ON THE INCOHERENCE OF AGNOSTICISM

[Preliminary Draft: Comments Welcome. Please do not cite without permission.]

1. Introduction.

Almost everyone hedges bets on God's existence. It is difficult to find a theist that is absolutely certain that God exists or an atheist that is absolutely certain that God does not exist. Most believers and non-believers place some positive probability on the existence of God and some positive probability on the non-existence of God. If square theism is the view that it is absolutely certain that God exists, then most theists are not square theists. And if square atheism is the view that it is absolutely certain that God does not exist, then most atheists are not square atheists.

This is not at all surprising, given our evidential situation relative to the existence of God. Square theism has the unwarranted implication that no new evidence could diminish your belief in the existence of God. But it certainly seems like there at least might be such evidence. The epistemic situation is the same for square atheism. It too has the unwarranted implication that, no matter what new information becomes available, atheism is certain. These extreme positions are not epistemically defensible.

In section (2) I argue that basic principles of epistemic probability entail that, for any modal proposition $\Box p$ or $\Diamond p$, if the epistemic probability of $\Box p$ or $\Diamond p$ is positive, then the epistemic probability of $\Box p$ or $\Diamond p$ is 1.

If it is not epistemically certain that $\sim\Diamond p$ then it is epistemically certain that $\Diamond p$. Further, for any proposition q that entails a modal proposition $\Box p$ or $\Diamond p$, if the epistemic probability of q is positive, then the epistemic probability of $\Box p$ or $\Diamond p$ is 1. Finally, for any modal proposition $\Box p$, if $\Box p$ has a positive epistemic probability, then the epistemic probability of p is 1.

In section (3) I provide an argument that epistemic agents cannot hedge their bets on the existence of God. An agent that assigns a positive epistemic probability to the existence of God on their evidence E must assign epistemic probability 1 to the existence of God. The argument assumes that epistemic agents know the axioms of probability and know the axioms of S5. It is also assumed that they know the basic principles of epistemic probability T4, T4.1, T5, and T6. The unsettling conclusion is that theism requires full commitment to the existence of God and atheism requires full commitment to the non-existence of God. There is no coherent epistemic attitude to the existence of God between these extreme attitudes.

2. Epistemic Probability

The thesis in T1 expresses what seems to be a trivial relation between modal propositions and objective probabilities.

T1. If a proposition p is metaphysically or logically impossible then the probability that p is true is 0.

The impossibility in T1 is metaphysical impossibility. A proposition is impossible if and only if the proposition is false in every possible world. There are no possible circumstances or conditions under which it is true. Propositions that are impossible standardly (sometimes exclusively) have 0 probability.

The probability in T1 is objective probability or chance. Objective probabilities are typically characterized as real features of the world. We find them, for instance, in how often people contract diseases, how probable it is that a coin will fall heads and how probable it is that an atom of radium will decay in the next 1600 years. Whether or not we come to know or believe the objective probability of contracting a disease, the chances of contracting that disease remains the same.[1] The objective probability of p is 0 if p is metaphysically or logically impossible.

It is similarly trivial that metaphysically or logically necessary propositions are certain to be true. If every impossible proposition p has probability 0 then every necessary proposition ~p has probability 1.

T2. If a proposition p is metaphysically or logically necessary then the probability that p is true is 1.

A proposition is metaphysically necessary if and only if it is true in every possible world. There are no possible circumstances or conditions under which it is false. The probability in T2 is also objective probability or chance.

The contrapositive of T1 states that any proposition that has a positive probability is not impossible. T3 also concerns the relations between metaphysical possibility and objective probabilities.

T3. If the probability of p is positive, then p is (at least) possible.

The thesis in T3 tells us that any proposition p that has some positive chance of being true is true in some possible world. If there is no world in all of metaphysical space in which p is true, then it has no chance of being true.

