UNDAUNTED SETS
(Extended Abstract)*

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"Sets arise out of the interaction of thinking creatures and the world about them."
—Jon Barwise (1988)

Recently, there has been a great surge of interest in set theory. Space doesn't permit us to give a thorough overview but the following citations must be made. A remarkable invention of Parikh, modestly called "dumb-founded sets," is receiving considerable attention, especially in the logic of political sciences. Similarly, by now everybody knows that the "situations" of Barwise and Perry would be in real trouble were it not for the strong assistance of Aczel's non-well-founded sets or hypersets, as they are commonly known.

Encouraged by all these, we have come up with a new flavor of sets which we choose to call "undaunted sets" (UND). Our sets are undaunted in a rather strong sense, viz. they can model anything and everything; they are neither discouraged nor dismayed by the difficulty of the task at hand. (Aside: In fact, an anonymous reviewer of this short note reported that he has found an ingenious way of solving the "Yale Shooting Problem" via UND. He hopes to report this result in the next IJCAI.)

All this is not without a price though. In order to achieve this universality, we had to unify the theories of Parikh and Aczel, with some technical assistance from KPU (also known as Kripke's Plateau of Unknown) along the way. And boy, was that exciting! (Details to be reported in the full version of this paper...)

What is an undaunted set? Well, we have a FOCS (and another STOC) paper in progress, which makes this notion very precise but the reader will have to be content with the following taste of the real thing. An undaunted set is a situation which can answer any membership question. How does that work? Easy: a situation is, as you well know, a limited portion of the reality that you can "individuate." Thus, it is able to tell you what it includes as members. However, since a situation is also a dumb-founded set (a fact which Parikh seems

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*This piece was inspired by Rohit Parikh's elegant note "Dumb-founded Sets" which appeared in EATCS BULLETIN. The reader should probably be told that it compares very badly, both in content and in style, to that paper. Finally, repeating a remark of Parikh, we have no intention to devalue any of the various set theories mentioned, including the dumb-founded sets.

to be aware of but doesn't explicitly acknowledge), it may answer some of your questions in a nice, ordinary (some say, Reagan-like) manner by a simple "Gee, I don't know. Can we talk about something else?" (Parikh denoted this response by g, but we'll use Γ in order to avoid potential confusion with the gravitational constant.)

Clearly, undaunted sets are non-well-founded. (Proof [Sketch]: An undaunted set is a situation and Barwise shows somewhere that the reality, the biggest situation, is non-well-founded.) This means that, applying Aczel's Special Final Coalgebra Theorem, one can solve a homogeneous set of equations over undaunted sets for a given undaunted set of unknowns. This gives the UND-theorist a real "handle" on the reality. (The reader may notice that one may get imaginary realities, a.k.a. unworldly realities, as a by-product of Aczel's solution but we assume that there is a post-processor which gets rid of those. The post-processor should simply eliminate all solutions which include Γ as a primary or an immediate secondary constituent. Since undaunted sets are hardly hereditarily finite, this seems to pose some efficiency problems but we'll let that pass.)

A literally free lunch you get by having undaunted sets is that you can ask two of your undaunted sets to communicate with each other and they would do so and they would even get some useful computation done in that way but then again, you would probably have no way of obtaining their results. You see, true, they have a way of talking about various things, but you would have to be in "that" situation in order to "get" that. As a famous theorem of Barwise and Etchemendy shows, you can be in a situation but probably the situation is not about you, so you wouldn't have a way of "relating" to it, to quote a Western phrase. They showed this by giving the following interesting counter-example: Propositions $\{s; [\text{Has}, \text{Max}, A\spadesuit; 1]\}$ and $\{s; [\text{Has}, \text{Max}, A\spadesuit; 0]\}$ may both be false in a particular situation s, simply because s doesn't "know" about Max, e.g., Max may not be in that situation. (So, you see, B&E were probably thinking of their situations as undaunted sets, too. Small world!)

Where does the KPU come into play in all this? Well, fortunately, we don't need any "deep" KPU here. All we need is undaunted sets with urelements and that is easy to get by just adding a U, viz. UNDU. A less general version of UND is known as UND- and is obtained by simply removing the urelements from UNDU, e.g., UNDU = UND- + "There are urelements." The practicing mathematician would have felt comfortable had we remarked that there is no essential difference between UND- and UNDU. Yes Ferdinand, there is! But there is also a catch. This difference makes itself manifest only in "independence" proofs and the practicing mathematician will agree that that is hardly his line of work.

In this extended abstract, we have introduced the concept of undaunted sets and discussed some of their properties. Various issues remain unsettled. We cite a few. Is UND finitely axiomatizable? Are NBG (von Neumann-Bernays-Goedel) and MK (Morse-Kelley) systems conservative extensions of UND, or vice versa? What about the good old ZF? Finally, how about Quine's New (now Old) Foundations (NF) vs. UND? Since NF has a universal set, we believe that there should be some connections. Note however that it is not known whether NF is consistent, even assuming Con(ZF); obviously, this is not true for UND, or UNDU for that matter. They are undoubtedly consistent.