New Prospects for a Causally Local Formulation of Quantum Theory

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It is difficult to extract reliable criteria for causal locality from the limited ingredients found in textbook quantum theory. In the end, Bell humbly warned that his eponymous theorem was based on criteria that "should be viewed with the utmost suspicion." Remarkably, by stepping outside the wave-function paradigm, one can reformulate quantum theory in terms of old-fashioned configuration spaces together with 'unistochastic' laws. These unistochastic laws take the form of directed conditional probabilities, which turn out to provide a hospitable foundation for encoding microphysical causal relationships. This unistochastic reformulation provides quantum theory with a simpler and more transparent axiomatic foundation, plausibly resolves the measurement problem, and deflates various exotic claims about superposition, interference, and entanglement. Making use of this reformulation, this paper introduces a new principle of causal locality that is intended to improve on Bell's criteria, and shows directly that systems that remain at spacelike separation cannot exert causal influences on each other, according to that new principle. These results therefore lead to a general hidden-variables interpretation of quantum theory that is arguably compatible with causal locality.

I. INTRODUCTION

In physics, 'locality' can refer to any of several distinguishable concepts. What follows is a non-exhaustive list of historically important examples.

- In physical theories like Newtonian mechanics that involve forces, one can ask whether those forces are limited by the speed of light, or instead consist of faster-than-light *action at a distance*. A well-known case of action at a distance is the Newtonian gravitational force $F_{\text{g}} = Gm_1m_2/|\mathbf{r}_1 - \mathbf{r}_2|^2$ between two spherically symmetric bodies with respective masses m_1 and m_2, and with respective centers of mass located at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, where G is Newton's constant. The status of this form of nonlocality is somewhat murkier in textbook formulations of quantum theory, in which forces do not appear to play a fundamental role.

- A physical theory is *signal-local* [1, 2] if it does not permit the transmission of controllable signals or messages faster than light. In principle, there are no constraints in Newtonian mechanics that would preclude sending superluminal signals—say, by exploiting the action-at-a-distance features of Newtonian gravitational forces. Newtonian mechanics is therefore

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presumably *signal-nonlocal*. By contrast, the aptly named *no-communication theorem* [3, 4] ensures that appropriately defined quantum systems—such as local quantum fields—cannot be used to send superluminal signals, so these quantum systems are signal-local.

- The *cluster decomposition principle* [5, 6] is the condition that correlation functions for a physical system consisting of widely separated constituent subsystems should factorize into a product of correlation functions for each of those individual subsystems. This condition ensures that the statistical behavior of nearby physical systems does not depend on the inaccessible details of other systems that are very far away, assuming the absence of any initial correlations between the nearby and faraway systems.

- For a local quantum field theory, one typically imposes *microcausality conditions* [6], which require that bosonic field operators should commute at spacelike separation, and that fermionic field operators should anticommute at spacelike separation. Among other consequences, these microcausality conditions ensure that local observables at spacelike separation are capable of being statistically uncorrelated.

- At the level of mereology, a spatially extended physical entity that is fully reducible to spatially local parts is said to be *separable*, and is otherwise said to be *nonseparable* or *holistic* [7, 8].

This paper will be concerned with a different type of locality, called *causal locality*, which will be taken to consist of the following statement:

} Causal influences should not be able to propagate faster than light. } (1)

Going back at least to the work of Albert Einstein, Boris Podolsky, and Nathan Rosen in 1935 [9], and continuing through the work of John Bell in the 1960s and beyond [10–16], there has been an ongoing debate over whether quantum theory is causally local in this sense. A major challenge for all such arguments is that causal locality expressly depends on the notion of a 'causal influence,' which is a notoriously difficult concept to define rigorously. One of the main goals of this paper will be to address this difficulty directly, as a stepping stone toward arguing that a specific new formulation of quantum theory [17, 18] is, in fact, causally local.

After a high-level overview of the Einstein-Podolsky-Rosen (EPR) argument and Bell's subsequent work in Section II, Section III will continue with a detailed analysis of Bell's results and their assumed criteria for causal locality. Section IV will then review a new *unistochastic formulation* of quantum theory, based on 'unistochastic' microphysical laws [17, 18]. Section V will introduce salient topics related to causality from the theory of Bayesian networks, and then, inspired in part by those ideas, Section VI will recast the unistochastic formulation in causal terms. Section VII will show that this overall approach makes possible an improved criterion for causal locality. Section VIII will then argue that the unistochastic formulation is causally local according that improved criterion. Section IX will conclude with a summary and a discussion of relevant implications for the interpretation of quantum theory.

II. EINSTEIN, PODOLSKY, ROSEN, AND BELL

The EPR argument [9] was based on a rudimentary version of *quantum steering*, a term introduced by Erwin Schrödinger shortly thereafter [19, 20].

In quantum steering, two observers, Alice and Bob, split a pair of quantum systems described by an entangled wave function, and then move a large distance apart. If Bob decides to carry out a local measurement on his system, then his choice of measurement basis will appear to 'steer' Alice's system to collapse to a corresponding basis. However, Bob will not be able to control which specific wave function Alice's system selects in that basis, nor will Alice be aware that anything strange has happened until she later confers with Bob. (Note that this paper will use the terms 'wave function' and 'state vector' interchangeably.)

Nonetheless, the overall behavior of the entangled pair of systems looks suspiciously like a form of causal nonlocality—a concrete manifestation of what Einstein in 1947 called "spooky action at a distance" ("*spukhafte Fernwirkung*") [21]. Following the EPR paper's publication, Schrödinger described the situation in the following way:

> It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it. [19]

The EPR paper took for granted that causal nonlocality should be impossible, and argued that the only available alternative was to assert that the faraway system should already know what measurement result it would reveal according to any hypothetical choice of measurement basis. Because this information was not encoded in the system's overall wave function, the authors of the EPR paper concluded that quantum theory was incomplete. Indeed, the EPR paper was titled "Can [the] Quantum-Mechanical Description of Physical Reality Be Considered Complete?"

If one were to regard the EPR paper's reasoning as sound, then one would seemingly be confronted with the following logical fork: either accept causal nonlocality in quantum theory, or instead assert both the incompleteness of quantum theory and the existence of a causally local way for measurement results to be "predetermined," in the language of John Bell's 1964 paper "On the Einstein-Podolsky-Rosen Paradox" [10]. Writing about the EPR argument in a 1981 paper, Bell described this logical fork in the following way:

> For after observing only one particle[,] the result of subsequently observing the other (possibly at a very remote place) is immediately predictable. Could it be that the first observation somehow fixes what was unfixed, or makes real what was unreal, not only for the near particle[,] but also for the remote one? For EPR[,] that would be an unthinkable 'spooky action at a distance.' To avoid such action at a distance[,] they have to attribute, to the space-time [sic] regions in question, *real* properties in advance of observation, correlated properties, which *predetermine* the outcomes of these particular observations. Since these real properties, fixed in advance of observation, are not contained in [the] quantum formalism, that formalism for EPR is *incomplete*." [Emphasis in the original.] [13]

In his 1964 paper [10], Bell argued that this logical fork was, in the end, a mirage, and that quantum theory unavoidably entailed causal nonlocality. To set up his argument, Bell considered general reformulations of quantum theory involving 'hidden variables' that uniquely predetermined measurement outcomes. Bell's goal was to show that any such *measurement-deterministic* hidden-variables theory would have to involve causally nonlocal effects.

As Bell noted in his 1964 paper, one such measurement-deterministic hidden-variables theory was already known, at least for the case of nonrelativistic systems of finitely many particles. Called the *de Broglie-form pilot-wave formulation* of quantum theory, or *Bohmian mechanics* [22–24], this theory featured faster-than-light action at a distance, which Bell called "a grossly nonlocal structure."

