Friendships of Scale:
Applying Scaling Laws to Interpersonal Relationships

Walter Barta
University of Houston
December, 2018 (DRAFT)

> "Give people the power to build community and bring the world closer together."
> Mark Zuckerberg, Facebook CEO

> "A wish for friendship arises swiftly, but friendship itself does not."
> Aristotle, *Nicomachean Ethics*, 1156b32

1. Introduction

Here we will argue that different kinds of friendships follow different kinds of scaling laws. First, we will review Aristotle's three types of friendship. Second, we will review three different types of scaling laws. Third, we will show how the three types of friendship roughly map on to the three types of scaling laws. After this, we will discuss some of the consequences of the scaling laws of friendship. We hope that the use of these abstract scaling laws to describe social values will convince computer programmers and policy makers to temper their would-be ambitions as social engineers.

2. Friendships

According to Aristotle, friendship one of the goods that we live for, perhaps even the chief good that we live for, since, as he puts it, "without friends, no one would choose to live, even if he possessed all other goods" (1155a5). Aristotle lists three kinds of friendship. Aristotle deduces the number three because he identifies "what is lovable" as "what is good, pleasant, or useful" (1155b20) and suggests that, therefore, "The forms of friendship, then, are three, equal in number to the things that are lovable" (1156a7-8). The three types of friendship then are friendships of usefulness, friendships of pleasure, and friendships of virtue (or genuine friendships), and are generally valuable according to this ascending order according to Aristotle's own ranking of things to be valued.

Notably, others have given different accounts of friendship from Aristotle (Helm, 2022), although it is usually "common to follow Aristotle" in making one's distinctions.

a. Usefulness

The first type of friendship is one of usefulness. As Aristotle describes it, a friendship of usefulness is composed of "Those who love each other on account of usefulness, then, do not love each other in themselves, but only insofar as they come to have something good from the other" (10-12). Aristotle sees this as the lowest form of friendship because it consists of loving a

person only for their indirect usefulness, not directly for any of their qualities. A relationship between coworkers who help each other at their job might be such a friendship

b. Pleasure

The second type of friendship is one of pleasure. As Aristotle describes it, a friendship of pleasure is composed of "those who love on account of pleasure, for people are fond of those who are witty, not because they are of a certain sort, but because they are pleasant to them." (12-14). Aristotle sees this as a middling form of friendship because it consists of loving a person for their pleasant effects, but not directly as a person. A relationship between lovers who enjoy the *experiential* effects that they have each other might be such a friendship.

c. Virtue

The third type of friendship is one of virtue, also described as a genuine friendship or complete friendship. As Aristotle describes it, "complete friendship is the friendship of those who are good and alike in point of virtue" (1156b8-9). Aristotle sees this as the highest form of friendship because consists of loving a person for their good qualities directly, not indirectly. Aristotle also sees this as the only form of "genuine" or "complete" friendship in the sense that the love between persons is direct love for the good qualities that the persons respectively possess, not indirectly for other goods indirectly obtained through their relationship. According to Aristotle's analysis, loving for these other reasons cannot be genuine because the friendship dissolves when its effects are gone, and it can be dissolved in advance in favor of other means of obtaining the same effects. Such a friendship would have to both value a person as a person and for their good qualities such that their flourishing is *interdependent* on the other's flourishing, perhaps like the friendship between two people who have known each other very closely for a very long time.

3. Scaling Laws

Scaling laws are mathematical descriptions of how dependent variables of processes change when independent variables change in scale. Scaling laws are used for determining whether specific properties of a systems can be expected to scale as other properties are. For example, how must the size of a building change the more occupants you attempt to fit within it? Can you build a building taller and taller indefinitely or will you eventually meet limits on scale (e.g., the force of gravity). Three kinds of scaling law are commonly discussed as describing different kinds of scalability limitations in processes: the Linear Scaling Law, for non-serial non-interdependent processes; the Asymptotic Scaling Law (or Amdahl's Scaling Law), for serialized, non-interdependent processes; the Retrograde Scaling Law (or Gunther's Scaling Law), for serialized, interdependent processes.

