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NeutroOrderedAlgebra: Applications to Semigroups

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Abstract. Starting with a partial order on a NeutroAlgebra, we get a NeutroStructure. The latter if it satisfies the conditions of NeutroOrder, it becomes a NeutroOrderedAlgebra. In this paper, we apply our new defined notion to semigroups by studying NeutroOrderedSemigroups. More precisely, we define some related terms like NeutrosOrderedSemigroup, NeutroOrderedIdeal, NeutroOrderedFilter, NeutroOrderedHomomorphism, etc., illustrate them via some examples, and study some of their properties.

Keywords: NeutroAlgebra, NeutroSemigroup, NeutroOrderedAlgebra, NeutrosOrderedSemigroup, NeutroOrderedIdeal, NeutroOrderedFilter, NeutroOrderedHomomorphism, NeutroOrderedStrongHomomorphism.
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1. Introduction

Neutrosophy, the study of neutralities, is a new branch of Philosophy initiated by Smarandache in 1995. It has many applications in almost every field. Many algebraists worked on the connection between neutrosophy and algebraic structures. Fore more details, we refer to [1–3]. Unlike the idealistic or abstract algebraic structures, from pure mathematics, constructed on a given perfect space (set), where the axioms (laws, rules, theorems, results etc.) are totally (100%) true for all spaces elements, our world and reality consist of approximations, imperfections, vagueness, and partialities. Starting from the latter idea, Smarandache introduced NeutroAlgebra. In 2019 and 2020, he [11–13] generalized the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) whose operations and axioms are partially true, partially indeterminate, and partially false as extensions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) whose operations and axioms are totally false. And in general, he extended any classical Structure, in no matter what field of knowledge, to a

NeutroStructure and an AntiStructure. A Partial Algebra is an algebra that has at least one Partial Operation, and all its Axioms are classical. Through a theorem, Smarandache [11] proved that a NeutroAlgebra is a generalization of Partial Algebra and gave some examples of NeutroAlgebras that are not Partial Algebras. Many researchers worked on special types of NeutroAlgebras and AntiAlgebras by applying them to different types of algebraic structures such as groups, rings, BE-Algebras, BCK-Algebras, etc. For more details, we refer to [4–6, 9, 10, 14, 15].

Inspired by NeutroAlgebra and ordered Algebra, our paper introduces and studies NeutroOrderedAlgebra. And it is constructed as follows: After an Introduction, in Section 2, we introduce NeutroOrderedAlgebra and some related terms such as NeutroOrderedSubAlgebra and NeutroOrderedHomomorphism. And in Section 3, we apply the concept of NeutroOrderedAlgebra to semigroups and study NeutroOrderedSemigroups by presenting several examples and studying some of their interesting properties.

2. NeutroOrderedAlgebra

In this section, we combine the notions of ordered algebraic structures and NeutroAlgebra to introduce *NeutroOrderedAlgebra*. Some new definitions related to the new concept are presented. For details about ordered algebraic structures, we refer to [7, 8].

Definition 2.1. [11] A non-empty set A endowed with n operations "$\star_i$" for $i = 1, \ldots, n$, is called *NeutroAlgebra* if it has at least one NeutroOperation or at least one NeutroAxiom with no AntiOperations nor AntiAxioms.

Definition 2.2. [8] Let A be an Algebra with n operations "$\star_i$" and "$\leq$" be a partial order (reflexive, anti-symmetric, and transitive) on A. Then $(A, \star_1, \ldots, \star_n, \leq)$ is an Ordered Algebra if the following conditions hold.
If $x \leq y \in A$ then $z \star_i x \leq z \star_i y$ and $x \star_i z \leq y \star_i z$ for all $i = 1, \ldots, n$ and $z \in A$.

Definition 2.3. Let A be a NeutroAlgebra with n (Neutro) operations "$\star_i$" and "$\leq$" be a partial order (reflexive, anti-symmetric, and transitive) on A. Then $(A, \star_1, \ldots, \star_n, \leq)$ is a *NeutroOrderedAlgebra* if the following conditions hold.

(1) There exist $x \leq y \in A$ with $x \neq y$ such that $z \star_i x \leq z \star_i y$ and $x \star_i z \leq y \star_i z$ for all $z \in A$ and $i = 1, \ldots, n$. (This condition is called degree of truth, "T".)
(2) There exist $x \leq y \in A$ and $z \in A$ such that $z \star_i x \nleq z \star_i y$ or $x \star_i z \nleq y \star_i z$ for some $i = 1, \ldots, n$. (This condition is called degree of falsity, "F".)
(3) There exist $x \leq y \in A$ and $z \in A$ such that $z \star_i x$ or $z \star_i y$ or $x \star_i z$ or $y \star_i z$ are indeterminate, or the relation between $z \star_i x$ and $z \star_i y$, or the relation between $x \star_i z$

and $y \star_i z$ are indeterminate for some $i = 1, \ldots, n$. (This condition is called degree of indeterminacy, "I".)

Where (T, I, F) is different from $(1, 0, 0)$ that represents the classical Ordered Algebra as well from $(0, 0, 1)$ that represents the AntiOrderedAlgebra.

Definition 2.4. Let $(A, \star_1, \ldots, \star_n, \leq)$ be a NeutroOrderedAlgebra. If "$\leq$" is a total order on A then A is called *NeutroTotalOrderedAlgebra*.

Definition 2.5. Let $(A, \star_1, \ldots, \star_n, \leq_A)$ be a NeutroOrderedAlgebra and $\emptyset \neq S \subseteq A$. Then S is a *NeutroOrderedSubAlgebra* of A if $(S, \star_1, \ldots, \star_n, \leq_A)$ is a NeutroOrderedAlgebra and there exists $x \in S$ with $(x] = \{y \in A : y \leq_A x\} \subseteq S$.

