What is it Like to be a Relativistic GRW Theory?
Or: Quantum Mechanics and Relativity, Still in Conflict After All These Years

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Abstract

The violation of Bell's inequality has shown that quantum theory and relativity are in tension: reality is nonlocal. Nonetheless, many have argued that GRW-type theories are to be preferred to pilot-wave theories as they are more compatible with relativity: while relativistic pilot-wave theories require a preferred slicing of space-time, foliation-free relativistic GRW-type theories have been proposed. In this paper I discuss various meanings of 'relativistic invariance,' and I show how GRW-type theories, while being more relativistic in one sense, are less relativistic in another. If so, the initial claim that GRW-type theories have a greater compatibility with relativity is unwarranted: both type of theories violate relativity, one way or another.

Keywords: spontaneous collapse theories, relativistic invariance, nonlocality

1. Introduction

One of the fundamental problems in the philosophy of quantum mechanics is the measurement problem: how to eliminate the unobserved macroscopic superpositions predicted by quantum theory. It is often claimed that when solving this problem, one can make quantum theory compatible with scientific realism, the view that scientific theories teach us about the nature of the world. The most promising solutions of this problem are the pilot-wave theory (also known as Bohmian Mechanics, or de Broglie-Bohm theory),[2] the spontaneous localization theory (or spontaneous collapse, dynamical reduction, or GRW theory), and the many-worlds theory (also known as Everettian Mechanics).[3] However, there is another challenge to face: how to construct a relativistic quantum theory. The trouble with this is the violation of Bell's inequality, which shows that quantum theory is nonlocal, therefore against the 'spirit' of relativity. Leaving many-worlds theories aside, GRW-type theories have been argued to be preferable to the pilot-wave theory for several reasons. In this paper I focus on the claim that GRW-type theories are more amenable to a relativistic extension than pilot-wave theories. That is, I compare relativistic pilot-

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[2] For sake of precision, however, notice that the theory originally proposed by Bohm (1952) is arguably not the same theory as the one developed by Dürr, Goldstein and Zanghí (1992), and now known as Bohmian mechanics. In fact, both are theories of particles; however, in Bohm's theory is a first-order theory in which the wavefunction is considered a real physical field in space, and there is a quantum potential, while nothing of the sort is present in Bohmian mechanics (see Dürr *et al.* for a comparison). Nonetheless, it is common practice among physicists interested in foundations and philosophers of physics to use the locutions 'Bohmian mechanics' and 'de Broglie-Bohm pilot-wave theory' interchangeably (see e.g., Bricmont 2016b, Norsen 2016). In the following, I will not consider Bohm's theory.

[3] Also in this case, it has been argued that Everett did not endorse a many-worlds interpretation of his formulation of quantum mechanics (see Barrett and Byrne, 2012). Nonetheless, it seems common practice among philosophers of physics and physicists to understand 'Everettian mechanics' to denote the many-worlds theory.

wave theories with the various proposals for relativistic GRW-type theories. Notice that, since the most promising relativistic versions of GRW have been developed within the so-called 'primitive ontology approach,' [4] I explicitly address the superiority claim in this framework, but I will also argue that the same conclusions hold outside of it. First, I present the non-relativistic quantum theories in the next section. Then in section 3 I review Bell's inequality and the consequence of its violation. In section 4, I quickly discuss relativistic pilot-wave theories and relativistic GRW-type theories, while in section 5 I argue that there seems to be two ways of understanding 'the spirit of relativity' (in addition of Lorentz invariance of the laws): either *locality of the interaction* or preservation of the *relativistic spacetime structure*. Relativistic GRW-type theories are relativistic in the latter sense, while relativistic pilot-wave theories are relativistic in the former. The natural question is then what makes relativistic GRW-type theories respect the relativistic spatiotemporal structure, and in section 6 I show that it is their stochasticity. I remark in section 7 that this result is compatible with the often-quoted distinction between parameter independence (PI) and outcome independence (OI), and the claim that theories violating OI are more compatible with relativity than theories violating PI. Also, I show that to preserve relativistic spatiotemporal structure you need stochasticity: relativistic deterministic theories have to have a preferred foliation. In section 8 I move to the discussion of the costs of being relativistic in the spatiotemporal sense: I show that, on top of having an unfamiliar ontology and stochastic laws, in GRW-type theories cause and effect are symmetric. Also, these theories are 'supernonlocal,' as they display nonlocal behavior when pilot-wave theories do not. This is crucial for my conclusion in section 9: if stochasticity gets us closer to relativity in the sense of permitting us to have only relativistic spatiotemporal structure, nonetheless, by entailing supernonlocality and symmetry between cause and effect, it also brings us further away in terms of locality of interaction. *This seems to undermine the reason to favor GRW-type theories over pilot-wave theories in the first place as the best balance between quantum mechanics and relativity: neither of these types of theories is fundamentally relativistic because each of them respects one spirit of relativity by violating the other.* So, the famous 'peaceful coexistence' between quantum mechanics and relativity seems farther than ever.

2. Non-Relativistic Quantum Theories

Quantum theory contains an evolution equation, namely the Schrödinger equation, of an object called the wavefunction. Because this equation is linear, superpositions of possible solutions are also solutions, namely they describe possible states of affair of the worlds. However, these predicted macroscopic superpositions are never observed, so that if a theory aims at describing the world it needs to deal with them. That is, it needs to solve the measurement problem. If one wants the theory not to rely on vague notions such as 'measurement,' then one has three broad options. First, deny that there is the need to suppress anything. This is the strategy of many-worlds theories, according to which the branching structure of possibilities is real. This is a bold and to some extent fascinating idea.[5] However, I do not discuss it further here because I wish to

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[4] For other attempts, see Bassi and Ghirardi (2020) and references therein.

[5] For more on this theory, see Wallace (2012). See also Barrett (2018), Vaidman (2014), and references therein. For an assessment of its compatibility with relativity, see Myrvold (2021).

focus on the other alternatives: since in many-worlds theories measurements do not have definite results, arguably Bell's inequality does not apply straightforwardly to them.[6] One other possibility is to 'build' the collapse into the dynamical law: this is what the GRW theory does. Otherwise, one might dispute that the wavefunction completely describes physical systems: in the pilot-wave theory physical objects are characterized by their positions, which evolve in time guided by a law determined by a Schrödinger-evolving wavefunction, called the guidance equation.[7]

The pilot-wave theory is a theory of particles, whose deterministic evolution is governed by a Schrödinger-evolving wavefunction.[8] In the literature people have called 'pilot-wave-type theory' (or Bohmian theory) any theory with a Schrödinger-evolving wavefunction.[9] Then one can construct pilot-wave theories with a non-particle ontology. For instance, Sf has a flash ontology,[10] while Sm has a matter density field ontology.[11] Also, one could combine the Schrödinger evolution with a stochastic law for the particle primitive ontology. One such example is stochastic mechanics.[12]

In the GRW theory the wavefunction evolves according to the Schrödinger equation up to a random time, at which the wavefunction centers around a random location. The rate of collapse is proportional to the number of 'particles' in the system so that macroscopic objects localize almost immediately.[13]

While initially people have thought of GRW as a theory about the wavefunction, this idea has been challenged by the so-called primitive ontology approach. It has been argued that the wavefunction is not the right kind of object to represent physical systems, and only three-

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[6] See, e.g., Goldstein *et al.* (2011), Vaidman (2016). However, see also Norsen (2016). For an explicitly nonlocal many-worlds theory within the primitive ontology framework see Allori *et al.* (2014). In addition, let me notice that, if one takes Bell's theorem as a general argument for nonlocality, every theory will have to be nonlocal, including the many-worlds theory.

[7] Among these theories one also finds the so-called modal interpretation. Since it has been argued (e.g., Myrvold 2002a) that they suffer from the same difficulties as the pilot-wave theory, in this paper I will only discuss the latter because it is simpler. For additional discussion on the modal interpretation and its compatibility with relativity, see also Myrvold (2009, 2021).

[8] See Goldstein (2017) and references therein.

[9] See, e.g., Allori *et al.* (2008).

[10] Allori *et al.* (2008).

[11] Allori *et al.* (2011). Another example of a Bohmian theory of this type is BQFT (Bohmian quantum field theory), which has deterministically evolving fields (Bohm 1952, Struyve and Westman 2006).

