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Neutrosophic quadruple algebraic hyperstructures

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Abstract. The objective of this paper is to develop neutrosophic quadruple algebraic hyperstructures. Specifically, we develop neutrosophic quadruple semihypergroups, neutrosophic quadruple canonical hypergroups and neutrosophic quadruple hyperrings and we present elementary properties which characterize them.

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1. Introduction

The concept of neutrosophic quadruple numbers was introduced by Florentin Smarandache [18]. It was shown in [18] how arithmetic operations of addition, subtraction, multiplication and scalar multiplication could be performed on the set of neutrosophic quadruple numbers. In [1], Akinleye et.al. introduced the notion of neutrosophic quadruple algebraic structures. Neutrosophic quadruple rings were studied and their basic properties were presented. In the present paper, two hyperoperations $\hat{+}$ and $\hat{\times}$ are defined on the neutrosophic set NQ of quadruple numbers to develop new algebraic hyperstructures which we call neutrosophic quadruple algebraic hyperstructures. Specifically, it is shown that $(NQ, \hat{\times})$ is a neutrosophic quadruple semihypergroup, $(NQ, \hat{+})$ is a neutrosophic quadruple canonical hypergroup and $(NQ, \hat{+}, \hat{\times})$ is a neutrosophic quadruple hyperrring and their basic properties are presented.

Definition 1.1 ([18]). A neutrosophic quadruple number is a number of the form (a, bT, cI, dF) where T, I, F have their usual neutrosophic logic meanings and $a, b, c, d \in \mathbb{R}$ or $\mathbb{C}$. The set NQ defined by

$$NQ = \{(a, bT, cI, dF) : a, b, c, d \in \mathbb{R} \text{ or } \mathbb{C}\} \tag{1.1}$$

is called a neutrosophic set of quadruple numbers. For a neutrosophic quadruple number (a, bT, cI, dF) representing any entity which may be a number, an idea, an object, etc, a is called the known part and (bT, cI, dF) is called the unknown part.

Definition 1.2. Let $a = (a_1, a_2T, a_3I, a_4F), b = (b_1, b_2T, b_3I, b_4F) \in NQ$. We define the following:

$$a + b = (a_1 + b_1, (a_2 + b_2)T, (a_3 + b_3)I, (a_4 + b_4)F), \tag{1.2}$$

$$a - b = (a_1 - b_1, (a_2 - b_2)T, (a_3 - b_3)I, (a_4 - b_4)F). \tag{1.3}$$

Definition 1.3. Let $a = (a_1, a_2T, a_3I, a_4F) \in NQ$ and let α be any scalar which may be real or complex, the scalar product $\alpha.a$ is defined by

$$\alpha.a = \alpha.(a_1, a_2T, a_3I, a_4F) = (\alpha a_1, \alpha a_2T, \alpha a_3I, \alpha a_4F). \tag{1.4}$$

If $\alpha = 0$, then we have $0.a = (0, 0, 0, 0)$ and for any non-zero scalars m and n and $b = (b_1, b_2T, b_3I, b_4F)$, we have:

$$\begin{aligned} (m+n)a &= ma + na, \\ m(a+b) &= ma + mb, \\ mn(a) &= m(na), \\ -a &= (-a_1, -a_2T, -a_3I, -a_4F). \end{aligned}$$

Definition 1.4 ([18])**.** [Absorbance Law] Let X be a set endowed with a total order $x < y$, named " x prevailed by y" or "x less stronger than y" or "x less preferred than y". $x \leq y$ is considered as "x prevailed by or equal to y" or "x less stronger than or equal to y" or "x less preferred than or equal to y".

For any elements $x, y \in X$, with $x \leq y$, absorbance law is defined as

$$x.y = y.x = \text{absorb}(x, y) = \max\{x, y\} = y \tag{1.5}$$

which means that the bigger element absorbs the smaller element (the big fish eats the small fish). It is clear from (1.5) that

$$x.x = x^2 = \text{absorb}(x, x) = \max\{x, x\} = x \quad \text{and} \tag{1.6}$$

$$x_1.x_2 \cdots x_n = \max\{x_1, x_2, \cdots, x_n\}. \tag{1.7}$$

Analogously, if $x > y$, we say that "x prevails to y" or "x is stronger than y" or "x is preferred to y". Also, if $x \geq y$, we say that "x prevails or is equal to y" or "x is stronger than or equal to y" or "x is preferred or equal to y".

Definition 1.5. Consider the set $\{T, I, F\}$. Suppose in an optimistic way we consider the prevalence order $T > I > F$. Then we have:

$$TI = IT = \max\{T, I\} = T, \tag{1.8}$$

$$TF = FT = \max\{T, F\} = T, \tag{1.9}$$

$$IF = FI = \max\{I, F\} = I, \tag{1.10}$$

$$TT = T^2 = T, \tag{1.11}$$

$$II = I^2 = I, \tag{1.12}$$

$$FF = F^2 = F. \tag{1.13}$$

Analogously, suppose in a pessimistic way we consider the prevalence order $T < I < F$. Then we have:

$$TI = IT = \max\{T, I\} = I, \tag{1.14}$$
$$TF = FT = \max\{T, F\} = F, \tag{1.15}$$
$$IF = FI = \max\{I, F\} = F, \tag{1.16}$$
$$TT = T^2 = T, \tag{1.17}$$
$$II = I^2 = I, \tag{1.18}$$
$$FF = F^2 = F. \tag{1.19}$$

Except otherwise stated, we will consider only the prevalence order $T < I < F$ in this paper.