The result of Bell's 1964 paper was the first version of what is now called *Bell's theorem*, which implied that if a measurement-deterministic hidden-variables theory were based on *causally local* dynamics, according to Bell's criteria, then the theory should satisfy an inequality that is violated in quantum theory. The 2022 Nobel Prize in Physics [25] was awarded to Alain Aspect, John Clauser and Anton Zeilinger for their experiments verifying that quantum systems indeed violate Bell's inequality, fully in accord with the predictions of quantum theory.

Importantly, Bell's 1964 paper assumed the soundness of the EPR argument, which, in turn, implicitly relied on several contestable principles. These included appealing to an explicit form of wave-function collapse, as well as treating measurement interventions as primitive axiomatic ingredients of quantum theory.

At an even deeper level, the EPR argument depended on an *interventionist conception of causation*, in which causation is supposed to be explicated in terms of abstract *agents* carrying out formal *interventions* on one set of variables that then imply changes in another set of variables. (For a review of interventionist accounts of causation, see [26].) It is not obvious how to express the EPR argument more fundamentally in terms of the constituent atoms that make up measuring devices and embodied observers, all undergoing some global physical process. Nor is it clear that the EPR argument would be applicable to any formulation of quantum theory that foregoes not only primitive measurement interventions, but also lacks unique measurement outcomes, such as Hugh Everett's 'many worlds' interpretation [27–29].

Given these substantive reasons for doubting the EPR argument, Bell's 1964 results could not be taken to imply that quantum theory *necessarily* involved causal nonlocality. His 1964 results instead reduced to the more modest consequence of only ruling out measurement-deterministic hidden-variables theories obeying causally local dynamics.

Putting aside several other potential loopholes (see [30] for a review), Bell's 1964 paper therefore left open three possibilities: measurement-deterministic hidden-variables theories with *nonlocal* dynamics, hidden-variables theories with *stochastic* measurement outcomes, and formulations of quantum theory that eschewed hidden variables altogether.

In 1975 [12], Bell updated his theorem to encompass the second and third of these classes of possibilities, where the third class includes 'textbook' quantum theory itself. (For pedagogical reviews of textbook quantum theory, see [31–33].) Crucially, extending his theorem in this way required introducing a controversial new criterion for causal locality, a principle that Bell called

"local causality." Bell was able to show that all formulations of quantum theory satisfying his principle of local causality should obey a generalization of his inequality originally derived in 1969 by John Clauser, Michael Horne, Abner Shimony, and Richard Holt [11]. This generalized inequality is likewise violated by quantum theory.

In keeping with the terminology of [30], this paper will distinguish 'local causality' from the more basic condition of 'causal locality' defined in (1). In short, 'causal locality' means that any causal influences that *happen to occur* in a given scenario should not propagate faster than light, whereas 'local causality' *positively asserts the existence* of local causal relationships in specific situations.

There are several incorrect ways to read the stronger 1975 version of Bell's theorem. One is that the theorem rules out hidden variables altogether. Another false reading is that one can avoid violating Bell's principle of local causality merely by avoiding the introduction of hidden variables—but this reading confuses the weaker 1964 version of Bell's theorem with the stronger 1975 version, which applies even to theories that do not include hidden variables at all, like textbook quantum theory itself. The correct reading of Bell's theorem is to stay close to what Bell himself wrote and conclude that his principle of local causality is violated by all empirically adequate formulations of quantum theory, including the textbook version of the theory, again putting aside various potential loopholes.

It is far from clear, however, that the principle of local causality that Bell used to prove the stronger version of his theorem was the correct way to formulate the more basic condition of causal locality in the first place. Bell himself warned against taking his principle of local causality too seriously. Indeed, in a 1990 lecture [15], he cautioned that his principle "should be viewed with the utmost suspicion."

Bell had good reasons for being skeptical of his own theorem's premises, due to his history with an older theorem proved by John von Neumann decades before. That earlier theorem had been widely viewed as *completely* ruling out the possibility of hidden variables [34–36]. Already in 1935, Grete Hermann had determined that von Neumann's theorem depended on an assumption about expectation values that was too narrow [37, 38]. Bell essentially discovered the same flaw in von Neumann's proof decades later [39]. (For an excellent historical discussion of von Neumann's theorem, its shortcomings, and its critics, see [40].)

III. BELL'S PRINCIPLE OF LOCAL CAUSALITY

To lay the groundwork for the discussion ahead, it will be important to begin with a brief presentation of the 1964 and 1975 versions of Bell's theorem, with a focus on their key implicit assumptions. It is precisely these implicit assumptions that will be challenged in this paper, for the eventual purpose of developing a better criterion for causal locality.

In his 1990 lecture, Bell noted the limitations of textbook quantum theory, which lacked any notion of "local *be*ables"—meaning *actual* properties possessed by localized physical systems—as opposed to the theory's more austere and instrumentalist notions of *observables*, *measurement settings*, and *measurement outcomes*:

> Even then, we are frustrated by the vagueness of contemporary quantum mechanics. You will hunt in vain in the text-books [sic] for the local *be*ables of the theory. What you may find there are the so-called 'local observables'. It is then implicit that the apparatus of 'observation', or, better, of experimentation, and the experimental results, are real and localized. We will have to do as best we can with these rather ill-defined local beables, while hoping always for a more serious reformulation of quantum mechanics where the local beables are explicit and mathematical rather than implicit and vague. [Emphasis in the original.] [15]

In setting up the 1964 version of his theorem [10], Bell resorted to a pair of bivalent measurement outcomes $A = \pm 1$ and $B = \pm 1$ at far separation in space, together with their respective local measurement settings $\mathbf{a}$ and $\mathbf{b}$, with the special feature that if $\mathbf{a} = \mathbf{b}$, then $A = -B$. Bell then imagined a measurement-deterministic hidden-variables theory containing a set of hidden variables λ, and supposed that these hidden variables λ, together with the measurement settings $\mathbf{a}$ and $\mathbf{b}$, fully predetermined the values of the measurement outcomes A and B:

$$A = A(\mathbf{a}, \mathbf{b}, \lambda) = \pm 1, \quad B = B(\mathbf{a}, \mathbf{b}, \lambda) = \pm 1. \tag{2}$$

Following the terminology of [30], this assumption will be called *Outcome Determinism*.

In that 1964 paper, Bell's causal-locality assumptions included the condition that the measurement outcome A should not depend on the faraway measurement setting $\mathbf{b}$, and, similarly, that the measurement outcome B should not depend on the faraway measurement setting $\mathbf{a}$. Bell concluded that A should be a function $A(\mathbf{a}, \lambda)$ of $\mathbf{a}$ and λ alone, and that B should be a function $B(\mathbf{b}, \lambda)$ of $\mathbf{b}$ and λ alone:

$$A(\mathbf{a}, \mathbf{b}, \lambda) = A(\mathbf{a}, \lambda), \quad B(\mathbf{a}, \mathbf{b}, \lambda) = B(\mathbf{b}, \lambda). \tag{3}$$

Today these assumptions are known as *Parameter Independence* [41].

Crucially, Bell's proof also relied on a special implication of Outcome Determinism and Parameter Independence. Letting $\rho(\lambda)$ denote an assumed probability distribution for the hidden variables, Outcome Determinism (2) and Parameter Independence (3) suggested that the expectation value of the product of the measurement outcomes A and B over many runs of the experiment should be given by

$$P(\mathbf{a}, \mathbf{b}) = \int d\lambda \, \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda). \tag{4}$$

(As an aside, notice that the very existence of the probability distribution $\rho(\lambda)$ for the hidden variables was yet one more implicit assumption in Bell's proof.)