[12] Nelson (1985). Otherwise, in Bell-type quantum field theory, a stochastic evolution of the particles allows for creation and annihilation (see Bell 1986, Dürr *et al.* 2004; 2005; nonetheless, one could arguably describe particle creations and annihilations even with a deterministic dynamics, see Colin 2003, Colin and Struyve 2007; see also Nikolić 2010; for discussion, see also Oldofredi 2020).

[13] There are two new constants of nature, the localization accuracy $d = 10^{-7}$ m, and the localization frequency $f = 10^{-16}\ s^{-1}$, so that microscopic systems localize on average every hundred million years, while macroscopic systems every 10^{-7} seconds. There is a more general class of GRW-like theories, namely theories in which the wavefunction spontaneously collapses, which goes under the name of CSL, continuous spontaneous localization, which is an extension of the GRW logic (see Bassi and Ghirardi 2020). I will include these theories under the label 'GRW-type theories.'

dimensional ontologies (or four-dimensional, if in space-time) can provide a satisfactory explanation of the phenomena. This low-dimensional ontology has been dubbed the primitive ontology of the theory.[14] According to the primitive ontology approach, every quantum theory needs to be reformulated as a theory with a primitive ontology: for GRW there are different alternatives, depending on the choice of the primitive ontology, and they have been called GRWf, GRWm, and GRWp. In these theories the wavefunction evolves according to the same stochastic equation of the original GRW proposal but now it governs respectively a four-dimensional event ontology ('flashes'),[15] a three-dimensional matter density field ontology,[16] and a particle ontology.[17] The evolution of the primitive ontology is stochastic as well in all three theories. In fact, in GRWf the probabilistic distribution of the flashes is determined by the wavefunction; in GRWm the matter density evolution inherits the stochasticity of the evolution of the wavefunction; and in GRWp particles evolve according to the same guidance equation of the pilot-wave theory but every time the wavefunction collapses, they are 'displaced' at random to make the theory empirically adequate.[18]

Independently of whether one thinks that the primitive ontology approach is needed, I will assume it in this paper because the most promising relativistic GRW theories which have been proposed have a primitive ontology. Moreover, as I will discuss in Section 6, my conclusions will hold regardless of whether one endorses this approach or not.

3. Bell's Inequality

Now that we have seen how one can make a realist sense of quantum theories, one needs to construct a relativistic quantum theory. However, there is a tension between quantum mechanics and relativity which comes from the violation of Bell's inequality. Let's briefly summarize how the problem comes about.[19] The first part of the derivation Bell's inequality is the Einstein, Podolsky and Rosen (EPR) argument.[20] EPR wanted to show that quantum theory

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[14] See Allori *et al.* (2008), Allori (2013a). See also Allori (2019a) and references therein. For criticism, see e.g., Belot (2012), Ney and Phillips (2013), Albert (2015), Lewis (2015), Myrvold (2015, 2019), Wallace (2020), Egg (2021), Egg and Saatsi (2021), and references therein.

[15] Bell (1987).

[16] The matter density function has been introduced in Benatti *et al.* (1995). It is considered as a possible primitive ontology explicitly in Allori *et al.* (2008).

[17] Allori *et al.* (2008, 2014); Allori (2020a).

[18] Notice that a stochastically evolving primitive ontology (a 'stochastic primitive ontology') has been combined with a deterministically evolving wavefunction (a 'deterministic wavefunction'), like in Sf, stochastic mechanics or certain Bell-type quantum field theories. Also, a 'stochastic primitive ontology' has been paired with a stochastically evolving wavefunction (a 'stochastic wavefunction'), as for instance in all GRW-type theories. Instead, a deterministically evolving primitive ontology (a 'deterministic primitive ontology') has only been successfully combined with a deterministically evolving wavefunction (a 'deterministic wavefunction'), as in the original de Broglie-Bohm pilot-wave theory.

[19] In this reconstruction I follow Goldstein *et al.* (2011), Maudlin (2011), Norsen (2016), Bricmont (2016a). I assume that this reconstruction is correct, as my main goal in this paper is to show that, even granting that Bell has shown that reality is nonlocal, it is still problematical to think that GRW theories are more compatible with relativity than the pilot-wave theory. Indeed, most people who argue that GRW theories are more compatible with relativity than pilot-wave theories accept this reconstruction.

[20] Einstein, Podolsky and Rosen (1935). See also Eisntein (1948).

is incomplete: if it were complete then there would be an instantaneous action at a distance (nonlocal correlations), which was considered prohibited by relativity theory.

This idea of locality is that physical influences propagate continuously through distances, and arguably it has always been a desideratum for fundamental physical theories.[21] The special theory of relativity poses an additional constraint on this idea, namely that physical influences travel slower than the velocity of light.[22] Going back to EPR, in the reformulation of their experiment as proposed by Bell, one considers a particle source which emits pairs of spin-correlated particles in opposite directions. Two experimenters, Alice and Bob, may measure one of the spin components on the particle coming towards them using detectors which can be set in different ways. Alice and Bob record their results, and when they compare them, they discover that they are perfectly anti-correlated.[23] According to quantum theory, each particle in the pair has no definite spin-in-that-direction value, but it acquires one only upon measurement. So, suppose Alice finds spin up along one direction. Since the particles are entangled, Bob's particle acquires the opposite property in virtue of Alice's measurement. This means that the theory violates the above-mentioned locality condition: Bob's outcomes are altered by what happens in Alice's space-like separated region.

EPR believed that this nonlocality of quantum theory is artificial: it is evidence of the fact that the theory is incomplete. EPR concluded that the only other option is the existence of 'hidden variables,' which quantum theory does not specify: the devices are revealing a pre-existing (existing before the measurement) value of that property, contrary to what prescribed by quantum theory, so that the anti-correlations in the results are explained in terms of the anti-correlations at the source. We therefore need to supplement the description of quantum theory with these spin values to avoid nonlocality.[24]

Bell continued EPR's reasoning, since he wanted to see whether one could in principle locally complete quantum theory by adding hidden variables as suggested. First, he proposed a formal, more precise definition of locality in terms of probability of outcomes: "A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a spacetime region 3."[25] Notice that here Bell mentions local beables: roughly, this terminology should be understood as denoting some spatiotemporal entities which we could identify as physically real.[26] So, to check whether the condition of local

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[21] Think for instance about the objections to Newton's theory of gravitation. Newton agreed it was a problem and replied that his theory was incomplete, and that a future, better, theory should make this action at a distance go away (see Norsen 2011).

[22] This is due to the relativity of simultaneity: since two spacelike separated observers, namely observers such that a signal connecting them would have to propagate faster than light, will disagree on the temporal ordering of events, to avoid causal chains to go backwards one requires physical influences to travel slower than the speed of light.

[23] When Alice measures spin up in one direction, Bob will measure spin down in that direction.

[24] It is interesting to notice, as emphasized by Norsen (2011), that EPR's conclusion that quantum theory is incomplete is similar in spirit to Newton's reply to the objection that his theory requires action at a distance (see footnote 21).

[25] Bell (1990).

[26] For more on Bell's notion of local causality and on local beables, see Norsen (2011).

causality is satisfied by a given theory, we should first identify its local beables. For the purpose of this paper, one can identify the notion of local beables with the one of primitive ontology,[27] so that it will be meaningful to consider the question of whether a given theory violates local causality.[28] Then, Bell computed the restrictions such a local theory would have to have, and he arrived at an inequality, which instead would be violated by quantum mechanics. Therefore, one could use Bell's inequality as a crucial test to rule out one of the two theories. Subsequent experiments established convincingly that the quantum mechanical predictions are correct,[29] which implies that the type of theory Bell showed must respect the inequality is not empirically viable.

Bell took himself to have proven that any theory which makes the same predictions of quantum theory has to be nonlocal, as the local causality condition was the only assumption made in the derivation.[30] Locality can be seen as the prohibition of super-luminal causation, which is usually taken to be an implication of special relativity, as mentioned earlier. Therefore, a violation of local causality is in conflict with relativity.[31]

One possible reaction to this result is to scrutinize Bell's proof to find whether there is some hidden assumption and dispute it. Indeed, the so-called hypothesis of statistical independence, namely that the experimental settings do not depend on the distribution of the additional variables, was taken for granted. One way to make this false (and thus invalidate the nonlocality conclusion) is allowing for superdeterministic theories, which however Bell disregarded as conspiratorial.[32] Another way is to have retrocausal theories, in which the cause does not come before its effects.[33] I do not consider these routes here, as I wish to compare two options, GRW theories and the de Broglie-Bohm pilot-wave theory, in which statistical independence is true.

