Branching-Time and Doomsday*

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Abstract

Branching-time is a popular theory of time that is intended to account for the openness of the future. Generally, branching-time models the openness of the future by positing a multiplicity of concrete alternative futures mirroring all the possible ways the future could unfold. A distinction is drawn in the literature among branching-time theories: those that make use of moment-based structures and those that employ history-based ones. In this paper, I introduce and discuss a particular kind of openness relative to the possibility that time ends (Doomsday). I then show that whereas moment-based branching structures cannot represent this kind of openness, history-based structures can account for it. The conclusion is that history-based structures score a point over moment-based ones.

Key words: Time, Branching-Time, Open Future, Doomsday.

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1 Introduction

The future, it is commonly thought, is open in a way the past isn't. For instance, it is normally believed that the future, unlike the past, holds various alternative possibilities. When thinking about the next days, weeks or years, it is natural to assume that there are multiple possible ways our collective and individual futures might be, whereas the same does not hold for the past. Moreover, it is commonplace to hold that the future, unlike the past, is not completely decided yet. Whereas it is now established what happened one year ago, it is not decided yet *which* future, among the possible ones, will turn out to be our future. Although the vast majority of philosophers of time tend to agree with the idea that the future is *in some way or another* open, there is disagreement about how the openness of the future should exactly be understood, and what metaphysical theories of time best characterize this openness. The various theories of time in the literature model differently the openness of the future and have different understanding of what the openness itself amounts to (see Torre 2011 and Grandjean 2021 for an overview of the debate). Here, I will focus just on one of those theories of time, viz. a branching conception of time.

Branching-time holds, pictorially, that our world has the shape of a tree, as it branches from a single trunk in the direction of the future. In such a view, and from a perspective of a moment m, the past of m is represented as a single line of moments, whereas ahead of m we have a *future of possibilities*—several alternative futures that all stem from the unique past of m. One of the distinctive features of branching-time is that these alternative futures are taken to be concrete—according to branching-time, the alternative futures *exist* as concrete entities and are ontologically on a par with the past (and with respect to each other). The resulting picture is one where what is temporally possible does exist somewhere in the tree-like structure which is the world. Accordingly, branching-time models the openness of the future via the *existence* of this multiplicity of alternative futures, all belonging to the same tree-shaped world.[1]

Branching-time theories can be divided in two categories: those that use history-based structures and those that use moment-based structures. (I borrow the terminology from Grandjean & Pascucci 2021.) In a nutshell, the two approaches differ on how we treat one of the essential concepts of branching-time, viz. that of a history. Roughly, a history represents a single possible course of events within the many given by the branching structure. A history can be thought of as one of the complete developments of the world—pictorially, one linear path within the branching tree. In the case of moment-based branching structures, histories are not a fundamental aspect of reality. Rather, they supervene on the set of moments composing the tree and the fundamental earlier-later relation over them. In the case of history-based branching structures, on the other hand, what histories there are is one of the primitive features of a branching world. That is, in the context of history-based structures, histories are seen as one of the fundamental entities that compose temporal reality.

There is a debate about whether one should prefer moment-based or history-

[1] Branching-time can be cashed out in different alternative versions. Here I am going to use the label 'branching-time' for any theory of time that posits the existence of a multiplicity of alternative futures. In this broad sense, branching-time includes: i) standard B-theoretical branching-time (e.g., Thomason 1970, Belnap et al. 2001, and MacFarlane 2003), ii) A-theoretic versions of branching-time where the passing of time eliminates some branches (McCall 1976), and iii) theories where one of the alternative futures is metaphysically privileged, viz. it is the Thin Red Line (e.g., Øhrstrøm 2009, Malpass & Waver 2012, and Borghini & Torrengo 2013). What I am going to argue in this paper applies to all these versions of branching-time.

based structures. Part of the debate revolves around which option fares better in representing metaphysical possibilities with respect to temporal openness. In this paper, I will consider a special case of temporal openness related to the possibility that time itself ends (Doomsday). I will argue that history-based structures score a point over moment-based one, insofar as the former but not the latter can represent the doomsday scenario I will discuss.[2]