Invoking the formula (4) for the expectation value $P(\mathbf{a}, \mathbf{b})$, the end-result of the 1964 paper was the well-known *Bell inequality*:

$$1 + P(\mathbf{b}, \mathbf{c}) \geq |P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c})|. \tag{5}$$

Here $\mathbf{c}$ is an alternative choice of measurement setting. Quantum theory predicts violations of this inequality, and, again, the 2022 Nobel Prize in Physics [25] was awarded for the experimental confirmation of those violations.

Given Bell's criticism [39] of von Neumann's hidden-variables theorem over its assumptions about expectation values, as described earlier in this paper, it is ironic that Bell's own theorem likewise hinged on a statement about how expectation values were supposed to work. Without Outcome Determinism and Parameter Independence, the formula (4) is not the correct way to calculate the necessary expectation value.

To see why, consider a theory with stochastic measurement outcomes, as in Bell's 1975 paper, with some set of variables λ representing beables, whether hidden variables or not. (As noted by Bell in [13], one could even try to regard wave functions themselves as 'spatially nonseparable beables.') For this more general case, in the formula (4) for the expectation value, one then needs to replace the product

$$A(\mathbf{a}, \lambda)B(\mathbf{b}, \lambda) \tag{6}$$

with the statistical average

$$\sum_{A,B} \rho(A, B|\mathbf{a}, \mathbf{b}, \lambda)AB, \tag{7}$$

where $\rho(A, B|\mathbf{a}, \mathbf{b}, \lambda)$ is some joint probability distribution conditioned on the measurement settings $\mathbf{a}$ and $\mathbf{b}$, as well as conditioned on the variables λ representing the theory's beables. It follows that (4) should be replaced with

$$P(\mathbf{a}, \mathbf{b}) = \int d\lambda\, \rho(\lambda) \sum_{A,B} \rho(A, B|\mathbf{a}, \mathbf{b}, \lambda)AB. \tag{8}$$

In place of Outcome Determinism (2) and Parameter Independence (3), one then needs new assumptions in order to derive something like the Bell inequality (5).

From the standard rules for working with conditional probabilities, one can always write down the decomposition

$$\rho(A, B|\mathbf{a}, \mathbf{b}, \lambda) = \rho(A|\mathbf{a}, \mathbf{b}, \lambda, B)\rho(B|\mathbf{a}, \mathbf{b}, \lambda). \tag{9}$$

For a given measurement-stochastic theory, Bell's new *principle of local causality* was the condition that the theory should contain variables λ representing a sufficiently rich collection of beables localized in the overlap of the past light cones of the measurement outcomes A and B that λ screens off B and $\mathbf{b}$ from A, and also screens off $\mathbf{a}$ from B, in the sense that

$$\rho(A|\mathbf{a}, \mathbf{b}, \lambda, B) = \rho(A|\mathbf{a}, \lambda), \quad \rho(B|\mathbf{a}, \mathbf{b}, \lambda) = \rho(B|\mathbf{b}, \lambda). \tag{10}$$

Looking back at the decomposition (9), it is clear that this new assumption (10) is *equivalent*

to requiring that conditioning on the variables λ representing beables localized in the overlap of the past light cones of A and B leads to the following factorization condition:

$$\rho(A, B|\mathbf{a}, \mathbf{b}, \lambda) = \rho(A|\mathbf{a}, \lambda)\rho(B|\mathbf{b}, \lambda). \tag{11}$$

Indeed, in his 1981 paper [13], Bell took this latter formula to be his basic principle of local causality, and attempted to justify it on its own merits.

The factorization version (11) of Bell's principle of local causality is, in turn, *also* equivalent to the conjunction of two other assumptions.

The first assumption is the following *weaker* factorization condition:

$$\rho(A, B|\mathbf{a}, \mathbf{b}, \lambda) = \rho(A|\mathbf{a}, \mathbf{b}, \lambda)\rho(B|\mathbf{a}, \mathbf{b}, \lambda). \tag{12}$$

This property is now called *Outcome Independence* [41].

The other assumption is a generalization of Parameter Independence (3) to mean that the conditional probabilities for the measurement outcome A do not depend on the measurement setting $\mathbf{b}$, and that the conditional probabilities for the measurement outcome B do not depend on the measurement setting $\mathbf{a}$:

$$\rho(A|\mathbf{a}, \mathbf{b}, \lambda) = \rho(A|\mathbf{a}, \lambda), \quad \rho(B|\mathbf{a}, \mathbf{b}, \lambda) = \rho(B|\mathbf{b}, \lambda). \tag{13}$$

Assuming Outcome Independence (12) together with the updated version of Parameter Independence (13), one obtains Bell's factorization (11), where again λ denotes variables representing a sufficiently rich collection of beables localized in the overlap of the past light cones of the measurement results A and B. The expectation value (8) then becomes

$$P(\mathbf{a}, \mathbf{b}) = \int d\lambda\, \rho(\lambda) \left(\sum_A \rho(A|\mathbf{a}, \lambda) A \right) \left(\sum_B \rho(B|\mathbf{b}, \lambda) B \right), \tag{14}$$

which closely resembles the 1964 version (4) of the same expectation value. This formula thereby makes it possible to derive a more general form of the Bell inequality, as first obtained in 1969 by Clauser, Horne, Shimony, and Holt [11]. This inequality is violated by all theories that are empirically equivalent to textbook quantum theory—including the textbook theory itself—so all such theories must also violate Bell's principle of local causality.

Bell's principle of local causality—in either of its equivalent forms (10) or (11)—implicitly depends on an assumption that goes beyond questions of locality. That implicit assumption is called *Reichenbach's principle of common causes*. (For a review, see Section 19 of Hans Reichenbach's book [42], and also [43].)

Reichenbach's principle of common causes states that if two variables A and B are correlated, in the sense that their joint probability $P(A, B)$ fails to factorize as the product of their standalone

probabilities $P(A)$ and $P(B)$,

$$P(A, B) \neq P(A)P(B), \tag{15}$$

and if A and B do not causally influence each other, then there should exist some other variable C such that conditioning on C leads to the following factorization:

$$P(A, B|C) = P(A|C)P(B|C). \tag{16}$$

That is, Reichenbach's principle *positively asserts the existence* of a 'common-cause' variable C for A and B. In this way, the variable C is said to 'explain' or 'account for' the correlation between A and B.[1]

Bell's principle of local causality—again in either of its equivalent forms (10) or (11)—clearly invokes Reichenbach's principle, with the role of the asserted common-cause variable C played by the variables λ representing beables localized in the overlap of the past light cones of the measurement results A and B.

Reichenbach's principle of common causes may seem sensible and intuitive in the context of everyday experience, but those are far from definitive reasons to take it to be a fundamental requirement for causal locality. In particular, embedded in both Reichenbach's principle of common causes and Bell's principle of local causality is the assumption that the asserted common causes in question must specifically take the form of variables that can be conditioned on and then summed or integrated over.

Just as a formulation of quantum theory that violates von Neumann's assumptions about expectation values can evade von Neumann's theorem and thereby admit hidden variables, a formulation of quantum theory that fails to adhere to the strictures of Reichenbach's principle of common causes could violate Bell's principle of local causality without necessarily entailing nonlocal causation—as was pointed out, for example, by William Unruh:

> It is true that this common cause cannot be stated in exactly the form which for example Reichenbach set up to describe common causes for a classical statistical system. But that is not surprising. Quantum mechanics is not classical mechanics. The structure of the correlations in a quantum system differ from those in a classical system, as Bell so succinctly showed. But those correlations do not arise mysteriously somehow in the development of a widely spaced system. Those correlations do not require some mysterious non-local [sic] action to be explained. They are simply there, as are correlations in a classical system, due to the evolution from a common (quantum) cause in the past. [44]

Returning once again to Bell's 1990 lecture [15], Bell actually formulated *two* versions of his principle of local causality.

Bell identified the first version as the following statement:

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[1] Note that this presentation of the principle is slightly generalized from Reichenbach's original formulation, which assumed that A and B were *positively* correlated, so that $P(A, B) > P(A)P(B)$.