Assuming therefore that Bell's proof goes through, one has to face the fact that the world is nonlocal: Bell's local causality condition is independent of what the theory claims to exist, as emphasized by Norsen (2011), so every theory which aims at being empirically adequate has to be nonlocal. So, the crucial question is: What does it mean for the prospects of finding a quantum relativistic invariant theory? One possibility is the one proposed by pilot-wave theories, namely to think of relativity as 'merely' empirical, in a Lorentzian way: all relativistic effects emerge at the level of predictions.[34] As I discuss in section 4.1, this is achieved by

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[27] However, see Allori (2021) for a distinction.

[28] Notice, in passing, that this requirement seems to rule out the many-worlds theory: since it is only about the evolution of the wavefunction, it does not have any local beables.

[29] Aspect *et al.* (1982).

[30] For discussions of Bell's proof in general, see e.g., the contributions to Gao and Bell (2016).

[31] Bell (1987).

[32] See Caluser *et al*. (1985). See also Norsen (2009), Bricmont (2016a), Chen (2021) for further criticisms. For a defense of superdeterminism, see Hossenfelder and Palmer (2020) while for a recent discussion, see Baas and Le Bihan (forthcoming).

[33] For more on retrocausality in quantum mechanics, see e.g., Freidman (2019) and references therein.

[34] At some point, this is what Bell seemed to have argued, at least according to Norsen (2011).

constructing a nonlocal deterministic theory of particles guided by a Dirac evolving wavefunction, and suitably adding a preferred slicing of spacetime (a foliation) which provides an undetectable frame for absolute simultaneity.

Regardless, many have resisted this. As anticipated, one could simply reject Bell's proof. Also, many who accept Bell's theorem, and therefore accept nonlocality, are looking for ways to 'peacefully combine' quantum nonlocality and relativity. This has appeared to be a real possibility starting with Bell, who noticed a singular way of making the GRW theory relativistic by introducing a flash ontology.[35] As discussed in section 4.2, several GRW-type theories have been proposed which are advertised as relativistic even if nonlocal: in contrast with the proposed relativistic pilot-wave theories, these theories postulate no preferred foliation of spacetime. In virtue of this, many have claimed that *GRW-type theories are more compatible with relativity than the pilot-wave theory.*[36] I present proposals of relativistic quantum theories in the next section. I evaluate the claim that GRW-type theories are more compatible with relativity than pilot-wave theories in section 8. Before that, however, one has to figure out what the spirit of relativity is, as discussed starting in section 5.

4. **Relativistic Quantum Theories**

People agree that nonrelativistic quantum theories are in tension with relativity in the following respects:

a) The dispersion relation $E = p^2/2m$ in the Schrödinger equation is nonrelativistic;
b) The wavefunction is a function of (a single) time;
c) There is the collapse of the wavefunction.[37]

The first problem can be overcome by having a Dirac or Klein-Gordon equation for the evolution of the wavefunction. The second point is an obstacle because the wavefunction somehow mediates the interaction, and it is in configuration space. If the wavefunction describes arbitrarily distant systems with the very same time variable, then it suggests the existence of an absolute simultaneity. In other words, in order to make sense of claims like the following: "what happens here now, instantaneously influences what happens there now," one needs to be able to talk about the same 'now' here and there. Nonetheless, this problem can be cured by introducing a multi-time wavefunction, in which there is a time variable for each system, each evolving according to its own equation of motion.[38] Finally, the wavefunction collapse is instantaneous, and therefore problematical from a relativistic point of view because it affects systems independently of their relative distance, thereby threatening locality. This problem is absent in the case of the pilot-wave theory, while collapse theories may have different solutions (see section 4.2). In the next two subsections, I survey the current literature on proposed relativistic invariant quantum theories.

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[35] Bell (1987).

[36] Following Bell, see Tumulka (2006). See also e.g., Ghirardi (2012), Myrvold (2002, 2016, 2021), and references therein.

[37] Some have also maintained that there is the additional problem that a relativistic quantum theory needs to be a field theory (Malament 1996; see also Myrvold 2021). I will not consider this difficulty in this paper because, as I will be evident in the text, the point I wish to make arises independently of this issue.

[38] See Leinert *et al.* (2020), and references therein.

4.1. Relativistic Pilot-Wave-type Theories

All pilot-wave theories are explicitly nonlocal, given that the wavefunction is in configuration space.[39] Moreover, since each particle configuration is taken at the same time, these theories require *a notion of absolute simultaneity*. So, while the Lorentz invariance of the law can be taken care of by having a Dirac evolving wavefunction, one will also need a global spacetime structure. One possibility is to simply postulate one.[40]

There are at least two concerns with this. The first is that such a foliation is unobservable: no one can empirically detect the preferred frame. The worry is that entities which are not detectable in principle are (suitably) irrelevant and can be eliminated. For instance, Galilean spacetime (arguably) comes from the observation that absolute velocities are not detectable. However, in reply I would notice that there is a sense in which the case of the pilot-wave theory resembles more the case of absolute acceleration. Newton argued that, in contrast with absolute velocity, absolute acceleration is needed to explain some observable effects (the water's curved surface in a rotating bucked), even if it is not detectable. Similarly, here we need the 'absolute' foliation to account for relativistic effects. This undetectability comes from not having access to particles' configurations: if one could determine the exact trajectories of particles, then one also could determine the foliation. So, one could simply accept that there are things which are physically inaccessible to us.[41]

The second objection is that this addition seems *ad hoc*: it is added for the only purpose of 'saving the theory,' rather than 'arising 'naturally' from it.[42] Proposals have been put forward to respond to this challenge, the most promising of which argues that the foliation can be suitably extracted from the wavefunction.[43] It is claimed that this way of introducing the foliation is more natural, because all quantum theories, in virtue of having the wavefunction, have this foliation too. However, one could reply that it does not follow that they all have such a preferred frame: the fact that some mathematical object X (in this case the foliation) can be extracted from the wavefunction does not mean that X has some physical significance.

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[39] Notice that this type of theories is nonlocal regardless of the ontological status of the wavefunction. Nonlocality is particularly explicit if one thinks of the wavefunction as a field in configuration space (see e.g., Albert 2015, Ney 2020), or as a part of the structure of the law of interaction among of the particles (Goldstein and Zanghì 2013, Allori 2020b). It is less explicit if one considers the wavefunction to be a multi-filed in three-dimensional space (Forrest 1988, Belot 2012, Hubert and Romano 2017), or tentatively eliminated (Norsen 2015) but the *nonlocality of the interaction* is still there.

[40] Berndl *et al.* (1996), Dürr *et al.* (1999).

[41] See also Maudlin (1996).

[42] Technically, the worry is that one could make anything relativistic invariant (or invariant with respect to any transformation) by adding suitable structure. Take a non-Lorenz invariant theory with primitive ontology P and law L. That means that the 'trajectories' of P (i.e., their worldlines), when transformed according to the Lorentz group, are no longer solutions (that is, they are no longer possible states of affairs of the world). However, we can always add something to the primitive ontology, so that $P' = (P, X)$, in such a way that by stipulation the new law L' would transform solutions into solutions under the Lorentz group. Such theory would be Lorentz invariant, but not genuinely so (see Bell 1987, Berndl *et al.* 1996).

[43] Dürr *et al.* (2013).

Let me make a remark about another pilot-wave-type theory which will be relevant later (section 6). Notice that for Sf (the theory mentioned in section 4.1 with a flash ontology and a Schrödinger evolving wavefunction) one can also construct a Lorentz invariant flash distribution for non-interacting particles whose motion is governed by multi-time Dirac evolving wavefunctions.[44] However, unlike rGRWf (a relativistic GRW-type theory with a flash ontology, see next section) the flashes have a temporal ordering, defining which of two flashes at spacelike distance is earlier and which is later.[45] Thus, this theory also contains some foliation-like structure.[46]

Notice that these theories are relativistic in the sense that their laws are Lorentz invariant (thereby providing the correct dispersion relation), but the *interaction is nonlocal* (which is unavoidable, due to Bell's theorem) and there is a *nonrelativistic spatiotemporal structure* (the preferred foliation), as it reintroduces absolute simultaneity. Because of this, many have argued that these theories are not genuinely relativistic, and GRW-type theories instead provide an improvement in this respect. Let us see what relativistic GRW-type theories look like in the next section.

