

Beyond Diminishing Marginal Utility

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Introduction: Assuming Diminishing Marginal Utility

Diminishing Marginal Utility is widely accepted as a law of human action, and therefore has become one of the primary premises of ethics, economics, and politics. In popular parlance, “diminishing returns” has entered into the clichés of common sense; in philosophical argument, it has achieved the status of an axiomatic assumption; and indeed, in terms of personal experience or folk psychology, it seems to largely hold true for goods in general over a range of consumption. However, a theory of diminishing marginal utility rests on some assumptions that we ignore at our peril. The a priori and a posteriori proofs of diminishing marginal utility, as developed by the Austrian economists like Ludwig von Mises and Hermann Heinrich Gossen, should be reassessed and critiqued from their fundamental principles to illuminate the theory’s pitfalls. Once these premises and their conclusions are not taken for granted, a more comprehensive and conscientious theory of value can be proposed: Complex or Inflecting Marginal Utility.

Defining Diminishing Marginal Utility

Any philosophical system that considers human actions, economic goods and utilities would have to establish some stance on the relationship between quantities of goods consumed and quantities of utility—for example, how much food consumed leads to how much pleasure? Any such stance would naturally entail a stance on the marginal utility: the relationship between additional goods consumed and additional utility—for example, how much *more* food eaten leads to how much *more* pleasure? The Marginal Utility function would be the slope of the Utility function. Diminishing Marginal Utility is often assumed to account for the relationship between goods and utility.

Diminishing Marginal Utility (DMU): the utility of a good varies on margin by diminishing.

The DMU curve has at least two different sections. The sections of the curve may be described as follows:

Table 1: Diminishing Marginal Utility

#	Propositions	Function
1	As goods increase from zero towards satiety (G_b), utility increases and marginal utility is positive. This roughly describes the goodness of eating enough to be satisfied.	If $0 < G_b$ then $u_m = +$

		and $u_i = +$
2	As goods increase from satiety towards infinity, utility decreases and marginal utility is negative. This roughly describes the badness of eating too much, past the point of satisfaction.	If $G_b < G_i < \infty$ then $u_m = -$ and $u_i = \pm$
3	Therefore, utility has two discrete sections on the scale of goods, and marginal utility is Diminishing.	