Definition 1.6. Let $a = (a_1, a_2T, a_3I, a_4F), b = (b_1, b_2T, b_3I, b_4F) \in NQ$. Then

$$\begin{aligned} a.b &= (a_1, a_2T, a_3I, a_4F).(b_1, b_2T, b_3I, b_4F) \\ &= (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2)T, (a_1b_3 + a_2b_3 + a_3b_1 + a_3b_2 + a_3b_3)I, \\ &\quad (a_1b_4 + a_2b_4, a_3b_4 + a_4b_1 + a_4b_2 + a_4b_3 + a_4b_4)F). \end{aligned} \tag{1.20}$$

Theorem 1.7 ([1])**.** *$(NQ, +)$ is an abelian group.*

Theorem 1.8 ([1])**.** *$(NQ, .)$ is a commutative monoid.*

Theorem 1.9 ([1])**.** *$(NQ, .)$ is not a group.*

Theorem 1.10 ([1])**.** *$(NQ, +, .)$ is a commutative ring.*

Definition 1.11. Let NQR be a neutrosophic quadruple ring and let NQS be a nonempty subset of NQR. Then NQS is called a neutrosophic quadruple subring of NQR, if $(NQS, +, .)$ is itself a neutrosophic quadruple ring. For example, $NQR(n\mathbb{Z})$ is a neutrosophic quadruple subring of $NQR(\mathbb{Z})$ for $n = 1, 2, 3, \cdots$.

Definition 1.12. Let NQJ be a nonempty subset of a neutrosophic quadruple ring NQR. NQJ is called a neutrosophic quadruple ideal of NQR, if for all $x, y \in NQJ, r \in NQR$, the following conditions hold:

(i) $x - y \in NQJ$,
(ii) $xr \in NQJ$ and $rx \in NQJ$.

Definition 1.13 ([1])**.** Let NQR and NQS be two neutrosophic quadruple rings and let $\phi : NQR \to NQS$ be a mapping defined for all $x, y \in NQR$ as follows:

(i) $\phi(x + y) = \phi(x) + \phi(y)$,
(ii) $\phi(xy) = \phi(x)\phi(y)$,
(iii) $\phi(T) = T$, $\phi(I) = I$ and $\phi(F) = F$,
(iv) $\phi(1, 0, 0, 0) = (1, 0, 0, 0)$.

Then ϕ is called a neutrosophic quadruple homomorphism. Neutrosophic quadruple monomorphism, endomorphism, isomorphism, and other morphisms can be defined in the usual way.

Definition 1.14. Let $\phi : NQR \to NQS$ be a neutrosophic quadruple ring homomorphism.

(i) The image of ϕ denoted by $Im\phi$ is defined by the set

$$Im\phi = \{y \in NQS : y = \phi(x), \text{for some } x \in NQR\}.$$

(ii) The kernel of ϕ denoted by $Ker\phi$ is defined by the set

$$Ker\phi = \{x \in NQR : \phi(x) = (0, 0, 0, 0)\}.$$

Theorem 1.15 ([1]). *Let $\phi : NQR \to NQS$ be a neutrosophic quadruple ring homomorphism. Then:*

(1) *$Im\phi$ is a neutrosophic quadruple subring of NQS,*
(2) *$Ker\phi$ is not a neutrosophic quadruple ideal of NQR.*

Theorem 1.16 ([1]). *Let $\phi : NQR(\mathbb{Z}) \to NQR(\mathbb{Z})/NQR(n\mathbb{Z})$ be a mapping defined by $\phi(x) = x + NQR(n\mathbb{Z})$ for all $x \in NQR(\mathbb{Z})$ and $n = 1, 2, 3, \ldots$. Then ϕ is not a neutrosophic quadruple ring homomorphism.*

Definition 1.17. Let H be a non-empty set and let $+$ be a hyperoperation on H. The couple $(H, +)$ is called a canonical hypergroup if the following conditions hold:
(i) $x + y = y + x$, for all $x, y \in H$,
(ii) $x + (y + z) = (x + y) + z$, for all $x, y, z \in H$,
(iii) there exists a neutral element $0 \in H$ such that $x + 0 = \{x\} = 0 + x$, for all $x \in H$,
(iv) for every $x \in H$, there exists a unique element $-x \in H$ such that $0 \in x + (-x) \cap (-x) + x$,
(v) $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$, for all $x, y, z \in H$.
A nonempty subset A of H is called a subcanonical hypergroup, if A is a canonical hypergroup under the same hyperaddition as that of H that is, for every $a, b \in A$, $a - b \in A$. If in addition $a + A - a \subseteq A$ for all $a \in H$, A is said to be normal.

Definition 1.18. A hyperring is a tripple $(R, +, .)$ satisfying the following axioms:
(i) $(R, +)$ is a canonical hypergroup,
(ii) $(R, .)$ is a semihypergroup such that $x.0 = 0.x = 0$ for all $x \in R$, that is, 0 is a bilaterally absorbing element,
(iii) for all $x, y, z \in R$,

$$x.(y + z) = x.y + x.z \text{ and } (x + y).z = x.z + y.z.$$

That is, the hyperoperation . is distributive over the hyperoperation $+$.

Definition 1.19. Let $(R, +, .)$ be a hyperring and let A be a nonempty subset of R. A is said to be a subhyperring of R if $(A, +, .)$ is itself a hyperring.

Definition 1.20. Let A be a subhyperring of a hyperring R. Then

(i) A is called a left hyperideal of R if $r.a \subseteq A$ for all $r \in R, a \in A$,
(ii) A is called a right hyperideal of R if $a.r \subseteq A$ for all $r \in R, a \in A$,
(iii) A is called a hyperideal of R if A is both left and right hyperideal of R.

Definition 1.21. Let A be a hyperideal of a hyperring R. A is said to be normal in R, if $r + A - r \subseteq A$, for all $r \in R$.

For full details about hypergroups, canonical hypergroups, hyperrings, neutrosophic canonical hypergroups and neutrosophic hyperrings, the reader should see [3, 14]

2. Development of neutrosophic quadruple canonical hypergroups and neutrosophic quadruple hyperrings

In this section, we develop two neutrosophic hyperquadruple algebraic hyperstructures namely neutrosophic quadruple canonical hypergroup and neutrosophic quadruple hyperring . In what follows, all neutrosophic quadruple numbers will be real neutrosophic quadruple numbers i.e $a, b, c, d \in \mathbb{R}$ for any neutrosophic quadruple number $(a, bT, cI, dF) \in NQ$.