The point I am going to make, viz. that moment-based structures cannot represent a particular kind of Doomsday scenario while history-based ones can, might seem too modest. However, it has its own philosophical significance. First of all, branching-time in general is a widespread approach which is used to model both technically and metaphysically the openness of the future. Secondly, the notion of a history plays a crucial role within branching-time. Moreover, the distinction between moment-based structures and history-based ones is not just a technical one. On the contrary, the two approaches have different metaphysical and ontological commitments. The way we understand histories—either as fundamental and primitive aspects of reality or as derivative entities that entirely depend on moments of time and the relation among them—does make a difference in our understanding of what constitutes temporal reality (see also Zanardo 2006: 381 on this point). Finally, my point has to do with the notion of descriptive adequacy, as long as I will argue that history-based structures can represent a kind of Doomsday scenario while moment-based ones cannot. When trying to establish what views of reality one should hold, descriptive adequacy is a fundamental parameter both from a technical and a metaphysical viewpoint. That is, we want our theoretical

[2] Loss (2019) argues that Doomsday scenarios can be problematic for another theory of time that is intended to account for the openness of the future, viz. the growing block theory of time.

models to be able to represent scenarios that seem to be genuine possibilities. If I am right in what follows, this paper shows that moment-based structures have a problem with descriptive adequacy with respect to the Doomsday case I will discuss, whereas history-based structures do not.

Roadmap. In section 2, I will briefly recap some technicalities of branching-time, as well as the distinction between history-based and moment-based branching structures. In section 3, I will present and discuss some literature about the debate on moment-based structures versus history-based ones. Next, in section 4, I will present my example about Doomsday and show why this Doomsday scenario favors history-based structures over moment-based ones.

2 Some Preliminaries

Throughout the paper, I will make use of the temporal Ockhamist semantics (Prior 1967). Although the points I am going to make do not depend on the nature of the Ockhamist semantics, I will adopt the Ockhamist semantics as it is a common and convenient way to formalize modal-temporal claims in the context of branching time. In this section, I will quickly recap some of its main tenets, as well as two important principles of branching-time structures, viz. no backward branching and historical connectedness. I will first introduce the Ockhamist semantics in the context of moment-based branching structures, as moment-based branching structures are somewhat the standard ones, whereas toward the end of the section I will illustrate the difference between moment-based structures and history-based ones. What follows draws heavily from Belnap et al. (2001) and Øhrstrøm & Hasle (2020). The reader already familiar with these formalities can skip this section.

To start, let us introduce a temporal language L which includes an infinite set of non-tensed atomic sentence letters $p, q, r \ldots$, the standard connectives $\rightarrow$ and $\neg$, the temporal operators P ('it was the case that...') and F ('it will be the case that...'), and the necessity operator $\Box$. The dual operators H ('it was always the case that...'), G ('it will always be the case that...'), and the possibility operator $\Diamond$ are defined in the usual manner as $\neg P \neg$, $\neg F \neg$, and $\neg \Box \neg$ respectively. The grammar is defined recursively in a standard way. Next, a *Branching-Time Model (BTM)* is defined as an ordered triple, $\langle T, \leq, TRUE \rangle$, where T is a non-empty set of moments, $\leq$ is a binary, transitive, and reflexive *at-least-as-earlier-than* relation over T, and $TRUE$ is a two place-function that assigns either 1 (true) or 0 (false) to couples of moments/atomic sentence letters. Once we defined the reflexive at-least-as-earlier-than relation, we can define the irreflexive *earlier-than* relation ($<$) as follows: for any m, m' in T, $m < m'$ iff $m \leq m'$ and $m \neq m'$. (In the rest of the article, I will mostly use the $<$-relation). *Histories* are then defined as maximally ordered sets of $<$-related moments of T—intuitively, a history is one of the many possible *complete* developments of the world. To ensure no backward branching, it is imposed that the relation $\leq$ satisfies the condition that for any moment m, m', m'' in T, if $m \leq m''$ and $m' \leq m''$ then either $m \leq m'$ or $m' \leq m$. To grant connectedness across the structure, it is imposed that for any moments m, m' there is a moment m'' such that $m'' \leq m$ and $m'' \leq m'$—intuitively, this guarantees that if you go back enough from any two distinct branches, you will find at some point a shared trunk.

