, are being joined with other fluids. A consequence of fermentation's seething is that homogeneous fluid bodies are united. The fermentation process, whether in the case of human sweat, cloud formation, or, as Hobbes mentions, young wine, need not be caused by fire (OL I.264). Such heat is produced because of the circular motion involved in the perpetual change of place.

Hobbes uses "possible" to signal that the sun may not *actually* cause felt heat by means of simple circular motion. We find the same level of certainty in the current explanation as we did with the explanation of sense: we know that *if* the sun moves the air around it by simple circular motion, causing fermentation, then the fermentation motion continues to the human body and causes felt heat in the sentient body when the fluid parts leave and the non-fluid parts swell to prevent vacant spaces. Thus, the supposition that the sun moves the ether around it by simple circular motion makes this a possible cause.²⁶

²⁶ Elsewhere, Hobbes identifies the simple motion of the sun as a "supposition" (OL I.351; cf. Horstmann 2001, 494-495).

3.3 De corpore Part IV Reconsidered

These two natural-philosophical explanations from *De corpore* Part IV display Hobbes's use of experience and borrowed maker's knowledge from geometry. When borrowing a geometrical principle in natural philosophy, we must *suppose* that bodies move according to that principle. This borrowing from geometry would be difficult to explain on the deductivist interpretation of the relation between Hobbes's geometry and natural philosophy, for in neither explanation do we find a deduction nor the suggestion that one could be provided from geometry to these explanations. Instead, I have argued that the borrowing in these explanations, and elsewhere in *De corpore* Part IV (see fn. 5), should be understood as mixed mathematical explanations, making them part of the "true physics" described in *De homine* 10.5.

We find additional support for the view that "true physics" is mixed-mathematics in Hobbes's comments about failed or limited explanations. In some explanations, Hobbes thinks that since the 'that' are insufficiently known it is useless to search for the 'why'. For example, even though Kepler posits a cause for the eccentricity of the earth's orbit, Hobbes holds that the 'that' is insufficiently known and that a search for causes would be in vain: "But since the *hoti* is not yet evident, it is in vain for the *dioti* to be searched for" (Hobbes 1655, 254).²⁷ Such a demonstration of the 'that' for Hobbes must precede any search for causes, and without it the search for causes may not begin.

²⁷ This is present in the first edition of *De corpore* (1655) and transmitted to the English edition of *Concerning Body* (1656, 329) and to the *English Works* edition (EW I.443). However, the *Latin Works* edition (OL I.361) does not contain this, following the 2nd Latin edition of *De corpore* (1668); Schumann follows the OL (Hobbes 1999, 301).

4. Conclusion

I have argued that Hobbes's use of geometry within the natural-philosophical explanations of *De corpore* Part IV is best understood as the borrowing of causal principles. Hobbes saw this borrowing as legitimate because natural philosophy is mixed mathematics, which is "true physics." As such, my goal has been to place Hobbesian natural philosophy within the trajectory of Aristotelian mixed mathematics and show its affinity to Barrow's broadened purview for mixed mathematics.

Since geometrical principles are grounded in maker's knowledge, they transfer some measure of certainty to natural-philosophical explanations. Nevertheless, natural-philosophical explanations do not carry the complete certainty ascribed to maker's knowledge, insofar as Hobbes holds that we cannot know whether the causes supposed are "actually the causes of things" (OL I.531). Since we have maker's knowledge of these causal principles, we can be certain that *if* natural bodies behaved in the manner supposed then the behavior consequent from that type of motion would result. In the end, explanations in Hobbesian natural philosophy have an epistemic standing that is between the certainty of maker's knowledge and the limited prudence characterizing those who have no causal knowledge and rely solely on memory and associations.

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