Definition 2.1. Let $+$ and $.$ be hyperoperations on $\mathbb{R}$ that is $x+y \subseteq \mathbb{R}, x.y \subseteq \mathbb{R}$ for all $x, y \in \mathbb{R}$. Let $\hat{+}$ and $\hat{\times}$ be hyperoperations on NQ. For $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F) \in NQ$ with $x_i, y_i \in \mathbb{R}, i = 1, 2, 3, 4$, define:

$$\begin{aligned} x\hat{+}y \quad = \quad & \{(a, bT, cI, dF) : a \in x_1 + y_1, b \in x_2 + y_2, \\ & c \in x_3 + y_3, d \in x_4 + y_4\}, \end{aligned} \tag{2.1}$$

$$\begin{aligned} x\hat{\times}y \quad = \quad & \{(a, bT, cI, dF) : a \in x_1.y_1, b \in (x_1.y_2) \cup (x_2.y_1) \cup (x_2.y_2), c \in (x_1.y_3) \\ & \cup(x_2.y_3) \cup (x_3.y_1) \cup (x_3.y_2) \cup (x_3.y_3), d \in (x_1.y_4) \cup (x_2.y_4) \\ & \cup(x_3.y_4) \cup (x_4.y_1) \cup (x_4.y_2) \cup (x_4.y_3) \cup (x_4.y_4)\}. \end{aligned} \tag{2.2}$$

Theorem 2.2. *$(NQ, \hat{+})$ is a canonical hypergroup.*

Proof. Let $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F), z = (z_1, z_2T, z_3I, z_4F) \in NQ$ be arbitrary with $x_i, y_i, z_i \in \mathbb{R}, i = 1, 2, 3, 4$.

(i) To show that $x\hat{+}y = y\hat{+}x$, let

$x\hat{+}y = \{a = (a_1, a_2T, a_3I, a_4F) : a_1 \in x_1 + y_1, a_2 \in x_2 + y_2, a_3 \in x_3 + y_3,$
$a_4 \in x_4 + y_4\}$,

$y\hat{+}x = \{b = (b_1, b_2T, b_3I, b_4F) : b_1 \in y_1 + x_1, b_2 \in y_2 + x_2, b_3 \in y_3 + b_3,$
$b_4 \in y_4 + x_4\}$.

Since $a_i, b_i \in \mathbb{R}, i = 1, 2, 3, 4$, it follows that $x\hat{+}y = y\hat{+}x$.

(ii) To show that that $x\hat{+}(y\hat{+}z) = (x\hat{+}y)\hat{+}z$, let

$y\hat{+}z = \{w = (w_1, w_2T, w_3I, w_4F) : w_1 \in y_1 + z_1, w_2 \in y_2 + z_2,$
$w_3 \in y_3 + z_3, w_4 \in y_4 + z_4\}$. Now,

$$\begin{aligned} x\hat{+}(y\hat{+}z) &= x\hat{+}w \\ &= \{p = (p_1, p_2T, p_3I, p_4F) : p_1 \in x_1 + w_1, p_2 \in x_2 + w_2, p_3 \in x_3 + w_3, \\ &\quad p_4 \in x_4 + w_4\} \\ &= \{p = (p_1, p_2T, p_3I, p_4F) : p_1 \in x_1 + (y_1 + z_1), p_2 \in x_2 + (y_2 + z_2), \\ &\quad p_3 \in x_3 + (y_3 + z_3), p_4 \in x_4 + (y_4 + z_4)\}. \end{aligned}$$

Also, let $x\hat{+}y = \{u = (u_1, u_2T, u_3I, u_4F) : u_1 \in x_1 + y_1, u_2 \in x_2 + y_2, u_3 \in x_3 + y_3, u_4 \in x_4 + y_4\}$ so that

$$\begin{aligned} (x\hat{+}y)\hat{+}z &= u\hat{+}z \\ &= \{q = (q_1, q_2T, q_3I, q_4F) : q_1 \in u_1 + z_1, q_2 \in u_2 + z_2, q_3 \in u_3 + z_3, \\ &\quad q_4 \in u_4 + z_4\} \\ &= \{q = (q_1, q_2T, q_3I, q_4F) : q_1 \in (x_1 + y_1) + z_1, q_2 \in (x_2 + y_2) + z_2, \\ &\quad q_3 \in (x_3 + y_3) + z_3, q_4 \in (x_4 + y_4) + z_4\}. \end{aligned}$$

Since $u_i, p_i, q_i, w_i, x_i, y_i, z_i \in \mathbb{R}, i = 1, 2, 3, 4$, it follows that $x\hat{+}(y\hat{+}z) = (x\hat{+}y)\hat{+}z$.

(iii) To show that $0 = (0, 0, 0, 0) \in NQ$ is a neutral element, consider

$$\begin{aligned} x\hat{+}(0,0,0,0) &= \{a = (a_1, a_2T, a_3I, a_4F) : a_1 \in x_1 + 0, a_2 \in x_2 + 0, a_3 \in x_3 + 0, \\ &\quad a_4 \in x_4 + 0\} \\ &= \{a = (a_1, a_2T, a_3I, a_4F) : a_1 \in \{x_1\}, a_2 \in \{x_2\}, a_3 \in \{x_3\}, \\ &\quad a_4 \in \{x_4\}\} \\ &= \{x\}. \end{aligned}$$

Similarly, it can be shown that $(0, 0, 0, 0)\hat{+}x = \{x\}$. Hence $0 = (0, 0, 0, 0) \in NQ$ is a neutral element.