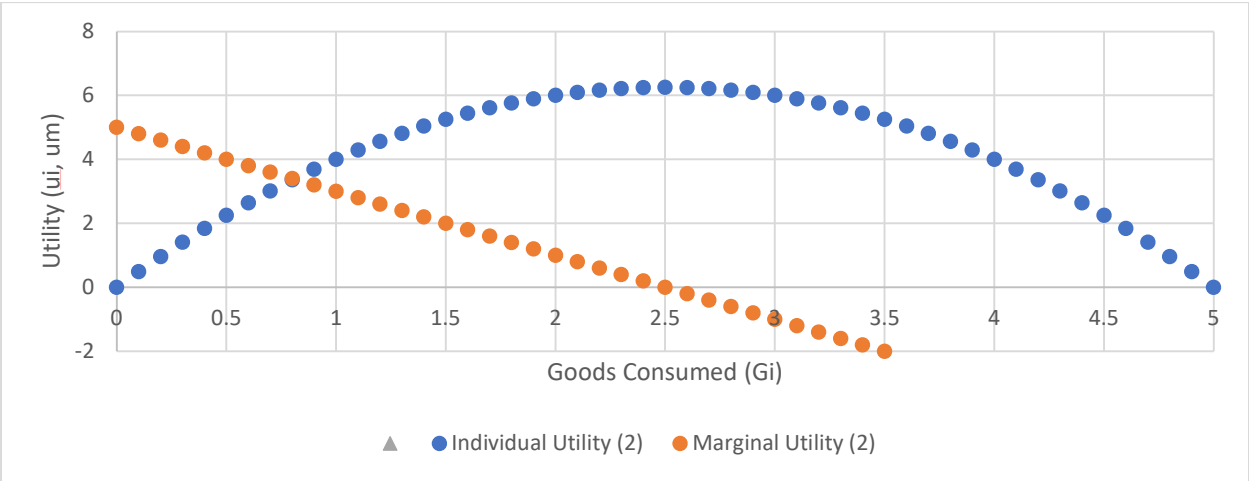


Figure 1: Diminishing Marginal Utility

DMU has abundant prima facie plausibility: per experience, pleasure lasts until satisfaction, after which it diminishes—food is tasty until we are full, after which stuffing ourselves hurts. In addition to this, the DMU also has two theoretical arguments in its favor: 1) a psychological a posteriori argument put forth by Hermann Heinrich Gossen; 2) and an axiomatic a priori argument put forth by Ludwig von Mises.

Gossen’s Empirical Argument

One of the first historically important and theoretically comprehensive defenses of DMU in economic thought was formulated by Hermann Heinrich Gossen in his work *The Development of the Laws of Human Intercourse and the Consequent Rules of Human Action* (Gossen). In that work, Gossen suggests an a posteriori argument that the DMU is an empirically verifiable relationship between consumption and satisfaction. In his model, DMU is a psychological phenomenon readily appreciable by any consumptive creature. W. S. Jevons later championed Gossen’s argument in his own work, *The Theory of Political Economy*. Jevons summarizes Gossen’s argument as follows:

The natural law of pleasure is then clearly stated, somewhat as follows: Increase of the same kind of consumption yields pleasure continuously diminishing up to the point of satiety . . . Hence he [Gossen] draws the practical conclusion that each

person should so distribute his resources as to render the final increments of each pleasure-giving commodity of equal utility for him.” (Jevons xxxvi-xxxvii)

This argument has withstood the test of time, as the cornerstone of most economic thought. Any attempt to offer an alternative theory would first have to contend with Gossen’s fundamental contention.

Empirical Counterargument

However, the a posteriori argument proposed by Gossen is vulnerable to empiricism. The critical empirical counterargument for Gossen’s DMU follows from the possibility of observing counterexamples:

#	Propositions
1	Gossen’s a posteriori proof of Diminishing Marginal Utility rests upon the empirical assumption that marginal utility decreases across quantities of goods consumed.
2	Empirical evidence suggests counterexamples in which marginal utility can increase across quantities of goods consumed.
3	Therefore, Diminishing Marginal Utility is not proven a posteriori.

Finding counterexamples in which marginal utility increases across increasing quantities of goods quite straightforward. To elaborate upon one counterexample, consider the following:

Coconut Tree Island: A single castaway is marooned on a desert island with a single palm tree. For the sake of simplicity of argument, this is a one person one good economy, thus eliminating the complexities of a multi-person and multi-good marketplace. Although the castaway can tell that there might be coconuts in the tree, he cannot tell whether there are or how many. By his reckoning, climbing the tree and fetching the food may not be worth the effort. Indeed, before ever tasting the coconut and increasing his utility, by tasting of the coconut, there is the condition that he decrease his utility, by climbing the tree for the coconut, cracking the coconut, preparing the coconut for consumption, and otherwise harming himself via production—chipping his tooth, giving himself blisters, etc.—...

Indeed, in such cases as climbing coconut trees, a posteriori DMU does not hold. This is empirically true for any commodities that require labor to produce (i.e., all material commodities). Negative utility is entailed by the production and consumption of the good, and thus imbedded into the continuum of human action of acquiring utility. This, roughly speaking, corresponds to von Mises’s concept of the disutility of labor, which is a precondition of the production of a good.¹ The only true Gossen Goods, those goods which

¹ “People work only when they value the return of labor higher than the decrease in satisfaction brought about by the curtailment of leisure. To work involves disutility.” (132)

satisfy Gossen's DMU, are goods obtained for free, without the potential negative utility intervening as entailed in production—this rules out crawfish, pitted olives, hot peppers, old beaters, updates on operating system, etc.. Indeed, to characterize the general counterexample: empirically speaking, when acquiring goods, there are often observable conditional and contingent decreases in utility preceding and intervening in increases in utility. Of course, Gossen himself should be aware of this, since we are obviating his "same kind of consumption" clause (a *ceteris paribus* condition), but violating this clause is precisely the step we would want to take to move beyond DMU toward more complex analysis of utility arrangements.