T1 – T3 do not directly govern epistemic probability or the probability agents assign to a proposition on the basis of her evidence E. The epistemic probability of p on E is one's estimate of how probable p is given the data in E and it can be true that your estimate of the probability of p on E violates T1 – T3. Suppose that E is your evidence for a false mathematical proposition p, for instance. E might include your (mistaken) proof that p. You might assign a positive epistemic probability to p, given your evidence E, despite the fact that p is impossible. Suppose E is the evidence against a true mathematical proposition p. E might include your (mistaken) disproof that p. You might assign no positive probability to p given E, despite the fact that p is necessarily true.

The epistemic probability that a proposition p is true is not in general p's objective probability. Once a coin has landed heads, the objective probability that a coin has landed heads is extremely high—some say 1—but your epistemic probability might not be high at all. You might have little or no evidence of how the coin landed. It might be true that the objective probability or chance that the universe had a beginning is 0, if it never had a beginning. Still, on the evidence we possess, there is a reasonable probability that the universe had a beginning. The probabilities of the rival hypotheses of 'big bang and 'steady state', for instance, are epistemic probabilities. These epistemic probabilities are a measure of the confirmation current evidence provides for these alternative hypotheses.

An agent's epistemic probability for p is also not an agent's credence or degree of belief for p.[2] An agent's credence that p is not in general the probability of p relative to the agent's evidence. It is not a measure of the confirmation current evidence provides for p. You might, for instance, believe it will probably rain tonight, and that credence might not be based on having checked the weather forecast or even having been outside. Your credence that it will rain tonight needn't be based on any evidence at all—it might just be a hunch—and so is not the epistemic probability that it will rain.

The epistemic probability that p is true is the probability of p given an agent's evidence E. It is a measure of confirmation and not a measure of degree of belief or confidence. Nonetheless it's frequently true that an epistemic agent must estimate the degree of confirmation E provides for p. And epistemic agents can disagree on that issue. What degree of confirmation does our evidence provide for an oscillating universe? Epistemic agents offer various estimates, each of which provides the agent's epistemic probability of an oscillating universe. What degree of confirmation does the discovery of iconography provide for a particular religious practice? Epistemic agents will again vary in reasonable estimates. The epistemic probability that p is the probability that p given evidence E. But nothing about epistemic probability requires that the degree of confirmation should be obvious or that the degree confirmation should not be a matter of dispute.

It's worth noting that the theses in T1 – T3 are *indirectly* relevant to epistemic probability. It is true for instance, that if you know that proposition p is impossible, then your epistemic probability for p should be 0. That impossible propositions have probability 0 is a consequence of the probability axioms.[3] The probability axioms restrict the assignment of epistemic probability to the extent that we know, for instance, that p entails q, or that p and q are mutually exclusive, or that p is necessary or impossible. If you know that p is necessary, then your epistemic probability for p should be 1, since again, that is a requirement of the probability axioms. There are additional principles that directly govern epistemic probability.

T4. $P(\Box p \mid E)$ is positive only if $P(p \mid E) = 1$

What does T4 state? According to T4, if you assign some positive probability to the modal proposition $\Box p$ on the evidence you have E, then you must assign the proposition p probability 1 on your evidence.[4] But what makes T4 true? If $P(\Box p \mid E)$ is positive—as stated in the antecedent of T4—then, of course, $P(p \mid E)$ is also positive. This follows from the theorem in M.

M. $\Box(p \longrightarrow q) \longrightarrow (P(p \mid E) \leq P(q \mid E))$[5]

According to M, if p entails q, then the probability of q on E is at least as high as the probability of p on E. Since it is true that $\Box(\Box p \longrightarrow p)$, and we know that $P(p \mid E)$ is positive. Still, it is reasonable to ask why we can't assign $P(p \mid E)$ some value greater than 0 but less than 1.