> The direct causes (and effects) of events are near by [sic], and even the indirect causes (and effects) are no further away than permitted by the velocity of light. (17)

This first version is very close in spirit to the condition of causal locality introduced at the beginning of this paper in (1), and is merely a locality condition on whatever causal influences *happen to occur*.

However, Bell then stated that "The above principle of local causality is not yet sufficiently sharp and clean for mathematics," followed by "Now it is precisely in cleaning up intuitive ideas for mathematics that one is likely to throw out the baby with the bathwater. So the next step should be viewed with the utmost suspicion." It was at this point that Bell turned to the second version of his principle of local causality, which *positively asserted the existence* of common causes and became the mathematical statement (10).

This paper is hardly the first written argument to claim that Bell's principle of local causality is not the correct way to capture causal locality in a formulation of quantum theory. Beyond implicitly depending on Reichenbach's principle of common causes, one should also note that some readings of Bell's theorem, like several related theorems [11, 14], assume a notion of causation based on treating measurement settings and measurement results as primitive interventions by abstract agents. That is, these theorems depend on an interventionist conception of causation, as defined earlier in this paper. It is therefore not clear whether the theorems would make sense if one were instead to work at the level of the constituent atoms of the relevant measuring devices and physically embodied observers, all as parts of some sort of global probabilistic process.

Indeed, when thinking in terms of a global probabilistic process, without abstract agents and primitive interventions, it is far from obvious how to identify causal influences or even nonlocal interactions, especially without concrete notions like Newtonian forces that are capable of establishing definitive physical linkages between systems. (For an introduction to some of the challenges that arise when attempting to make sense of causation in physics, see [45].)

Other theorems, such as [46], depend on strong assumptions about the existence of theoretical joint probability distributions involving the measurement results of subsystems at *intermediate* times during an overall unitary process. The new formulation of quantum theory to be reviewed shortly provides principled reasons why such theoretical joint probability distributions should not be assumed to exist—the formulation simply does not supply them in its microphysical laws, due in part to *indivisibility*, a concept that will turn out to play a central role.

IV. THE UNISTOCHASTIC FORMULATION OF QUANTUM THEORY

As described in [17, 18], one can reformulate quantum theory in terms of a sufficiently general theory of stochastic processes, working entirely outside the traditional 'wave-function paradigm.' Note that this approach is not continuous with older attempts to formulate quantum theory in stochastic terms [47–52], all of which assumed a fundamental Markov condition, nor is it connected with stochastic-collapse approaches to quantum theory [53], which treat wave functions or density matrices as basic ingredients of physical reality.

The necessary axioms for this stochastic formulation are much simpler and more transparent than for traditional textbook treatments of quantum theory, without any need for metaphysically opaque postulates about wave functions in abstract Hilbert spaces over the complex numbers.[2]

At the level of kinematics, one assumes a system with a set of configurations, forming an old-fashioned configuration space $\mathcal{C}$. The specific choice of configuration space depends on the particular kind of system one is modeling, just like in classical physics, so $\mathcal{C}$ could consist of arrangements of particle positions, or of local field intensities, or of digital bits, or of some other physical ingredients altogether.

Sticking for simplicity to the discrete case, perhaps after a suitable degree of coarse-graining, the configuration space then consists of a collection of configurations $i = 1, \ldots, N$. (One can generalize the analysis ahead to the continuous case by introducing a measure on the configuration space and by replacing summations with integrations.)

At the level of dynamics, the microphysical laws consist of conditional or transition probabilities of the form

$$\Gamma_{ij}(t) \equiv p(i, t|j, 0) \quad [\text{for } i, j = 1, \ldots N], \tag{18}$$

each of which supplies the probability for the system to be in its ith configuration at a continuously variable time t, given that the system is in its jth configuration at a suitable initial time 0. (No assumption is made here that $t > 0$ or $t < 0$.) Introducing standalone probability distributions at the initial time 0 and at arbitrary times t,

$$p_j(0) \equiv p(j, 0), \quad p_i(t) \equiv p(i, t) \quad [\text{for } i, j = 1, \ldots N], \tag{19}$$

the conditional or transition probabilities (18) that make up the basic microphysical laws give a simple linear relationship between the standalone probabilities $p(j, 0)$ at the initial time 0 and the standalone probabilities $p(i, t)$ at the final time t, in accordance with the standard rules for conditional probabilities and marginalization:

$$p(i, t) = \sum_{j=1}^{N} p(i, t|j, 0)p(j, 0) \quad [\text{for } i = 1, \ldots N]. \tag{20}$$

Following the somewhat more succinct notation introduced above, this linear relationship becomes

$$p_i(t) = \sum_{j=1}^{N} \Gamma_{ij}(t)p_j(0) \quad [\text{for } i = 1, \ldots N]. \tag{21}$$

Working in terms of matrices, one can write the standalone probability distributions here as

[2] Technically speaking, the Hilbert spaces of quantum theory are defined not over the complex numbers alone, but over the *pseudo-quaternions* [54], which are a Clifford algebra generated by 1, the imaginary unit i, and the complex-conjugation operator K. This operator K is needed for implementing time-reversal transformations, and satisfies $K^2 = 1$ together with the anticommutation relation $Ki = -iK$. Altogether, the elementary pseudo-quaternions 1, i, K, and iK satisfy the basic relations $-i^2 = K^2 = (iK)^2 = (i)(K)(iK) = 1$.

$N \times 1$ column vectors,

$$p(0) \equiv \begin{pmatrix} p_1(0) \\ \vdots \\ p_N(0) \end{pmatrix}, \quad p(t) \equiv \begin{pmatrix} p_1(t) \\ \vdots \\ p_N(t) \end{pmatrix}, \tag{22}$$

and one can write the collection of transition probabilities as an $N \times N$ *transition matrix*,

$$\Gamma(t) \equiv \begin{pmatrix} \Gamma_{11}(t) & \Gamma_{12}(t) & & \\ \Gamma_{21}(t) & \ddots & & \\ & & & \Gamma_{NN}(t) \end{pmatrix}. \tag{23}$$

One can then naturally express the basic linear relationship (21) as an elementary matrix product:

$$p(t) = \Gamma(t)p(0). \tag{24}$$

The $N \times N$ transition matrix $\Gamma(t)$ consists of non-negative entries, and its columns each sum to 1:

$$\left.\begin{array}{l} \Gamma_{ij}(t) \geq 0 \quad [\text{for } i,j = 1,\ldots,N], \\ \displaystyle\sum_{i=1}^{N} \Gamma_{ij}(t) = 1 \quad [\text{for } j = 1,\ldots N]. \end{array}\right\} \tag{25}$$

Mathematically speaking, these properties identify $\Gamma(t)$ as a *(column) stochastic matrix*.

An important concept here is the historically recent notion of *divisibility* [55, 56], which is loosely related to the well-known *Markov property*. For a *divisible* transition matrix $\Gamma(t)$ with a variable time t, and given an intermediate time t' between 0 and t, there always exists a valid stochastic matrix $\Gamma(t \leftarrow t')$ such that one can 'divide' the dynamics from 0 to t into subintervals from 0 to t', and then from t' to t, as ordinary matrix multiplication:

$$\underbrace{\Gamma(t)}_{0 \text{ to } t} = \underbrace{\Gamma\big(t \leftarrow t'\big)}_{t' \text{ to } t} \underbrace{\Gamma\big(t'\big)}_{0 \text{ to } t'}. \tag{26}$$

By contrast, for the kind of stochastic process that is equivalent to a quantum system, the transition matrix will generically be *indivisible*, meaning that no valid such stochastic matrix $\Gamma(t \leftarrow t')$ satisfying the divisibility property (26) will exist. A stochastic process based on a potentially indivisible transition matrix will be called a *generalized stochastic system* or *process*.