Defining Complex Marginal Utility

As shown, empirical evidence suggests that DMU is a simplification of a more comprehensive theory of marginal utility. The assumption of empirical DMU rests in the assumption of the "same kind of consumption". In the modern world of mass-manufactured products and highly efficient markets, we might even be convinced that a homogenous supply and rational action is attainable, if only via the advent of modernity: interchangeable parts, reproducible commodities, price transparencies, etc.. However, the world is not necessarily either. Many natural goods might demonstrate complications to the DMU. Consider again the case of the coconut:

Coconut Tree Island Continued: ...the single castaway is still marooned on a desert island with a single palm tree decides to eat a coconut. Although the castaway's supply may seem homogeneous enough, that assumption fades away quickly when considering the process of producing and consuming a supply of coconuts. Before anything, the man must climb the coconut tree to access the coconuts, decreasing his individual utility. Then, the man must crack and prepare a single coconut, decreasing his individual utility. Next, the man may consume the single coconut, increasing his individual utility. Finally, the man may become stuffed with coconut, decreasing his individual utility. With each subsequent coconut the process may repeat... As shown by its irregular process of consumption, the coconut is a particularly heterogeneous and/or irrational good. The irregularity of the good is obvious in the case of the coconut, as typified in its out-of-reach growth and hard-to-crack shell: the consumption of the good is laden with implied labors which precede and intercede in the enjoyment of the good.

What the realistic Coconut Tree Island case shows is that any given relationship of goods and utilities is riddled with empirical specifics of the case. That coconuts have tall trees and hard shells are merely contingent facts. But what this shows is that any simple version of marginal utility for a given good can be complicated by further considerations, leading to a Complex Marginal Utility.

Complex Marginal Utility (CMU): the utility of a good varies on margin depending upon the specifics of the case of the goods involved (Table 2, Figure 2):

Table 2: Complex Marginal Utility

#	Propositions	Function
1	...Preceding conditions specific to a good may increase or decrease marginal utility...	If $0 < G_b$ Then $u_m = ?$ And $u_i = ?$
2	As goods increase from zero towards satiety (G_b), utility increases and marginal utility is positive.	If $0 < G_b$ then $u_m = +$ and $u_i = \pm$
3	...Intervening specific conditions may increase or decrease marginal utility...	If $0 < G_i < \infty$ Then $u_m = ?$ And $u_i = ?$
4	As goods increase from satiety towards infinity, utility decreases and marginal utility is negative.	If $G_b < G_i < \infty$ then $u_m = -$ and $u_i = \mp$
5	...Succeeding specific conditions may increase or decrease marginal utility...	If $G_b < G_i < \infty$ Then $u_m = ?$ And $u_i = ?$
6	Therefore, utility is complex and related to the specifics of the case of the given good.	

