IEEE International Workshop on Intelligent Motion Control
Boğaziçi University, Istanbul, 20-22 August 1990.
Edited by Okyay KAYNAK. IEEE Cat. No: 90TH0272-5

MENTAL MODELS OF FORCE AND MOTION

Varol Akman, Deniz Ede
Department of Computer Engineering and Information Sciences
Bilkent University, P. O. Box 8, Maltepe, 06572 Ankara, Turkey

Wm. Randolph Franklin
Electrical, Computer, and Systems Engineering Department
Rensselaer Polytechnic Institute, Troy, New York 12180-3590, USA

Paul J. W. ten Hagen
Center for Mathematics and Computer Science
Kruislaan 413, 1098 SJ Amsterdam, The Netherlands

Abstract Future robots should have common sense about the world in order to handle the problems they will encounter. A large part of this commonsense knowledge must be naive physics knowledge, since carrying out even the simplest everyday chores requires familiarity with physics laws. But how should one start codifying this knowledge? What kind of skills should be elicited from the experts (each and every one of us)? This paper will attempt to provide some hints by studying the mental models of force and motion.

Keywords naive physics, human problem solving, envisioning, qualitative reasoning, classical mechanics, mental models

> "Perhaps the most exciting discovery that you can make about the physical world is that all the diverse phenomena of nature are tied together by surprisingly few relationships."
> --*P. G. Hewitt* [11]

1. Introduction

The laws of physics play too important a part in our daily activities. This is true regardless of the possibility that we may not even be fully aware of how these laws work. This paper will try to explain the reasoning strategies of some subjects whom we asked to solve simple physics problems. It will concentrate on classical mechanics problems. Explanations as to *why the subjects think what they think* will be presented. Specifically, the questions it will tackle are as follows:

(1) What kind of naive physics knowledge is necessary to solve physics problems?
(2) Usually, people solve physics problems using either a qualitative or a quantitative approach, or a combination of these. How much of the knowledge they use is quantitative and how much qualitative?
(3) Why do people have misconceptions about the motion of objects?

In our simplified *roller coaster world*, objects are point masses and surfaces are frictionless. We are mainly interested in *envisionment* where one starts with a structural description of a scene and determines all possible sequences of behavior [7, 10].

1.1. NEWTON and MECHO

Implementations that can perform the qualitative and the quantitative analyses of the roller coaster world are NEWTON [6] and MECHO [2]. In NEWTON, four phases of problem-solving are identified as question answering, envisioning, planning, and quantitative reasoning. *Question answering* involves the identification of a question which is given by the user in a structured notation. *Envisioning* is intuitively visualizing the order of occurrence of possible events. In the *planning* phase, a plan is produced for quantitative reasoning and is executed by a collection of mathematically expert routines. The explicit values for the desired variables of the given problem are instantiated through a sequence of calls to procedure-like bodies. The surface on which the objects move is divided into segments that are of a consistent type. There are some production rules to perform envisioning; these are examined to produce the envisioning tree. The process of envisioning continues until either some oscillatory movements are observed or landmark events such as fall, collision, etc. are detected.

MECHO can handle paths more complex than NEWTON's. To carry out a qualitative analysis, MECHO searches the domain of possible events in a goal-directed manner, i.e. unlike NEWTON it does not keep the *full* envisionment tree [2].

1.2. Physical representations

There are some views which regard the framework of programs like NEWTON and MECHO flawed. Central among them is Larkin's argument [12]; also cf. [1]. She points out that NEWTON has a naive way of representing physical knowledge. In particular, NEWTON's internal representation contains direct representations of the *visible entities* mentioned in the problem description. As a result, NEWTON follows the direction of time flow. Larkin, on the other hand, argues that:

(1) Physicists solve problems with a recourse to *fictitious entities* (forces, momenta, etc.) instead of familiar entities (springs, pulleys, etc.) appearing in the problem.
(2) They usually use constraints whereas NEWTON follows the direction of time flow.

Larkin's objections are grounded in two assumptions. First, using the entities in the problem representation directly may be making life more difficult. Second, since the envisionment tree is developed by unidirectional operators that codify new information consistent with the passing of time, the timeless nature of physics laws is forgotten.