Descriptions of these three scaling laws can be found in this section, mathematical equations for them can be found in the appendix, and graphical depictions of them can be found in the next section.

a. Linear Scaling Law

A linear scaling law assumes that the capacity (X) is some function of the number of components (N) multiplied by some constant factor gamma (γ) (Edwards, 1995).

b. Asymptotic Scaling Law

An Asymptotic Scaling Law (or Amdahl's Scaling Law) is the linear scaling law but with an extra term in the denominator that discounts for serialization (Amdahl, 1967). This term is the non-concurrent (serialized) fraction (α) of the process times the number of steps in the series of components, the number of components minus one. When the serialized fraction is zero ($\alpha = 0$), an asymptotic scaling law becomes a linear scaling law.

c. Retrograde Scaling Law

A Retrograde Scaling Law (or Gunther's Universal Scaling Law) is an Asymptotic Scaling Law but with an extra term in the denominator that discounts for interdependence (Gunther, 1993). This term is the cross-talking (interdependent) fraction (β) of the process times the number of unique relations between components, the number of components times the number of components minus one. When the cross-talking fraction is zero ($\beta = 0$), the Retrograde Scaling Law becomes an asymptotic scaling law (Gunther, 2002).

4. Friendship and Scaling Laws

Scaling laws of this kind are typically used to describe the scalability of manufacturing and information systems that are processing jobs composed of components, in series or parallel with running times and waiting times as a function of the number of jobs processed; however, they can be used to describe social processes as well. To the extent that we can think of human relationships as processes with jobs composed of components and running times and waiting times, the scaling laws can be used analogously to describe these relationships. Indeed, scaling laws frequently use the independent variable (N) to signify humans (users, customers, or clients) processed, so the description is more than just analogy, but is more just a common way in which scaling laws are applied to systems in which humans are variables (like the example of housing humans in a building, used above).

Accepting this application of scaling laws to human systems, each different type of friendship (usefulness, pleasure, virtue) corresponds to a different type of scaling law (linear, asymptotic, retrograde).