4.2. Relativistic GRW-type Theories

Bell (1987) noted that GRWf theory was 'as relativistic invariant as it could be' because it displayed relative time-translation invariance: two distant systems can get shifted in time by different amounts.[47] GRWf is relative time-translation invariant, while GRWm is not. In fact, according to GRWf, in an EPR-type of experiment each flash is a spacetime point, which correlates with nothing on the other side. In general, there is *no time ordering* between the events in the two wings: there is only a probability distribution of flashes, which does not depend on whether the events were simultaneous or not. Instead, in GRWm absolute simultaneity can be read off from the matter density: the moment in which the matter density disappears in one location is the same moment in which the detection happens at the other side.[48]

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[44] Tumulka (2007).

[45] This is because the flashes are constructed here in generations, and the distribution of a flash depends upon which of the other flashes belong to the same or the previous generation.

[46] Other relativistic pilot-wave theories have been proposed, but are not viable, as they have no equivariant measure and therefore they have predictions which are inconsistent with quantum mechanics. They all use the multi-time formalism and Lorentz invariant equations for the wavefunction. For instance, in one proposal (Berndl *et al*. 1996, Dewdney and Horton 2001, Nikolić 2005) a current vector defined by the wavefunction generates spatiotemporal paths parametrized by a common parameter. Moreover, another proposal is Lorentz invariant without a foliation, as it uses the light-cone structure as simultaneity structure (Goldstein and Tumulka 2001, Tumulka 2007) but it has a backward microscopic arrow of time (a theories defined on the past light-cone would have been local). This theory does not describe interactions and has no equivariant measure. However, it constitutes an example on how one can use the spacetime structure to achieve nonlocality, by allowing for retrocausality.

[47] This can be seen as the non-relativistic shadow of Lorentz invariance, because it is equivalent to assuming that absolute simultaneity plays no role in the theory: if it would, then the two times in two distant systems could not be shifted independently.

[48] See also Maudlin (2011).

In constructing relativistic GRW-type theories, people used a Dirac multi-time wavefunction to take care of the energy dispersion and the unique time variable in the wavefunction.[49] The remaining challenge is to implement the collapse using only relativistic spacetime structure. This was first successfully achieved in rGRWf.[50] The theory specifies the joint probability distribution of all future flashes where the initial conditions are a wavefunction defined on an arbitrary spacetime slicing and a 'seed flash' for each 'particle.' Instead of universal time, one takes hyperboloids foliating the interior of the light-cone of a seed flash, which assign a proper time to each event. The hyperboloid structure also defines the collapse mechanism, as one specifies a Gaussian function relative to it. This theory is nonlocal: for two 'particles' in an entangled state, the collapse associated with a flash on one particle can change the probability distribution for the flashes of the other particle, regardless of their spacelike separation.

People have also proposed rGRWm, a Lorentz invariant collapse theory where the matter density ontology is defined at each point with respect to the past light-cone at that point. So, now it has become relative time-translation invariant, as it is Lorentz invariant, and foliation-free. This theory is also nonlocal, for similar reasons as rGRWf.

There is no relativistic GRW particle theory yet. However, there seems to be no reason to believe that such an extension cannot be constructed. For instance, one could use N Dirac evolving multi-time wavefunctions and light-cone structures, just like in the case of rGRWm, or hyperboloids like in GRWf. The recent proposal by Tumulka (2020) for relativistic GRW theories with interaction is said to be suitable both for rGRWm and rGRWf, and presumably also for a rGRWp. Similarly, also rGRWp would be nonlocal.[51]

5. The Spirits of Relativity

As we have seen, both relativistic pilot-wave theories and relativistic GRW-type theories have Lorentz invariant laws, but the former introduce a nonrelativistic spatiotemporal structure, while the latter do not. It is in virtue of this fact that it is claimed that GRW-type theories are more compatible with relativity than the pilot-wave theory. In the rest of the paper, I argue that this is not so obviously the case. I start by showing in this section that it is controversial what

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[49] Some other attempts to relativistic GRW-type theories have been proposed which do not use the multi-time wavefunction but still have a Lorentz invariant stochastic evolution (see for instance, Bedingham 2011, Pearle 2015). This is of no consequence for the main conclusion of this paper, as I will show in Section 6, because the stochasticity of the evolution of the wavefunction, regardless of how this stochasticity is implemented, is the relevant ingredient.

[50] Tumulka (2006a). See also Tumulka (2020) for a theory with interaction.

[51] For completeness, let me mention other proposals for relativistic spontaneous collapse theories. Some early proposals are found in Pearle (1990); Ghirardi, Grassi and Pearle (1990) but have been recognized to be problematical (see Bassi and Ghirardi 2020). Dowker and Henson (2004) construct a spontaneous collapse theory which is very similar to rGRWm, but it is defined on a lattice spacetime. It is a theory with a primitive ontology of field values at the lattice sites, and it is Lorentz invariant in the 'correct lattice sense.' Dove and Squires (1995) extend a previous proposal by Hellwig and Kraus (1970) developed in the context of ordinary quantum theory to a flash GRW theory. In this model, the wavefunction collapses along the past light-cone of the spacetime point at which a measurement takes place. However, the theory involves retrocausation, as explained in Tumulka (2009). Also, they only propose a Lorentz-invariant collapse rule for the wavefunction given the flashes, but no distribution law for the flashes. For more discussion on these theories, see Ghirardi and Bassi (2020), Tumulka (2006a,b, 2007).

the spirit of relativity is supposed to be: aside from Lorentz invariant laws, different people have provided different answers.

For some, the spirit of relativity is simply *spacetime.* Thus, if a theory contains an object not in spacetime, such as the wavefunction, then the theory is already in tension with relativity because the wavefunction lives in an abstract, high-dimensional space, rather than space-time. On this reading, only a theory of 'exclusively local beables,' namely a theory in which matter is represented by spatiotemporal entities, could qualify as a candidate for relativistic status.[52] In this sense, neither pilot-wave nor GRW theories (and every other theory which has in its ontology something like a wavefunction) is compatible with relativity.[53]

But this is a radical proposal, even by admission of those who entertain it.[54] Alternatively, one may think that the main ingredient of relativity is the *locality of the interaction*: the fact that there is a limit to the maximum velocity implies that there is no simultaneous action at a distance.[55] Some may think that this has to do with signaling or communication, which cannot exceed the velocity of light. However, this is an anthropomorphic way to talk about locality.[56] In any case, one can understand locality more precisely as discussed in the previous section in terms of Bell's notion of local causality, which captures the idea that physical influence travels continuously and slower than light. Obviously we cannot have this, as all theories violate the local causality condition. Nonetheless, one may still want to make this violation as small as possible, as it is already problematical as it is. Notice that in this understanding of the fundamental feature of relativity, one poses no constraints on spatiotemporal structure. That is, a relativistic theory, in this 'local interaction first' sense, may have a preferred foliation because what matters for a theory to be relativistic is for it to have *interactions which are as local as possible, or the least nonlocal.* This seems to be the sense in which the pilot-wave theories described in section 4.1 are relativistic.[57] In a terminology that I will introduce later, one would want the (nonlocal) theory not to be 'supernonlocal.' In other words, the idea is that, since one cannot have a local theory because suitable two-particle systems would display nonlocal correlations, the less the interaction is nonlocal, the better. Since in pilot-wave theories there is no nonlocality in one-particle systems, while in GRW-type theory there is (see Section 8.4), the former is more compatible with relativity, if one considers the locality of interaction as the true spirit of relativity.

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[52] Norsen (2010a).