Remark 2.6. A NeutroOrderedAlgebra has at least one NeutroOrderedSubAlgebra which is itself.

Definition 2.7. Let $(A, \star_1, \ldots, \star_n, \leq_A)$ and $(B, \circledast_1, \ldots, \circledast_n, \leq_B)$ be NeutroOrderedAlgebras and $\phi : A \to B$ be a function. Then

(1) ϕ is called *NeutroOrderedHomomorphism* if there exist $x, y \in A$ such that for all $i = 1, \ldots, n$, $\phi(x \star_i y) = \phi(x) \circledast_i \phi(y)$, and there exist $a \leq_A b \in A$ with $a \neq b$ such that $\phi(a) \leq_B \phi(b)$.
(2) ϕ is called *NeutroOrderedIsomomorphism* if ϕ is a bijective NeutroOrderedHomomorphism. In this case, we write $A \cong_I B$.
(3) ϕ is called *NeutroOrderedStrongHomomorphism* if for all $x, y \in A$ and for all $i = 1, \ldots, n$, we have $\phi(x \star_i y) = \phi(x) \circledast_i \phi(y)$ and $a \leq_A b \in A$ is equivalent to $\phi(a) \leq_B \phi(b)$ for all $a, b \in A$.
(4) ϕ is called *NeutroOrderedStrongIsomomorphism* if ϕ is a bijective NeutroOrderedStrongHomomorphism. In this case, we write $A \cong_{SI} B$.

Example 2.8. Let $(A, \star_1, \ldots, \star_n, \leq_A)$ be a NeutroOrderedAlgebra, B a NeutroOrderedSubAlgebra of A, and $\phi : B \to A$ be the inclusion map ($\phi(x) = x$ for all $x \in B$). Then ϕ is a NeutroOrderedStrongHomomorphism.

Example 2.9. Let $(A, \star_1, \ldots, \star_n, \leq_A)$ be a NeutroOrderedAlgebra and $\phi : A \to A$ be the identity map ($\phi(x) = x$ for all $x \in A$). Then ϕ is a NeutroOrderedStrongIsomomorphism.

Remark 2.10. Every NeutroOrderedStrongHomomorphism (NeutroOrderedStrongIsomorphism) is a NeutroOrderedHomomorphism (NeutroOrderedIsomorphism).

Theorem 2.11. *The relation "$\cong_{SI}$" is an equivalence relation on the set of NeutroOrderedAlgebras.*

Proof. By taking the identity map and using Example 2.9, we can easily prove that "$\cong_{SI}$" is a reflexive relation. Let $A \cong_{SI} B$. Then there exist a NeutroOrderedStrongIsomorphism $\phi : (A, \star_1, \ldots, \star_n, \leq_A) \to (B, \circledast_1, \ldots, \circledast_n, \leq_B)$. We prove that $\phi^{-1} : B \to A$ is a NeutroOrderedStrongIsomorphism. For all $b_1, b_2 \in B$, there exist $a_1, a_2 \in A$ with $\phi(a_1) = b_1$ and $\phi(a_2) = b_2$. For all $i = 1, \ldots, n$, we have:

$$\phi^{-1}(b_1 \circledast_i b_2) = \phi^{-1}(\phi(a_1) \circledast_i \phi(a_2)) = \phi^{-1}(\phi(a_1 \star_i a_2)) = a_1 \star_i a_2 = \phi^{-1}(b_1) \star_i \phi^{-1}(b_2).$$

Moreover, having $a_1 \leq_A a_2 \in A$ equivalent to $\phi(a_1) \leq_B \phi(a_2) \in B$ and ϕ an onto function implies that $b_1 = \phi(a_1) \leq_B \phi(a_2) = b_2 \in B$ is equivalent to $a_1 = \phi^{-1}(b_1) \leq_A a_2 = \phi^{-1}(b_2) \in A$. Thus, $B \cong_{SI} A$ and hence, "$\cong_{SI}$" is a symmetric relation. Let $A \cong_{SI} B$ and $B \cong_{SI} C$. Then there exist NeutroOrderedStrongIsomorphisms $\phi : A \to B$ and $\psi : B \to C$. One can easily see that $\psi \circ \phi : A \to C$ is a NeutroOrderedStrongIsomorphism. Thus, $A \cong_{SI} C$ and hence, "$\cong_{SI}$" is a transitive relation. $\square$

Remark 2.12. The relation "$\cong_I$" is a reflexive and symmetric relation on the set of NeutroOrderedAlgebras. But it may fail to be a transitive relation.

3. NeutroOrderedSemigroup

In this section, we use the defined notion of NeutroOrderedAlgebra in Section 2 and apply it to semigroups. As a result, we define NeutroOrderedSemigroup and other related concepts. Moreover, we present some examples of finite as well as infinite NeutroOrderedSemigroups. Finally, we study some properties of NeutroOrderedSubSemigroups, NeutroOrderedIdeals, and NeutroOrderedFilters.

Definition 3.1. [8] Let $(S, \cdot)$ be a semigroup ("$\cdot$" is an associative and a binary closed operation) and "$\leq$" a partial order on S. Then $(S, \cdot, \leq)$ is an *ordered semigroup* if for every $x \leq y \in S$, $z \cdot x \leq z \cdot y$ and $x \cdot z \leq y \cdot z$ for all $z \in S$.

Definition 3.2. [8] Let $(S, \cdot, \leq)$ be an ordered semigroup and $\emptyset \neq M \subseteq S$. Then

(1) M is an *ordered subsemigroup* of S if $(M, \cdot, \leq)$ is an ordered semigroup and $(x] \subseteq M$ for all $x \in M$. i.e., if $y \leq x$ then $y \in M$.
(2) M is an *ordered left ideal* of S if M is an ordered subsemigroup of S and for all $x \in M$, $r \in S$, we have $rx \in M$.
(3) M is an *ordered right ideal* of S if M is an ordered subsemigroup of S and for all $x \in M$, $r \in S$, we have $xr \in M$.
(4) M is an *ordered ideal* of S if M is both: an ordered left ideal of S and an ordered right ideal of S.