If we assign a positive probability to $P(\Box p \mid E)$—as in the antecedent of T4—then we must believe that $\sim\Box(E \longrightarrow \sim\Box p)$. That is, we must believe that our evidence E does not entail $\sim\Box p$. If we believed that our evidence E entailed $\sim\Box p$, (and that E was consistent) then we would believe that $P(\sim\Box p \mid E) = 1$, and so that $P(\Box p \mid E) = 0$.[6] By a simple logical transformation, $\sim\Box(E \longrightarrow \sim\Box p)$ entails $\Diamond(E \,\&\, \Box p)$. But the formula $\Diamond(E \,\&\, \Box p)$ entails $\Diamond\Box p$, and in S5 $\Diamond\Box p$ entails $\Box\Box p$. So we know via the probability axioms that $P(\Box p \mid E) = 1$. And it follows by M that $P(p \mid E) = 1$.[7] That establishes T4. T4.1 follows from T4 and M directly, since $\Box(\Box p \longrightarrow \Box p)$.

T4.1 $P(\Box p \mid E)$ is positive only if $P(\Box p \mid E) = 1$

The argument briefly is that we believe that $P(\Box p \mid E)$ is positive only if we believe that $\Box p$ is consistent with E, or that $\Diamond(E \,\&\, \Box p)$. If we did not believe that $\Box p$ is consistent with E, we would be committed to holding that $P(\Box p \mid E)$

$= 0$ or that $P(\Box p \mid E)$ is undefined.[8] But by hypothesis we do believe that $P(\Box p \mid E)$ is positive. So, we should conclude that $P(\Box p \mid E) = 1$.

It's important to note that the argument does not aim to show that epistemic agents who believe that $P(\Box p \mid E)$ is positive will in general conclude that $P(p \mid E) = 1$. It is not assumed that epistemic agents are perfectly rational. But it does require a mistake in reasoning to believe that $P(\Box p \mid E)$ is positive and not arrive at the conclusion that $P(p \mid E) = 1$. It also requires a mistake in reasoning to believe that $P(\Box p \mid E)$ is positive and not arrive at the conclusion that $P(\Box p \mid E) = 1$. It is in fact a very common mistake to believe that we can assign a modal proposition some positive epistemic probability less than 1.

It's a direct consequence of the thesis in T4 and T4.1 that if I assign any modal proposition—any propositions such as $\Diamond p$ or $\Box q$ that includes a (wide scope) modal operator—a positive epistemic probability, then I must assign it probability 1. I can assign $\Diamond p$ or $\Box q$ an epistemic probability n ($0 < n < 1$) only if I make a mistake in reasoning. For instance, I might fail to see that $\Diamond p$ or $\Box q$ entail in S5 that $\Box\Diamond p$ or $\Box\Box q$, or fail to note that modal inferences are governed by S5, or fail to observe the probability axioms. S5 axioms entail that every modal proposition is necessarily true or necessarily false and the probability axioms require that modal propositions take value 1 or 0. For any evidence E and any modal proposition p, the epistemic probability of p on E is 1 or 0. This follows from T4 and T4.1.

There are of course modal propositions whose truth-value we do not know. Famous examples include Goldbach's Conjecture (GC) and the Continuum Hypothesis (CH). If either of these mathematical propositions has a positive epistemic probability, then it has epistemic probability 1. No doubt some are inclined to assign CH a positive epistemic probability less than 1 on the evidence E. But if P(CH | E) is assigned, say, .5, then we must believe that CH is consistent with E. That is, we must believe that $\Diamond$(CH & E). But if $\Diamond$(CH & E) then $\Box$CH. That is a consequence of S5 and the fact that ($\Box$CH v $\Box$~CH). CH is either necessarily true or necessarily false. But then it follows on the probability axioms that P(CH| E) = 1. On the other hand, if we do not believe that $\Diamond$(CH & E), then we believe that $\Box$(E $\longrightarrow$ ~CH) and we cannot assign CH a positive probability. If we believe that $\Box$(E $\longrightarrow$ ~CH), then we easily derive that either P(CH | E) = 0 or P(CH| E) is undefined. Neither of these is consistent with assigning CH some positive probability on our evidence E.