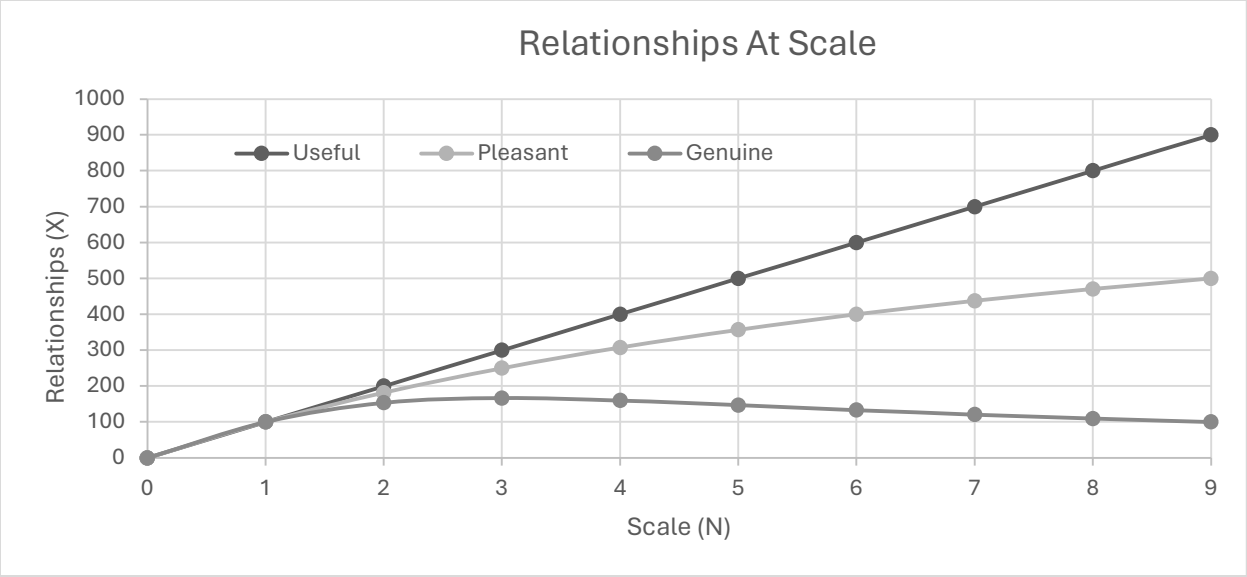

Figure 1: Relationships at Scale (Useful, Linear; Pleasant, Asymptotic; Genuine, Retrograde) where the system of friendships is described by the dependent variable (X) representing the number of freindships, the independent variable (N) representing the number of persons, the linear scaling factor (γ=100), the serialized fraction of the system (α=0.1), and the interdependent fraction of the system (β=0.1).

a. Linear Usefulness

First, we can observe that friendships of usefulness can scale linearly.

The linear scalability of usefulness can be deduced from the observation that usefulness does not seem to diminish at scale.

For example, if subscribers are providing monetary gain, having one subscriber is not as good as having ten subscribers which is not as good as having one hundred subscribers (etc.).

Because there is no obvious upper limit to the scalability of usefulness, and if there is some theoretical limit, it is not within realistic horizons, friendships of usefulness clearly follow a linear scaling law.

There may be specific forms of usefulness that fail to scale indefinitely, perhaps leveling at some asymptote (e.g., the world only uses so many paperclips), but acknowledging these cases we may set them aside as exceptions to the more general rule (Gelles & Mitchell, 1996).

b. Asymptotic Pleasure

Second, we can observe that friendships of pleasure can scale asymptotically.

The asymptotic scalability of pleasure can be deduced from the observation that experience levels out at scale:

> **An Experiential Constraint**: new experiences (like pleasures) grow linearly according to duration and intensity towards some maximum.

Each person has a finite number of experiences, which is limited by their finite amount of time and space. One can have twenty-four hours of pleasure per day inside a human skull, but no more than this, which puts a hard cap on the amount of pleasures experienceable.

For example, viewing one hour of pleasurable content per day is not as good as viewing ten hours of pleasurable content per day; however, viewing one hundred hours of pleasurable content per day is no better than viewing twenty-four hours of pleasurable content per day, because one cannot view any more than twenty-four.

Because there is an obvious upper limit to the scalability of pleasure as a function of experiential time due to the Experiential Constraint, friendships of pleasure must follow Amdahl's (asymptotic) Scaling Law.

This means that, to the extent that experiential time is non-concurrent, which is to say necessarily serialized processes, pleasurable friendships are not indefinitely scalable.

There may be specific forms of pleasure that fail to asymptote at scale, perhaps degrading after some peak (e.g., eating cake until stuffed), but acknowledging these cases we may set them aside as exceptions to the more general rule (Shephard, 1970).

c. Retrograde Virtue

Third, we can observe that friendships of virtue can scale retrogressively.

The retrograde scalability of virtue can be deduced from the observation that genuine interdependent relations seem to diminish at scale:

> **An Interdependent Constraint**: new interdependencies (like genuine friendships) grow quadratically according to the number of persons in a population.

Genuine friendships require interpersonal dependencies of emotional commitment between persons in a network; each person's experience depends upon the experiences of their friends, in an inextricably cross-talking network. As the number of connections grows the number of interdependencies grows quadratically.

For example, having one good friend may create one interpersonal interdependency, but having a group of ten good friends creates ninety interpersonal interdependencies.

Thus, the obvious upper limit to the scalability of genuine friendship is the capacity for growth of interdependency to remain undiminished as the quadratic growth of interconnection dilutes any individual connection due to the Interdependence Constraint, friendships of virtue clearly follow Gunther's (retrograde) Scaling Law.

This means that, to the extent that interpersonal relationships are cross-talking, which is to say necessarily interdependent processes, genuine friendships are eventually regressively scaling as the interdependencies slow the ability to scale. Indeed, the retrograde scaling of genuine friendships might be thought of as a kind of impossibility theorem for amicability at scale.

