PROCESS RELIABILISM, PRIME NUMBERS AND THE GENERALITY PROBLEM

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ABSTRACT: This paper aims to show that Selim Berker's widely discussed *prime number case* is merely an instance of the well-known *generality problem* for process reliabilism and thus arguably not as interesting a case as one might have thought. Initially, Berker's case is introduced and interpreted. Then the most recent response to the case from the literature is presented. Eventually, it is argued that Berker's case is nothing but a straightforward consequence of the generality problem, i.e., the problematic aspect of the case for process reliabilism (if any) is already captured by the generality problem.

KEYWORDS: Berker's prime number case, process reliabilism, the generality problem

1. Introduction

In recent debate on *process reliabilism*[1] a specific case crafted by Selim Berker has gained attention.[2] The case, which concerns cognitive processes and prime numbers, is meant as a challenge leveled against reliabilism:[3]

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[1] In rough outline *process reliabilism* as proposed by Alvin Goldman is the following view: A belief-token *b* is epistemically justified if and only if *b* is caused/sustained by a reliable process. Here, a *reliable process* is a process of belief formation that (would) produce(s) a sufficiently high ratio of true to false beliefs, given a specified set of circumstances and a domain of application. Process reliabilism was first proposed and defended by Alvin Goldman. See for example Alvin Goldman, "What Is Justified Belief?," in *Justification and knowledge*, ed. G. Pappas (Springer, 1979), 1-23; Alvin Goldman, *Epistemology and Cognition* (Harvard University Press, 1986).

[2] The case was originally stated in Selim Berker, "The Rejection of Epistemic Consequentialism," *Philosophical Issues* 23 (2013): 363-387. It was then criticized in Alvin Goldman, "Reliabilism, Veritism, and Epistemic Consequentialism," *Episteme* 12, 2 (2015): 131-143. Berker responded in Selim Berker, "Reply to Goldman: Cutting Up the One to Save the Five in Epistemology," *Episteme* 12 (2015): 145-153. Recently, Berker's case was further criticized in Jeffrey Dunn and Kristoffer Ahlstrom-Vij, "Is Reliabilism a Form of Consequentialism?" *American Philosophical Quarterly* 54 (2) (2017): 183-194.

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> **Prime numbers**. Suppose the following is true of me: whenever I contemplate whether a given natural number is prime, I form a belief that it is not. 'Is 25 prime? No, it is not.' 'Is 604 prime? No, it is not.' 'Is 7 prime? No, it is not.' Let us also stipulate that this is the only cognitive process by which I form beliefs about the primeness of natural numbers [...]. Since the ratio of prime to composite numbers less than n approaches 0 as n approaches infinity, my belief-forming process tends to yield a ratio of true to false beliefs that approaches 1. Therefore process reliabilists are forced to say that, because my belief-forming process is almost perfectly reliable, any belief formed on its basis is justified. But that's crazy! When I form a belief that 7 is not prime, it is simply not correct to say that, although that belief is false, it is epistemically redeemed by the truth of the other beliefs which would be formed via the process that led to it.[4]

Berker claims that a cognitive process like *whenever I contemplate whether a given natural number is prime, I form a belief that it is not* must be deemed reliable by the process reliabilist and that this is absurd. Berker is basing his claim on the observation that "Since the ratio of prime to composite numbers less than n approaches 0 as n approaches infinity, my belief-forming process tends to yield a ratio of true to false beliefs that approaches 1."[5] This will allegedly have the consequence that whenever an agent considers whether a given natural number n is prime or not, the agent will be epistemically justified in believing that it is not, even in cases where this is obviously false (e.g., in the case of 7). According to Berker this leads to a specific type of problematic trade-off between propositions for the reliabilist. Since the epistemic status of a concrete belief-token under evaluation, say the belief that *7 is not prime*, will be determined by the ratio of true to false beliefs – some of which will have different propositional content from the belief under evaluation – which the prime number-process outputs, it is claimed that in such cases process reliabilism sacrifices one proposition for other propositions, or that one instance of epistemic wrongdoing is perpetrated for the sake of a greater epistemic good, or, echoing Rawls, that the "separateness of propositions" is not respected.[6]

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[3] Note that we use the terms 'process reliabilism' and simply 'reliabilism' interchangeably throughout the paper.