An $N \times N$ matrix Γ is called a *unistochastic matrix* if there exists a (generally non-unique) $N \times N$ unitary matrix U such that the individual entries of Γ are each the modulus-squares of the corresponding entries of U:

$$\Gamma_{ij} = |U_{ij}|^2 \quad [\text{for } i,j = 1,\ldots,N]. \tag{27}$$

In [57], Alfred Horn originally called such matrices "ortho-stochastic," but that term is now reserved for the special case in which U can be taken to be a real-orthogonal matrix. The term "unistochastic" appears to have first been introduced by Robert Thompson in [58].

Crucially, notice that the equalities appearing in (27) hold *entry-by-entry*. That is, Γ is not given by a simple matrix product like $U^\dagger U$, which would just give the identity matrix $\mathbb{1}$, due to the unitarity of U. In particular, the overall relationship between Γ and U does not commute with matrix multiplication.

A generalized stochastic system with a unistochastic transition matrix $\Gamma(t)$ will be called a *unistochastic system* or *process*. As proved in [18], one can always assume that a generalized stochastic system is, in fact, a unistochastic system, by slightly enlarging or *dilating* the configuration space if necessary, and invoking the *Stinespring dilation theorem* [59]. It therefore suffices to focus one's attention on unistochastic systems.

Reconstructing quantum theory from the set of unistochastic systems is then an extended mathematical exercise.

Given the $N \times N$ unistochastic transition matrix $\Gamma(t)$ for a given unistochastic system, one starts by taking the quantum system's unitary *time-evolution operator* to be a (generally not-uniquely) associated $N \times N$ time-dependent unitary matrix $U(t)$:

$$\Gamma_{ij}(t) = |U_{ij}(t)|^2 \quad [\text{for } i, j = 1, \ldots, N]. \tag{28}$$

Unlike the underlying unistochastic transition matrix $\Gamma(t)$, this unitary time-evolution operator $U(t)$ satisfies a divisibility condition in the form of the usual composition law

$$\underbrace{U(t)}_{0 \text{ to } t} = \underbrace{U\big(t \leftarrow t'\big)}_{t' \text{ to } t} \underbrace{U\big(t'\big)}_{0 \text{ to } t'}, \tag{29}$$

where the *relative* time-evolution operator $U(t \leftarrow t')$ is defined by

$$U\big(t \leftarrow t'\big) \equiv U(t) U^\dagger\big(t'\big) \tag{30}$$

and is guaranteed to be unitary. The fact that modulus-squaring the entries of a matrix, as in (28), does not commute with matrix multiplication accounts for the failure of $\Gamma(t)$ likewise to be divisible.

Indeed, if one attempts to define a unistochastic transition matrix $\Gamma(t \leftarrow t')$ from t' to t based on the relative time-evolution operator (30),

$$\Gamma_{ij}\big(t \leftarrow t'\big) \equiv \big|U_{ij}\big(t \leftarrow t'\big)\big|^2, \tag{31}$$

then one ends up with a discrepancy between the *actual-indivisible* dynamical evolution $\Gamma(t)$ from 0 to t and the *nearest-divisible* dynamical evolution $\Gamma(t \leftarrow t')\Gamma(t')$:

$$\Gamma(t) \neq \Gamma\big(t \leftarrow t'\big)\Gamma\big(t'\big). \tag{32}$$

From the standpoint of regarding the quantum system as a unistochastic system, the well-known *interference effects* of quantum theory merely reflect this discrepancy:

$$\Gamma(t) - \Gamma\big(t \leftarrow t'\big)\Gamma\big(t'\big) \neq 0 \quad \text{[interference effects]}. \tag{33}$$

Writing the initial standalone probability distribution $p_j(0)$ as the diagonal entries of an $N \times N$ initial *density matrix* $\rho(0)$ whose other entries are all 0s,

$$\rho(0) \equiv \mathrm{diag}(p_1(0), \ldots, p_N(0)) \equiv \begin{pmatrix} p_1(0) & 0 & \\ 0 & \ddots & \\ & & p_N(0) \end{pmatrix}, \tag{34}$$

the quantum system's density matrix at all other times is defined by the usual similarity transformation given by the time-evolution operator $U(t)$:

$$\rho(t) \equiv U(t)\rho(0)U^{\dagger}(t). \tag{35}$$

Observe that the resulting time-dependent density matrix $\rho(t)$ is not generally diagonal for times $t \neq 0$.

Notice also that the famous linearity of the time evolution of quantum theory, as exhibited by relationship between $\rho(t)$ and $\rho(0)$, is not a mystery, but ultimately descends from the linearity of the basic relationship (21), which again follows directly from the standard rules for conditional probabilities and marginalization.

Assuming sufficient smoothness in t, so that *Stone's theorem* applies [60], one can define the system's self-adjoint *Hamiltonian* $H(t)$ as the infinitesimal generator of time translations,

$$H(t) \equiv i\hbar \frac{\partial U(t)}{\partial t} U^{\dagger}(t) = H^{\dagger}(t), \tag{36}$$

in which case the system's density matrix $\rho(t)$ satisfies the *von Neumann equation*,

$$i\hbar \frac{\partial \rho(t)}{\partial t} = [H(t), \rho(t)], \tag{37}$$

where the brackets denote the usual matrix commutator (not a Poisson bracket):

$$[X, Y] \equiv XY - YX. \tag{38}$$

If the system's density matrix $\rho(t)$ is rank-one, then it can be factorized in terms of a complex-valued $N \times 1$ *state vector* or *wave function* $\Psi(t)$,

$$\rho(t) = \Psi(t)\Psi^{\dagger}(t) \quad \text{[if rank-one]}, \quad \Psi(t) \equiv \begin{pmatrix} \Psi_1(t) \\ \vdots \\ \Psi_N(t) \end{pmatrix}, \tag{39}$$

in which case the state vector $\Psi(t)$ evolves according to the *Schrödinger equation*,

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H(t)\Psi(t). \tag{40}$$

It is notable that these familiar quantum-theoretic equations emerge from an underlying stochastic process, which ultimately consists of a system moving along some trajectory in a prosaic configuration space according to (indivisible) stochastic transition probabilities.

Observe that the state vector or wave function $\Psi(t)$ appears here as just a convenient piece of secondary, derived mathematics, rather than as anything like a primary or fundamental physical object. In the context of this overall stochastic picture, the wave function is not a piece of ontological furniture, but instead encodes epistemic information—the system's probabilities—as well as nomological information—the system's unistochastic microphysical dynamics.

Given a *random variable* $A(t)$ on the system's configuration space, meaning a spectrum of magnitudes $a_1(t), \ldots, a_N(t)$ that depend on the system's configuration $i = 1, \ldots, N$ and that generically also depend explicitly on the time t, the statistical expectation value of $A(t)$ is defined as

$$\langle A(t)\rangle \equiv \sum_{i=1}^{N} a_i(t)p_i(t). \tag{41}$$

In terms of the system's density matrix $\rho(t)$, as defined in (35), and introducing a diagonal matrix $A(t)$ according to

$$A(t) \equiv \mathrm{diag}(a_1(t), \ldots, a_N(t)) \equiv \begin{pmatrix} a_1(t) & 0 & \\ 0 & \ddots & \\ & & a_N(t) \end{pmatrix}, \tag{42}$$

one can rewrite the expectation value (41) in the equivalent form

$$\langle A(t)\rangle = \mathrm{tr}(A(t)\rho(t)), \tag{43}$$

which looks just like the standard formula from quantum theory.