(iv) To show that that for every $x \in NQ$, there exists a unique element $\hat{-}x \in NQ$ such that $0 \in x\hat{+}(\hat{-}x) \cap (\hat{-}x)\hat{+}x$, consider

$$\begin{aligned} x\hat{+}(\hat{-}x) \cap (\hat{-}x)\hat{+}x &= \{a = (a_1, a_2T, a_3I, a_4F) : a_1 \in x_1 - x_1, a_2 \in x_2 - x_2, \\ &\quad a_3 \in x_3 - x_3, a_4 \in x_4 - x_4\} \cap \{b = (b_1, b_2T, b_3I, b_4F) : \\ &\quad b_1 \in -x_1 + x_1, b_2 \in -x_2 + x_2, b_3 \in -x_3 + x_3, b_4 \in -x_4 + x_4\} \\ &= \{(0, 0, 0, 0)\}. \end{aligned}$$

This shows that for every $x \in NQ$, there exists a unique element $\hat{-}x \in NQ$ such that $0 \in x\hat{+}(\hat{-}x) \cap (\hat{-}x)\hat{+}x$.

(v) Since for all $x, y, z \in NQ$ with $x_i, y_1, z_i \in \mathbb{R}, i = 1, 2, 3, 4$, it follows that $z \in x\hat{+}y$ implies $y \in \hat{-}x\hat{+}z$ and $x \in z\hat{+}(\hat{-}y)$. Hence, $(NQ, \hat{+})$ is a canonical hypergroup. □

Lemma 2.3. *Let $(NQ, \hat{+})$ be a neutrosophic quadruple canonical hypergroup. Then*

(1) $\hat{-}(\hat{-}x) = x$ *for all* $x \in NQ$,
(2) $0 = (0, 0, 0, 0)$ *is the unique element such that for every* $x \in NQ$*, there is an element* $\hat{-}x \in NQ$ *such that* $0 \in x\hat{+}(\hat{-}x)$,
(3) $\hat{-}0 = 0$,
(4) $\hat{-}(x\hat{+}y) = \hat{-}x\hat{-}y$ *for all* $x, y \in NQ$.

Example 2.4. Let $NQ = \{0, x, y\}$ be a neutrosophic quadruple set and let $\hat{+}$ be a hyperoperation on NQ defined in the table below.

$\hat{+}$	0	x	y
0	0	x	y
x	x	$\{0, x, y\}$	y
y	y	y	$\{0, y\}$

Then $(NQ, \hat{+})$ is a neutrosophic quadruple canonical hypergroup.

Theorem 2.5. $(NQ, \hat{\times})$ *is a semihypergroup.*

Proof. Let $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F), z = (z_1, z_2T, z_3I, z_4F) \in NQ$ be arbitrary with $x_i, y_i, z_i \in \mathbb{R}, i = 1, 2, 3, 4$.

(i)

$$\begin{aligned} x\hat{\times}y &= \{a = (a_1, a_2T, a_3I, a_4F) : a_1 \in x_1y_1, a_2 \in x_1y_2 \cup x_2y_1 \cup x_2y_2, a_3 \in x_1y_3 \\ &\quad \cup x_2y_3 \cup x_3y_1 \cup x_3y_2 \cup x_3y_3, a_4 \in x_1y_4 \cup x_2y_4 \\ &\quad \cup x_3y_4 \cup x_4y_1 \cup x_4y_2 \cup x_4y_3 \cup x_4y_4\} \\ &\subseteq NQ. \end{aligned}$$

(ii) To show that $x\hat{\times}(y\hat{\times}z) = (x\hat{\times}y)\hat{\times}z$, let

$$\begin{aligned} y\hat{\times}z &= \{w = (w_1, w_2T, w_3I, w_4F) : w_1 \in y_1z_1, w_2 \in y_1z_2 \cup y_2z_1 \cup y_2z_2, \\ &\quad w_3 \in y_1z_3 \cup y_2z_3 \cup y_3z_1 \cup y_3z_2 \cup y_3z_3, w_4 \in y_1z_4) \cup y_2z_4 \\ &\quad \cup y_3z_4 \cup y_4z_1 \cup y_4z_2 \cup y_4z_3 \cup y_4z_4\} \end{aligned} \tag{2.3}$$

so that

$$\begin{aligned} x\hat{\times}(y\hat{\times}z) &= x\hat{\times}w \\ &= \{p = (p_1, p_2T, p_3I, p_4F) : p_1 \in x_1w_1, p_2 \in x_1w_2 \cup x_2w_1 \cup x_2w_2, \\ &\quad p_3 \in x_1w_3 \cup x_2w_3 \cup x_3w_1 \cup x_3w_2 \cup x_3y_3, p_4 \in x_1w_4 \cup x_2w_4 \\ &\quad \cup x_3w_4 \cup x_4w_1 \cup x_4w_2 \cup x_4w_3 \cup x_4w_4\}. \end{aligned} \tag{2.4}$$

Also, let

$$\begin{aligned} x\hat{\times}y &= \{u = (u_1, u_2T, u_3I, u_4F) : u_1 \in x_1y_1, u_2 \in x_1y_2 \cup x_2y_1 \cup x_2y_2, u_3 \in x_1y_3 \\ &\quad \cup x_2y_3 \cup x_3y_1 \cup x_3y_2 \cup x_3y_3, u_4 \in x_1y_4 \cup x_2y_4 \\ &\quad \cup x_3y_4 \cup x_4y_1 \cup x_4y_2 \cup x_4y_3 \cup x_4y_4\} \end{aligned} \tag{2.5}$$

so that

$$\begin{aligned} (x\hat{\times}y)\hat{\times}z &= u\hat{\times}z \\ &= \{q = (q_1, q_2T, q_3I, q_4F) : q_1 \in u_1z_1, q_2 \in u_1z_2 \cup u_2z_1 \cup u_2z_2, \\ &\quad q_3 \in u_1z_3 \cup u_2z_3 \cup u_3z_1 \cup u_3z_2 \cup u_3z_3, q_4 \in u_1z_4 \cup u_2z_4 \\ &\quad \cup u_3z_4 \cup u_4z_1 \cup u_4z_2 \cup u_4z_3 \cup u_4z_4\}. \end{aligned} \tag{2.6}$$

Substituting w_i of (2.3) in (2.4) and also substituting u_i of (2.5) in (2.6), where $i = 1, 2, 3, 4$ and since $p_i, q_i, u_i, w_i, x_i, z_i \in \mathbb{R}$, it follows that $x\hat{\times}(y\hat{\times}z) = (x\hat{\times}y)\hat{\times}z$. Consequently, $(NQ, \hat{\times})$ is a semihypergroup which we call neutrosophic quadruple semihypergroup. □

Remark 2.6. $(NQ, \hat{\times})$ is not a hypergroup.