[4] Berker, "The Rejection of Epistemic Consequentialism," 374-375.

[5] Berker, "The Rejection of Epistemic Consequentialism," 374-375.

[6] Berker, "Reply to Goldman," 145-153.

2. Recent Response by Dunn and Ahlstrom-Vij

Let us now consider the most recent response[7] to **Prime Numbers** by Jeffery Dunn and Kristoffer Ahlstrom-Vij (henceforth 'D & A').[8] D & A defend process reliabilism by observing that Berker helps himself to certain crucial implicit assumptions to make his case work, i.e., to make the prime number-process seem plausible and reliable:

> [...] Berker wants us to assume that (*a*) there are processes dedicated to generating beliefs about primehood, (*b*) he is relying on such a dedicated process – let us refer to it as *P* – in the scenario imagined, (*c*) for any number queried, *P* generates the output that it is not prime, and (*d*) numbers are queried in some quasi-random way among the natural numbers. Under those assumptions, *P* is reliable. And if so, reliabilism should say that the belief *that 7 is not prime*, generated by way of *P*, is justified. That, we claim, is the correct verdict. Any feeling that this is a counter-intuitive verdict should be traced, not to reliabilism, but to the psychological implausibility of (*a*), especially when paired with (*d*). For note that it is implausible indeed that some agent is as likely to contemplate whether 73,046,482,192,753 is prime as whether 53 is prime.[9]

Thus, D & A admit that given assumptions (a)-(d) stated above, the prime number-process will turn out to be reliable from the perspective of the reliabilist. However, they also point to an explanation of the counter-intuitiveness in **Prime Numbers**. They submit that the counter-intuitiveness is not due to process reliabilism *per se*, but to the psychological implausibility of assumption (a), and especially in conjunction with (d). It just seems implausible for there to be a human cognitive process dedicated to contemplating whether (more or less) random natural numbers are prime. Hence, D & A do appear to have an appealing defence of process reliabilism against Berker's case.

3. The Generality Problem

Even though we side with D & A with respect to **Prime Numbers**, we want to add some important qualifications in this section. In fact, we believe that D & A give Berker too much credit in their response. For it is clear, with or without assumptions (a)-(d) explicated, that Berker's proposed prime number-process has

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[7] Due to the limitations of space we have decided solely to present the newest response to Berker's case. Goldman gives three objections to the case in Goldman, "Reliabilism, Veritism, and Epistemic Consequentialism," 131-143. Note that none of Goldman's responses foresees our point about the generality problem below (cf. section 3).

[8] Dunn and Ahlstrom-Vij, "Is Reliabilism a Form of Consequentialism," 183-194.

[9] Dunn and Ahlstrom-Vij, "Is Reliabilism a Form of Consequentialism," 187.

nothing to do with the standard cognitive human processes that are of interest to process reliabilists.[10] Cardinal examples of such processes are *visual perception*, *(long-term) memory*, *competent deduction* etc. Thus, we find it unnecessary for D & A to admit that the prime number-process will be deemed reliable by the process reliabilist under any circumstances. Berker simply misconstrues the reliabilist.

To give an extreme example illustrating why one should not give Berker any wriggle room in making a strawman out of reliabilism, consider the following cognitive process: *Every time I see a living individual on the streets of London, I will form the belief that this individual is not a crocodile*. Given various assumptions, e.g., a restriction to normal worlds, we could make this process look extremely reliable because only very rarely (if ever) crocodiles are seen on the streets of London, but this is still no reason for the process reliabilist to admit that this is a reliable cognitive process. An argument for this is straightforward. If one were to use the crocodile-process on a sample with 50% humans and 50% crocodiles, then the process would only result in true beliefs half of the time. Likewise, if we were to use the process on a sample with only 10% humans and 90% crocodiles, the process would be unreliable. As the crocodile-process has exactly the same structure as the prime number-process, we can conclude that the reliability of such processes is determined by the specific sample it processes. If **Prime Numbers** had been concerned with a sample of the natural numbers with 50% primes and 50% composite numbers, then the reliability of the prime number-process would have been fifty-fifty. Of course, this is not the kind of processes that the reliabilist accepts. That would indeed be absurd! In contrast, processes such as competent deduction do produce reliable results across various samples (in normal worlds), e.g., it does not matter whether a subject assesses a sample of 90% valid arguments and 10% invalid ones or *vice versa*; competent deduction would yield a reliable output of belief-tokes concerning the validity of the arguments in any case.