1.3. Other related research

People have fuzzy and frequently, wrong ideas about the physics of everyday life. DiSessa found out that a group of elementary school students learning to control a computer-simulated Newtonian object invariably had the wrong Aristotelian expectation that bodies move in the direction they are last pushed [8]. Another study by McCloskey reports that assumptions of the naive theories of motion are quite consistent across college students [14, 15]. It turns out that the theories developed by different individuals can best be described as different forms of the same basic theory. What is striking is that this basic theory is inconsistent with the fundamental principles of classical physics. McCloskey shows that this naive theory of motion is reminiscent of a pre-Newtonian theory: the medieval impetus theory (to be explined in the sequel) that setting a body in motion "imprints" in the body a force, or impetus, that keeps the body in motion. Clement also presented data indicating that many students have a significantly different view of certain aspects of mechanics than the Newtonian view [4, 5].

2. Sample problems

Five physics problems have been prepared. Each problem was chosen with a special purpose in mind, i.e. to provide answers to the questions stated in the Introduction. The problems are introductory physics problems; they are not difficult or tricky. Excluding the last one, each problem can be solved with high school physics knowledge [9].

Our research has been initiated by asking these five problems to ten persons and letting them to think aloud. Out of these ten, four were high school students, two were university physics students, and four were university students from other departments (but they took at least one physics course). The subjects' answers will not be listed individually for each subject. Rather, they will be classified by looking at similarities. Thus, only the generic answers will be given.

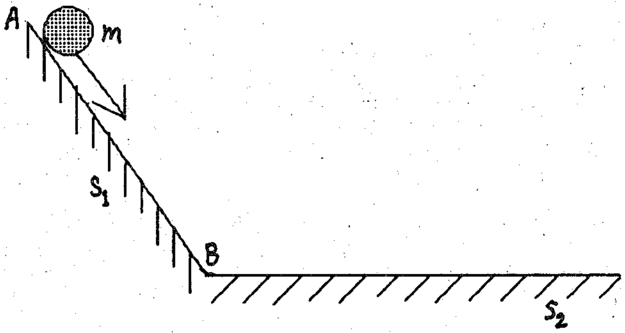

Figure 1. Scene for Problem 1

Problem 1 S_1 and S_2 are frictionless surfaces. A ball m slides, starting from rest, along S_1. Describe its motion (cf. Figure 1).
Answer m will slide down, because of the gravity, gaining speed. When it comes to point B it will have the maximum speed. With this constant speed it will continue to roll until infinity.

The answers are as follows:

(1) m gains speed, reaches B, and then with this velocity goes to infinity, for S_2 is frictionless (three subjects).
(2) m gains speed and reaches B. At B it jumps and reaches height h, drops again (cf. Figure 2). In this way it continues until infinity (three subjects).
(3) m gains speed and reaches B. It will stop somewhere along S_2 (four subjects).

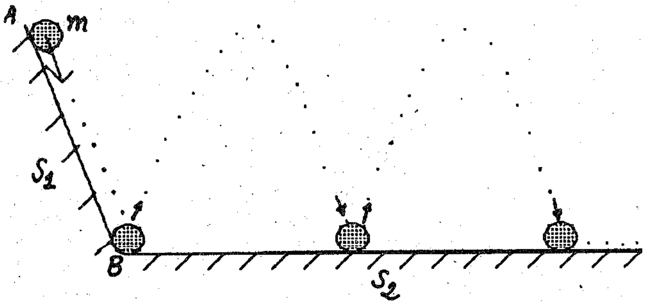

Figure 2. A solution to Problem 1, proposed by three subjects

Problem 2 There is no air resistance. Which block will hit the ground first? (cf. Figure 3)
Answer 2m has twice the weight and hence twice the accelerating force. If a_1 (resp. a_2) is the acceleration of the first (resp. second) block, then $a_1=F/m$ (resp. $a_2=2F/2m$). Since the accelerations are the same, they will hit the ground at the same time.

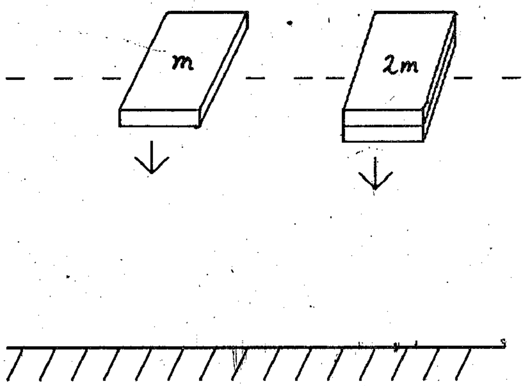

Figure 3. Scene for Problem 2

The answers are:

(1) They hit the ground at the same time, because they both have the same acceleration, e.g. the gravitational acceleration g. From the formula $F=mg$, the accelerations of the blocks are found as $a_1=mg/m=g$ and $a_2=2mg/2m=g$ (three subjects).
(2) The same answer as in (1). In this case there were two reasons: (a) "I know Galileo's experiment" and (b) "I performed Galileo's experiment from our balcony when I was in high school" (two subjects).
(3) Since gravitational force depends on the mass, 2m will hit the ground first (four subjects).
(4) We use the kinematics formula $D=V_it+(1/2)gt^2$ (D distance, V_i initial velocity, g gravitational acceleration, t time). Since $V_i=0$, D depends only on t, so they will hit the ground at the same time (one subject).

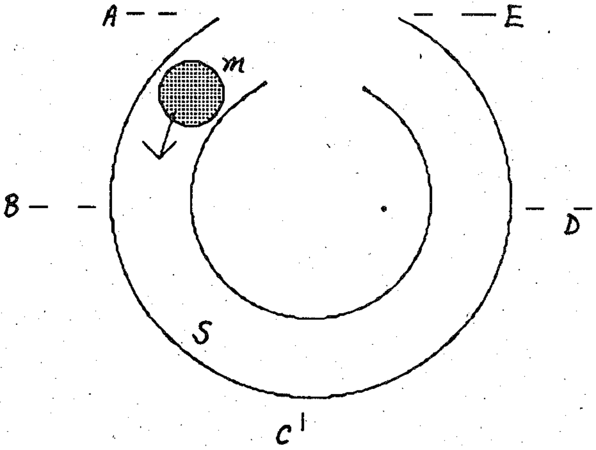

Figure 4. Scene for Problem 3

Problem 3 A mass m is shot along a tube. The inside surface S of the tube (cf. Figure 4) is frictionless. Will m reach point E? If yes, describe the motion of m after E. If not, which height can it reach?
Answer At A, m has potential energy mgh (h is the height). Since S is frictionless, m won't lose any energy and will reach E with that potential energy. At E velocity becomes 0, so m turns back. Between points A and E it continues to oscillate until infinity.

The answers are:

(1) Since S is frictionless and energy must be conserved, m reaches E. It turns back and reaches A with the same reasoning. Between A and E it continues to slide until infinity (three subjects).
(2) From the conservation of energy m reaches E. Since it began to make a circular motion, it tries to complete this circular motion, so it jumps from E to A (cf. Figure 5). It

continues to make this movement until infinity (two subjects).
(3) m can absolutely reach C, can come near D, but not E. It turns back and reaches near B. Again it turns back and moving back and forth like this it stops in the middle (three subjects).
(4) From the conservation of energy m reaches E. Since it gained speed, it leaves the tube as in Figure 6 (two subjects).

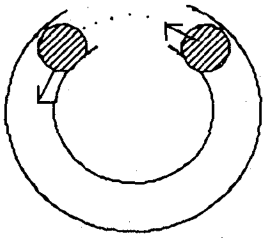

Figure 5. A solution for Problem 3, proposed by two subjects

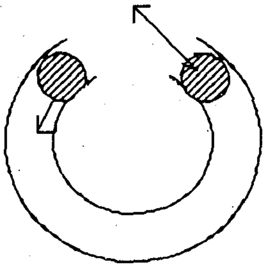

Figure 6. A solution for Problem 3, proposed by two subjects

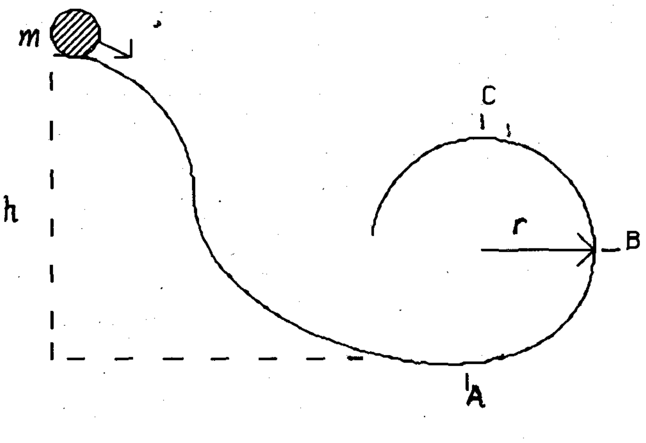

Figure 7. Scene for Problem 4

Problem 4 Mass m starts from rest. To be able to pass from point C what should be the height h?
Answer In order not to fall off at C the centrifugal and gravitational forces must be equal: $mg=mV^2/r$ and hence $V^2=gr$. Using the conservation of energy principle: $(1/2)mV^2 + 2mgr = mgh$ or $(1/2)r + 2r = h$. Hence $h=(5/2)r$.