(5) M is an *ordered filter* of S if $(M, \cdot)$ is a semigroup and for all $x, y \in S$ with $x \cdot y \in M$, we have $x, y \in M$ and $[y) \subseteq M$ for all $y \in M$. i.e., if $y \in M$ with $y \leq x$ then $x \in M$.

Definition 3.3. Let $(S, \cdot)$ be a NeutroSemigroup and "$\leq$" be a partial order (reflexive, anti-symmetric, and transitive) on S. Then $(S, \cdot, \leq)$ is a *NeutroOrderedSemigroup* if the following conditions hold.

(1) There exist $x \leq y \in S$ with $x \neq y$ such that $z \cdot x \leq z \cdot y$ and $x \cdot z \leq y \cdot z$ for all $z \in S$. (This condition is called degree of truth, "T".)

(2) There exist $x \leq y \in S$ and $z \in S$ such that $z \cdot x \nleq z \cdot y$ or $x \cdot z \nleq y \cdot z$. (This condition is called degree of falsity, "F".)

(3) There exist $x \leq y \in S$ and $z \in S$ such that $z \cdot x$ or $z \cdot y$ or $x \cdot z$ or $y \cdot z$ are indeterminate, or the relation between $z \cdot x$ and $z \cdot y$, or the relation between $x \cdot z$ and $y \cdot z$ are indeterminate. (This condition is called degree of indeterminacy, "I".)

Where (T, I, F) is different from $(1, 0, 0)$ that represents the classical Ordered Semigroup, and from $(0, 0, 1)$ that represents the AntiOrderedSemigroup.

Definition 3.4. Let $(S, \cdot, \leq)$ be a NeutroOrderedSemigroup . If "$\leq$" is a total order on A then A is called *NeutroTotalOrderedSemigroup*.

Definition 3.5. Let $(S, \cdot, \leq)$ be a NeutroOrderedSemigroup and $\emptyset \neq M \subseteq S$. Then

(1) M is a *NeutroOrderedSubSemigroup* of S if $(M, \cdot, \leq)$ is a NeutroOrderedSemigroup and there exist $x \in M$ with $(x] = \{y \in S : y \leq x\} \subseteq M$.

(2) M is a *NeutroOrderedLeftIdeal* of S if M is a NeutroOrderedSubSemigroup of S and there exists $x \in M$ such that $r \cdot x \in M$ for all $r \in S$.

(3) M is a *NeutroOrderedRightIdeal* of S if M is a NeutroOrderedSubSemigroup of S and there exists $x \in M$ such that $x \cdot r \in M$ for all $r \in S$

(4) M is a *NeutroOrderedIdeal* of S if M is a NeutroOrderedSubSemigroup of S and there exists $x \in M$ such that $r \cdot x \in M$ and $x \cdot r \in M$ for all $r \in S$.

(5) M is a *NeutroOrderedFilter* of S if $(M, \cdot, \leq)$ is a NeutroOrderedSemigroup and there exists $x \in S$ such that for all $y, z \in S$ with $x \cdot y \in M$ and $z \cdot x \in M$, we have $y, z \in M$ and there exists $y \in M$ $[y) = \{x \in S : y \leq x\} \subseteq M$.

Proposition 3.6. *Let $(S, \cdot, \leq)$ be a NeutroOrderedSemigroup and $\emptyset \neq M \subseteq S$. Then the following statements are true.*

(1) *If S contains a minimum element (i.e. there exists $m \in S$ such that $m \leq x$ for all $x \in S$.) and M is a NeutroOrderedSubSemigroup (or NeutroOrderedRightIdeal or NeutroOrderedLeftIdeal or NeutroOrderedIdeal) of S then the minimum element is in M.*

(2) *If If S contains a maximum element (i.e. there exists $n \in S$ such that $x \leq n$ for all $x \in S$.) and M is a NeutroOrderedFilter of S then M contains the maximum element of S.*

Proof. The proof is straightforward. □

Remark 3.7. Let $(S, \cdot, \leq)$ be a NeutroOrderedSemigroup. Then every NeutroOrderedIdeal of S is NeutroOrderedLeftIdeal of S and a NeutroOrderedRightIdeal of S. But the converse may not hold. (See Example 3.16.)

Definition 3.8. Let $(A, \star, \leq_A)$ and $(B, \circledast, \leq_B)$ be NeutroOrderedSemigroups and $\phi : A \to B$ be a function. Then

(1) ϕ is called *NeutroOrderedHomomorphism* if $\phi(x \star y) = \phi(x) \circledast \phi(y)$ for some $x, y \in A$ and there exist $a \leq_A b \in A$ with $a \neq b$ such that $\phi(a) \leq_B \phi(b)$.
(2) ϕ is called *NeutroOrderedIsomomorphism* if ϕ is a bijective NeutroOrderedHomomorphism.
(3) ϕ is called *NeutroOrderedStrongHomomorphism* if $\phi(x \star y) = \phi(x) \circledast \phi(y)$ for all $x, y \in A$ and $a \leq_A b \in A$ is equivalent to $\phi(a) \leq_B \phi(b) \in B$.
(4) ϕ is called *NeutroOrderedStrongIsomomorphism* if ϕ is a bijective NeutroOrderedStrongHomomorphism.

Example 3.9. Let $S_1 = \{s, a, m\}$ and $(S_1, \cdot_1)$ be defined by the following table.