Definition 2.7. Let $(NQ, \hat{+})$ be a neutrosophic quadruple canonical hypergroup. For any subset NH of NQ, we define

$$\hat{-}NH = \{\hat{-}x : x \in NH\}.$$

A nonempty subset NH of NQ is called a neutrosophic quadruple subcanonical hypergroup, if the following conditions hold:

(i) $0 = (0, 0, 0, 0) \in NH$,

(ii) $x\hat{-}y \subseteq NH$ for all $x, y \in NH$.

A neutrosophic quadruple subcanonical hypergroup NH of a netrosophic quadruple canonical hypergroup NQ is said to be normal, if $x\hat{+}NH\hat{-}x \subseteq NH$ for all $x \in NQ$.

Definition 2.8. Let $(NQ, \hat{+})$ be a neutrosophic quadruple canonical hypergroup. For $x_i \in NQ$ with $i = 1, 2, 3 \ldots, n \in \mathbb{N}$, the heart of NQ denoted by NQ_ω is defined by

$$NQ_\omega = \bigcup \sum_{i=1}^{n} (x_i \hat{-} x_i).$$

In Example 2.4, $NQ_\omega = NQ$.

Definition 2.9. Let $(NQ_1, \hat{+})$ and $(NQ_2, \hat{+}')$ be two neutrosophic quadruple canonical hypergroups. A mapping $\phi : NQ_1 \to NQ_2$ is called a neutrosophic quadruple strong homomorphism, if the following conditions hold:

(i) $\phi(x \hat{+} y) = \phi(x) \hat{+}' \phi(y)$ for all $x, y \in NQ_1$,
(ii) $\phi(T) = T$,
(iii) $\phi(I) = I$,
(iv) $\phi(F) = F$,
(v) $\phi(0) = 0$.

If in addition ϕ is a bijection, then ϕ is called a neutrosophic quadruple strong isomorphism and we write $NQ_1 \cong NQ_2$.

Definition 2.10. Let $\phi : NQ_1 \to NQ_2$ be a neutrosophic quadruple strong homomorphism of neutrosophic quadruple canonical hypergroups. Then the set $\{x \in NQ_1 : \phi(x) = 0\}$ is called the kernel of ϕ and it is denoted by $Ker\phi$. Also, the set $\{\phi(x) : x \in NQ_1\}$ is called the image of ϕ and it is denoted by $Im\phi$.

Theorem 2.11. *$(NQ, \hat{+}, \hat{\times})$ is a hyperring.*

Proof. That $(NQ, \hat{+})$ is a canonical hypergroup follows from Theorem 2.2. Also, that $(NQ, \hat{\times})$ is a semihypergroup follows from Theorem 2.4.

Next, let $x = (x_1, x_2T, x_3I, x_4F) \in NQ$ be arbitrary with $x_i, y_i, z_i \in \mathbb{R}, i = 1, 2, 3, 4$. Then

$$\begin{aligned}
x \hat{\times} 0 &= \{u = (u_1, u_2T, u_3I, u_4F) : u_1 \in x_1.0, u_2 \in x_1.0 \cup x_2.0 \cup x_2.0, u_3 \in x_1.0 \\
&\quad \cup x_2.0 \cup x_3.0 \cup x_3.0 \cup x_3.0, u_4 \in x_1.0 \cup x_2.0 \cup x_3.0 \cup x_4.0 \cup x_4.0 \\
&\quad \cup x_4.0 \cup x_4.0\} \\
&= \{u = (u_1, u_2T, u_3I, u_4F) : u_1 \in \{0\}, u_2 \in \{0\}, u_3 \in \{0\}, u_4 \in \{0\}\} \\
&= \{0\}.
\end{aligned}$$

Similarly, it can be shown that $0 \hat{\times} x = \{0\}$. Since x is arbitrary, it follows that $x \hat{\times} 0 = 0 \hat{\times} x = \{0\}$, for all $x \in NQ$. Hence, $0 = (0, 0, 0, 0)$ is a bilaterally absorbing element.

To complete the proof, we have to show that $x \hat{\times} (y \hat{+} z) = (x \hat{\times} y) \hat{+} (x \hat{\times} z)$, for all $x, y, z \in NQ$. To this end, let $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F), z = (z_1, z_2T, z_3I, z_4F) \in NQ$ be arbitrary with $x_i, y_i, z_i \in \mathbb{R}, i = 1, 2, 3, 4$. Let

$$\begin{aligned}
y \hat{+} z &= \{w = (w_1, w_2T, w_3I, w_4F) : w_1 \in y_1 + z_1, w_2 \in y_2 + z_2, w_3 \in y_3 + z_3, \\
&\quad w_4 \in y_4 + z_4\}
\end{aligned} \tag{2.7}$$

so that

$$\begin{aligned} x\hat{\times}(y\hat{+}z) &= x\hat{\times}w \\ &= \{p=(p_1,p_2T,p_3I,p_4F): p_1\in x_1w_1, p_2\in x_1w_2\cup x_2w_1\cup x_2w_2, \\ &\quad p_3\in x_1w_3\cup x_2w_3\cup x_3w_1\cup x_3w_2\cup x_3y_3, p_4\in x_1w_4\cup x_2w_4 \\ &\quad \cup x_3w_4\cup x_4w_1\cup x_4w_2\cup x_4w_3\cup x_4w_4\}. \end{aligned} \tag{2.8}$$