Most of the students didn't want to solve this problem. They told us that they couldn't think of anything and remember any formula. None of them got the correct answer. Most of them claimed that h must be greater than 2r. Two reasons for this claim were:

(1) Energy must be conserved.
(2) h must be greater than 2r so that m can gain more velocity.

Figure 8. Scene for Problem 5

Problem 5 A man is running. While running he drops a ball. Draw the path that the ball will follow.
Answer It will follow the path shown in Figure 9.

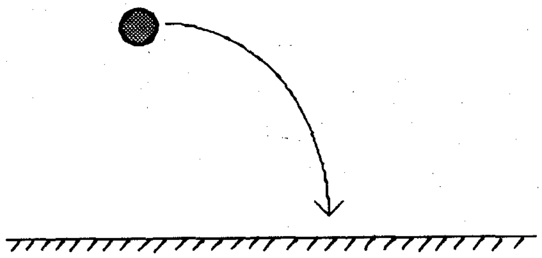

Figure 9. Solution for Problem 5

Three types of paths that the subjects drew are:

(1) The ball follows the path shown in Figure 10 (five subjects).
(2) The correct solution as in Figure 9 (three subjects).
(3) The ball follows the path shown in Figure 11 (two subjects).

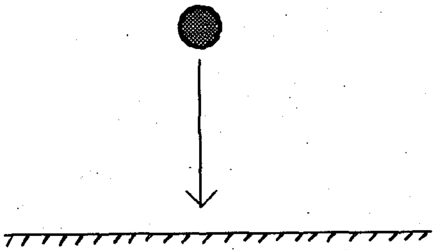

Figure 10. A solution for Problem 5, suggested by five subjects

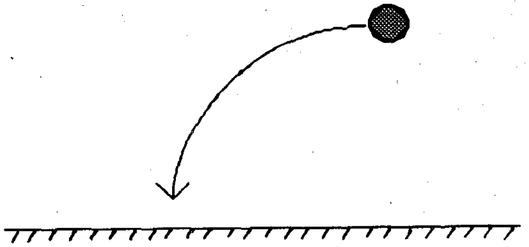

Figure 11. A solution for Problem 5, suggested by two subjects

3. Analysis

We observed that people generally solve physics problems either by intuition or some quantitative knowledge they have, or both. A good problem solver should combine his qualitative and quantitative knowledge to solve a problem. Qualitative knowledge represents the scene in terms of visual, gross features of the problem. The qualitative argument does not need a fully described scene.

When faced with a problem some people first try to predict the solution. This important part of qualitative knowledge can be termed *envisioning*. For example, when one drops a ball, one simply predicts that the ball will fall down. Envisioning is pre-physics knowledge. Using envisioning and quantitative knowledge people can solve physics problems quickly. For example, an easy problem can be solved without using any mathematical formulas. Even if we are only interested in problems requiring equations, the qualitative analysis still plays a crucial role. Although the qualitative analysis of a problem may fail, necessitating a quantitative analysis, it determines the kind of event happening; thus, providing a concise suggestion as to which equations are relevant. Summing up, it can be stated that a problem solver should be able to employ multiple representations to solve physics problems.

It has been stated in the Introduction that each of five sample problems were chosen for a specific purpose. To get an overall idea about the different representations that has been discussed above, let's reconsider three problems:

First problem Envisioning plays little role here and no quantitative knowledge is needed. The only envisioning that can be made here is to describe the path to B and then to predict the movement after B. Everybody can predict that m rolls down the inclined plane. The answers given by the subjects differ about motion after B. In the answers which describe the motion after B, envisioning can be seen easily. Predictions were such that either m would stop or would continue until infinity either by sliding or jumping.

Third problem We can see the role of envisioning here better. Since there are more possibilities, more predictions are made. The possible path to reach E is described: m will slide down the curved surface. After reaching the bottom, m starts to climb the right curve. At this point, predictions start to differ (all subjects knew that m would reach the bottom). Almost all subjects predict that m would reach D. After D, m may change its direction. Either it continues to climb or turns back. If m turns back what will be the next movement? How far can it go? There were various opinions on these questions.