This said, the Ockhamist evaluation function V assigns truth values to well-formed formulas relative to a BTM-model. More precisely, truth values are assigned relative to couples of a moment m *and* a history h passing through m. Where ϕ and ψ are any wff of L:

- if ϕ is an atomic sentence letter, $V(\phi) = 1$ at m/h iff $TRUE(\phi, m) = 1$
- $V(\neg\phi) = 1$ at m/h iff $V(\phi) = 0$ at m/h.
- $V(\phi \to \psi) = 1$ at m/h iff $V(\phi) = 0$ at m/h or $V(\psi) = 1$ at m/h.
- $V(F\phi) = 1$ at m/h iff $V(\phi) = 1$ at m'/h for some $m' \in h$ with $m < m'$.
- $V(P\phi) = 1$ at m/h iff $V(\phi) = 1$ at m'/h for some $m' \in h$ with $m' < m$.
- $V(\Box\phi) = 1$ at m/h iff for all h' such that $m \in h'$, $V(\phi) = 1$ at m/h'.

In short, formulas that do not feature the necessity or the possibility operator get their moment/history evaluation based on what happens only on that history. Formulas that instead do feature a reference to what is *possible* or *necessary*—the kind of claims I will here be mostly interested in—require one to check what happens in other histories passing through the moment of evaluation.

What said so far summarizes the Ockhamist semantics in the context of moment-based branching-time structures. In the case of history-based structures, what changes is how we construe the branching-time models. Whereas in the case of moment-based structures the models are defined as an ordered triple $\langle T, <, TRUE \rangle$, in the case of history-based structures we have a fundamental parameter that represents the set of all histories. An example of a history-based approach is that of *Bundled Trees* (Zanardo 2006 and 2006a for an overview). In bundled tree approaches, histories are taken to be primitive entities. One fundamental parameter of the model, viz. the bundle $\mathcal{B}$, specifies what histories are admitted in a given branching structure. A bundle $\mathcal{B}$ on T is usually described as the set of histories with the property that, for every m in T, there is a history h such that $m \in h$ (Zanardo 2006a: 489).

To sum up, in moment-based structures, what histories there are completely depends on the set of moments T together with the $<$-relation among moments, as histories are defined as maximally ordered sets of $<$-related moments. That is, once the set T and the $<$-relation are defined in a given Branching-Time Model, we automatically acquire the set of histories in the model. In history-based structures, on the other hand, histories need not be maximally ordered sets of $<$-related moments, and what histories there are is specified by the model. Crucially, in the context of history-based structures, one can omit from the set of histories some sets of moments that would normally count as histories in the moment-based approach. Here ends this section, as this should be enough for our purposes.

3 History-based versus moment-based structures

There is an open debate on what should be preferred between moment-based and history-based branching structures. One of the aspects of the debate revolves around the following aspect, viz. which one between the two structures fares better with respect to representing scenarios that, from an intuitive viewpoint, constitute genuine metaphysical possibilities.

For instance, Øhrstrøm and Hasle (1994: 268-269) argue that moment-based structures face problems with respect to a particular scenario.[3] Consider the following two sentences.

[3] Their example is based on that of Nishimura (1979).

(1) Inevitably, if today there is life on earth, then either this is the last day (of life on earth) or the last day will come.

(2) At any possible day at which there is life on earth, it is possible that there will be life on earth the following day.

(1) conveys the idea that life on earth cannot go on forever—the last day of life on earth should eventually come—whereas (2) states that if there is life, there is a possibility for the continuation of life at least until the next day. Next, with q standing for 'there is life on earth' and with days as units of time, (1) can be formalized as $q \to (G\neg q \vee F(q \wedge Hq \wedge G\neg q))$ for all m/h (given the 'inevitably' operator), and (2) as $q \to \Diamond F(1)q$ for all m/h (given that it holds at all days).[4] Øhrstrøm and Hasle judge (1) and (2) to be compatible, i.e. *not* contradictory. If the two are indeed compatible and hold at all m/h pairs, there could then be a branching structure where (1) and (2) are both true at all moment/history pairs. Consider such a structure and imagine that at m_0/h_1 q and $\neg G\neg q$ are the case. It follows, in virtue of (1), that there is another day later than m_0 where the last day on earth comes, i.e. a moment m_1 where, relative to the pair m_1/h_1, $q \wedge Hq \wedge G\neg q$ is the case—life on earth ends at m_1 on the history h_1. However, as q is the case at m_1, it is also the case that $\Diamond F(1)q$, in virtue of (2). This implies that there must be a day following m_1, on a further history h_2, where q.[5] Let us name this moment m_2'. But as q is the case at m_2', it follows via (1) that at the pair m_2'/h_2, $G\neg q \vee F(q \wedge Hq \wedge G\neg q)$. Hence either m_2' is the last day of life on earth relative to

[4]Notice that the formalization of (2) features the metric version of the temporal operator F. $F(n)p$ must be read as 'it will be the case in n units of time that p'. I will use the metric temporal opearor in other examples throughout the paper. For the sake of brevity, I will not here recap how a metric can be imposed on a branching structure. The reader interested in the details of this can look at, among others, Belnap et al. (2001: 195-6) and Spolaore and Gallina (2020: 102-3).