$\cdot_1$	s	a	m
s	s	m	s
a	m	a	m
m	m	m	m

Since $s \cdot_1 (s \cdot_1 s) = s = (s \cdot_1 s) \cdot_1 s$ and $s \cdot_1 (a \cdot_1 m) = s \neq m = (s \cdot_1 a) \cdot_1 m$, it follows that $(S_1, \cdot_1)$ is a NeutroSemigroup.
By defining the total order

$$\leq_1 = \{(m, m), (m, s), (m, a), (s, s), (s, a), (a, a)\}$$

on S_1, we get that $(S_1, \cdot_1, \leq_1)$ is a NeutroTotalOrderedSemigroup. This is easily seen as: $m \leq_1 s$ implies that $m \cdot_1 x \leq_1 s \cdot_1 x$ and $x \cdot_1 m \leq_1 x \cdot_1 s$ for all $x \in S_1$. And having $s \leq_1 a$ but $s \cdot_1 s = s \not\leq_1 m = a \cdot_1 s$.

Example 3.10. Let $S_2 = \{0, 1, 2, 3\}$ and $(S_2, \cdot_2)$ be defined by the following table.

$\cdot_2$	0	1	2	3
0	0	0	0	3
1	0	1	1	3
2	0	3	2	2
3	3	3	3	3

Since $0 \cdot_2 (0 \cdot_2 0) = 0 = (0 \cdot_2 0) \cdot_2 0$ and $1 \cdot_2 (2 \cdot_2 3) = 1 \neq 3 = (1 \cdot_2 2) \cdot_2 3$, it follows that $(S_2, \cdot_2)$ is a NeutroSemigroup.
By defining the total order

$$\leq_2 = \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$$

on S_2, we get that $(S_2, \cdot_2, \leq_2)$ is a NeutroTotalOrderedSemigroup. This is easily seen as: $0 \leq_2 3$ implies that $0 \cdot_2 x \leq_2 3 \cdot_2 x$ and $x \cdot_2 0 \leq_2 x \cdot_2 3$ for all $x \in S_2$. And having $1 \leq_2 2$ but $2 \cdot_2 1 = 3 \not\leq_2 2 = 2 \cdot_2 2$.

We present examples on NeutroOrderedSemigroups that are not NeutroTotalOrderedSemigroups.

Example 3.11. Let $S_2 = \{0, 1, 2, 3\}$ and $(S_2, \cdot_2')$ be defined by the following table.

$\cdot_2'$	0	1	2	3
0	0	0	0	0
1	0	1	1	1
2	0	1	3	2
3	0	1	3	2

Since $0 \cdot_2' (0 \cdot_2' 0) = 0 = (0 \cdot_2' 0) \cdot_2' 0$ and $2 \cdot_2' (2 \cdot_2' 3) = 3 \neq 2 = (2 \cdot_2' 2) \cdot_2' 3$, it follows that $(S_2, \cdot_2')$ is a NeutroSemigroup.
By defining the partial order (which is not a total order)

$$\leq_2' = \{(0,0), (0,1), (0,2), (1,1), (2,2), (3,3)\}$$

on S_2, we get that $(S_2, \cdot_2', \leq_2')$ is a NeutroOrderedSemigroup (that is not a NeutroTotalOrderedSemigroup). This is easily seen as:
$0 \leq_2' 1$ implies that $0 \cdot_2' x = x \cdot_2' 0 = 0 \leq_2' 1 = 1 \cdot_2' x = x \cdot_2' 1$. And having $0 \leq_2' 2$ but $2 \cdot_2' 0 = 0 \not\leq_2' 3 = 2 \cdot_2' 2$.

Example 3.12. Let $S_3 = \{0, 1, 2, 3, 4\}$ and $(S_3, \cdot_3)$ be defined by the following table.

$\cdot_3$	0	1	2	3	4
0	0	0	0	3	0
1	0	1	2	1	1
2	0	4	2	3	3
3	0	4	2	3	3
4	0	0	0	4	0

Since $0 \cdot_3 (0 \cdot_3 0) = 0 = (0 \cdot_3 0) \cdot_3 0$ and $1 \cdot_3 (2 \cdot_3 1) = 1 \neq 4 = (1 \cdot_3 2) \cdot_3 1$, it follows that $(S_3, \cdot_3)$ is a NeutroSemigroup.
By defining the partial order

$$\leq_3 = \{(0,0), (0,1), (0,3), (0,4), (1,1), (1,3), (1,4), (2,2), (3,3), (3,4), (4,4)\}$$

on S_3, we get that $(S_3, \cdot_3, \leq_3)$ is a NeutroOrderedSemigroup that is not NeutroTotalOrdered-Semigroup as "$\leq_3$" is not a total order on S_3. This is easily seen as:
$0 \leq_3 4$ implies that $0 \cdot_3 x \leq_3 4 \cdot_3 x$ and $x \cdot_3 0 \leq_3 x \cdot_3 4$ for all $x \in S_3$. And having $0 \leq_3 1$ but $0 \cdot_3 2 = 0 \nleq_3 2 = 1 \cdot_3 2$.

Example 3.13. Let $\mathbb{Z}$ be the set of integers and define "$\odot$" on $\mathbb{Z}$ as follows: $x \odot y = xy - 1$ for all $x, y \in \mathbb{Z}$. Since $0 \odot (1 \odot 0) = -1 = (0 \odot 1) \odot 0$ and $0 \odot (1 \odot 2) = -1 \neq -3 = (0 \odot 1) \odot 2$, it follows that $(\mathbb{Z}, \odot)$ is a NeutroSemigroup. We define the partial order "$\leq_{\mathbb{Z}}$" on $\mathbb{Z}$ as $-1 \leq_{\mathbb{Z}} x$ for all $x \in \mathbb{Z}$ and for $a, b \geq 0$, $a \leq_{\mathbb{Z}} b$ is equivalent to $a \leq b$ and for $a, b < 0$, $a \leq_{\mathbb{Z}} b$ is equivalent to $a \geq b$. In this way, we get $-1 \leq_{\mathbb{Z}} 0 \leq_{\mathbb{Z}} 1 \leq_{\mathbb{Z}} 2 \leq_{\mathbb{Z}} \ldots$ and $-1 \leq_{\mathbb{Z}} -2 \leq_{\mathbb{Z}} -3 \leq_{\mathbb{Z}} \ldots$. Having $0 \leq_{\mathbb{Z}} 1$ and $x \odot 0 = 0 \odot x = -1 \leq_{\mathbb{Z}} x - 1 = 1 \odot x = x \odot 1$ for all $x \in \mathbb{Z}$ and $-1 \leq_{\mathbb{Z}} 0$ but $(-1) \odot (-1) = 0 \nleq_{\mathbb{Z}} -1 = 0 \odot (-1)$ implies that $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$ is a NeutroOrderedSemigroup with -1 as minimum element.