[53] It seems important to notice that the situation may be made less severe in this respect by considering the wavefunction as a multi-field in three-dimensional space, or not as part of the ontology of matter (for instance, one could consider the wavefunction part of the law of these theories). Similarly, Rovelli's relational interpretation (1996) or Bub and Pitowsky information-theoretic account of quantum theory (2010), seem to be in a better position in this respect (even if by considering the wavefunction as epistemic they run into other problems, see e.g. Norsen 2016). Nonetheless, in addition to the ontology being local, one would also have to face the fact that the interaction is nonlocal, and this is what Bell has shown. Therefore, the advantage of these formulations seems minimal in this respect.

[54] Norsen, p.c.

[55] That is, no 'spooky action at a distance' (Einstein, Born 1971).

[56] This idea has been criticized by Maudlin (2011).

[57] Dürr *et al.* (2013).

Another possibility, which seems to be the most widely entertained, is to think of the essence of relativity as its *spacetime structure*: "a theory is compatible with Relativity if it can be formulated without ascribing to space-time any more or different intrinsic structure than the (special or general) relativistic metric."[58] So, a theory is relativistic if it uses only relativistic spacetime structure, for instance the light-cone structure. A nonlocal theory can be relativistic in this way if the nonlocal interaction happens *via* that spacetime structure. Let's call this view 'spacetime structure first' view of relativity. In this approach, notions like absolute simultaneity for instance find no place in a relativistic theory, and therefore also a preferred spatiotemporal foliation, which amounts to reintroduce absolute simultaneity. Arguably, as anticipated, this is what people have in mind when they claim that GRW-type theories are more compatible with relativity than pilot-wave theory: unlike pilot-wave theories, these theories do not have a preferred spatiotemporal foliation.[59]

Let me make this point in another way. One may be puzzled by the distinction between respecting local interactions and preserving relativistic spacetime structure: isn't locality, the idea that what happens here does not instantaneously and discontinuously influence what happens there, captured by having the light-cone structure? The answer is negative. In fact, while it is true that the influence between two systems is local if it remains confined in the light-cone, one may still have nonlocal influences. That is, in some relativistic pilot-wave theories one has nonlocal interactions on top of a non-relativistic spacetime structure, such as a spatiotemporal foliation. Also, there are theories, like relativistic GRW-type theories, which are nonlocal but also have a relativistic spacetime structure like the light-cone structure. In other words, one could in principle have the following possible options:

1) A theory with local interaction and exclusively relativistic spatiotemporal structure;
2) A theory with local interaction and nonrelativistic spatiotemporal structure;
3) A theory with nonlocal interaction and exclusively relativistic spatiotemporal structure (e.g. relativistic GRW-type theories);
4) A theory with nonlocal interaction and nonrelativistic spatiotemporal structure (e.g. relativistic pilot-wave theories).

Because of Bell's theorem, only 3 and 4 are living options, and in this context, it seems natural to think that 3 is to be preferred over 4. Indeed, this is what people have been insisting on: one cannot have locality of interaction, so one would like to have a theory which is 'as relativistic as possible,' meaning it is nonlocal but at least it does not possess any nonrelativistic spatiotemporal structure.

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[58] Maudlin (1996); see also Maudlin (2011). This understanding of 'spirit of relativity' is shared also by Tumulka (see, e.g., 2006, 2009) and Myrvold (see, e.g., 2016, 2021).

[59] A remark on the meaning of 'foliation-free theory.' This terminology should be taken to mean that there is no preferred foliation with a dynamical role. That is, the dynamical laws do not involve any such structure. In particular, as mentioned earlier, the fact that one could extract a preferred foliation from the wavefunction, does not imply that every quantum theory, in virtue of having a wavefunction, will also have a preferred foliation, because such foliation could be not dynamical.

Instead, in the rest of the paper I argue that one can have a relativistic spatiotemporal structure only if one *also* allows for a more nonlocal type of interaction (the theories are supernonlocal, in which one sees nonlocality also with one particle). So, the true options are not 3 and 4 but rather:

3*) A theory with *supernonlocal* interaction and exclusively relativistic spatiotemporal structure (relativistic GRW theories);

4*) A theory with nonlocal interaction and *nonrelativistic* spatiotemporal structure (relativistic pilot-wave theories).

It is not obvious to say which one is to be preferred because, as I argue later, the supernonlocality which arises by only allowing for relativistic spatiotemporal structure, goes hand in hand with the disappearance of cause and effect. To understand this, in the next sections I explore the reason why relativistic GRW-type theories manage to require no preferred foliation.

Before continuing, let me stress that here I have just examined two common ideas about what relativity really requires, namely (i) a constraint on spacetime structure (namely, that nothing beyond the relativistic structure is involved) and (ii) some notion of local interaction. Nonetheless, the conclusions of this paper should be taken in light of the fact that it is unclear what relativity really requires.[60]

6. Why Do Relativistic GRW-type Theories Require No Foliation?

The interesting question now is: *What exactly allows GRW-type theories to accomplish the task of being nonlocal but only use relativistic spacetime structure* in their formulation?

Let's explore this by systematically list the various aspects of rGRWf and rGRWm and see if they can be held individually responsible. First, is it the multi-time formalism? The answer is negative: some foliation-types theories use it too (see, e.g., the relativistic extension of Sf mentioned at the end of section 4.1).[61] Is it the flash ontology? Because of the discreteness of the flash ontology the temporal ordering of events is not fundamental.[62] Nonetheless, this ontology is not necessary: rGRWm needs no foliation too. And it is not sufficient either: the relativistic extension proposed for Sf has a foliation-like structure. Also, it cannot be the type of relativistic spacetime structure used: rGRWf uses hyperboloids, while rGRWm uses past light-cones. Is it because in rGRWm and in rGRWf the primitive ontology is defined in terms of the wavefunction, in contrast with pilot-wave theories, in which the particle ontology is defined independently?[63] In this regard, it would be nice to see how a Lorentz invariant GRWp could be

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[60] See e.g., Dürr *et al.* (2016) on this point.

[61] Also, as mentioned already, some relativistic GRW-type theories do not use this formalism, see Bedingham (2011), Pearle (2015).

[62] Tumulka (2009).

[63] In GRWf and GRWm, as well as in rGRWf and rGRWm, the flashes are distributed as dictated by the collapses of the wavefunction and the matter density is defined as
$m(x,t)=\sum_{i=1}^{N} m_i \int dq_1 \ldots dq_N \delta^3(q_i-x)|\psi_t(q_1 \ldots q_N,t)|^2$, where N is the number of 'particles,' and m_i are their masses.

explicitly constructed, as in this case the primitive ontology of particle would not be defined in terms of the wavefunction. However, there is no obvious reason to think that the foliation-free schema to generalize rGRW-type theories with interaction, which works for rGRWm and rGRWf, would not work for rGRWp. If so, the trick of using only relativistic spatiotemporal structure is not accomplished by the nature of the primitive ontology.

Is it the stochasticity of the law? All rGRW theories considered here are doubly stochastic (both in the evolution of the wavefunction and of the primitive ontology), so which stochastic law may be doing the trick? For doubly deterministic theories, like the pilot-wave theory,[64] it turns out that one gets a nonlocal theory only if one uses the future light-cones (rather than past light-cones), and therefore it is retrocausal.[65] One could try to use deterministic wavefunction evolution and stochastic primitive ontology, like in Sf, but there would still be a foliation-like structure. So, this gives support to the idea that *stochasticity is the key ingredient to get a relativistic foliation-free theory*. A proof that stochasticity is required for preserving relativistic spatiotemporal structure is reported in the next section.

Before we see that though, let us see if the guidance law for the primitive ontology can give us some further insight. In rGRWm and rGRWf, this law is stochastic too. One may wonder whether one could dispense of the double stochasticity of these theories but, as we have seen, a wavefunction evolving stochastically (a 'stochastic wavefunction') only combines with a primitive ontology evolving stochastically (a 'stochastic primitive ontology'): flashes are intrinsically stochastic, the matter density in GRWm inherits the stochasticity of its evolution from the stochasticity of the evolution of the wavefunction, while the particles in GRWp have to evolve stochastically to preserve equivariance.[66] So, our current GRW-type theories are doubly stochastic. Notice that if one does not endorse the primitive ontology approach, their task is easier in this respect, as they do not have to deal with the evolution equation for the primitive ontology.