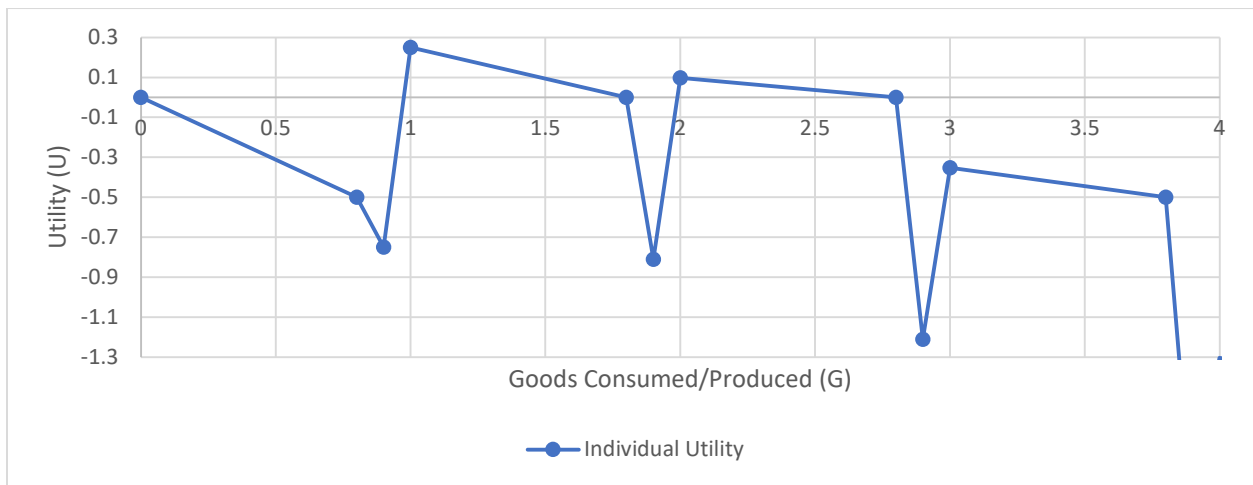


Figure 2: Complex Marginal Utility (for Coconuts)

In other words, while consumption is roughly modeled by the DMU, it fails to account for the embeddedness of production in the process of human action. It is not clear in any sense that production/consumption are coherently separable. Even in the simple case of eating fruit, it is not clear that the bite (production) is separable from the taste (consumption). So, the process of production/consumption is not necessarily modeled by DMU because the rational actor cannot necessarily arrange the utilities of actions ordinally, largest first and smallest last, nor insure a guarantee of homogeneous supply. Instead, the rational actor is forced, by irrational and heterogeneous factors of the nature of the good consumed (e.g., a coconut), to accept decreases in utility before ever encountering increases in utility. In other words, the supply is heterogeneous, the order is non-ordinal, and the good as actor is irrational. DMU falls short of adequately accounting for the case of a given good (e. g. coconut) and thus fails the universalizability principle. Instead, the coconut follows a Complex Marginal Utility.

Of course, by its very nature, CMU is case-specific and difficult to model as a mathematically continuous function. But, for many cases, a complex curve might be approximated as a higher order polynomial. In some cases, the cases of what we might dub Gossen Goods, where goods are free, the DMU may suffice. But in other cases, those of non-Gossen Goods (e.g., coconuts), a theory of Inflecting Marginal Utility might better describe the realities of human experience and human consumption, as an alternative to the DMU. Indeed, for most goods, the CMU may be approximated as Inflecting Marginal Utility without much theoretical trouble.

Mises's Analytical Argument

A second important defense of DMU in modern economic thought was formulated by Ludwig von Mises in his work "Human Action". Ultimately, Mises agrees with Gossen's conclusions but disagrees with Gossen's empirical premises. Mises suggests that:

The law of marginal utility and decreasing marginal value is independent of Gossen's law of the saturation of wants (first law of Gossen) (Mises 124).

Fundamentally, Mises believes that Gossen's argument is too empirical, and could be instead merely analytical. Thus, Mises suggest an a priori argument that DMU is analytically provable, without any appeal to empirical facts of the matter. In his model, DMU is a logically deducible course of action readily apprehended by rational actors. As he puts it:

... we do not need any physiological or psychological experience, knowledge, or reasoning. It [DMU] follows necessarily from our assumptions that people act (choose) and that in the first case acting man has n units of a homogeneous supply and in the second case $n - 1$ units. Under these conditions no other result is thinkable. Our statement is formal and aprioristic and does not depend on any experience. (Mises 124)

Unlike Gossen's argument, Mises's argument is not merely amenable via counterexamples, but withstands empirical data. Any attempt to offer an alternative theory would first have to refute the fundamental premise of rational actors and homogenous goods, or else the mathematical proof that any quantity minus one is less than the quantity itself.