[5]The example assumes for the sake of simplicity that time is discrete.

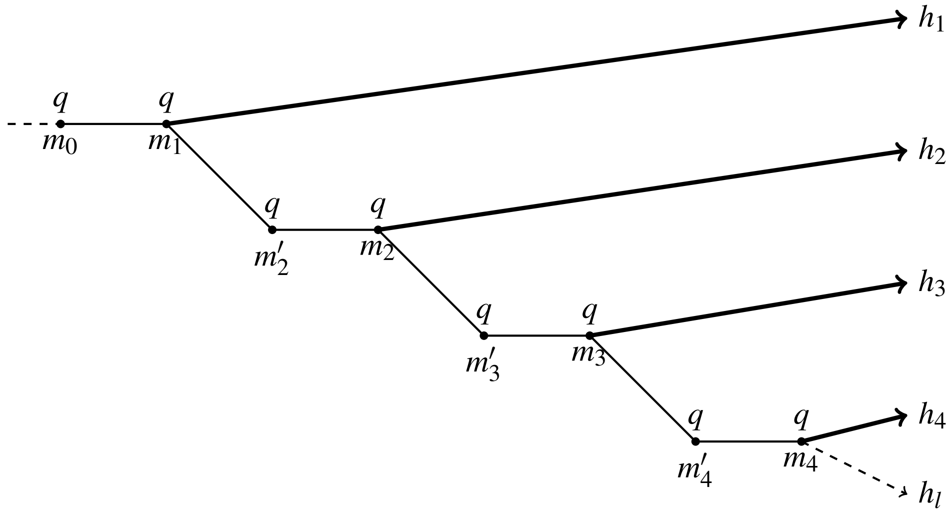

Figure 1 – The thickest parts of the tree are parts at which there is no life on earth, i.e. where $\neg q$ is the case.

h_2 or down the road of h_2 we have a moment m_2 where q and, relative to the pair m_2/h_2, $q \wedge Hq \wedge G\neg q$ is the case, as well as $\Diamond F(1)q$ (because of (2)). In virtue of the truth of the latter formula, there must be a day following m_2 on a further history h_3, where q…It is easy to see that this process can just be applied over and over (see fig. 1).

Consider then the set of moments in the lower part of the structure in the figure. This set is constituted only by moments at which there is life on earth. Moreover, this set is a maximally ordered set of $<$-related moments, and hence in the context of moment-based structures it constitutes a *history*—let us name it h_l. By adopting moment-based structures it thus follows that, contra the initial assumption, (1) is not true at the pair m_0/h_l, as q is the case at m_0, yet life on earth never ends on h_l. We thus have a contradiction, as we started by assuming that (1) and (2) were true at all moment/history pairs, whereas we end up having that (1) is not true at m_0/h_l. This, in Øhrstrøm and Hasle's view, shows that moment-based structures fail to represent the possibility of a branching world where (1) and (2) are both the case. On the other hand, in history-based structures, the set of $<$-related moments that we are calling h_l can be excluded by the set of histories, thereby avoiding the contradiction and making (1) and (2) compatible.

Belnap et al. (2001: 199-201) discuss a similar example and draw a different conclusion. They start by considering a branching world where the following is the case at all moment/history pairs.

(3) As long as there is life on earth, (i) life on earth might end before the next day, and (ii) life on earth might not end before the next day.

This means that if there is life on earth at a given day, it is open whether life on earth will continue the next day.[6] We can formalize (3) as $q \to (\Diamond F(1)q \wedge \Diamond F(1)\neg q)$. They then argue that from (3), it follows

(4) every life-on-earth chain of length n can be extended to a life-on-earth chain of length $n+1$.

Next Belnap et al. consider the following claim

(5) At m_0, it is necessary that life on earth will end after a finite number of days.