Substituting $w_i, i=1,2,3,4$ of (2.7) in (2.8), we obtain the following:

$$p_1\in x_1(y_1+z_1), \tag{2.9}$$

$$p_2\in x_1(y_2+z_2)\cup x_2(y_1+z_1)\cup x_2(y_2+z_2), \tag{2.10}$$

$$p_3\in x_1(y_3+z_3)\cup x_2(y_3+z_3)\cup x_3(y_1+z_1)\cup x_3(y_2+z_2)\cup x_3(y_3+z_3), \tag{2.11}$$

$$\begin{aligned} &p_4\in x_1(y_4+z_4)\cup x_2(y_4+z_4)\cup x_3(y_4+z_4)\cup x_4(y_1+z_1)\cup x_4(y_2+z_2), \\ &\cup x_4(y_3+z_3)\cup x_4(y_4+z_4). \end{aligned} \tag{2.12}$$

Also, let

$$\begin{aligned} x\hat{\times}y &= \{u=(u_1,u_2T,u_3I,u_4F): u_1\in x_1y_1, u_2\in x_1y_2\cup x_2y_1\cup x_2y_2, \\ &\quad u_3\in x_1y_3\cup x_2y_3\cup x_3y_1\cup x_3y_2\cup x_3y_3, u_4\in x_1y_4\cup x_2y_4 \\ &\quad \cup x_3y_4\cup x_4y_1\cup x_4y_2\cup x_4y_3\cup x_4y_4\} \end{aligned} \tag{2.13}$$

$$\begin{aligned} x\hat{\times}z &= \{v=(v_1,v_2T,v_3I,v_4F): v_1\in x_1z_1, v_2\in x_1z_2\cup x_2z_1\cup x_2z_2, \\ &\quad v_3\in x_1z_3\cup x_2z_3\cup x_3z_1\cup x_3z_2\cup x_3z_3, v_4\in x_1z_4\cup x_2z_4 \\ &\quad \cup x_3z_4\cup x_4z_1\cup x_4z_2\cup x_4z_3\cup x_4z_4\} \end{aligned} \tag{2.14}$$

so that

$$\begin{aligned} (x\hat{\times}y)\hat{+}(x\hat{\times}z) &= u\hat{+}v \\ &= \{q=(q_1,q_2T,q_3I,q_4F): q_1\in u_1+v_1, q_2\in u_2+v_2, \\ &\quad q_3\in u_3+v_3, q_4\in u_4+v_4\}. \end{aligned} \tag{2.15}$$

Substituting u_i of (2.13) and v_i of (2.14) in (2.15), we obtain the following:

$$q_1\in u_1+v_1\subseteq x_1y_1+x_1z_1\subseteq x_1(y_1+z_1), \tag{2.16}$$

$$\begin{aligned} &q_2\in u_2+v_2\subseteq (x_1y_2\cup x_2y_1\cup x_2y_2) \\ &+(x_1z_2\cup x_2z_1\cup x_2(z_2) \\ &\subseteq x_1(y_2+z_2)\cup x_2(y_1+z_1)\cup x_2(y_2+z_2), \end{aligned} \tag{2.17}$$

$$\begin{aligned} &q_3\in u_3+v_3\subseteq (x_1y_3\cup x_2y_3\cup x_3y_1)\cup x_3y_2\cup x_3y_3) \\ &+(x_1z_3\cup x_2z_3\cup x_3z_1)\cup x_3z_2\cup x_3z_3) \\ &\subseteq x_1(y_3+z_3)\cup x_2(y_3+z_3)\cup x_3(y_1+z_1)\cup x_3(y_2+z_2)\cup x_3(y_3+z_3). \end{aligned} \tag{2.18}$$

$$\begin{aligned} &q_4\in u_4+v_4\subseteq (x_1y_4\cup x_2y_4\cup x_3y_4)\cup x_4y_1\cup x_4y_2)\cup x_4y_3\cup x_4y_4) \\ &+(x_1z_4\cup x_2z_4\cup x_3z_4)\cup x_4z_1\cup x_4z_2)\cup x_4z_3\cup x_4z_4) \\ &\subseteq x_1(y_4+z_4)\cup x_2(y_4+z_4)\cup x_3(y_4+z_4)\cup x_4(y_1+z_1)\cup x_4(y_2+z_2) \\ &\cup x_4(y_3+z_3)\cup x_4(y_4+z_4). \end{aligned} \tag{2.19}$$

Comparing (2.9), (2.10), (2.11) and (2.12) respectively with (2.16), (2.17), (2.18) and (2.19), we obtain $p_i=q_i, i=1,2,3,4$. Hence, $x\hat{\times}(y\hat{+}z)=(x\hat{\times}y)\hat{+}(x\hat{\times}z)$, for all

$x, y, z \in NQ$. Thus, $(NQ, \hat{+}, \hat{\times})$ is a hyperring which we call neutrosophic quadruple hyperring. □

Theorem 2.12. *$(NQ, \hat{+}, \circ)$ is a Krasner hyperring where $\circ$ is an ordinary multiplicative binary operation on NQ.*

Definition 2.13. Let $(NQ, \hat{+}, \hat{\times})$ be a neutrosophic quadruple hyperring. A nonempty subset NJ of NQ is called a neutrosophic quadruple subhyperring of NQ, if $(NJ, \hat{+}, \hat{\times})$ is itself a neutrosophic quadruple hyperring.

NJ is called a neutrosophic quadruple hyperideal if the following conditions hold:

(i) $(NJ, \hat{+})$ is a neutrosophic quadruple subcanonical hypergroup.

(ii) For all $x \in NJ$ and $r \in NQ$, $x\hat{\times}r, r\hat{\times}x \subseteq NJ$.

A neutrosophic quadruple hyperideal NJ of NQ is said to be normal in NQ, if $x\hat{+}NJ\hat{-}x \subseteq NJ$, for all $x \in NQ$.