Analytic Counterargument

However, the a priori argument proposed by Mises is vulnerable to rejecting his premises. Indeed, the critical analytic counterargument follows from the inevitability of interventions in the ordering of goods:

#	Propositions
1	Von Mises's a priori proof of Diminishing Marginal Utility rests upon the assumption that rational actors arrange homogenous goods in ordinal order of utility.
2	But irrational, heterogeneous actors in the world, whether societal or individual, biological or chemical, physical or metaphysical, necessarily arrange utilities in their own orders, including non-ordinal orders.
3	Therefore, Diminishing Marginal Utility is not proven a priori.

Although quantities of goods themselves have the extrinsic property of diminishing when consumed; utilities of goods are non-extrinsic properties of goods and therefore do not necessarily diminish when consumed but may rather follow relationships of their own. In other words, postulating ordinality, largest first to smallest last orderings, by assuming rational actors and homogenous goods and by assuming the absence of irrational actors and heterogeneities, should not be accepted lightly. Utilities are not necessarily ordinally arrangeable, but may proceed from non-human, non-ordinal arrangements. Because rational actors always and necessarily operate on and are operated on by irrational actors, factors preceding and exceeding the domain of human action, they cannot necessarily assume rational ordinal order for goods. Because it is possible for non-rational processes to intervene, non-ordinality necessarily follows. This is logically true for all actions that operate amongst non-rational objects. For Mises's statement to be true then, the universe must be completely composed of ordinal actors, which is quite dubious. In fact, the non-ordinality of utilities should be consistent with Mises own account, in that Mises considers at length the "disutility of labor", the negative utility empirically associated with the production of goods.²

All that Mises must do to reject DMU is provide that production and consumption, the disutility of labor and utility of goods, exist on an intermingled continuum of human action. But, in doing so, Mises becomes Gossen-like, his analytic argument collapsing back into empirical arguments about specific goods. Indeed, to better characterize the analytical counterargument: logically speaking, when acquiring goods, there may be necessary decreases in utility preceding increases in utility, because the inevitability of non-ordinality is imbedded into the arrangements of objects in the world.

² "The disutility of labor is not of a categorial and aprioristic character . . . But the real world is conditioned by the disutility of labor. Only theorems based on the assumption that labor is a source of uneasiness are applicable for the comprehension of what is going on in this world." (Mises 65)

Furthermore, working within Mises own definition of human action, we can infer that acts of ordering, that is the preconditions of ordinality, are themselves actions in the ordering of action. As Murray Rothbard puts it in his *Man, Economy and State* human actions include the “ideas”, “ways”, and “plans” for achieving further actions.³ In this important sense, any DMU as an ordinal arrangement must necessarily presume a prerequisite action, the action of ordering.

There are two ways to accommodate this prerequisite action under Mises’ own account. The rational agent must always:

- 1) Either Precede their sequence of actions by rank-ordering the sequence in order of greatest to least utility, then proceeding down. The rank-ordering is a comparison sort algorithm. The algorithms’ computational cost is a function of the length of the sequence, in the best case linear (if the universe happens to be pre-ordered) and in the worst case an $x \cdot \log x$ (if the universe is arbitrarily unordered) (Cormen, 2009, p. 174).
- 2) Or, precede each step in their sequence of actions by finding the action of greatest utility in the unordered sequence. The finding is a search algorithm. The algorithms’ computational cost is a function of the remaining length of the sequence, in the best case is a constant function (if the next greatest utility is next in the sequence) and in the worst case a linear (if the next greatest utility is arbitrarily placed in the sequence) (Cormen, 2009, p. 185-6).

In either case, the rank-ordered diminishing marginal utility is interceded by the disutility of the computational costs, either as an initial condition or as a continuous discount, costs which are a necessary feature of any ordering procedure. This allows us, even within Mises own model, to expand beyond the DMU.

Defining Inflecting Marginal Utility

As shown, the analytic argument suggests that DMU does not factor in the important role that initial ordering has to play in the ordinality of action. The assumption of DMU rests in the assumption of “homogeneous supply” and “rational action” that Mises acknowledges explicitly. And we can accept these assumptions, but can still consider that the universe does not come pre-ordered of itself, before human action. Therefore, any given rational ordering of goods requires the action of ordering itself, as a prerequisite. In as much as ordering itself entails work and work entails disutility, any analytic marginal utility curve should start negative. Furthermore, in as much as each act of ordinal consumption may require a re-ordering of the arrangement of goods, each further act of consumption is depressed though on a diminishing basis, such that the last good requires no ordering at all.