7. Can We Have a Relativistic, Deterministic, and Foliation-Free Theory?

In the previous section we saw that stochasticity is the relevant ingredient in our current relativistic, foliation-free theories. At this point one may ask *whether we can do better than this*: Could a deterministic theory violate Bell's inequality and use only relativistic spatiotemporal structure? We do not have such a theory yet, but maybe in the future we will.
Notice that noticing the connection between stochasticity and relativistic invariance understood in the spatiotemporal sense is not something new. For a very long time, people have argued that there are two ways of violating Bell's inequality. One can rewrite the locality condition in Bell's inequality in terms of two conditions which have been called 'parameter independence' (PI) and

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[64] They are doubly deterministic in the sense that both the particles and the wavefunction evolve deterministically (respectively according to the guidance equation and the Schrödinger equation).

[65] As discussed in Goldstein and Tumulka (2001), and in Tumulka (2007), a theory using past light-cones is local.

[66] See Allori (2020a) and references therein.

'outcome independence' (OI).[67] PI states that in an EPR-type experiment the probability of a given outcome depends on the settings of the apparatus that measured it, and not on the settings of the other experiment. Instead, OI states that the probability of one outcome is independent of the other outcome.

In deterministic theories, like the pilot-wave theory, the locality condition reduces to PI, because there is only one outcome. So, given the violation of Bell's inequality, they respect OI and must violate PI. Instead, GRW-like theories respect PI and violate OI by construction: the probability of outcomes on one side does not depend on whether on the other side a measurement is performed or not.[68] Some have argued that violating OI rather than PI allows for a 'peaceful coexistence' between nonlocality and relativity. The main idea is that violating OI preserves the spatiotemporal structure, in contrast with violating PI.[69]

The notions of PI and OI have been criticized as too operationalists or arbitrary to play a central role in answering foundational questions about what a theory should look like.[70] While I agree with that, however, this seems too much of a coincidence: deterministic theories 'just happen' to violate PI *and* have a foliation, and stochastic theories 'just happen' to violate OI *and* require no foliation. One is led to wonder whether there is a connection between violating PI and needing a foliation, and violating OI and not needing one, and what this connection is. It turns out that this connection is given by a theorem by Ghirardi and Grassi (1996), which shows that any theory violating PI must have a preferred foliation. Here is a brief sketch of the proof. Take an EPR experiment and a frame O in which the experiment on the right, R, is performed earlier than the experiment on the left, L. If we have parameter dependence, then we can only have it on L, because R, being in the past, cannot be influenced by the future. Therefore, the probabilities of the outcomes of the left depend on the settings on the right while the probabilities of the outcomes of the right do not depend on the settings on the left. However, if we take another frame O' in which the order of L and R is reversed, and thus their mutual dependencies are opposite. However, assuming that probabilities are objective, in the sense that the experimenters agree on them (the probabilities of the outcomes in O' should be the same as in O), we have contradiction: in O, L-outcomes depend R-settings, but R-outcomes do not depend on L-settings; in O' the opposite is true. Therefore, the only option to get out of the contradiction is to add a preferred frame.[71] Notice that the dynamics of the primitive ontology does not change the situation, as it is the wavefunction which determines the probability of the experimental outcomes, hence whether a theory violates PI or OI.

Anyway, if this argument is sound, then one can conclude that all deterministic theories need a preferred foliation, and therefore the route for a foliation-free, Lorentz invariant nonlocal theory

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[67] The first to propose to rewrite the locality condition in terms of two different conditions was Jarrett (1984), and he distinguished between 'simple locality' and 'completeness.' His conclusions have been criticized especially by Norsen (2009) and Maudlin (2011). The terminology 'parameter independence' and 'outcome independence' is by Shimony (1984) and seem to better characterize the content of these conditions.

[68] See, e.g., the first part of the proof in Butterfield *et al.* (1993) as reported in Norsen (2010b).

[69] Jarrett (1984), Shimony (1984); see also, e.g., Redhead (1987), Cushing and McMullin (1989), Butterfield (1992), Ghirardi (2012), Myrvold (2016), Bassi and Ghirardi (2020), and references therein.

[70] Maudlin (2011), Goldstein p.c., Norsen p.c.

[71] Assuming there is no retrocausation. See also Ghirardi *et al.* (1993).

is to violate OI. That is, given Ghirardi and Grassi's result, we can read this to mean that *only stochastic theories may be foliation free*. In other words, one can translate the various 'macroscopic' violations of Bell's inequality, usually cashed out in terms of violation of PI or OI, in a more fundamental, or 'microscopic,' terminology, in terms of what the dynamics, the ontology and the spatiotemporal structure are. In fact, one could say that all deterministic theories violate Bell's inequality by introducing a preferred foliation (this corresponds to a violation of PI). Instead, some stochastic theories (like GRW-type theories) violate Bell's inequality without the need of a preferred foliation in virtue of their stochastic laws (they violate OI).

8. Comparison

The analysis in the previous sections strongly suggests that a foliation-free relativistic theory can be implemented with any primitive ontology but has to be stochastic. That is, relativistic GRW-type theories are the prototype of such a theory.

In this section, I discuss some features of GRW-type theories, and I argue that the stochasticity of these theories brings with it many unwelcome consequences: in addition to possibly providing an unfamiliar ontology, the biggest drawbacks are that the distinction between cause and effect seems to vanish, and they show a greater degree of nonlocality in the interaction. Because of these features, I believe it is premature to claim that GRW-type theories should be preferred to pilot-wave theories, and perhaps we should give more credit to the possibility of thinking of relativity as nonfundamental.

8.1. Unfamiliar Ontology

Both rGRWf and rGRWm provide us with an unfamiliar ontology.[72] In fact, if the world is described by rGRWm (or GRWm), then it is somewhat 'too continuous:' the matter density, being defined in terms of the wavefunction, inherits its tails, and therefore macroscopic solid objects have low-matter density tails which other macroscopic solid objects could cross without being subject to any actual interaction, contrary to expectations.[73] If instead rGRWf (or GRWf) is true, the world is instead 'too discrete:' material objects are made up of spatiotemporal events, and therefore are mostly 'empty.'[74] Thus, the flash ontology clashes with our intuitive idea that macroscopic or even mesoscopic objects, such as large molecules, exists continuously. In the GRWf picture, they do not: the world is discrete not only in space, like in the case of the atomic picture, but also in time.

One could reply to this charge by pointing out the need of balancing common sense with the theory explanatory power: sometimes a satisfactory explanation leads us far from what we intuitively believe to exist. For instance, atomic theory provides a better explanation than any more commonsensical continuous matter theory.[75] However, in the case of GRWf it is still

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[72] Lewis (2020) and Esfeld (2020), for instance, have argued that their primitive ontology is not as familiar as a particle ontology, regardless of the status of the wavefunction.

[73] See also Lewis (2003), McQuinn (2015) and references therein for more.

[74] For a small macroscopic object made of roughly 10^{19} 'particles,' if each 'particle' undergoes a collapse every 10^{16} seconds, there is a flash roughly once every 10^{-3} seconds, and *nothing at all* in between.

[75] See Allori (2020a) for more, and Esfeld (2020) for a defense of the flash ontology.

puzzling that while matter is discrete, the gravitational and electromagnetic fields will be instead continuous in time.[76] Moreover, while in the case of a matter density ontology one can still think of having approximate 'particle' trajectories in some suitable mesoscopic regime, this seems more difficult in the case of the flashes, given the numbers above.[77]

While these theories have an unfamiliar ontology, the pilot-wave theory has the simplest ontology ever, namely particles. However, a GRW particle theory does exist, namely GRWp. If it can be made relativistic, and I see no reason why one could not, then this unfamiliar ontology objection would dissipate.

As a final remark, let me add that even if one does not endorse the primitive ontology approach and believes that GRW-type theories are theories of the wavefunction, the unfamiliarity charge still stands, as the wavefunction is an object in a high dimensional space, rather than in the more commonsensical three-dimensional space.[78]

8.2. Stochasticity of the Laws

If the analysis in the previous session is correct, then the price to pay for having a nonlocal, Lorentz invariant, foliation-free theory is its stochasticity. This seems a serious disadvantage over pilot-wave theories, which are deterministic.[79] Having deterministic laws seems better because, apart from being simpler, they straightforwardly allow for mechanistic (or causal or in general reductionist) explanations in which the macroscopic phenomena are understood in terms of the microscopic dynamics.