Fourth problem This illustrates envisioning best. The problem requests an exact answer, i.e. h. But h won't get a numerical value, for it depends only on r which is not given a numerical value. The problem is different from the first and the third. To find the exact solution an expert should use his quantitative knowledge efficiently. But before applying this knowledge to the problem he must make some predictions. These predictions may be as follows: m sliding down the plane will reach the bottom. In order not to fall off at C centrifugal force must be equal to the gravitational force. The profit of these predictions can be summarized as follows: if one intuitively believes that the object will reach C, he can restrict his domain of physics knowledge. For example, he chooses formulas that are applicable at C. Two such formulas are $F=mV^2/r$ and $F=mg$. On the other hand if one thought that mass m would not be able to reach C (because it cannot go beyond B), then we cannot hope to get any other insight from him.

4. Theories of motion: a brief history

Aristotle divided motion into two classes [8]:

> (1) Natural motion: Every body in the universe had a proper place and any body not in its proper place would strive to get there. Larger bodies were expected to strive harder. Accordingly, bodies in the same medium were thought to fall with speeds proportional to their weights: the heavier a body the faster it fell.
> (2) Violent motion: Violent motion was imposed motion. The essential thing about violent motion was that it was externally caused and was imparted to objects.

It was Galileo [5] who gave credence to the Copernican view of a moving earth. He accomplished this by discrediting the Aristotelian ideas about motion. Aristotle's falling-body hypothesis was demolished by Galileo who dropped objects of various weights from the top of the Leaning Tower of Pisa. Contrary to Aristotle's assertion, he found that a stone twice as heavy did not fall twice as fast. Except for the small effects of air resistance, Galileo saw that objects of various weights, when released at the same instant, fell together. It is rumored that many observers of this demonstration who witnessed the objects hit the ground at the same time laughed at young Galileo and continued to stick to their Aristotelian teachings. After debunking the falling body theory, Galileo went further and denied the basic principle of Aristotle---that a body requires a push or pull to keep it moving. According to Galileo, if there is no interference with a moving body, it will keep moving in a straight line forever; no push, pull, or force of any kind is necessary. Finally, Newton's laws of motion were to complete the overthrow of the Aristotelian ideas:

> (1) A body continues in its state of rest or uniform motion along a straight line, unless it is compelled to change that state by forces impressed upon it.
> (2) The acceleration of a body is directly proportional to the net force acting on it and inversely proportional to its mass. A body is accelerated in the direction of the force acting on it. Applied in the direction of the body's motion, a force will increase the body's speed. Applied in the opposite direction, it will decrease the speed. Forces other than the applied force may act on the object. Usually these are frictional forces. The direction of the frictional force is always in a direction opposing the motion.
> (3) To every action force there is an equal and opposite reaction force. Action force and reaction force act on different bodies. When a net force is applied along a body's direction of motion, the body travels in a straight line and its motion is linear. When a net force acts in any other direction, the body travels in a curve and its motion is nonlinear.

5. Performance evaluation

First problem To be able to correctly answer this question the only thing that must be known is Newton's first law. Knowing it is not enough because we will see in the Conclusion that although some people know this law they do not believe it. Four of our subjects believed that m would eventually stop. When they were asked why they thought that this should be the case, almost all said: "It must stop. I don't know the reason. But it cannot go further because everything stops eventually and there is no force acting on the object." Clearly, it is incorrect to believe rolling objects stop because of the lack of force acting on it. The reason should be thus: "There is friction or some other force acting in the opposite direction to the motion."

Second problem Since there is no air resistance, using Newton's second law, they will hit the ground at the same time. Most of the subjects had a strong intuition that the heavier one would hit first: "I don't know the exact reason. But it must be so because 2m is heavier." One of the subjects who reasoned as above after a while hesitated and decided to make an experiment. He took in one hand a duster, in the other a small dictionary and released them from the same height. After he saw that they both hit the ground at the same time he looked surprised; he could not give a meaning to this situation.

Fifth problem Newtonian mechanics explains that when the stone is dropped, it continues to move forward at the same speed as the running person because no force is acting to change its horizontal velocity (ignoring air resistance). As the stone travels forward, it also moves down at a steady increasing speed. The forward and downward motions combine in a path that closely approximates a parabola. Reason given by a subject to support Figure 10: "There is no other possibility. Since there is gravitational force, it will fall directly to the ground." Reason given by another subject to support Figure 11: "The man has a speed. When he drops the ball he continues to run and ball is left behind."