Consider the special case in which $A = P_i$ is a rank-one projector consisting of a matrix with a 1 in its ith diagonal entry and 0s in all its other entries:

$$P_i \equiv \mathrm{diag}(0, \ldots, 0, \underset{\substack{\uparrow \\ i\text{th entry}}}{1}, 0, \ldots, 0). \tag{44}$$

It follows that if $\rho(t)$ is similarly rank-one, in the sense of being factorizable according to (39) in terms of a state vector $\Psi(t)$, then the expectation value (43) reduces to the simplest version of the Born rule:

$$p_i(t) = |\Psi_i(t)|^2. \tag{45}$$

Random variables on the unistochastic system's configuration space have the status of *beables*, in Bell's terminology. By modeling the measurement process in detail—treating measurement devices as mundane stochastic systems in their own right—one can show that non-diagonal self-adjoint operators represent observables that are *emergent phenomena* at the level of measurements, and so are called *emergeables* in [17]. A unistochastic system's beables and emergeables together comprise the system's full noncommutative algebra of *observables*.

Just as one can represent a stochastic process in the Hilbert-space formalism familiar from quantum theory, one can take any quantum system in its Hilbert-space formalism and turn the relationship (28) around to define a corresponding stochastic process. This *stochastic-quantum correspondence* is a many-to-one relationship in both directions—a single stochastic process will generally have many different-looking Hilbert-space representations, and a given quantum system in its Hilbert-space formalism may represent many different-looking stochastic processes. The relationship between a stochastic process and its corresponding Hilbert-space representation is therefore analogous to the relationship between a classical-deterministic system described by second-order differential equations of motion and its corresponding Hamiltonian phase-space representation, a relationship that is likewise many-to-one in both directions.

At a practical level, one can therefore regard the Hilbert-space formalism as a form of 'analytical mechanics' for highly general stochastic processes, just as the Hamiltonian phase-space formalism provides an analytical mechanics for a second-order classical-deterministic system. Like any form of analytical mechanics, the Hilbert-space formalism provides a powerful set of mathematical tools for specifying microphysical laws in a systematic manner, for studying dynamical symmetries, for proving theorems, and for calculating predictions.

The fact that one can reformulate a given quantum system as a unistochastic system deflates much of the exotic talk about quantum theory. As spelled out in [17], from the standpoint of this *unistochastic formulation* of quantum theory, the *measurement problem* arguably disappears, because measuring devices are now to be modeled as ordinary (if complicated) subsystems of an overall stochastic process, and one can show that they end up in measurement-outcome configurations probabilistically in accord with the usual predictions of the Born rule. Moreover, superposition is no longer a literal smearing of configurations, interference is just a breakdown (33) in divisible dynamics, and decoherence is merely the leakage of statistical correlations out into the larger environment.

In particular, as explained in [17], decoherence automatically generates *division events*, which are new times t' at which the microphysical transition matrix $\Gamma(t)$ *does* divide, in the sense of (26). A division event t' is therefore a time that can serve in place of the initial time 0 in the unistochastic system's microphysical conditional probabilities.

If t' is a division event, then the unistochastic system contains genuine microphysical conditional probabilities of the form

$$\Gamma_{ii'}\big(t \leftarrow t'\big) \equiv p\big(i, t | i', t'\big), \tag{46}$$

which are conditioned on the system's configuration i' at the division event t', where $\Gamma(t \leftarrow t')$ is a

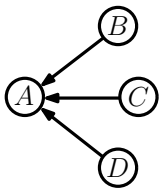

Figure 1. A simple Bayesian network with four random variables A, B, C, and D denoted by nodes, with directed edges pointing to A from B, C, and D.

valid stochastic matrix satisfying the divisibility condition (26). One expects that for a macroscopic system in strong contact with a noisy environment that eavesdrops on the system's configuration over a characteristic time scale δt, the system's microphysical laws will become effectively Markovian for time steps of duration δt.

V. BAYESIAN NETWORKS AND CAUSATION

As explained earlier, the traditional textbook formulation of quantum theory does not provide a hospitable domain for a non-interventionist account of causation, making it very difficult to devise clear statements about causal influences in general or causal locality in particular. By replacing the Hilbert-space axioms with a true set of microphysical laws consisting of conditional probabilities $\Gamma_{ij}(t) \equiv p(i,t|j,0)$, as introduced in (18), the new unistochastic formulation of quantum theory reviewed in this paper opens up an important connection with the literature on *Bayesian networks* [61], which provide a much more amenable foundation for a non-interventionist causal account.

In simple terms, a Bayesian network is a model that consists of a set of random variables connected by a collection of conditional probabilities. Displayed graphically, a Bayesian network will typically denote the random variables by *nodes*, and will denote the conditional probabilities by directed line segments or *edges* connecting some of those nodes together.

For example, if a node representing a random variable A is at the pointed end of directed edges from nodes representing random variables B, C, and D, as in Figure 1, then the Bayesian network must supply a basic conditional probability distribution $p(a|b,c,d)$ among its laws, where lowercase letters denote the possible values of the corresponding random variables:

$$p(a|b,c,d) \equiv p(A=a|B=b,C=c,D=d). \tag{47}$$

This conditional probability is the probability that the random variable A has the value a, given that the random variables B, C, and D have the respective values b, c, and d. In other words, the values of B, C, and D determine the conditional probability distribution for the values of A.

It follows that if the random variables B, C, and D were to develop *contingent* joint probabilities $p(b,c,d)$ in some concrete, real-life instantiation of the Bayesian network, then the random variable A would automatically inherit a contingent standalone probability distribution $p(a)$ of its own

according to the standard multilinear rule

$$p(a) = \sum_{b,c,d} p(a|b,c,d)p(b,c,d). \tag{48}$$

Said in another way, the basic conditional probabilities $p(a|b,c,d)$, together with the contingent joint probabilities $p(b,c,d)$ for B, C, and D, dictate the contingent standalone probabilities $p(a)$ for A, and they do so in a multilinear way.

Importantly, the basic conditional probability distribution $p(a|b,c,d)$ supplied by the Bayesian network in the present example is *directed*, in the sense that the value a of the random variable A appears to the *left* of the 'given' symbol |, whereas the respective values b, c, and d of the random variables B, C, and D appear to the *right*. To understand the significance of this directedness, it will be worthwhile to construct a different conditional probability for comparison.

To that end, notice that if one were to combine the Bayesian network's *basic* conditional probability distribution $p(a|b,c,d)$ with the *contingent* joint probability distribution $p(b,c,d)$, then one could formally define a joint probability distribution $p(a,b,c,d)$ for all four of the random variables A, B, C, and D by invoking the standard rule for conditional probabilities:

$$p(a,b,c,d) \equiv p(a|b,c,d)p(b,c,d). \tag{49}$$

Defining a joint probability distribution $p(a,c,d)$ for A, C, and D alone by marginalizing the joint probability distribution $p(a,b,c,d)$ over B,

$$p(a,c,d) \equiv \sum_{b} p(a,b,c,d), \tag{50}$$

and assuming that this joint probability $p(a,c,d) \neq 0$ were nonzero, one could then formally condition on A, C, and D to obtain the conditional probability

$$p(b|a,c,d) \equiv \frac{p(a,b,c,d)}{p(a,c,d)} \quad [\text{if } p(a,c,d) \neq 0]. \tag{51}$$

Writing out this conditional probability in more detail, one would obtain the formula

$$p(b|a,c,d) = \frac{p(a|b,c,d)p(b,c,d)}{\sum_{b'} p(a|b',c,d)p(b',c,d)}, \tag{52}$$

which makes clear that $p(b|a,c,d)$ would depend on the contingent joint probability distribution $p(b,c,d)$—and in a nonlinear manner. Hence, although $p(b|a,c,d)$ might exist, it would be a *derived* conditional probability distribution that depended on the contingencies of the given concrete instantiation of the Bayesian network, and would therefore have a different physical status from the *basic, nomological* conditional probability distribution $p(a|b,c,d)$ supplied by the Bayesian network's laws.