Definition 2.14. Let $(NQ_1, \hat{+}, \hat{\times})$ and $(NQ_2, \hat{+}', \hat{\times}')$ be two neutrosophic quadruple hyperrings. A mapping $\phi : NQ_1 \to NQ_2$ is called a neutrosophic quadruple strong homomorphism, if the following conditions hold:

(i) $\phi(x\hat{+}y) = \phi(x)\hat{+}'\phi(y)$, for all $x, y \in NQ_1$,

(ii) $\phi(x\hat{\times}y) = \phi(x)\hat{\times}'\phi(y)$, for all $x, y \in NQ_1$,

(iii) $\phi(T) = T$,

(iv) $\phi(I) = I$,

(v) $\phi(F) = F$,

(vi) $\phi(0) = 0$.

If in addition ϕ is a bijection, then ϕ is called a neutrosophic quadruple strong isomorphism and we write $NQ_1 \cong NQ_2$.

Definition 2.15. Let $\phi : NQ_1 \to NQ_2$ be a neutrosophic quadruple strong homomorphism of neutrosophic quadruple hyperrings. Then the set $\{x \in NQ_1 : \phi(x) = 0\}$ is called the kernel of ϕ and it is denoted by $Ker\phi$. Also, the set $\{\phi(x) : x \in NQ_1\}$ is called the image of ϕ and it is denoted by $Im\phi$.

Example 2.16. Let $(NQ, \hat{+}, \hat{\times})$ be a neutrosophic quadruple hyperring and let NX be the set of all strong endomorphisms of NQ. If $\oplus$ and $\odot$ are hyperoperations defined for all $\phi, \psi \in NX$ and for all $x \in NQ$ as

$$\begin{aligned} \phi \oplus \psi &= \{\nu(x) : \nu(x) \in \phi(x)\hat{+}\psi(x)\}, \\ \phi \odot \psi &= \{\nu(x) : \nu(x) \in \phi(x)\hat{\times}\psi(x)\}, \end{aligned}$$

then $(NX, \oplus, \odot)$ is a neutrosophic quadruple hyperring.

3. Characterization of neutrosophic quadruple canonical hypergroups and neutrosophic hyperrings

In this section, we present elementary properties which characterize neutrosophic quadruple canonical hypergroups and neutrosophic quadruple hyperrings.

Theorem 3.1. *Let NG and NH be neutrosophic quadruple subcanonical hypergroups of a neutrosophic quadruple canonical hypergroup $(NQ, \hat{+})$. Then*

(1) *$NG \cap NH$ is a neutrosophic quadruple subcanonical hypergroup of NQ,*

(2) *$NG \times NH$ is a neutrosophic quadruple subcanonical hypergroup of NQ.*

Theorem 3.2. *Let NH be a neutrosophic quadruple subcanonical hypergroup of a neutrosophic quadruple canonical hypergroup $(NQ, \hat{+})$. Then*

(1) *$NH\hat{+}NH = NH$,*
(2) *$x\hat{+}NH = NH$, for all $x \in NH$.*

Theorem 3.3. *Let $(NQ, \hat{+})$ be a neutrosophic quadruple canonical hypergroup. NQ_ω, the heart of NQ is a normal neutrosophic quadruple subcanonical hypergroup of NQ.*

Theorem 3.4. *Let NG and NH be neutrosophic quadruple subcanonical hypergroups of a neutrosophic quadruple canonical hypergroup $(NQ, \hat{+})$.*

(1) *If $NG \subseteq NH$ and NG is normal, then NG is normal.*
(2) *If NG is normal, then $NG\hat{+}NH$ is normal.*

Definition 3.5. Let NG and NH be neutrosophic quadruple subcanonical hypergroups of a neutrosophic quadruple canonical hypergroup $(NQ, \hat{+})$. The set $NG\hat{+}NH$ is defined by

$$NG\hat{+}NH = \{x\hat{+}y : x \in NG, y \in NH\}. \tag{3.1}$$

It is obvious that $NG\hat{+}NH$ is a neutrosophic quadruple subcanonical hypergroup of $(NQ, \hat{+})$.

If $x \in NH$, the set $x\hat{+}NH$ is defined by

$$x\hat{+}NH = \{x\hat{+}y : y \in NH\}. \tag{3.2}$$

If x and y are any two elements of NH and τ is a relation on NH defined by $x\tau y$ if $x \in y\hat{+}NH$, it can be shown that τ is an equivalence relation on NH and the equivalence class of any element $x \in NH$ determined by τ is denoted by $[x]$.

Lemma 3.6. *For any $x \in NH$, we have*

(1) *$[x] = x\hat{+}NH$,*
(2) *$[\hat{-}x] = \hat{-}[x]$.*

Proof. (1)

$$\begin{aligned} [x] &= \{y \in NH : x\tau y\} \\ &= \{y \in NH : y \in x\hat{+}NH\} \\ &= x\hat{+}NH. \end{aligned}$$

(2) Obvious. □

Definition 3.7. Let NQ/NH be the collection of all equivalence classes of $x \in NH$ determined by τ. For $[x], [y] \in NQ/NH$, we define the set $[x]\hat{\oplus}[y]$ as

$$[x]\hat{\oplus}[y] = \{[z] : z \in x\hat{+}y\}. \tag{3.3}$$

Theorem 3.8. *$(NQ/NH, \hat{\oplus})$ is a neutrosophic quadruple canonical hypergroup.*

Proof. Same as the classical case and so omitted. □

Theorem 3.9. *Let $(NQ, \hat{+})$ be a neutrosophic quadruple canonical hypergroup and let NH be a normal neutrosophic quadruple subcanonical hypergroup of NQ. Then, for any $x, y \in NH$, the following are equivalent:*

(1) $x \in y\hat{+}NH$,
(2) $y\hat{-}x \subseteq NH$,
(3) $(y\hat{-}x) \cap NH \neq \varnothing$

Proof. Same as the classical case and so omitted. □

Theorem 3.10. *Let $\phi : NQ_1 \to NQ_2$ be a neutrosophic quadruple strong homomorphism of neutrosophic quadruple canonical hypergroups. Then*

(1) *$Ker\phi$ is not a neutrosophic quadruple subcanonical hypergroup of NQ_1,*
(2) *$Im\phi$ is a neutrosophic quadruple subcanonical hypergroup of NQ_2.*

Proof. (1) Since it is not possible to have $\phi((0,T,0,0)) = \phi((0,0,0,0))$, $\phi((0,0,I,0)) = \phi((0,0,0,0))$ and $\phi((0,0,0,F)) = \phi((0,0,0,0))$, it follows that $(0,T,0,0), (0,0,I,0)$ and $(0,0,0,F)$ cannot be in the kernel of ϕ. Consequently, $Ker\phi$ cannot be a neutrosophic quadruple subcanonical hypergroup of NQ_1.