Of course, we could also refuse to commit on an epistemic probability for P(CH| E). This is the position that we cannot estimate the confirmation that our evidence E provides for CH. It is important that this is not an agnostic attitude toward CH. It is not the position that the evidence E provides some confirmation for CH and some (perhaps equal) confirmation for ~CH. It is rather the position that we have no idea whether evidence E confirms, disconfirms, or is irrelevant to CH.

It's also important to note that assigning epistemic probability 1 to proposition p on E is consistent with p being metaphysically impossible and having objective probability 0. T4 tells us only that an epistemic agent is committed to assigning p probability 1 on E, if she assigns $\Box p$ some positive probability on E. The fact is that an epistemic agent might be mistaken in her assignment of positive probability to $\Box p$ on E. Epistemic agents can and do wrongly estimate the epistemic probability of propositions on their evidence. Epistemic agents might also fail to notice that $\Box p$ is true only if $\Box\Box p$ is true or that $P(\Box p \mid E)$ is positive only if $\Box\Box p$. Given a sufficient number of errors an epistemic agent might find himself with an agnostic attitude toward CH.

The thesis in T5 also governs epistemic probability and follows directly from T4. From T4 and the fact that $P(\Box{\sim}p \mid E)$ is positive it follows that $P({\sim}p \mid E) = 1$, and so that $P(p \mid E) = 0$.

T5. $P(\Box{\sim}p \mid E)$ is positive only if $P(p \mid E) = 0$

Again, T5 makes no claim about the actual metaphysical necessity of ~p. T5 states that if your estimate of the epistemic probability of $\Box{\sim}p$ on evidence E is positive—if you believe, on the evidence, that there's some positive probability that $\Box{\sim}p$ is true—then the epistemic probability of p on the same evidence E is

0. The argument is perfectly analogous to the argument for T4. If you believe that the probability of $\Box{\sim}p$ on E is positive, then you must also believe that your evidence E is consistent with the modal proposition $\Box{\sim}p$ or that $\Diamond(E \,\&\, \Box{\sim}p)$. Alternatively, if you believe that $\Box(E \longrightarrow {\sim}\Box{\sim}p)$, then you would have to assign $P(\Box{\sim}p \mid E)$ either 0 or undefined.

Since in S5 $\Diamond(E \,\&\, \Box{\sim}p)$ entails $\Box\Box{\sim}p$, the value of $P(\Box{\sim}p \mid E) = 1$—all necessary truths take probability 1. And it then follows from M that $P({\sim}p \mid E) = 1$, and so that $P(p \mid E) = 0$. So, we should conclude that $P(\Box{\sim}p \mid E)$ is positive only if $P(p \mid E) = 0$. Of course, as we noted above, all of this is consistent with p in fact being necessarily true and the objective probability of p being 1.

The thesis in T6 states that if the epistemic probability of p on evidence E is positive, then the epistemic probability of $\Diamond p$ on the same evidence E is 1.

T6. $P(p \mid E)$ is positive only if $P(\Diamond p \mid E) = 1$

Why is my epistemic probability for $\Diamond p$ be 1 if my epistemic probability for p given E is positive? If you believe that the probability of p on E is positive, then you must believe that ${\sim}\Box(E \longrightarrow {\sim}p)$. That is, you must believe that your evidence E does not entail ~p. If you believed that $\Box(E \longrightarrow {\sim}p)$, the only epistemic value for $P(p \mid E)$ consistent with your beliefs is 0 (or undefined).

If, on the other hand, $\sim\Box(E \longrightarrow \sim p)$, then we can quickly derive $\Diamond(E \ \&\ p)$. And it is true in S5 that $\Diamond(E \ \&\ p)$ entails $\Box\Diamond p$. But if $\Box\Diamond p$—that is, if $\Diamond p$ is necessarily true—then $P(\Diamond p \mid E) = 1$. So, $P(p \mid E)$ is positive only if $P(\Diamond p \mid E) = 1$. If you assign p a positive epistemic probability on evidence E, then you should assign $\Diamond p$ epistemic probability 1 on E.