There exists a reading of a Bayesian network as a model of causal relationships, with causal influences manifesting as the Bayesian network's directed conditional probabilities. That is, if the

Bayesian network supplies a directed conditional probability distribution $p(a|b,c,d)$ in its basic laws, then one should read the Bayesian network as implying that the random variables B, C, and D causally influence the random variable A.

Although the causal influences encoded in Bayesian networks can be given an interventionist cast, a non-interventionist interpretation is available as well, with stochastic fluctuations in B, C, and D dictating stochastic fluctuations in A through the directed conditional probability distribution $p(a|b,c,d)$.[3]

Notice how the directedness of the conditional probability distributions supplied by a Bayesian network captures the inherently asymmetric nature of cause-and-effect relationships.

Interestingly, this connection between the directedness of a Bayesian network's basic conditional probabilities and the asymmetry of cause-and-effect also sheds light on why causal language is so fraught in the context of theories that are based on microphysical laws that are deterministic and reversible. In a deterministically reversible theory, if a value a of a variable A implies a corresponding value b of another variable B, then $p(b|a) = 1$, and, in addition, any contingent standalone probability $p(a)$ assigned to a will necessarily equal the contingent standalone probability $p(b)$ assigned to b. It follows immediately from Bayes' theorem that $p(a|b) = p(b|a) = 1$, so these conditional probabilities are not directed, and the asymmetry of cause-and-effect relationships is lost.

VI. A MICROPHYSICAL ACCOUNT OF CAUSATION

As reviewed in this paper, one can reformulate a quantum system in terms of an underlying unistochastic system. The microphysical laws of that unistochastic system consist of directed conditional probabilities (18), $\Gamma_{ij}(t) \equiv p(i,t|j,0)$, which are very much like the directed conditional probabilities that define the basic laws of a Bayesian network. Taking this resemblance seriously, one can read the microphysical laws of the unistochastic system as providing a microphysical notion of causal influences.

To make things more concrete, suppose that the unistochastic system consists of two subsystems Q and R, in the sense that $i = (q_t, r_t)$ and $j = (q_0, r_0)$, where lowercase letters denote specific configurations of the corresponding subsystems. One can then write the directed conditional probabilities (18) for the overall system as

$$p((q_t, r_t), t|(q_0, r_0), 0). \tag{53}$$

To say that the subsystem Q is free of causal influences from the subsystem R over the time interval from 0 to t would then be the statement that after marginalizing over the configuration r_t of R, the resulting conditional probability distribution no longer depends on r_0:

$$p(q_t, t|(q_0, r_0), 0) = p(q_t, t|q_0, 0). \tag{54}$$

[3] Note that this conception of causation as corresponding to directed conditional probability distributions is fundamentally distinct from *probability-raising theories of causation*. In particular, no assumption is made here that the directed conditional probabilities specifically raise any standalone probabilities.

VII. AN IMPROVED PRINCIPLE OF CAUSAL LOCALITY

One can now formulate an improved *principle of causal locality*:

> A theory with microphysical directed conditional probabilities is causally local if any pair of localized systems Q and R that remain at spacelike separation for the duration of a given physical process do not exert causal influences on each other during that process, in the sense that the directed conditional probabilities for Q are independent of R, and vice versa. (55)

Having stated this new principle of causal locality, one can show that quantum theory, formulated as a theory of unistochastic processes, indeed satisfies it.

For that purpose, consider a unistochastic system consisting of a pair of localized subsystems Q and R that remain at spacelike separation during a given physical process. The overall system's unistochastic transition matrix $\Gamma_{QR}(t)$ has a corresponding unitary time-evolution operator $U_{QR}(t)$ in the sense of (28). Invoking the spacelike separation of Q and R together with the usual assumptions employed in textbook quantum theory, the overall time-evolution operator $U_{QR}(t)$ tensor-factorizes into respective unitary time-evolution operators $U_Q(t)$ for Q and $U_R(t)$ for R individually:

$$U_{QR}(t) = U_Q(t) \otimes U_R(t). \tag{56}$$

In contrast with matrix multiplication, tensor products *do* commute with modulus-squaring the entries of a matrix, so the overall system's unistochastic transition matrix $\Gamma_{QR}(t)$ likewise tensor-factorizes:

$$\Gamma_{QR}(t) = \Gamma_Q(t) \otimes \Gamma_R(t). \tag{57}$$

Here $\Gamma_Q(t)$ is the unistochastic transition matrix for the subsystem Q corresponding to $U_Q(t)$ in the sense of the modulus-squaring relationship (28), and $\Gamma_R(t)$ is similarly the unistochastic transition matrix for the subsystem R corresponding to $U_R(t)$.

It follows immediately from the tensor-factorization (57), together with the definition (18) of the entries of a transition matrix as conditional probabilities, that the overall system's directed conditional probabilities factorize as

$$p((q_t, r_t), t|(q_0, r_0), 0) = p(q_t, t|q_0, 0)p(r_t, t|r_0, 0). \tag{58}$$

Hence, marginalizing over r_t leaves a conditional probability for Q that does not depend on r_0, precisely as in (54), and a similar statement holds with Q and R switched. One can therefore conclude that the principle of causal locality stated above in (55) is satisfied within this unistochastic formulation of quantum theory.

By contrast, suppose that the two subsystems Q and R are *not* kept at spacelike separation during the physical process in question, but locally interact at some intermediate time t' between

0 and t. Then, again following standard textbook arguments, the overall system's unitary time-evolution operator $U_{QR}(t)$ will fail to tensor-factorize at t':

$$U_{QR}(t') \neq U_Q(t') \otimes U_R(t'). \tag{59}$$

Because the corresponding transition matrix $\Gamma_{QR}(t)$ encodes *cumulative* statistical effects starting at the initial time 0, the transition matrix will *continue* to fail to tensor-factorize for all times $t \geq t'$ (at least until the next division event):

$$\Gamma_{QR}(t) \neq \Gamma_Q(t) \otimes \Gamma_R(t) \quad \big[\text{for } t \geq t'\big]. \tag{60}$$

The breakdown (60) in tensor-factorization for $t \geq t'$ is precisely *entanglement*, as manifested at the level of the underlying indivisible stochastic process. The factorization (58) therefore also breaks down, and so one can conclude that the two subsystems Q and R exert causal influences on each other, stemming from their local interaction at the time t'.

Notice that this local interaction, despite being the 'common cause' of the correlations between Q and R, is not the sort of 'variable' that can be plugged into the unistochastic theory's microphysical conditional probabilities. Reichenbach's principle of common causes (16) therefore does not hold.

VIII. REVISITING THE EINSTEIN-PODOLSKY-ROSEN ARGUMENT

The stage is now set for revisiting the EPR argument. Referring to Figure 2, suppose that an observer A ('Alice') has local access to the first subsystem Q, and that an observer B ('Bob') has local access to the second subsystem R, with no assumption that A and B are in local contact with each other. Treating A and B as ordinary (if complicated) subsystems of the overall stochastic process, one now has a transition matrix of the form

$$\Gamma_{QRAB}(t), \tag{61}$$

with individual entries consisting of directed conditional probabilities of the form

$$p((q_t, r_t, a_t, b_t), t | (q_0, r_0, a_0, b_0), 0). \tag{62}$$

(Note that A and B here do not denote random variables or observables, but refer to subsystems.)

The calculations ahead, which will be closely related to the no-communication theorem [3, 4], will show that the observer-subsystem B does not exert a causal influence on the observer-subsystem A. By symmetry, it will also follow that A does not exert a causal influence on B.