According to them, (4) and (5) are incompatible. That is, one cannot hold both without contradicting themselves. This, in fact, is the case if the branching

[6]Their example is actually about a radium atom that might or might not decay. I illustrate their argument in terms of life on earth possibly ending to highlight the parallels with the previous case. Moreover, their argument is explicitly intended against advocates of bundled trees (see end of section 2). I take it, though, that their argument can generalize towards any structure where the set of histories is a fundamental parameter of the model, i.e. history-based structures according to the terminology followed in this paper.

structure at play is a moment-based structure. This is because in the context of moment-based structures, the sequence of moments in the lower part of the structure (see the figure 2) constitutes a maximally ordered sets of $<$-related moments, i.e. a *history*, where there is always life on earth. This makes the claim (5) false—it is not inevitable at m_0 that life will end, as there is a history passing through m_0, viz. h_l, where life does *not* end. On the other hand, the claim (4) holds true in general in the structure. In fact, any chain of life on earth of length n can, in a temporal modal sense, extend to a chain of lenght $n+1$. That is, if you pick a life-on-earth chain reaching an arbitrary lenght n at a moment m, there always is in the structure a history passing through m where the life-on-earth chain has lenght $n+1$. This result, the fact that (4) turns out to be true, whereas (5) turns out false, is in accordance with the judgment that the two are contradictories. However, Belnap et. al note that if we switch to history-based structures, we could decide to ignore the history h_l, i.e. we could decide that the maximally ordered sets of moments that we are calling h_l is not one of the histories of the model. If so, (5) would be true, as in all histories stemming from m_0 life on earth ends after a finite number of days, and (4) would still be the case. That is, by switching to history-based structures, we could make (4) and (5) compatible, although, in their view, they should not and *cannot* be compatible. This, they argue, is a disadvantage of the history-based structures.

Before moving on, let us notice that one could have reasons to criticize both arguments. In the case of Øhrstrøm and Hasle's argument, one might object that the starting intuition on which the argument is built can be hard to accept. That is, one might argue that from an intuitive viewpoint, (1) and (2) should not be considered compatible. After all, if any day with life on earth *can* be followed

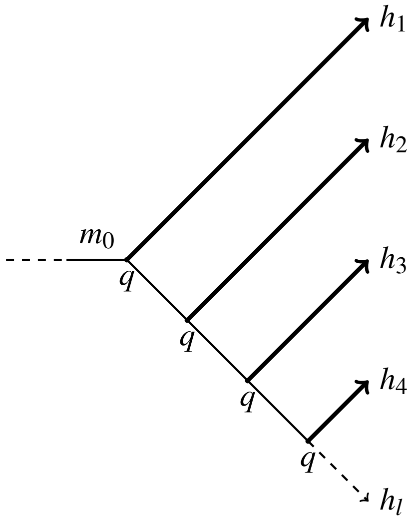

Figure 2 – The thickest parts of the tree are parts at which there is no life on earth, i.e. where $\neg q$ is the case.

by a further day where there is still life on earth, as per (2), it seems that there *can* be a complete development of the world where life never ends. This seems to imply, *contra* the intuition of Øhrstrøm and Hasle, that (1) simply cannot be true too, as it might be that the last day of life on earth will *not* come. On the other hand, one might also criticize the spirit of the argument put forward by Belnap et al.. For instance, Zanardo (2006: 394) argues that their intuition according to which (4) and (5) are incompatible is based on the presupposition of moment-based branching structures. Moreover, even if we grant them that (4) and (5) are truly intuitively incompatible and that this intuition is not affected by some former presupposition, it is unclear why this should constitute a conclusive objection to history-based structures. It is certainly correct that (4) and (5) *could* in principle be made compatible within history-based structures by deciding to not consider the lower set of $<$-related moments a history. However, it is unclear why one *should* not consider that set a history. After all, a supporter of history-based structures might very well say that, absent particular reasons, one should consider a maximal set of $<$-related moments a history, and that in the example by Belnap et al., there is no reason to *not* consider h_l a history. If that is the case, the argument by Belnap et al. fails to conclusively show that moment-based structures have an advantage here. At any rate, I do not want to further discuss the two arguments.[7] Rather, in the next section, I want to put forward a new argument that scores a point in favor of history-based structures.

4 Doomsday and Branching-time

In this section, I am going to provide an example that seems to score a clear point in favor of history-based structures. The scenario described in the example is not going to be about the end of life on earth, as the previous one, but about the end of time itself, i.e. Doomsday. I will argue that the scenario is a genuine metaphysical possibility, and I will show that it can be represented by history-based structures but not by moment-based ones. As the example involves a case of temporal openness with respect to Doomsday, I will first take a brief detour into two different general kinds of temporal openness with respect to the future.