It is worth underscoring that the argument does not aim to show that every epistemic agent will assign a positive epistemic probability to $P(p \mid E)$ or that every epistemic agent that assigns a positive epistemic probability to $P(p \mid E)$ will assign $P(\Diamond p \mid E) = 1$. The argument does aim to show that if an epistemic agent *does assign* a positive probability to $P(p \mid E)$, then she should assign $P(\Diamond p \mid E) = 1$. If she assigns a positive probability to $P(p \mid E)$ and does not assign $P(\Diamond p \mid E) = 1$, then she has mistakenly violated T6.

3. Against Agnosticism

Suppose that your evidence for God's existence is E. E might include the cumulative evidence you have from the well-known arguments for God's existence. E might also include the cumulative evidence you have against God's existence. William Rowe suggests that, if the aim is to determine the epistemic probability that God exists, then the information in E should be restricted to evidence that theists and non-theists share concerning the existence of God.

> I turn now to the background information k . . . What will k include? I take it as important here that k be restricted almost entirely to information that is shared by most theists and nontheists . . . To this end we will want to include in k our common knowledge of the occurrence of various evils in our world . . . as well as our knowledge that the world contains a great deal of evil. k will also include . . . our knowledge of many of the goods that occur and many of the goods that do not occur. . . [9]

If E includes the evidence shared by theists and nontheists—including the amount and distribution of evil, reports of religious experience, the total amount of good in the world, and so on—Rowe concludes that the probability that God exists should be about the same as the probability that God does not exist.

> If we conceive of k in the way just suggested, what assignment should be given to the probability that God exists given k, P(G | k)? . . . In order not to beg any . . . questions, I will assign 0.5 to P(G | k), and of course 0.5 to P(~G | k). We will say that k by itself makes neither God's existence nor his non-existence more likely than not.[10]

The evidence E for God's existence makes the epistemic probability of God's existence about as probable as not. An epistemic agent informed on the evidence for and against the existence of God, according to Rowe, should be agnostic.

But Rowe's conclusion is mistaken. It is not possible that P(God exists | E) = P(God does not exist | E), given the epistemic theses in T4 – T6. The thesis in T6 entails that if we assign some positive epistemic probability to the proposition that God exists, then the epistemic probability that it is possible that God exists is 1. This is just an instance of T6.

T6. $P(\text{God exists} \mid E)$ is positive only if $P(\Diamond(\text{God exists}) \mid E) = 1$

It follows from M that the epistemic probability of God's actual existence is at least as high as the probability of God's possible existence, if God's possible existence entails God's actual existence.

M. $\Box(\Diamond(\text{God exists}) \longrightarrow \text{God exists}) \longrightarrow (P(\Diamond(\text{God exists}) \mid E) \leq$

$P(\text{God exists} \mid E))$.

The argument depends on establishing the proposition in N, which is not entirely uncontroversial.

N. $\Box(\Diamond(\text{God exists}) \longrightarrow \text{God exists})$

According to N, necessarily, if it is possible that God exists, then God in fact exists. S5 ensures that every possible world is modally equivalent to every other possible world. Any modal proposition true in any world is true in every world. Let's assume a conception of God according to which God exemplifies necessary existence. Assume that God exists in some possible world or $\Diamond$(God exists). From the conception of God we are assuming, it follows that $\Diamond\Box$(God exists). There is some possible world in which God necessarily exists. But all worlds are modally equivalent. If there is some possible world in which God necessarily exists, then God necessarily exists in every world. We conclude that $\Box$(God exists). And it follows of course that God actually exists. The argument for N, therefore, depends on S5 and a conception of God according to which God is a necessarily existing being.

From T6 and the assumption that P(God exists) | E) is positive, we know that P($\Diamond$(God exists) | E) = 1. But from P($\Diamond$(God exists) | E) = 1, M and N we know that P(God exists) | E) = 1.