Example 3.14. Let "$\leq$" be the usual order on $\mathbb{Z}$ and "$\odot$" be the operation define on $\mathbb{Z}$ in Example 3.13. One can easily see that $(\mathbb{Z}, \odot, \leq)$ is not a NeutroTotalOrderedSemigroup as there exist no $x \leq y \in \mathbb{Z}$ (with $x \neq y$) such that $z \odot x \leq z \odot y$ for all $z \in \mathbb{Z}$. In this case and according to Definition 3.3, $(T, I, F) = (0, 0, 1)$.

Example 3.15. Let $\mathbb{Z}$ be the set of integers and define "$\otimes$" on $\mathbb{Z}$ as follows: $x \otimes y = xy + 1$ for all $x, y \in \mathbb{Z}$. Since $0 \otimes (1 \otimes 0) = 1 = (0 \otimes 1) \otimes 0$ and $0 \otimes (1 \otimes 2) = 1 \neq 3 = (0 \otimes 1) \otimes 2$, it follows that $(\mathbb{Z}, \otimes)$ is a NeutroSemigroup. We define the partial order "$\leq_{\otimes}$" on $\mathbb{Z}$ as $1 \leq_{\otimes} x$ for all $x \in \mathbb{Z}$ and for $a, b \geq 1$, $a \leq_{\otimes} b$ is equivalent to $a \leq b$ and for $a, b \leq 0$, $a \leq_{\otimes} b$ is equivalent to $a \geq b$. In this way, we get $1 \leq_{\otimes} 2 \leq_{\otimes} 3 \leq_{\otimes} 4 \leq_{\otimes} \ldots$ and $1 \leq_{\otimes} 0 \leq_{\otimes} -1 \leq_{\otimes} -2 \leq_{\otimes} \ldots$. Having $0 \leq_{\otimes} -1$ and $x \otimes 0 = 0 \otimes x = 1 \leq_{\otimes} -x + 1 = -1 \otimes x = x \otimes (-1)$ for all $x \in \mathbb{Z}$ and

$1 \leq_{\otimes} 0$ but $1 \otimes 1 = 2 \nleq_{\otimes} 1 = 0 \odot 1$ implies that $(\mathbb{Z}, \otimes, \leq_{\otimes})$ is a NeutroOrderedSemigroup with 1 as minimum element.

We present some examples on NeutroOrderedSubSemigroups, NeutroOrderedRightIdeals, NeutroOrderedLeftIdeals, NeutroOrderedIdeals, and NeutroOrderedFilters.

Example 3.16. Let $(S_3, \cdot_3, \leq_3)$ be the NeutroOrderedSemigroup presented in Example 3.12. Then $I = \{0, 1, 2\}$ is a NeutroSubSemigroup of S_3 as $(I, \cdot_3)$ is NeutroOperation (with no AntiAxiom as $0 \cdot_3 (0 \cdot_3 0) = (0 \cdot_3 0) \cdot_3 0$) and $0 \leq_3 1 \in I$ but $2 \cdot_3 0 = 0 \leq_3 4 = 2 \cdot_3 1$ is indeterminate over I as $4 \notin I$. Moreover, $(0] = \{0\} \subseteq I$. Since $g \cdot_3 0 = 0 \in I$ for all $g \in S_3$, it follows that I is a NeutroOrderedLeftIdeal of S_3. Moreover, having $1 \cdot_3 g \in \{0, 1, 2\} \subseteq I$ implies that I is a NeutroOrderedRightIdeal of S_3. Since there is no $g \in S$ satisfying $g \cdot_3 i \in I$ and $i \cdot_3 g \in I$ for a particular $i \in I$, it follows that I is not a NeutroOrderedIdeal of S_3.

Remark 3.17. Unlike the case in Ordered Semigroups, the intersection of NeutroOrdered-Subsemigroups may not be a NeutroOrderedSubsemigroup. (See Example 3.18.)

Example 3.18. Let $(S_3, \cdot_3, \leq_3)$ be the NeutroOrderedSemigroup presented in Example 3.12. One can easily see that $J = \{0, 1, 3\}$ is a NeutroOrderedSubsemigroup of S_3. From Example 3.16, we know that $I = \{0, 1, 2\}$ is a NeutroOrderedSubsemigroup of S_3. Since $(\{0, 1\}, \cdot_3)$ is a semigroup and not a NeutroSemigroup, it follows that $(I \cap J, \cdot_3, \leq_3)$ is not a NeutroOrdered-SubSemigroup of S_3. Here, $I \cap J = \{0, 1\}$.