What the prerequisite of ordering shows is that any given relationship of goods and utilities is interceded by acts of ordering. Thus, the analytic version of utility for a given

³ “He must have certain *ideas* about how to achieve his ends. Action thus consists of the behavior of individuals directed towards ends in ways that they believe will accomplish their purpose. Action requires an image of a desired end and “technological ideas” or plans on how to arrive at this end.” (Rothbard 3)

good is not just diminishing on margin according to human preference, but is also linearly depressed by the disutility of the labor of ordering itself: the combination of these two factors leads to an Inflecting Marginal Utility:

Inflecting Marginal Utility (IMU): the utility of a good varies on margin as increasing then decreasing.

In the IMU, instead of having a series of two different sections like the DMU, the curve may have three different sections. The sections of the curve may be described as follows:

Table 3: Argument for Inflecting Marginal Utility

#	Propositions	Function
1	As goods increase from zero towards sufficiency (G_a), utility briefly decreases, and marginal utility is negative. This roughly describes the badness of working, but not enough to produce food.	If $0 < G_i < G_a$ then $u_m = -$ and $u_i = +$
2	As goods increase from sufficiency towards satiety (G_b), utility increases and marginal utility is positive.	If $G_a < G_i < G_b$ then $u_m = +$ and $u_i = \pm$
3	As goods increase from satiety towards infinity, utility decreases and marginal utility is negative.	If $G_b < G_i < \infty$ then $u_m = -$ and $u_i = -$
4	Therefore, utility has three discrete sections on scale of goods, and marginal utility is not Diminishing, but Inflecting.	

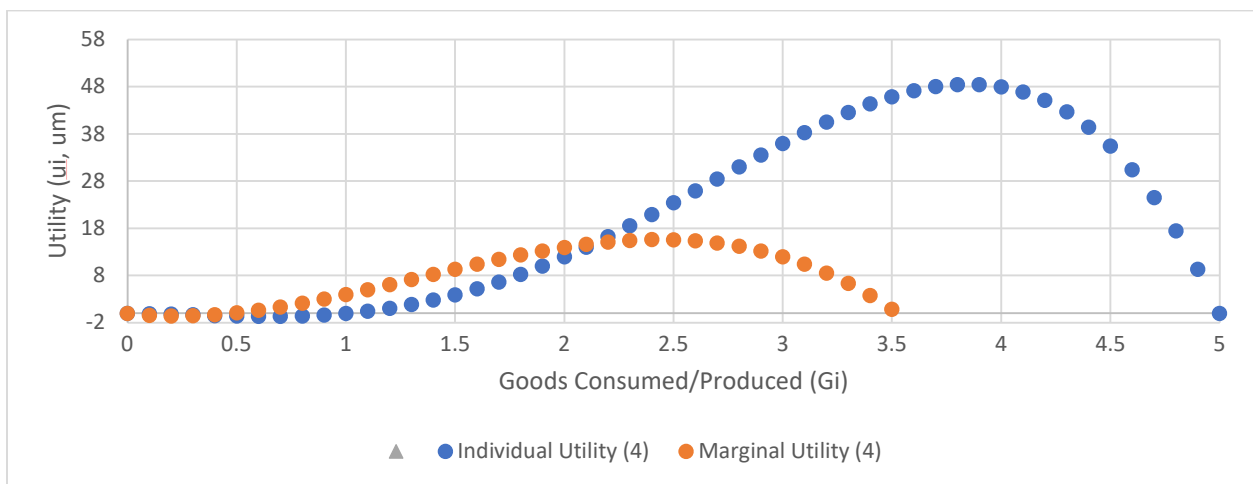


Figure 3: Inflecting Marginal Utility

As we can see, the IMU is consistent with our prima facie intuitions, just like the DMU, but more comprehensively describes the character of utility and real-life value judgments involving necessary non-ordinal orderings. Because goods in the IMU have four ranges of development, not two ranges of development, the nature of the utility function should be a fourth order polynomial, not a second order polynomial. This more complex curve describes a wider range of human action; and furthermore, because it was arrived at analytically, it should encompass all rational action.