It follows from the probability axioms that Rowe's probability assignment is mistaken. We cannot assign P(God exists) | E) some positive probability and also assign P(God does not exist) | E) some positive probability. If we assign P(God exists) | E) some positive probability then we know that P(God does not exist) | E) = 0.

We can put the argument from T6 in a slightly more formal way. The numbers to the left of each premise indicate the assumptions on which the line is based. Included among the assumptions in the argument are T6, M, and N.[11]

1	(1) P(God exists) | E) is positive	Assumption
1, T6	(2) P($\Diamond$(God exists) | E) = 1	1, T6
M	(3) $\Box$($\Diamond$(God exists) $\longrightarrow$ God exists) $\longrightarrow$ (P($\Diamond$(God exists)|E) $\leq$	

	P(God exists | E))	M
N	(4) $\Box(\Diamond(\text{God exists}) \longrightarrow \text{God exists})$	S5, N
M,N	(5) $P(\Diamond(\text{God exists}) \mid E) \leq P(\text{God exists} \mid E)$	3,4 MP
1, T6, M, N	(6) $\therefore P(\text{God exists} \mid E) = 1$	2,5 Prob. Axioms

The conclusion of the argument in (6) depends on assumptions (1), T6, M, and N. The assumption in (1) is uncontroversial, since it simply states that there is some positive epistemic probability that God exists. The argument simply does not apply to anyone who does not assign some positive epistemic probability to God's existence.

The remaining assumptions are T6, M, and N. But M is false only if $\Box(p \longrightarrow q)$, and $P(p \mid E) > P(q \mid E)$. Is it possible that p entails q and the probability of p is greater than the probability of q? Suppose $\Box(p \longrightarrow q)$. We know that $P(p \mid E) > P(q \mid E)$ is true only if some occurrences of p on E are not occurrences of q on E. But that is true only of $\Diamond(p \,\&\, {\sim}q)$. And that is inconsistent with our assumption $\Box(p \longrightarrow q)$. So, M appears uncontroversial.

If there is a problem with the argument, then it must be with the assumption T6 or N. We offered a proof of T6 in section (2). But let's consider a more formal version of the proof of T6. Recall that T6 states that $P(p \mid E)$ is positive only if $P(\Diamond p \mid E) = 1$. Let's assume that the antecedent of T6 is true.

1	(1) $P(p \mid E)$ is positive	Assumption

2	(2) $\Box(E \longrightarrow \sim p)$	Assumption
2	(3) $P(\sim p \mid E) = 1$	2, Prob. Axioms[12]
2	(4) $P(p \mid E)$ is not positive	3, Prob. Axioms
1,2	(5) $\curlywedge$	1,4 Negation Elim.[13]
1	(6) $\sim\Box(E \longrightarrow \sim p)$	2,5 Negation Intro.
1	(7) $\Diamond(E \;\&\; p)$	6, S5
1	(8) $\Diamond p$	7, S5
1	(9) $\Box\Diamond p$	8, S5
1	(10) $P(\Diamond p \mid E) = 1$	9, Prob. Axioms
	(11) $P(p \mid E)$ is positive only if $P(\Diamond p \mid E) = 1$	1, 10 $\longrightarrow$Intro

The inference to (3) does depend on the consistency of the evidence E. But $\Diamond E$ follows from the assumption in (1) that $P(p \mid E)$ is positive. $P(p \mid E)$ is undefined when $\sim\Diamond E$. Premises (3) and (4) follow from the assumption in (2), and the assumption in (2) is discharged in premise (6). The assumption in premise (1), finally, is discharged in premise (11). The proposition in line (11) is just T6, and it depends on no assumptions at all.