There may be specific forms of virtuous friendship that fail to peak at scale, though this seems largely inconceivable, since preserving moderation seems a necessary condition of such virtues (Aristotle, 1106a29-33). Therefore, the burden of proof lies on those attempting to characterize such a immoderate virtue.

5. Consequences

The assigning of different kinds of friends with different kinds of scaling laws has a several consequences for how we view friendship and scalability.

- First, if one is looking for friendships of usefulness, then one is looking for indefinite scale. To the extent that one attempts to focus on usefulness, one must embrace scale. The more useful the friendship, the more scale.
- Second, if one is looking for friendships of pleasure, then one is for scale only to up to a point. To the extent that one attempts to focus on pleasure, one must consider the eventual constraints of scale. The more pleasurable the relations, the closer to the constraints of scale one approaches.
- Third, if one is looking for genuine friendship, then one is not looking for indefinite scale. To the extent that one attempts to focus on friendships, one must avoid scale. The more genuine the friendship, the less scale. In the contrapositive, if one is looking for scale, then one is not looking for friendships. To the extent that one attempts to focus on scale, one must avoid friendships. The more extensive the scale, the less genuine the friendships. Ancient humans might have been able to accommodate interpersonal relations amongst the members of a small scale tribe, but modern humans cannot possibly accommodate the scale technologies enable them to socially navigate. Although digital technology has scaled up our possibilities of inter-human connections, our actual connections cannot possibly scale with the technology. Whereas we might have infinite possible connectivity, we still only have finite actual connectivity. Thus, we cannot possibly meet the social expectations made possible for us. (Yet, sadly, we often still feel obliged to accommodate infinity; and in turn we feel that infinity is obliged to accommodate to us, in vain.)
- Combining the above consequences, if usefulness and genuine friendship are both values, and the tradeoff of these two things at scale is real, then one encounters a real moral dilemma between the two (Williams, 1965). A usefulness-rich world may have to be one of inauthentic friendships, and a world of authentic friendships may have to have relative usefulness deficiencies; one cannot maximize one value without diminishing the other, and vice versa.

From these consequences, we can derive some conclusions about public policy and digital design:

- Regarding public policy, these consequences mean that, although we can expect societies at scale to have high usefulness (and pleasure to an extent), we should expect either a neutral or adverse effect on genuine friendships (Scott, 2020). Although public policy can scale the former, it cannot scale the latter. Furthermore, public policy and may do more harm than good if it tries, because, even if the metrics can be utilized, which is not guaranteed (Muller, 2018), scalability may simply produce mechanisms which favor usefulness and pleasure, which then outperform and outcompete genuine relationships. The most important things (like virtues and friendships) may ultimately be unscalable, or worse retrograde at scale.
- Regarding digital design, paradoxically, these consequences mean that perhaps the worst thing that one can do for developing genuine friendships is to focus on the enormous scales of social media (Turkle, 2011). Rather, social media companies, while claiming to want to "connect the world", may primarily be good at affording connections of usefulness (and pleasure to an extent) and diminishing connections of virtue (Miller, 2014). Indeed, although "informational commodities" may be highly useful, they may be minimally valuable (Lyotard, 1984, p. 5). The worry would be that the greatest possible

scale is a vanishing point for virtue, as useful and pleasurable relations crowd out genuine friendships.

- More generally, the problem may not be soluble in principle, for any persons in any networks. A network of robots operating at 1000 times human capacity might be able to have more friendships amongst themselves, but as long as interdependencies obtained, they would still be constrained by a retrograde scaling law at some level. It is not a flaw in the capacities of humans but in the quantitative behavior of interdependencies.

Notably, this is not to say that public policy or digital design and other economies of scale afford no values at all—they clearly do afford utility and pleasure to an extent (Gibson, 1966, 1979; Norman, 2013). Rather, their affordances of other values may either be independent of or in spite of the losses of genuine friendship at scale.

6. Conclusion

In summary, we should not be thinking of all types of friendships as equally scalable but should think of friendships of usefulness as linearly scalable, friendships of pleasure as asymptotically scalable, and friendships of virtue as retrogressively scalable. In other words, the more valuable the friendship (according to Aristotle), the less scalable it seems to be. This has strategic implications for how we should think about the value of scalability and the persistence of friendships in economies of scale, advising hesitation when attempting to use public policy or digital design to scale virtue.