6. Impetus theory

Why do people often misjudge the path of a moving object when they solve problems or carry out actions? The errors are not random but systematic. They arise from a general theory which is inconsistent with the principles of Newtonian mechanics. It is therefore the misconceptions embodied in an intuitive physical theory that occasionally give rise to errors of judgement about motion. Impetus theory is incompatible with Newtonian mechanics, for Newton's first law says that no force is required to keep an object at rest or at a constant velocity.

Impetus theory was a medieval correction to Aristotle's account of motion. It stated: "When a mover sets a body in motion, he implants into it a certain impetus, that is, a certain force enabling the body to move in the direction in which the mover starts it, be it upward, downward, sideward, or in a circle." Impetus theory asserts that motion must have a cause; to keep an object in motion we must continue to apply force on it. Since most moving objects eventually come to a stop an impetus theorist [sic] assumes that impetus, like energy, gradually dissipates.

We may ask: Why do people develop incorrect beliefs about the trajectories of moving objects that apparently conflict with everyday experience? Why, for example, do people come to believe that objects fall straight down when they are dropped? Below are some answers.

6.1. Perception of motion

Under some conditions the motion of objects is systematically misperceived. Objects dropped from a moving carrier are often perceived as falling straight down. Studies in perception of motion have shown that when something is viewed against a moving frame of reference, a visual illusion may arise, viz. the motion of the object relative to the moving frame of reference can be felt as absolute motion. Motion is the displacement of one object relative to others. Visually perceived, however, motion has no such relative aspect; it is an attribute of the moving object, even if only a temporary one. For example, even if a dot in the interior of a rectangle remains motionless as the rectangle moves to the right, the dot may be perceived as moving to the left.

6.2. Concept of friction

All three laws of Newton have a precondition such as: if the surface is frictionless and air resistance is neglected and ..., etc. However, it is difficult to observe a frictionless surface in the real world. People want to see prototypes to believe something. They cannot easily imagine frictionless surfaces. Although they learn about physics laws stated for such idealized objects, they cannot apply their knowledge to problems. Generally, they begin to solve problems by just reminiscing their experiences about motion from the real world.

6.3. Expert vs. novice performance

Six of our subjects were university students who took at least one physics course. We observed that performance is high in experienced persons. But experience must be used efficiently. The most obvious difference between an expert and a novice is that the expert knows many things the novice doesn't know and can rapidly evoke the particular items relevant to the problem at hand [3, 13].

7. Limitations of our study

First, we concerned ourselves with problems in mechanics [9]. Such an investigation can be made in another domain of physics but the contribution of that for robotics is probably less immediate [17]. Second restriction was inherent in the sample problems. Their numbers and types were limited. We tried to choose them as being appropriate for our goals. Finally, we could not quite capture the subjects' thinking although we told them to think aloud. It is difficult to transcribe thoughts at the time of thinking [16]. Furthermore, it is highly probable that there are things that the subjects thought but did not tell us.

8. Conclusion

We observe that physics is regarded as one of the most difficult courses by students. A student arrives with a vast amount of commonsense knowledge about mechanics. His competence comes from assimilating new information into an already existing framework. The pre-physics knowledge forms the basis and the bulk of a student's general physics knowledge. One of our high school subjects told us: "I don't like physics; it is the most troublesome course for me." When asked about the reason he replied: "Because the physics laws are not agreeable [sic] to my logic. While I'm solving physics problems there is on the one side physics laws, on the other side my opinions. I cannot believe that when I push an object on a frictionless surface, it will continue to move until infinity. My teacher repeated this lots of times, but my logic is not fit [sic] for it." Another interesting thing has been observed. One of the subjects (a physics student) was not thinking intuitively. He was just jumping into the problem and writing lots of formulas; but the result was always disappointing. He couldn't correctly answer any problem.

How can such difficulties be dispelled? An answer would be to teach Newtonian mechanics well. But studies by several researchers suggest that intuitive ideas are difficult to modify. Physics educators are seeking ways of designing computer-based instructional media to avoid such pitfalls. If this can be done then students will no more regard physics as a difficult course. Works by DiSessa [8] and others [18] suggest that experience with computer games, in which objects behave as if they move in a frictionless Newtonian world, may be helpful. Hence the study of intuitive theories and the processes by which they are acquired or modified holds promise for the development of improved educational methods.

ACKNOWLEDGMENT The first author thanks Özay Oral for moral support. The use of the pronoun "he" throughout the paper is for convenience.

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