The only assumption we have not considered from the argument above is N. According to N, it is possible that God exists only if God exists, or $\Box(\Diamond(\text{God exists}) \longrightarrow \text{God exists})$. The argument we are advancing assumes a concept of God according to which God exists as a matter of metaphysical necessity. The assumption is not that there exists a God with the divine attribute of necessary existence. Indeed, the argument does not assume that anything, in any possible world, is identical to God. The assumption is rather that if there is some x in any possible world such that x = God, then x necessarily exists. And if x necessarily exists, then of course x exists. The inference is simpler in S5. For any proposition $\Box p$, it is an S5 theorem that $\Diamond \Box p \longrightarrow p$. So, if it is possible that necessarily God exists, then God exists.

Still, it is possible to resist N. There are many alternative conceptions of God including pantheistic and panentheistic conceptions according to which God is not a necessarily existing being. There are Lewisian conceptions of God according to which there are infinitely many Gods in metaphysical space—none of them actual—all exemplifying various degrees of perfection. There are also quasi-traditional conceptions of God according to which God is a contingent being otherwise exemplifying most of the traditional divine attributes. Without N we cannot derive P(God exists) | E) = 1 from the assumption that P(God exists) | E) is positive. Agnosticism is a perfectly coherent attitude to take toward the existence of these non-traditional Gods.

But it is possible to avoid N altogether. There is an argument from T4 that if an epistemic agent places some positive probability on the proposition that God necessarily exists, then the agent must place probability 1 on the proposition that God exists. If you believe that the evidence you possess confirms to some degree the existence of a necessarily existing God, then the evidence you possess makes certain that God exists. Of course, if you believe that the evidence you possess confirms to some degree that God does not exist, then the evidence you possess makes certain that God does not exist. Consider a formal argument for a theistic instance of T4.

1 (1) $P(\Box(\text{God exists}) \mid E)$ is positive Assumption

(2) $P(\Box(\text{God exists}) \mid E)$ is positive $\longrightarrow \sim\Box(E \longrightarrow \Box(\text{God does not exist}))$. PA

(3) $\sim\Box(E \longrightarrow \Box(\text{God does not exist})) \longrightarrow \Diamond(E \;\&\; \Box(\text{God does not exist})$. S5

(4) $\Diamond(E \;\&\; \Box(\text{God does not exist}) \longrightarrow \Box\Box(\text{God exists})$. S5

(5) $\Box\Box(\text{God exists}) \longrightarrow P(\Box(\text{God exists}) \mid E) = 1$. PA

(6) $P(\Box(\text{God exists}) \mid E) = 1 \longrightarrow P(\text{God exists} \mid E) = 1$. M

(7) $P(\text{God exists} \mid E) = 1$. 1,2,3,4,5,6 MP (multiple applications),

(8) $\therefore P(\Box(\text{God exists}) \mid E)$ is positive $\longrightarrow P(\text{God exists} \mid E) = 1$. 1,7 $\longrightarrow$Intro.

The argument from T4 shows that….

The argument from T6 establishes that *if* an epistemic agent assigns some positive probability to God's existence *then* she should assign probability 1 to God's existence. Epistemic agents cannot coherently hedge their bets on God's existence. Indeed, epistemic agents cannot assign any probability n ($0 < n < 1$) to God's existence. So, of course, epistemic agents cannot assign any probability n ($0 < n < 1$) to God's non-existence.

It is true that few theists are absolutely certain that God exists. Most believers place some positive probability on the existence of God and some positive probability on the non-existence of God. But the argument from T6 shows that the only coherent epistemic attitude available to theists is unhedged belief in God. The alternative attitudes available are non-theistic attitudes. The only available atheistic attitude is unhedged disbelief in God. Epistemic agents otherwise must reject the suggestion that there is any epistemic relationship between their evidence E—the totality of evidence for and against the existence of God—and the proposition that God exists. These epistemic agents assign no probability at all to the proposition that God exists on the evidence E.

Notes

[1] See for instance D. H. Mellor, *The Matter of Chance* (Cambridge: Cambridge University Press, 1971), and his *Probability: A Philosophical Introduction* (New York: Routledge, 2005).