Appendix

Where the system is described by the dependent variable (X), the independent variable (N), the linear scaling factor (γ), the serialized fraction of the system (α), and the interdependent fraction of the system (β).

Linear Scaling Law

$$X(N) = \frac{\gamma N}{1}$$

(Gunther, 1993)

Asymptotic Scaling Law

$$X(N) = \frac{\gamma N}{1 + \alpha(N - 1)}$$

(from Amdahl, 1967, as adapted by Gunther, 2002)

Retrograde Scaling Law

$$X(N) = \frac{\gamma N}{1 + \alpha(N - 1) + \beta N(N - 1)}$$

(Gunther, 1993)

References

Amdahl GM (1967). “Validity of the single-processor approach to achieve large scale computing capabilities.” *AFIPS Joint Spring Conference Proceedings,* 30 (Atlantic City, NJ, Apr.

18–20), AFIPS Press, Reston VA, pp 483–485, At http://www-inst.eecs.berkeley.edu/~n252/paper/Amdahl.pdf

Aristotle (2011). *Nicomachean Ethics.* Translated by Bartlett, Robert C.; Collins, Susan D. Chicago: University of Chicago Press. *ISBN 978-0-226-02674-9*.

Cooper, John M. (1977). "Aristotle on the Forms of Friendship". *The Review of Metaphysics.* **30** (4): 619–648. *ISSN 0034-6632*. *JSTOR 20126987*.

Edwards, Harold M. (1995). *Linear Algebra.* Springer. p. 78. *ISBN 9780817637316*.

Gelles, Gregory M.; Mitchell, Douglas W. (1996). "Returns to Scale and Economies of Scale: Further Observations". Journal of Economic Education. **27** (3): 259–261. *doi*:*10.1080/00220485.1996.10844915*. *JSTOR 1183297*.

J. J. Gibson (1966). *The Senses Considered as Perceptual Systems*. Allen and Unwin, London.

J. J. Gibson (1979). 'The Theory of Affordances'. *The Ecological Approach to Visual Perception*. Houghton Mifflin Harcourt (HMH), Boston. p. 127.

Gunther, Neil J. (2002). "A New Interpretation of Amdahl's Law and Geometric Scalability". *arXiv*:*cs/0210017*.

Gunther, Neil J. (1993). ""A Simple Capacity Model for Massively Parallel Transaction Systems" in *Proc. CMG Conf*., San Diego, California" (PDF). *CMG, Pages 1035–1044.*

Helm, Bennett (2017). "Friendship", *The Stanford Encyclopedia of Philosophy* (Fall 2017 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/fall2023/entries/friendship/>.

Lyotard, Jean-Francois (1984). *The Postmodern Condition: A Report on Knowledge*, translated Geoff Bennington and Brian Massumi, Minneapolis, MN: Minnesota University Press.

Miller, Joe. “Zuckerberg: Facebook's mission is to 'connect the world'”. 24 February 2014. https://www.bbc.com/news/technology-26326844

Muller, Jerry Z. (2018). *The Tyranny of Metrics* (NED-New edition). Princeton University Press. https://doi.org/10.2307/j.ctvc7743t

Norman, D. A. (2013). *The design of everyday things*. MIT Press.

Scott, J. C. (1998). *Seeing Like a State*. Yale University Press.

Shephard, Ronald W. (1970-03-01). *"Proof of the law of diminishing returns"*. Zeitschrift für Nationalökonomie. **30** (1): 7–34. *doi*:*10.1007/BF01289990*. *ISSN 1617-7134*. *S2CID 154887748*.

Turkle, Sherry (2011). *Alone Together: Why We Expect More from Technology and Less from Each Other*. New York: Basic Books.

Williams, Bernard (1965). “Ethical Consistency,” *Proceedings of the Aristotelian Society* (Supplement), 39: 103–124; reprinted in Gowans (1987): 115–137.