Instead, the situation with indeterministic laws is more complex. What type of explanation does an indeterministic theory provide? Is it satisfactory? Most famously Einstein claimed that God "does not play dice,"[80] suggesting to some that a deterministic theory provides a better explanation of the phenomena. In any case, the intuition that deterministic theories are more explanatory is connected with the 'principle of uniformity of causation,' according to which the same cause produces the same effect. This principle does not hold for theories with a stochastic law: the same event, the cause, may be followed by many effects, each with a different probability. One open question is whether one can formulate the notion of causality to make sense of some sort of probabilistic causality.

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[76] However, within the primitive ontology framework some have maintained that fields should be seen as part of the law too, just as the wavefunction (Allori 2015). If so, the objection seems less severe.

[77] However, Allori (2018) has argued that this is still possible.

[78] See e.g., Monton (2002, 2006, 2013), Allori *et al.* (2008, 2011), Albert (2013, 2015), Allori (2013b), Emery (2017), Maudlin (2019). See replies in Albert (2015), Ney (2021) and references therein. Notice however, that these replies aim at recovering the appearances of macroscopic three-dimensional objects from the wavefunction, so they do not dispute the unfamiliarity of their ontology.

[79] At east most of them, if one does not consider, for instance, Bell-type quantum field theories.

[80] This is in a 1926 letter to Born; in Born and Einstein (1971).

A possible alternative would be to think of the indeterministic laws as ascribing 'physical,' ontic, probabilities to physical objects. For instance, one could understand probabilities as a kind of disposition, or a propensity, to explain things causally: what explains the localization of an object here is the disposition of its wavefunction to collapse here.[81]

8.3. Symmetry between Cause and Effect

Nonetheless, in relativistic GRW-type theories we have more than a violation of the principle of uniformity of causation. In fact, in GRW-type theories the distinction between cause and effect as we commonly understand it seems to vanish, so that this principle does not even apply. To see this, consider an EPR-type of experiment. In pilot-wave theories, since there is a preferred foliation, the settings on the side which measures first can be thought of as influencing the outcomes on the other side. In contrast, because of their stochasticity, in GRW-type theories cause and effect can be thought of as symmetrical: nothing can be regarded as the cause and the other as the effect in an obvious way. In fact, by no longer having continuous worldlines for the primitive ontology (for instance making the ontology discrete in both time and space as in GRWf, or having the primitive ontology jump, like in GRWm and GRWp) or in general by having the wavefunction jump (in theories without a primitive ontology) it is difficult to say in what sense an event has caused another.[82]

In addition of violating the principle of uniformity of causation, a symmetry between cause and effect seems to directly conflict with the ideas in both probabilistic causality and the propensity accounts, in which respectively, one still thinks of the cause as preceding the effect. Thus, it is unclear whether these approaches can survive relativistic GRW-type theories.
Alternatively, typicality approaches, which do not rely either on dispositions or probabilistic causality, have been proposed to explain macroscopic laws,[83] but more needs to be said to generalize these in the case of stochastic laws.[84]
Otherwise, one could argue that the type of explanation provided by relativistic GRW-type theories is more in the spirit of unification.[85] While deterministic theories are able to provide a reductive, constructive or dynamical explanation, relativistic GRW-type theories unify the phenomena, in virtue of their greater compatibility with relativity. While this idea may initially seem promising, the deductive character of current unification models clashes with the stochasticity of the laws.[86]

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[81] For propensities see Popper (1957). For more discussion, see also, among others, Frigg and Hoefer (2007), Dorato and Esfeld (2010), Esfeld and Gisin (2010), Monton (2013), Egg and Esfeld (2015), and Esfeld *et al.* (2017).

[82] See also Ghirardi and Grassi (1996), Esfeld and Gisin (2014), Myrvold (2016).

[83] Goldstein (2012), Lazarovoci and Reicher (2015), Wilhelm (2019), Bricmont (2020).

[84] However, see Allori (2019b).

[85] See also Einstein's distinction between constructive and principle theories (1919). Constructive theories are theories in which the macroscopic phenomena are accounted for in terms of the microscopic dynamics, while principle theories constrain the phenomena with principles which govern what can and cannot happen. Flores (1999) has argued that constructive theories are compatible with Salmon's mechanistic explanation (1987), principle theories can be tied to unification approaches such as the ones of Friedman (1974), Kitcher (1989). See also Felline (2011).

[86] The current models of explanatory unification generalize the idea that explanations are arguments (developed by the deductive-nomological model), and therefore are committed to the so-called 'expectability thesis:' a unifying

Anyway, it is important to notice the following: whether or not one can make sense of stochastic explanations, *the fact that in GRW-type theories the notions of cause and effect become symmetric threatens the very idea of influence.* In fact, how can we make sense of the notion of influence, if we cannot even identify which one is the cause and which one is the effect?

While it is true that Bell's definition of local causality is applicable to stochastic theories, as obvious given we say that GRW-type theories are nonlocal, still the intuitive notion of locality it is supposed to capture in terms of cause and effect is gone, and this should be recognized.

One could argue that there is still room for the notion of cause and effect, at least in some frame-relative sense. In fact, one could argue, take an EPR experiment where Alice measures her particle first. Her doing that can be seen as the cause of the result on Bob's side. However, in reply notice that, in relativistic GRW-type theories there is a perfectly legitimate frame in which Bob performs his experiment first, and thus it will be his measurement to be the cause of the result on Alice's side. In other words, there are frames in which one event is the cause and frames in which the same event is the effect. Since there is no privileged frame, these frames are equally legitimate and thus cause and effect have no absolute meaning.[87]

Moreover, it seems that even if we could save the notion of causation or influence, we have an instance of super-luminal causation, or super-luminal influence.[88] It has been argued instead that what happens in these theories is better understood as an instance of 'nonlocal probabilistic correlation.' As we have seen, in GRW-type theories frames may disagree about which event is the cause and which is the effect. If we consider the frame in which Alice measures her particle first, this event will be the cause of Bob's getting whatever result he got, and the opposite is true in the frame in which Bob measure first. Nonetheless, this implies that the probability distributions are frame-dependent: in the frame in which Alice measures first, then Bob's measurement is determined with probability 1; however, in the frame in which Bob measures first, then his result is not determined.[89] Also, it is argued that there is no conflict with relativity "so long as every reference frame agrees on the probability assigned to any set of outcomes of a set of spacelike-separated measurement event. This ensures that the statistics of a repeated series of measurements will not distinguish between reference frames (or between alternate foliations)."[90]

Notice however that these considerations are unhelpful from the point of view of the primitive ontology approach, as what matters in this framework is the actual distribution of the spatiotemporal events (the primitive ontology) which is supposed to account for the observed probability distributions. One could argue that this is exactly right, and that the primitive

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explanation must show how the explanandum is to be expected from the explanans. With stochastic laws, events with a low probability of happening cannot be expected. One possible reply is to use van Camp's proposal (2011) that principle theories unify in the sense that they provide the conceptual schema necessary to talk about explanation within a theory in the first place. The idea is that principle theories play a distinctive conceptual role as necessary preconditions for explanation (see also DiSalle 2006).

[87] Tumulka (2006b) similarly argues that the direction of influence is indeterminate.

[88] See e.g., Lewis (2021).

[89] See Myrvold (2002) for the original argument. See also Myrvold (2016, 2019, 2021).

[90] Myrvold (2002).

ontology approach is merely a burden in trying to find a relativistic quantum theory. Regardless, notice that even accepting this conclusion would not remove the fact that cause and effect are symmetric in these theories, and that the notion of local influence seem to have no room.

Arguably, therefore, in GRW-type theories *the interaction notion of locality is completely lost*. This goes beyond what happens in the pilot-wave theory, as effects always follow their causes. Instead, implementing relativistic spatiotemporal structure with stochastic laws as done by GRW-type theories seems to get local influence entirely out of the picture for the reason we have just seen. This has an important consequence: *if the original idea for considering GRW- type theories was to find a way in which quantum mechanics and relativity could peacefully coexist, then while these theories are relativistic in the sense of respecting relativistic spatiotemporal structure, they leave little room for the other spirit, namely the locality of the interaction.*

8.4. Supernonlocality

There is another way in which GRW-type theories show their greater departure from the ideal of a local interaction when compared to pilot-wave theories. Again, all these theories are nonlocal. However, I am going to show in this section, GRWm and GRWf (as well as their Lorentz invariant counterparts, and the other relativistic GRW-type theories) also show a feature which one can dub 'supernonlocality.' That is, they display nonlocal behavior for systems which would be local according to pilot-wave theories, like 'one particle' systems. This behavior has been noted by several commentators[91] but, quite surprisingly, only Bricmont (2016b) finds it problematical.