Conclusion: Improving Diminishing Marginal Utility

In summary, DMU correctly characterizes patterns of utility over ordinal iterations of consumption. However, consumed goods affect utility on a continuum of human action, from production to through consumption. And, because these actions are not necessarily ordinally arranged—even on Mises own account—utility has a more complex curve, not of a Diminishing nature, but of a Complex nature empirically and an Inflecting nature analytically. The DMU is still a great initial assumption, but both its empirical and analytical justifications may prove too simple for certain regimes of goods.

But after all, what is at stake in driving these considerations of marginal utility? For utilitarian theorists, these claims entail strong ethical stakes about domains of human wellbeing; but even for non-utilitarians, these claims affect real deliberations about economic states of affairs. Indeed, the CMU in general, and IMU in particular can avoid certain special problems in welfare ethics and economics, giving a positive reason to explore it. Possible implications of IMU are:

1. **Necessities and Luxuries:** The IMU would seem to suggest a more rigorous and useful way of differentiating necessities and luxuries. Anything beneath sufficiency might be thought of as necessity; anything above sufficiency might be thought of as a luxury. Thus, the IMU has the intrinsic benefit of providing both a lower limit on consumption, a sufficiency criterion, which implies that there is some minimum good; and an upper limit on consumption, a satiety criterion, some maximum good. In as much, the IMU would seem to describe a mixed economic structure, with lower bounds on sanctions for free markets and upper bounds on sanctions for safety nets, satisfying both capitalist and socialist intuitions. In contrast, DMU falls short of making this kind of necessity/luxury distinction because, in an economy governed by the DMU there is no sufficiency point.⁴
2. **Avoidance of Extremes:** The IMU also seems to offer a moderate solution to extreme problems in utilitarian theory. Derek Parfit's "Repugnant Conclusion" and Robert Nozick's "Utility Monster" are both extreme cases of population density that result from assumptions about marginal utility (Parfit, Nozick). In

⁴ "It is a mistake to assume that the desire to procure the bare necessities of life and health is more rational, natural, or justified than the striving after other goods or amenities." (Mises 19) "It refers to every kind of want-satisfaction, not only to the satisfaction of the vital necessities of mere survival." (Mises 484)

these cases, specifying no utility curve results in the extremes of utility as it approaches zero and infinity. The Repugnant Conclusion is a case in which there is no lower bound of minimal satisfaction.⁵ The Utility Monster is a case in which there is no upper bound of maximal satisfaction.⁶ The IMU avoids these problems by rendering both the high end and low end of the real utility curve untenable, while only the middle of the curve remains. In contrast, an underspecified DMU does not offer a solution to these ethical conundrums.

These extra benefits surely give some desirability to the rejection of the DMU. But, even without this added value, the DMU should be amended on the validity of the arguments and counterarguments made above and its theoretical shortcomings in representing real world consumption. Attributing our disagreements on many economic issues to our incomplete economic assumptions, Diminishing Marginal Utility is one such assumption and can be updated to a more complex consideration of real world utility.

⁵ “On some versions of the *Law of Diminishing Marginal Utility*, this is just what is implied. On these versions, each unit of resources produces more utility if it is given to people who are worse off, so that the most productive distribution will be the one where everyone's life is barely worth living. There is here an obvious oversight. Large amounts of resources are needed to make each person's life even reach the level where life begins to be worth living.” (Parfit 525)

⁶ “Utilitarian theory is embarrassed by the possibility of utility monsters who get enormously greater gains in utility from any sacrifice of others than these others lose. For, unacceptably, the theory seems to require that we all be sacrificed in the monster's maw, in order to increase total utility.” (Nozick 42)

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