The future can be open in at least two ways. For instance, say that Anne has never eaten frozen yogurt up to a moment m, and in some continuations of m she eats frozen yogurt at some point, whereas she never does in others. With respect to this case, we can say that it is open at m *whether* Anne will eat frozen yogurt. Anne might eat frozen yogurt, as she very well might not. Let us use the label of

[7] For another take on this example about life on earth, see Thomason (1984: 151-52). See also the already mentioned article by Grandjean & Pascucci (2021) for an application of the example to programming and computational contexts.

absolute openness for this kind of openness. As for the second kind of openness, consider Bob, who, unlike Anne, can't resist frozen yogurt. At m it is inevitable, hence not open, that sooner or later Bob will eat frozen yogurt again. However, in some continuations of m Bob eats frozen yogurt after one day, whereas in other continuations he does so after two days. In this case, although at m it is inevitable that Bob will eat frozen yogurt, it is open *when* this will happen. He might eat frozen yogurt in one day, as he might very well do it in two days instead. Let us use the label of *relative openness* for this second kind of openness.

It should be clear how branching-time can easily model the two frozen yogurt scenarios. For instance, in the case of absolute openness, one just needs at least one history passing trough m where Anne eats frozen yogurt and at least another one where she never does so. Similarly for the case of relative openness. We need that in all histories passing through m, Bob eats frozen yogurt after m. And, we also need at least one history passing through m where Bob eats frozen yogurt one day after m but not two days after m, and at least another history where the opposite takes place. This works fine independently of whether one assumes moment-based or history-based structures.

Let us now see what happens when we introduce the possibility that time ends (Doomsday). As far as relative and absolute openness are concerned, both moment-based and history-based structures can represent open doomsday scenarios. To do so, we need branching structures to contain *endpoints*, where a moment m is an endpoint iff for no moment m' in T, $m < m'$. We can take an endpoint to be a moment at which $G\perp$ is true (see Meyer 2015), as ending points are the only points where the formula would result (vacuously) true—$\perp$ stands for an arbitrarily chosen but fixed logical falsehood. With that in mind, consider a case of

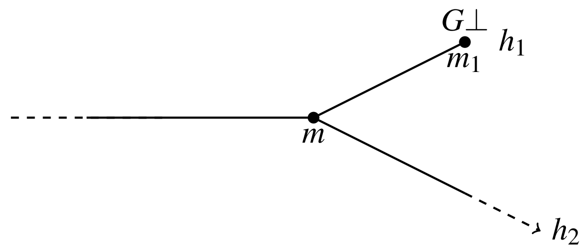

Figure 3 – *Doomsday and absolute openness.*

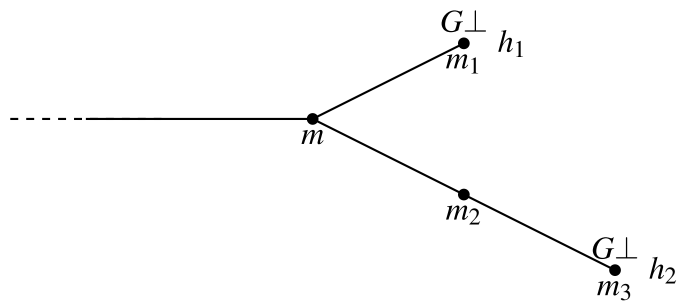

Figure 4 – *Doomsday and relative openness.*

absolute openness where it is open at m *whether* Doomsday will come. To represent this, one needs at least one history passing through m featuring an endpoint later than m and at least another one *not* featuring an endpoint (see fig. 3). In such a scenario, at m/h_1 (and at m/h_2 too) $\Diamond FG\bot$ and $\neg\Box FG\bot$ are both true—it is possible but not necessary that time will end. Likewise for relative openness. If at m it is inevitable that time will end but it is open *when*, we need that all histories passing through m have endpoints and those endpoints are at different temporal distances from m (see fig. 4 for an example). In the example, at m/h_1 (and at m/h_2 too) $\Box FG\bot$, $\neg\Box F(1)G\bot$, $\neg\Box F(2)G\bot$, $\Diamond F(1)G\bot$, and $\Diamond F(2)G\bot$ are all true—although it is inevitable that Doomsday will come, it is open *when* it will. Again, this works well independently of whether we adopt history-based or moment-based models, as both can feature endpoints.