Consider an experiment with a single 'particle' confined in a box. Someone inserts a barrier in the box, thus splitting the 'particle''s wavefunction in two. The two half-boxes are then separated, at which point one of them is opened.[92] According to the pilot-wave theory, in such a situation the 'particle' is simply in one of the half-boxes all the time.
In rGRWm, given it is only one 'particle,' no wavefunction collapse has happened and the matter density is half in one half-box and half in the other. Suppose now someone opens, say, the half-box on the right. The 'particle' gets coupled with the measuring device, and the wavefunction most likely collapses. That is, the matter density all jumps in one of the half-boxes. Since the process is random, either the part on the right jumps to the left (then nothing is found opening the right half-box); or it all jumps to the right (then the 'particle' is found). In both cases there is nonlocal matter transfer even with one 'particle.'[93] Nothing like this happens in the pilot-wave theory, hence the name 'supenonlocality:' to see nonlocal effects one needs at least to have two particles, while here we only need one particle.
In GRWf, given the discrete nature of the ontology, there is nothing in both half-boxes until someone opens one of them. At that time, the wavefunction gets coupled with the macroscopic

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[91] See e.g., Maudlin (2011), Bedingham *et al.* (2013), Bassi and Ghirardi (2020).

[92] This experiment has been first proposed by Einstein; see Norsen (2005).

[93] Bricmont (2016b).

system, which leads to the wavefunction collapse, hence generating a flash, which randomly appears in one of the half-boxes. Again, the theory is supernonlocal.[94]
Since the stochasticity of the evolution is at the root of supernonlocality, it should not be surprising that GRWp is supernonlocal too. In the Einstein boxes experiment, the particle is only in one of the half-boxes and the wavefunction has likely not collapsed until someone opens one of the half-boxes. When that happens, due to the coupling with the device, the wavefunction collapses and the particle follows (even if it is displaced at random, it jumps in the box in which the wavefunction has collapsed).

One could reply that these examples do not show that there is nonlocal matter transfer. One could say they are similar to shining a laser pointer across the face of the Moon: the spot can move faster than light, but 'the spot' is not really a thing that moves.[95] However, this is not an option available for these theories, as it implies that the matter density, the flashes and the particles are not really things that move, which is the opposite of what the primitive ontology approach prescribes. Notice also that theories without a primitive ontology will have the same supernonlocal feature: the wavefunction will jump, and it is the only thing there is to represent matter.

One could still insist that being supernonlocal is not making the situation worse for compatibility of relativity. In fact, either a theory is compatible with relativity, or it is not, and it is not a matter of how many times a theory shows such an incompatibility. Both theories are nonlocal, and the fact that a supernonlocal theory would display nonlocality in more situations does not make it worse.[96] In reply, I think this way of putting the situation is misleading. In fact, it is a matter of how many instances of nonlocal behavior one has in a theory. Rather, the problem has to do with the situations in which one has such a nonlocal behavior. While in the pilot-wave theory one needs at least two particles, in GRW-type theories are nonlocal even for one 'particle.' This is problematical because such nonlocality is not needed (as shown in the case of the pilot-wave theories), and it does not seem to be a good scientific practice to have theories which are more radical than it is necessary. In addition, nonlocality is commonly understood as a feature of composed systems: when you have two or more particles, they influence one another in a nonlocal way. Instead, since by definition supernonlocality happens when there is only one system, it is unclear what the notion of locality would even mean in these theories. It is not a matter of influence anymore, there is no interaction with one system. So, if locality is to be understood as a property of the type of interaction there is, GRW seems to undermine the very notion of locality understood in these terms, another instance of GRW undermining the notion of influence.

Otherwise, one could claim that superlocality is a small price to pay if, by allowing it, we get closer to a relativistic quantum theory. However, I think that, similarly to what I have

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[94] Maudlin (2011), Bricmont (2016b).

[95] Norsen p.c.

[96] I am grateful to an anonymous reviewer for pressing me on this point.

emphasized above in the case of stochasticity, *supernonlocality seem to undermine the whole motivation for GRW-type theories*. In fact, someone may be attracted to GRW-type theories to get quantum theory and relativity closer in the 'spacetime structure first' approach. However, again, since these theories are supernonlocal, they get quantum theory and relativity further apart in the other spirit of relativity, namely with respect to the locality of interaction. In other words, if the idea is to find the best balance between the various senses of 'relativistic theory' (locality of interaction and relativistic spacetime structure), then *relativistic GRW-type theories may gain in the latter but lose in the former*. Because of this reason, I believe that the alleged advantage of relativistic GRW theories in 'respecting relativity' rests on shaky grounds.

9. Assessment

Let's begin by summarizing what has been done. After briefly reviewing quantum theories, I have discussed what the spirit of relativity might be, and what could it mean for a quantum theory to be 'relativistic,' given the violation of Bell's inequality makes all options nonlocal. I have distinguished two, even if non-exhaustive, possibilities, over and above the Lorentz invariance of the laws. First, one could think that a theory is relativistic if it respects local causality. If so, since Bell has shown us that we cannot have that, relativistic effects have to suitably emerge in a world in which relativity is not fundamental. This leads to theories with a preferred foliation, such as the relativistic pilot-wave theories. These theories provide a rather straightforward framework: they have the simplest ontology, namely particles; they can be deterministic (both in the wavefunction, and the primitive ontology); they are nonlocal but not 'supernonlocal,' meaning that nonlocal effect show with at least two particles. That is, they are *as local as you can hope*, given that the world is nonlocal. Their weakness is that they have preferred foliation (either postulated or dynamical) which is in principle undetectable. However, adding a foliation, which amounts to add an absolute frame, is an actual cost only if you think of relativity as fundamental. If one is willing to accept instead that relativity is not fundamental, then these theories are very natural, as a preferred foliation is already in the cards due to nonlocality.

However, this is too much for most people, some of which then look at GRW-type theories with interest. They are nonlocal, stochastic, but also foliation-free. Because they do not need any nonrelativistic spatiotemporal structure, their main claimed advantage over pilot-wave theories is their deeper compatibility with relativity. However, I have shown how they symmetrize the notion of cause and effect, and they are supernonlocal. These features are particularly troublesome, I have argued, as they undermine the motivation of whole project, namely greater compatibility with relativity. In fact, the original idea was to bring quantum theory and relativity closer together. GRW-type theories do this by respecting relativistic spatiotemporal structure using stochastic laws. However, this unfortunately backfires, as it brings them further apart from the perspective of locality. So, while respecting the relativistic spacetime structure, they are *more nonlocal than needed.*

To conclude and summarize, I believe that thinking of relativistic pilot-wave theories as empirically relativistic and relativistic GRW-type theories as fundamentally relativistic, as sometimes suggested even by those who agree that both these theories are nonlocal, is misleading: it is unclear what ‘fully’ or ‘genuinely’ or ‘fundamentally’ relativistic means, and regardless both violate one important feature of relativity. Within our current understanding, it is like ‘genuine relativistic invariance’ is a blanket which is too short, given quantum nonlocality: *if you manage to make a theory as local as possible in the sense of the interaction, then you lose relativistic spacetime structure (like in relativistic pilot-wave theories), while if you preserve the relativistic spacetime structure then you lose all the locality you could have left in the interaction (like in relativistic GRW-type theories).* In other words, relativistic pilot-wave theories have a preferred frame, which is ‘against’ the relativistic spatiotemporal structure, but they are ‘as local as possible,’ in the sense that the interaction is not supernonlocal. Relativistic GRW-theories respect the relativistic spatiotemporal structure by making cause and effect symmetric, but they end up giving up on locality of interaction entirely, as also shown by their supernonlocality. While the fact that relativistic pilot-wave theories were not respecting the relativistic spatiotemporal structure was already well understood, the fact that relativistic GRW-type theories lose all the locality one can have in a nonlocal theory was not, and this is part of what I was clarifying in this paper.

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