One begins by expressing the directed conditional probabilities (62) in the usual Hilbert-space formalism as the following trace:

$$\mathrm{tr}(P_{q_t, r_t, a_t, b_t}\, \rho_{QRAB}(t)). \tag{63}$$

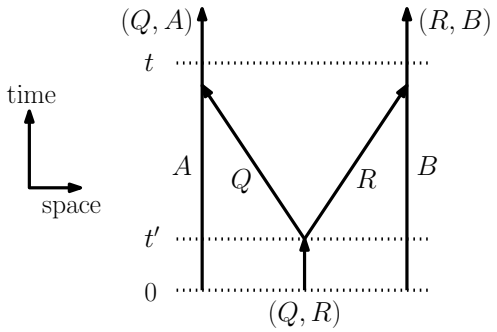

Figure 2. A spacetime diagram depicting an idealized version of the EPR thought experiment, with the two subsystems Q and R separating in space after they interact at the time t', and then respectively joining up with the two observer-subsystems A ('Alice') and B ('Bob'). The two observer-subsystems A and B are assumed to remain spacelike separated throughout the experiment.

Here P_{q_t,r_t,a_t,b_t} is a rank-one projector onto the state vector $|q_t, r_t, a_t, b_t\rangle$,

$$P_{q_t,r_t,a_t,b_t} \equiv |q_t, r_t, a_t, b_t\rangle\langle q_t, r_t, a_t, b_t|, \tag{64}$$

and $\rho_{QRAB}(t)$ is the overall system's density matrix at the time t,

$$\rho_{QRAB}(t) \equiv U_{QRAB}(t)\rho_{QRAB}(0)U^{\dagger}_{QRAB}(t), \tag{65}$$

with $\rho_{QRAB}(0)$ the initial density matrix at the time 0,

$$\rho_{QRAB}(0) \equiv |q_0, r_0, a_0, b_0\rangle\langle q_0, r_0, a_0, b_0|, \tag{66}$$

and with $U_{QRAB}(t)$ the unitary time-evolution operator for the overall system.

Suppose that the two subsystems Q and R locally interact only at a time $t' > 0$. Then one can rewrite the formula (65) for the overall system's density matrix at the later time $t \geq t'$ as

$$\rho_{QRAB}(t) \equiv U_{QRAB}\big(t \leftarrow t'\big)\rho_{QRAB}\big(t'\big)U^{\dagger}_{QRAB}\big(t \leftarrow t'\big). \tag{67}$$

Here $U_{QRAB}(t \leftarrow t')$ is the relative time-evolution operator for the time interval from t' to t, defined as in (30), and $\rho_{QRAB}(t')$ is the overall system's density matrix at the interaction time t',

$$\begin{aligned} \rho_{QRAB}\big(t'\big) &\equiv U_{QRAB}\big(t'\big)\rho_{QRAB}(0)U_{QRAB}\big(t'\big). \\ &= |\Psi_{QR}, a_0, b_0\rangle\langle\Psi_{QR}, a_0, b_0|, \end{aligned} \tag{68}$$

with Ψ_{QR} denoting the (now-entangled) wave function of the subsystem pair (Q, R).

By assumption, the relative time-evolution operator $U_{QRAB}(t \leftarrow t')$ from t' to t encodes local interactions between the two subsystems Q and A, as well as local interactions between the two subsystems R and B, but no local interactions between the subsystem pair (Q, A) and the

subsystem pair (R, B). Hence, the relative time-evolution operator tensor-factorizes as

$$U_{QRAB}\big(t \leftarrow t'\big) = U_{QA}\big(t \leftarrow t'\big) \otimes U_{RB}\big(t \leftarrow t'\big). \tag{69}$$

It follows from a straightforward calculation that the reduced density matrix for the subsystem pair (Q, A) at the later time $t \geq t'$ is given by

$$\begin{aligned}
&\rho_{QA}(t) \equiv \mathrm{tr}_{RB}(\rho_{QRAB}(t)) \\
&= \mathrm{tr}_{RB}\Big(\Big(U_{QA}\big(t \leftarrow t'\big) \otimes U_{RB}\big(t \leftarrow t'\big)\Big) \\
&\qquad \rho_{QRAB}\big(t'\big)\Big(U^{\dagger}_{QA}\big(t \leftarrow t'\big) \otimes U^{\dagger}_{RB}\big(t \leftarrow t'\big)\Big)\Big) \\
&= \mathrm{tr}_{R}\Big(\Big(U_{QA}\big(t \leftarrow t'\big) \otimes \mathbb{1}_R\Big) \\
&\qquad \Big(|\Psi_{QR}, a_0\rangle\langle\Psi_{QR}, a_0|\Big)\Big(U^{\dagger}_{QA}\big(t \leftarrow t'\big) \otimes \mathbb{1}_R\Big)\Big),
\end{aligned} \tag{70}$$

where $\mathbb{1}_R$ is the identity operator on the Hilbert space of the subsystem R. Notice that all the dependence on b_0 has disappeared. Thus, upon marginalizing over q_t, r_t, and b_t, one finds

$$\begin{aligned}
&p(a_t, t|(q_0, r_0, a_0, b_0), 0) \\
&= \sum_{q_t, r_t, b_t} p((q_t, r_t, a_t, b_t), t|(q_0, r_0, a_0, b_0), 0) \\
&= p(a_t, t|(q_0, r_0, a_0), 0),
\end{aligned} \tag{71}$$

where

$$\begin{aligned}
&p(a_t, t|(q_0, r_0, a_0), 0) \\
&\equiv \langle a_t|\mathrm{tr}_{QR}\Big(\Big(U_{QA}\big(t \leftarrow t'\big) \otimes \mathbb{1}_R\Big) \\
&\qquad \Big(|\Psi_{QR}, a_0\rangle\langle\Psi_{QR}, a_0|\Big)\Big(U^{\dagger}_{QA}\big(t \leftarrow t'\big) \otimes \mathbb{1}_R\Big)\Big)|a_t\rangle.
\end{aligned} \tag{72}$$

One sees explicitly that there is no causal influence on the observer-subsystem A from the observer-subsystem B, in the sense of causal influences used in this paper. The only causal influences on the observer-subsystem A are from the two subsystems Q and R, which both intersect the past light cone of A.

IX. CONCLUSION

The past century has seen the appearance of many interpretations of quantum theory, nearly all of which treat the wave function and the Schrödinger equation as the central entities of the theory, and differ on whether to regard the wave function as a physical object. As a purportedly

physical object, the wave function would presumably be understood to be some sort of field on a configuration space of very high dimension, as Schrödinger originally imagined in his early work on what he called 'undulatory mechanics' [62], or as existing in an abstract Hilbert space of some very high dimension, as might be more in keeping with Everett's 'many worlds' interpretation [27–29]. Other approaches either augment the wave function with additional ('hidden') variables, like the pilot-wave approach of de Broglie and Bohm [22–24], or insist that the wave function is merely an instrumentalist tool for encoding epistemic information about measurement settings and results, as in some versions of the Copenhagen interpretation [63].

None of these approaches provide a particularly hospitable domain for talking about causation. They either rely inextricably on an interventionist conception of causation, or they simply lack the kinds of microphysical ingredients that merit being given causal meanings.

As explained above, the *unistochastic formulation* of quantum theory reviewed in this paper lies outside the wave-function paradigm, and is based on treating every quantum system as a unistochastic process in disguise [17, 18], an approach that deflates a lot of the exotic talk about quantum phenomena. The laws of this unistochastic process take the form not of differential equations, but of directed conditional probabilities, which have a long history of admitting an interpretation as encoding causal relationships. From this perspective, quantum theory could be understood as a theory of microphysical causation *par excellence*.

By invoking this microphysical notion of causation, one can formulate a more straightforward criterion (55) for causal locality than Bell's principle of local causality—in either of its equivalent forms (10) or (11). As this paper has shown, quantum theory, regarded as a theory of unistochastic processes, satisfies this improved criterion, and is therefore arguably a causally local theory. Remarkably, one therefore arrives at what appears to be a causally local hidden-variables formulation of quantum theory, despite many decades of skepticism that such a theory could exist.

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