However, consider this further case of Doomsday openness. It might be the case that, at one specific moment, say *m*, it is open whether the world continues or, rather, ends right at *m*. People await in trepidation, wanting to know whether it is Doomsday or whether they will ever enjoy frozen yogurt again. It is possible that time ends right at *m*, but that is not necessary. The world *might* end right at *m*, as it might very well continue. At the moment *m*, it would be true to say something along the lines of

(6) this present moment *might* be Doomsday.

In this scenario, we are faced with a kind of temporal openness that seems to be distinct from what I am labeling absolute and relative openness. In the case of absolute and relative openness, the openness is completely future-oriented. It is open at some moment what *will* be the case— whether or when someone *will* eat frozen yogurt, whether or when it *will* be Doomsday, and so on. In the latter example instead, the openness is still future-oriented—*will* the world continue?—but it somewhat regards the present too. From the perspective of some moment *m*, it is open whether the present moment *m is* Doomsday—*m* might or might not *be* Doomsday. Moreover, this kind of openness has this further peculiarity. Normally, a moment *m* in a branching structure does not contain any type of indeterminacy. For instance, it would not make sense to say that it is open at a moment *m* whether Bob eats frozen yogurt *at m*. What events there are at *m* is completely determinate. In the case of the example at hand, though, it does make sense to say that it is open at *m* whether *m* is Doomsday. Thus, this seems to be a further kind of openness, which differs from what I am labeling absolute and relative openness. Let us then introduce the new label of *present openness* for it.

It should here be noted that this scenario of present openness seems to be a perfectly legitimate metaphysical possibility. Why could it not be the case that a world is such that at a very specific moment, or at several moments, it is an open possibility that the world itself ends? Perhaps we can think of worlds where the laws of nature make it the case that at some special moments it is indeterminate whether the world itself continues or not. It is then natural to expect that it is a duty of a satisfactory theory of the openness of the future to be able to account for such a scenario.[8]

So, how can branching-time models account for present openness and ground the truth of claims such as (6)? We might be tempted to say that given that we are trying to model the open possibility that time ends at m, i.e. the possibility that there is no time after m, we could use an *absence* of branches to represent the possibility that m has no continuation. We might then picture in our head a structure like the one represented in fig. 5. However, we should not be deceived by the impression of a missing branch stemming from m in fig. 5. If there is a missing branch at m, to use words in a way that Quine would have not liked, then there are missing branches at all other moments. Hence, it cannot be that the absence of a branch at m grounds the possibility that time ends at m, as this would be the case at moments other than m too—moments at which, let us assume, Doomsday is *not* a possibility. Hence resorting to absent branches cannot be of any help in modeling present openness.

[8]Notice also that a contender of branching-time, i.e. divergent possible worlds à la Lewis 1986, has no problem with accounting for this case of present openness. We just need two possible worlds that are qualitatively identical up to a moment m, and such that one ends at m whereas the other one continues afterwards. There thus seems to be some theoretical pressure toward the branching-theorist to find a way to account for the case of present openness with respect to Doomsday.

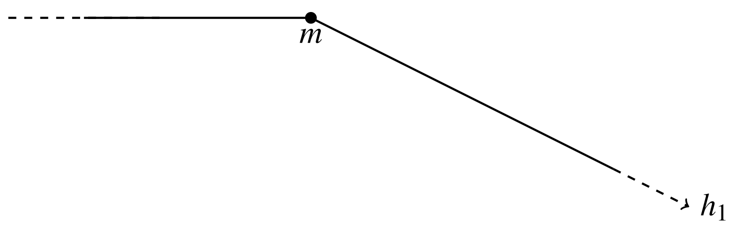

Figure 5 – A seemingly absent branch.

To have a grip on what is needed, let us notice that, in very general terms, branching-time models temporal openness by positing the existence of different histories where different things occur. Hence, it seems that in this case we need one history where m is the last moment of time (Doomsday), and a further history where the world continues after m. The difficulty of course is that m needs to be part of both those histories. It might then seem that we are running toward a contradiction where m both is and is not the last moment of time. The way out of the contradiction, and the way to model present openness too, is to resort to what we might call *broken* histories. Roughly, a broken history is a set of moments which somewhat has a last moment *and* also has later moments according to the fundamental earlier-than relation. More precisely, a *broken-at-m history* (in symbols, h^*_m) is a subset of T such that i) it is the union of m and *all* the moments earlier than m ii) it has upper bounds in T (i.e. moments later than m according to the fundamental earlier-than relation).[9] Once the possibility of broken histories is introduced, one can model the case of present openness as in fig. 6. The moment m belongs both to the history h_1 and to the broken history h^*_m. At the pair m/h_1, it is not Doomsday, $G\bot$ is false. At the pair m/h^*_m, on the other hand, it is Doomsday, $G\bot$ is true. So, at m/h_1 (and at m/h^*_m too) $\Diamond G\bot$ and $\neg\Box G\bot$ are both true—at m,