Example 3.19. Let $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$ be the NeutroOrderedSemigroup presented in Example 3.13. Then $I = \{-1, 0, 1, -2, -3, -4, \ldots\}$ is a NeutroOrderedIdeal of $\mathbb{Z}$. This is clear as:

(1) $0 \odot (1 \odot 0) = -1 = (0 \odot 1) \odot 0$ and $0 \odot (-1 \odot -2) = -1 \neq 1 = (0 \odot -1) \odot -2$;
(2) $g \odot 0 = 0 \odot g = -1 \in I$ for all $g \in \mathbb{Z}$;
(3) $-1 \in I$ and $(-1] = \{-1\} \subseteq I$;
(4) $0 \leq_{\mathbb{Z}} 1 \in I$ implies that $0 \odot x = x \odot 0 = -1 \leq_{\mathbb{Z}} x - 1 = x \odot 1 = 1 \odot x$ for all $x \in I$ and $-1 \leq_{\mathbb{Z}} 0 \in I$ but $-1 \odot -1 = 0 \nleq_{\mathbb{Z}} -1 = 0 \odot -1$.

Example 3.20. Let $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$ be the NeutroOrderedSemigroup presented in Example 3.13. Then $F = \{-1, 0, 1, 2, 3, 4, \ldots\}$ is a NeutroOrderedFilter of $\mathbb{Z}$. This is clear as:

(1) $0 \odot (1 \odot 0) = -1 = (0 \odot 1) \odot 0$ and $1 \odot (2 \odot 3) = 4 \neq 2 = (1 \odot 2) \odot 3$;
(2) $1 \in F$ and for all $x \in \mathbb{Z}$ such that $x - 1 = 1 \odot x = x \odot 1 \in F$, we have $x \in F$;
(3) $0 \in F$ and $[0) = \{0, 1, 2, 3, 4, \ldots\} \subseteq F$;
(4) $0 \leq_{\mathbb{Z}} 1 \in F$ and $0 \otimes (-1) = -1 \leq -2 = 1 \otimes (-1)$ is indeterminate in F.

Here, F is not a NeutroOrderedSubSemigroup of $\mathbb{Z}$ as there exist no $x \in F$ with $(x] \subseteq F$.

Example 3.21. Let $(S_2, \cdot_2, \leq_2)$ be the NeutroTotalOrderedSemigroup presented in Example 3.10. Then $F = \{1, 2, 3\}$ is a NeutroOrderedFilter of S_2. This is clear as:

(1) $2 \cdot_2 (2 \cdot_2 2) = (2 \cdot_2 2) \cdot_2 2$ and $1 \cdot_2 (2 \cdot_2 1) = 3 \neq 1 = (1 \cdot_2 2) \cdot_2 1$;

(2) $1 \cdot_2 x \in F$ and $z \cdot_2 1 \in F$ implies that $x, z \in F$;

(3) $3 \in F$ and $(3] = \{3\} \subseteq F$;

(4) $2 \leq_2 3 \in F$ implies that $2 \cdot_2 x \leq_2 3 \cdot_2 x$ and $x \cdot_2 2 \leq_2 x \cdot_2 3$ for all $x \in F$ and $1 \leq_2 2$ but $2 \cdot_2 1 = 3 \nleq_2 2 = 2 \cdot_2 2$.

Lemma 3.22. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. Then S is a NeutroTotalOrderedSemigroup if and only if S' is a NeutroTotalOrderedSemigroup.*

Proof. The proof is straightforward. □

Remark 3.23. Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedIsomorphism. Then Lemma 3.22 may not hold. (See Example 3.24.)

Example 3.24. Let $(S_2, \cdot_2, \leq_2)$ be the NeutroTotalOrderedSemigroup presented in Example 3.10, $(S_2, \cdot_2', \leq_2')$ be the NeutroOrderedSemigroup presented in Example 3.11, and $\phi : (S_2, \cdot_2, \leq_2) \to (S_2, \cdot_2', \leq_2')$ be defined as $\phi(x) = x$ for all $x \in S_2$. One can easily see that ϕ is a NeutroOrderedIsomorphism that is not NeutroOrderedStrongIsomorphism as: $\phi(0 \cdot_2 0) = \phi(0) = 0 = \phi(0) \cdot_2' \phi(0)$, $0 \leq_2 1$ and $\phi(0) = 0 \leq_2' 1 = \phi(1)$, $1 \leq_2 3$ but $\phi(1) = 1 \nleq_2' 3 = \phi(3)$. Having $(S_2, \cdot_2, \leq_2)$ a NeutroOrderedSemigroup that is not NeutroTotalOrderedSemigroup and $(S_2, \cdot_2', \leq_2')$ a NeutroTotalOrderedSemigroup illustrates our idea.

Lemma 3.25. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. Then S contains a minimum (maximum) element if and only if S' contains a minimum (maximum) element.*

Proof. The proof is straightforward. □

Remark 3.26. In Lemma 3.25, if $\phi : S \to S'$ is a NeutroOrderedIsomorphism that is not a NeutroOrderedStrongIsomorphism then S' may contain a minimum (maximum) element and S does not contain. (See Example 3.27.)

Example 3.27. With reference to Example 3.24, $(S_2, \cdot_2, \leq_2)$ has 0 as its minimum element whereas $(S_2, \cdot_2', \leq_2')$ has no minimum element.

Lemma 3.28. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $M \subseteq S$ is a NeutroOrderedSubsemigroup of S then $\phi(M)$ is a NeutroOrderedSubsemigroup of S'.*

Proof. First, we prove that $(\phi(M), \star)$ is a NeutroSemigroup. Since $(M, \cdot)$ is a NeutroSemigroup, it follows that $(M, \cdot)$ is either NeutroOperation or NeutroAssociative.