Now, let us make the following crucial observation. In so far as Berker's case is a problem for reliabilism at all, this is merely because of what follows from the *generality problem*,[11] which is already widely accepted as a genuine problem for

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[10] This point is also underscored by Goldman in his response to Berker: "No process should be confined to a single, specific subject-matter. Thus, what Berker calls a 'process' in his prime number example cannot really be accepted by process reliabilism. Agent S may be said to use a certain rule for deciding whether a particular natural number is prime (always saying 'no' for every natural number specified). But this rule should not be equated with a process." Goldman, "Reliabilism, Veritism, and Epistemic Consequentialism," 141-142.

[11] The generality problem was originally formulated by Goldman in "What Is Justified Belief?" 1-

process reliabilists. The generality problem points out the difficulties of individuating the relevant process in play in a given case. Suppose that you are glancing out of your bedroom window, forming the belief that *it is raining*. Presumably, you formed your belief via a reliable process – the question is what process exactly? Are you using *visual perception*, or *glancing*, or *glancing through a bedroom window*, or perhaps some fourth and somewhat different process? Goldman and Beddor describe the problem more generally as follows:

> Any particular belief is the product of a token causal process in the subject's mind/ brain, which occurs at a particular time and place. Such a process token can be 'typed,' however, in many broader or narrower ways. Each type will have its own associated level of reliability, commonly distinct from the levels of reliability of other types it instantiates. Which repeatable type should be selected for purposes of assigning a reliability number to the process token? If no (unique) type can be selected, what establishes the justificational status of the resulting belief?[12]

Berker's prime number case is merely an instantiation of this problem. Whenever the agent in the case comes across a natural number and wonders whether it is prime, she forms a particular negative belief-token, e.g., *7 is not prime*. If this can be counted as an acceptable cognitive process at all, it can plausibly be "typed" in different more or less broad ways. For example, the prime number-process might (less artificially) be seen as a narrow kind of heuristic, which is merely applied when one considers small natural numbers (say the numbers from 1 to 50). This narrowness would indeed impact the reliability associated with the process.[13] Hence, all Berker has shown with **Prime Numbers** is that it is possible to cook up a case individuating a very artificial cognitive process that yields a problem for the process reliabilist in terms of the process's associated level of reliability, but this is old news in epistemology as it is merely a consequence of the generality problem.

Finally, a quick remark on the kind of problematic, epistemic trade-offs Berker takes the reliabilist to face, violating the "separateness of propositions" (cf. section 1). Setting aside the difficulties of the generality problem and whether

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23, but was later developed in a more systematic way by Richard Feldman and Earl Conee. See, for example, Richard Feldman, "Reliability and Justification," *The Monist* 68 (1985): 159-174; Earl Conee and Richard Feldman, "The Generality Problem for Reliabilism," *Philosophical Studies* 89 (1998): 1-29.

[12] Alvin Goldman and Bob Beddor, "Reliabilist Epistemology," *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/ archives/win2016/entries/reliabilism/>.

[13] Something similar is suggested by Goldman in "Reliabilism, Veritism, and Epistemic Consequentialism," 141.

process reliabilism would ever be committed to various peculiar cognitive processes with high reliability, it is no secret that process reliabilism is going to allow that processes with a high truth ratio can once in a while produce false but justified beliefs. Thus, it should be of no surprise to the reliabilist that she can sometimes end up in situations where she holds a belief that is epistemically justified even if blatantly false.