[9] Notice that according to this definition, the histories h_1 in fig. 3, and the histories h_1 and h_2 in fig. 4 are not broken histories.

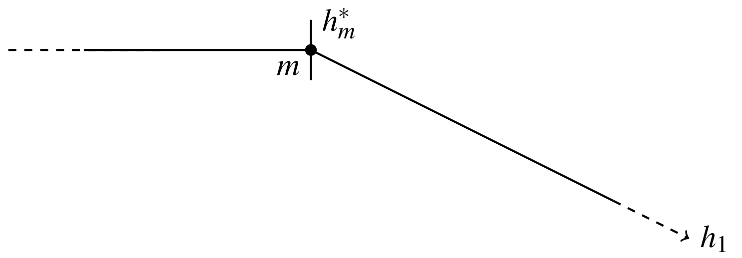

Figure 6 – Present openness and broken histories. The vertical line at m represents the end of the broken history h_m^*.

it is possible but not necessary that it *is* Doomsday. In other words, m might be Doomsday. The result is that by making use of broken histories, we can represent the scenario of present openness.

However, it should be noted that we can make theoretical room for broken histories only within the context of history-based structures. In fact, in the case of moment-based structures, something is a history only if it is a maximally ordered set of $<$-related moments in T. And, a broken-at-m history cannot be a *maximal* set of $<$-related moments, as by definition a broken-at-m history requires the existence of moments in T that are later than m. On the other hand, history-based structures have the leeway to include broken histories within a model, as in those structures what histories there are is a fundamental parameter of the model. Since the employment of broken histories is necessary to represent present openness, it follows that moment-based structures cannot represent the case of present openness, whereas history-based ones can.[10] Given that the case of present openness

[10]One might have some qualms toward the idea that a moment m of a broken-at-m history can rightly be considered a Doomsday moment. After all, by the very definition of a broken history, there must be moments after m. How can then m be a Doomsday moment? If one agrees with this point, then the case of present openness becomes an objection to branching-time itself, insofar as it would not matter whether we cash it out in terms of moment-based or history-based structures. Branching-time could simply not represent the case of present openness. Here I do not want to pursue this line of thought, as my general point is that *if* one adopts branching-time to model temporal openness, then the case of present openness favors history-based versions of branching-

is, I argued, a genuine metaphysical possibility, moment-based structures face a problem here. The genuine metaphysical possibility of present openness can be represented by history-based structures, but not by moment-based structures. Hence, history-based structures score a point over moment-based ones.

5 Conclusion

Before concluding, let us notice that the present openness scenario is different in nature from the example about life on earth discussed in section 3. In that discussion, one of the questions was whether or not we should *remove* from the set of histories of the model a set of moments that would normally count as a history, i.e. the two histories h_l. In the present openness scenario instead, what is required is that we *add* a history. That is, we need to add to the set of histories something that normally would *not* count as a history, i.e. a broken history. Moreover, the life on earth case does not seem to give a clear indication on what is preferable between history-based and moment-based structures. The debate features different positions, and, as I argued toward the end of section 3, it can be debatable whether either of the arguments considered scores a clear point in favor of one of the two structures over the other. On the other hand, the case of present openness I discussed in section 4 clearly suggests an advantage of history-based structures. The scenario of present openness seems to be a genuine metaphysical possibility. However, we need broken histories to model it, and broken histories can be admitted only if we adopt history-based structures. Moment-based structures sim-

time. It can be noted, though, that a possible line of reply from the branching theorist could be to argue that we should be careful and relativize our claims to histories. The moment m has no later moments (it is Doomsday) *relative to* the broken-at-m history, whereas it has later moments only relative to other histories passing through it.

ply cannot make room for broken histories. It follows that history-based structures can represent a genuine metaphysical possibility that cannot be represented in the framework of moment-based structures. If so, history-based structures score a clear point against moment-based structures.

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