- Case $(M, \cdot)$ is NeutroOperation. There exist $x, y, a, b \in M$ such that $x \cdot y \in M$ and $a \cdot b \notin M$ or $x \cdot y$ is indeterminate. The latter implies that there exist $\phi(x), \phi(y), \phi(a), \phi(b) \in \phi(M)$ such that $\phi(x) \star \phi(y) = \phi(x \cdot y) \in \phi(M)$ and $\phi(a) \star \phi(b) = \phi(a \cdot b) \notin \phi(M)$ or $\phi(x) \star \phi(y) = \phi(x \cdot y)$ is indeterminate.
- Case $(M, \cdot)$ is NeutroAssociative. There exist $x, y, z, a, b, c \in M$ such that $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ and $(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$. The latter implies that there exist $\phi(x), \phi(y), \phi(z), \phi(a), \phi(b), \phi(c) \in \phi(M)$ such that $(\phi(x) \star \phi(y)) \star \phi(z) = \phi(x) \star (\phi(y) \star \phi(z))$ and $(\phi(a) \star \phi(b)) \star \phi(c) \neq \phi(a) \star (\phi(b) \star \phi(c))$ (as ϕ is one-to-one.).

Since M is a NeutroOrderedSubsemigroup of S, it follows that there exist $x \in M$ such that $(x] \subseteq M$. It is easy to see that $(\phi(x)] \subseteq \phi(M)$ as for all $t \in S'$, there exist $y \in S$ such that $t = \phi(y)$. For $\phi(y) \leq_{S'} \phi(x)$, we have $y \leq_S x$. The latter implies that $y \in M$ and hence, $t \in \phi(M)$.

Since M is a NeutroOrderedSubsemigroup of S, it follows that:

(T) There exist $x \leq_S y \in M$ (with $x \neq y$) such that $z \cdot x \leq_S z \cdot y$ and $x \cdot z \leq_S y \cdot z$ for all $z \in M$;

(F) There exist $a \leq_S b \in M$ and $c \in M$ with $a \cdot c \nleq_S b \cdot c$ (or $c \cdot a \nleq_S c \cdot b$);

(I) There exist $x \leq_S y \in M$ and $z \in M$ with: $z \cdot x$ (or $x \cdot z$ or $y \cdot z$ or $z \cdot y$) indeterminate or $z \cdot x \leq_S z \cdot y$ (or $x \cdot z \leq_S y \cdot z$) indeterminate in M.

Where $(T, I, F) \neq (1, 0, 0)$ and $(T, I, F) \neq (0, 0, 1)$. This implies that

(T) There exist $\phi(x) \leq_{S'} \phi(y) \in \phi(M)$ (with $\phi(x) \neq \phi(y)$ as $x \neq y$) such that $\phi(z) \star \phi(x) \leq_{S'} \phi(z) \star \phi(y)$ and $\phi(x) \star \phi(z) \leq_{S'} \phi(y) \star \phi(z)$ for all $\phi(z) \in \phi(M)$;

(F) There exist $\phi(a) \leq_{S'} \phi(b) \in \phi(M)$ and $\phi(c) \in \phi(M)$ with $\phi(a) \star \phi(c) \nleq_{S'} \phi(b) \star \phi(c)$ (or $\phi(c) \star \phi(a) \nleq_{S'} \phi(c) \star \phi(b)$);

(I) There exist $\phi(x) \leq_{S'} \phi(y) \in \phi(M)$ and $\phi(z) \in \phi(M)$ with: $\phi(z) \star \phi(x)$ (or $\phi(x) \star \phi(z)$ or $\phi(y) \star \phi(z)$ or $\phi(z) \star \phi(y)$) indeterminate or $\phi(z) \star \phi(x) \leq_{S'} \phi(z) \star \phi(y)$ (or $\phi(x) \star \phi(z) \leq_{S'} \phi(y) \star \phi(z)$) indeterminate in $\phi(M)$.

Where $(T, I, F) \neq (1, 0, 0)$ and $(T, I, F) \neq (0, 0, 1)$. Therefore, $\phi(M)$ is a NeutroOrderedSubsemigroup of S'. □

Lemma 3.29. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $M \subseteq S$ is a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S then $\phi(M)$ is a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S'.*

Proof. We prove that if $M \subseteq S$ is a NeutroOrderedLeftIdeal of S then $\phi(M)$ is a NeutroOrderedLeftIdeal of T. For NeutroOrderedRightIdeal, it is done similarly. Using Lemma 3.28, it suffices to show that there exist $z \in \phi(M)$ such that for all $t \in S'$ $t \star z \in \phi(M)$. Since M is a NeutroOrderedLeftIdeal of S, it follows that there exist $m \in M$ such that $s \cdot m \in m$ for all $s \in S$. Having ϕ an onto function implies that for all $t \in S'$, there exist $s \in S$ with $t = \phi(s)$. By setting $z = \phi(m)$, we get that $t \star z = \phi(s) \star \phi(m) = \phi(s \cdot m) \in \phi(M)$. □

Lemma 3.30. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $M \subseteq S$ is a NeutroOrderedIdeal of S then $\phi(M)$ is a NeutroOrderedIdeal of S'.*

Proof. The proof is similar to that of Lemma 3.29. □

Example 3.31. Let $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$ and $(\mathbb{Z}, \otimes, \leq_{\otimes})$ be the NeutroOrderedSemigroups presented in Example 3.13 and Example 3.15 respectively, and $\phi : (\mathbb{Z}, \odot, \leq_{\mathbb{Z}}) \to (\mathbb{Z}, \otimes, \leq_{\otimes})$ be defined as $\phi(x) = x+2$ for all $x \in \mathbb{Z}$. One can easily see that ϕ is a NeutroOrderedStrongIsomorphism. By Example 3.19, we have $I = \{-1, 0, 1, -2, -3, -4, \ldots\}$ is a NeutroOrderedIdeal of $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$. Applying Lemma 3.30, we get that $\phi(I) = \{1, 2, 3, 0, -1, -2, \ldots\}$ is a NeutroOrderedIdeal of $(\mathbb{Z}, \otimes, \leq_{\otimes})$.