[2] The objective probability or chance that p is also not the probability of p relative to an agent's evidence E. See D. H. Mellor, *The Matter of Chance* (Cambridge: Cambridge University Press, 1971), and his *Probability: A Philosophical Introduction* (New York: Routledge, 2005). For views very close to Mellor's, see David Lewis, 'A Subjectivist's

Guide to Objective Chance' in his *Philosophical Papers II* (Oxford: Oxford University Press, 1986).

[3] The probability axioms are the following. (i) $1 \geq P(p) \geq 0$ for any proposition p; (ii) $P(q)=1$ for necessary truth q; (iii) $P(p \vee q)=P(p)+P(q)$ for mutually exclusive p and q, and (iv) $P(p \;\&\; q)=P(p) \cdot P(q|p)$ for any propositions p, q. These are equivalent to the Kolmogorov axioms.

[4] In cases where it is obvious that I'm mentioning a proposition—say, in this case, '$\Box p$'—I leave off use of single quotes and let the proposition name itself.

[5] This is the monotonicity requirement. If A is a subset of, or equal to B, then the probability of A is less than, or equal to the probability of B or $A \subseteq B$ only if $P(A) \leq P(B)$.

[6] We are assuming that our evidence E is not necessarily false. If E is necessarily false then it is true that $\Box(E \longrightarrow p)$, for any p, but $P(p \mid E)$ is undefined. Otherwise, $P(p \mid E) = 1$.

[7] Consider a more formal version of this argument that depends on no assumptions.

1. $P(\Box p \mid E)$ is positive $\longrightarrow \sim\Box(E \longrightarrow \sim\Box p)$, Prob. Axioms, S5

2. $\sim\Box(E \longrightarrow \sim\Box p) \longrightarrow \Diamond(E \;\&\; \Box p)$. S5

3. $\Diamond(E \;\&\; \Box p) \longrightarrow \Box\Box p$. S5

4. $\Box\Box p \longrightarrow P(\Box p \mid E) = 1$. Prob. Axioms

5. $P(\Box p \mid E) = 1 \longrightarrow P(p \mid E) = 1$. M

6. $\therefore P(\Box p \mid E)$ is positive $\longrightarrow P(p \mid E) = 1$. 1, 2, 3, 4, 5 H.S. (multiple applications)

[8] Compare endnote (12) below. Assume $\Box(E \longrightarrow \sim\Box p)$. We know that $P(\sim\Box p \mid E) = P(\sim\Box p) . P(E \mid \sim\Box p)/P(\sim\Box p)$. But given our assumption this is equivalent to $P(\sim\Box p \mid E) = P(\sim\Box p) /P(\sim\Box p) = 1$. Alternatively, we know that $P(\sim\Box p \mid E) = P(\sim\Box p \;\&\; E)/P(E) = P(E)/P(E) = 1$.

[9] William Rowe, 'The Evidential Argument from Evil: A Second Look' in Daniel Howard Snyder (ed.) *The Evidential Argument from Evil* (Bloomington: Indiana University Press, 1996). pp. 264-5.

[10] Ibid. p. 265.

[11] The inference rules are from the S5 modal extension of NK. See Graeme Forbes, *Modern Logic* (Oxford: Oxford University Press, 1994).

[12] Assume $\Box(E \longrightarrow \sim p)$. We know that $P(\sim p \mid E) = P(\sim p) . P(E \mid \sim p)/P(E)$. But given our assumption this is equivalent to $P(\sim p \mid E) = P(E) /P(E) = 1$. Alternatively, we know that $P(\sim p \mid E) = P(\sim p \;\&\; E)/P(E) = P(E)/P(E) = 1$.

[13] The rules of negation elimination and negation introduction operate together as a reductio ad absurdum rule. The absurdity $\curlywedge$ is derived on line (5) via negation elimination is applied to lines (1) and (4), and the assumption in line (2) is negated and discharged at line (6) via negation introduction.