(2) Obvious. □

Remark 3.11. If $\phi : NQ_1 \to NQ_2$ is a neutrosophic quadruple strong homomorphism of neutrosophic quadruple canonical hypergroups, then $Ker\phi$ is a subcanonical hypergroup of NQ_1.

Theorem 3.12. *Let $\phi : NQ_1 \to NQ_2$ be a neutrosophic quadruple strong homomorphism of neutrosophic quadruple canonical hypergroups. Then*

(1) *$NQ_1/Ker\phi$ is not a neutrosophic quadruple canonical hypergroup,*
(2) *$NQ_1/Ker\phi$ is a canonical hypergroup.*

Theorem 3.13. *Let NH be a neutrosophic quadruple subcanonical hypergroup of the neutrosophic quadruple canonical hypergroup $(NQ, \hat{+})$. Then the mapping $\phi : NQ \to NQ/NH$ defined by $\phi(x) = x\hat{+}NH$ is not a neutrosophic quadruple strong homomorphism.*

Remark 3.14. Isomorphism theorems do not hold in the class of neutrosophic quadruple canonical hypergroups.

Lemma 3.15. *Let NJ be a neutrosophic quadruple hyperideal of a neutrosophic quadruple hyperring $(NQ, \hat{+}, \hat{\times})$. Then*

(1) $\hat{-}NJ = NJ$,
(2) $x\hat{+}NJ = NJ$, *for all* $x \in NJ$,
(3) $x\hat{\times}NJ = NJ$, *for all* $x \in NJ$.

Theorem 3.16. *Let NJ and NK be neutrosophic quadruple hyperideals of a neutrosophic quadruple hyperring $(NQ, \hat{+}, \hat{\times})$. Then*

(1) *$NJ \cap NK$ is a neutrosophic quadruple hyperideal of NQ,*
(2) *$NJ \times NK$ is a neutrosophic quadruple hyperideal of NQ,*
(3) *$NJ\hat{+}NK$ is a neutrosophic quadruple hyperideal of NQ.*

Theorem 3.17. *Let NJ be a normal neutrosophic quadruple hyperideal of a neutrosophic quadruple hyperring $(NQ, \hat{+}, \hat{\times})$. Then*

(1) $(x\hat{+}NJ)\hat{+}(y\hat{+}NJ) = (x\hat{+}y)\hat{+}NJ$, *for all* $x, y \in NJ$,
(2) $(x\hat{+}NJ)\hat{\times}(y\hat{+}NJ) = (x\hat{\times}y)\hat{+}NJ$, *for all* $x, y \in NJ$,
(3) $x\hat{+}NJ = y\hat{+}NJ$, *for all* $y \in x\hat{+}NJ$.

Theorem 3.18. *Let NJ and NK be neutrosophic quadruple hyperideals of a neutrosophic quadruple hyperring $(NQ, \hat{+}, \hat{\times})$ such that NJ is normal in NQ. Then*
(1) *$NJ \cap NK$ is normal in NJ,*
(2) *$NJ\hat{+}NK$ is normal in NQ,*
(3) *NJ is normal in $NJ\hat{+}NK$.*

Let NJ be a neutrosophic quadruple hyperideal of a neutrosophic quadruple hyperring $(NQ, \hat{+}, \hat{\times})$. For all $x \in NQ$, the set NQ/NJ is defined as

$$NQ/NJ = \{x\hat{+}NJ : x \in NQ\}. \tag{3.4}$$

For $[x], [y] \in NQ/NJ$, we define the hyperoperations $\hat{\oplus}$ and $\hat{\otimes}$ on NQ/NJ as follows:

$$[x]\hat{\oplus}[y] = \{[z] : z \in x\hat{+}y\}, \tag{3.5}$$

$$[x]\hat{\otimes}[y] = \{[z] : z \in x\hat{\times}y\}. \tag{3.6}$$

It can easily be shown that $(NQ/NH, \hat{\oplus}, \hat{\otimes})$ is a neutrosophic quadruple hyperring.

Theorem 3.19. *Let $\phi : NQ \to NR$ be a neutrosophic quadruple strong homomorphism of neutrosophic quadruple hyperrings and let NJ be a neutrosophic quadruple hyperideal of NQ. Then*
(1) *$Ker\phi$ is not a neutrosophic quadruple hyperideal of NQ,*
(2) *$Im\phi$ is a neutrosophic quadruple hyperideal of NR,*
(3) *$NQ/Ker\phi$ is not a neutrosophic quadruple hyperring,*
(4) *$NQ/Im\phi$ is a neutrosophic quadruple hyperring,*
(5) *The mapping $\psi : NQ \to NQ/NJ$ defined by $\psi(x) = x\hat{+}NJ$, for all $x \in NQ$ is not a neutrosophic quadruple strong homomorphism.*

Remark 3.20. The classical isomorphism theorems of hyperrings do not hold in neutrosophic quadruple hyperrings.

4. Conclusion

We have developed neutrosophic quadruple algebraic hyperstrutures in this paper. In particular, we have developed new neutrosophic algebraic hyperstructures namely neutrosophic quadruple semihypergroups, neutrosophic quadruple canonical hypergroups and neutrosophic quadruple hyperrings. We have presented elementary properties which characterize the new neutrosophic algebraic hyperstructures.

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