Lemma 3.32. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $M \subseteq S$ is a NeutroOrderedFilter of S then $\phi(M)$ is a NeutroOrderedFilter of S'.*

Proof. Using Lemma 3.28, we get that $(\phi(M), \star)$ is a NeutroSemigroup and that $\leq_{S'}$ is NeutroOrder on $\phi(M)$. i.e., Conditions (1), (2), and (3) of Definition 3.3 are satisfied.
Since M is a NeutroOrderedFilter of S, it follows that there exist $x \in M$ such that $[x) \subseteq M$. It is easy to see that $[\phi(x)) \subseteq \phi(M)$ as for all $t \in S'$, there exist $y \in S$ such that $t = \phi(y)$. For $\phi(x) \leq_{S'} \phi(y)$, we have $x \leq_S y$. The latter implies that $y \in M$ and hence, $t \in \phi(M)$.
Since M is a NeutroOrderedFilter of S, it follows that there exist $x \in M$ such that for all $y, z \in S$ with $x \cdot y \in M$ and $z \cdot x \in M$ we have $y, z \in M$. The latter and having ϕ onto implies that there exist $t = \phi(x) \in \phi(M)$ such that for all $\phi(y), \phi(z) \in S'$ with $\phi(x) \star \phi(y) \in \phi(M)$ and $\phi(z) \star \phi(x) \in \phi(M)$ we have $\phi(y), \phi(z) \in \phi(M)$. □

Example 3.33. Let $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$ and $(\mathbb{Z}, \otimes, \leq_{\otimes})$ be the NeutroOrderedSemigroups presented in Example 3.13 and Example 3.15 respectively, and $\phi : (\mathbb{Z}, \odot, \leq_{\mathbb{Z}}) \to (\mathbb{Z}, \otimes, \leq_{\otimes})$ be the NeutroOrderedStrongIsomorphism defined as $\phi(x) = x + 2$ for all $x \in \mathbb{Z}$. By Example 3.20, we have $F = \{-1, 0, 1, 2, 3, 4, \ldots\}$ is a NeutroOrderedFilter of $(\mathbb{Z}, \odot, \leq_{\mathbb{Z}})$. Applying Lemma 3.32, we get that $\phi(F) = \{1, 2, 3, 4, 5, 6, \ldots\}$ is a NeutroOrderedFilter of $(\mathbb{Z}, \otimes, \leq_{\otimes})$.

Lemma 3.34. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $N \subseteq S'$ is a NeutroOrderedSubsemigroup of S' then $\phi^{-1}(N)$ is a NeutroOrderedSubsemigroup of S.*

Proof. Theorem 2.11 asserts that $\phi^{-1} : S' \to S$ is a NeutroOrderedStrongIsomorphism. Having $N \subseteq S'$ a NeutroOrderedSubsemigroup of S' and by using Lemma 3.28, we get that $\phi^{-1}(N)$ is a NeutroOrderedSubsemigroup of S. □

Lemma 3.35. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $N \subseteq S'$ is a NeutroOrderedSubsemigroup of S' then $\phi^{-1}(N)$ is a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S.*

Proof. Theorem 2.11 asserts that $\phi^{-1} : S' \to S$ is a NeutroOrderedStrongIsomorphism. Having $N \subseteq S'$ a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S' and by using Lemma 3.29, we get that $\phi^{-1}(N)$ is a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S. □

Lemma 3.36. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $N \subseteq S'$ is a NeutroOrderedSubsemigroup of S' then $\phi^{-1}(N)$ is a NeutroOrderedIdeal of S.*

Proof. Theorem 2.11 asserts that $\phi^{-1} : S' \to S$ is a NeutroOrderedStrongIsomorphism. Having $N \subseteq S'$ a NeutroOrderedIdeal of S' and by using Lemma 3.35, we get that $\phi^{-1}(N)$ is a NeutroOrderedIdeal of S. □

Lemma 3.37. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. If $N \subseteq S'$ is a NeutroOrderedFilter of S' then $\phi^{-1}(N)$ is a NeutroOrderedFilter of S.*

Proof. Theorem 2.11 asserts that $\phi^{-1} : S' \to S$ is a NeutroOrderedStrongIsomorphism. Having $N \subseteq S'$ a NeutroOrderedFilter of S' and by using Lemma 3.32, we get that $\phi^{-1}(N)$ is a NeutroOrderedFilter of S. □

We present our main theorems.

Theorem 3.38. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. Then $M \subseteq S$ is a NeutroOrderedSubsemigroup of S if and only if $\phi(M)$ is a NeutroOrderedSubsemigroup of S'.*

Proof. The proof follows from Lemmas 3.28 and 3.34. □

Theorem 3.39. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. Then $M \subseteq S$ is a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S if and only if $\phi(M)$ is a NeutroOrderedLeftIdeal (NeutroOrderedRightIdeal) of S'.*

Proof. The proof follows from Lemmas 3.29 and 3.35. □

Theorem 3.40. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. Then $M \subseteq S$ is a NeutroOrderedIdeal of S if and only if $\phi(M)$ is a NeutroOrderedIdeal of S'.*

Proof. The proof follows from Lemmas 3.30 and 3.36. □

Theorem 3.41. *Let $(S, \cdot, \leq_S)$ and $(S', \star, \leq_{S'})$ be NeutroOrderedSemigroups and $\phi : S \to S'$ be a NeutroOrderedStrongIsomorphism. Then $M \subseteq S$ is a NeutroOrderedFilter of S if and only if $\phi(M)$ is a NeutroOrderedFilter of S'.*

Proof. The proof follows from Lemmas 3.32 and 3.37. □

4. Conclusion

This paper contributed to the study of NeutroAlgebra by introducing, for the first time, NeutroOrderedAlgebra. The new defined notion was applied to semigroups and many interesting properties were proved as well illustrative examples were given on NeutroOrderedSemigroups.

For future research, it will be interesting to apply the concept of NeutroOrderedAlgebra to different algebraic structures such as groups, rings, modules, etc. and to study AntiOrderedAlgebra.

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