Synthese (2019) 196:247–263
https://doi.org/10.1007/s11229-017-1470-4

Optimal representations and the Enhanced Indispensability Argument

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Received: 3 January 2017 / Accepted: 15 June 2017 / Published online: 3 July 2017

Abstract The Enhanced Indispensability Argument (EIA) appeals to the existence of Mathematical Explanations of Physical Phenomena (MEPPs) to justify mathematical Platonism, following the principle of Inference to the Best Explanation. In this paper, I examine one example of a MEPP—the explanation of the 13-year and 17-year life cycle of magicicadas—and argue that this case cannot be used defend the EIA. I then generalize my analysis of the cicada case to other MEPPs, and show that these explanations rely on what I will call 'optimal representations', which are representations that capture all that is relevant to explain a physical phenomenon at a specified level of description. In the end, because the role of mathematics in MEPPs is ultimately representational, they cannot be used to support mathematical Platonism. I finish the paper by addressing the claim, advanced by many EIA defendants, that quantification over mathematical objects results in explanations that have more theoretical virtues, especially that they are more general and modally stronger than alternative explanations. I will show that the EIA cannot be successfully defended by appealing to these notions.

Keywords Enhanced Indispensability Argument · Mathematical explanations · Optimal representations · Magicicadas · Mathematical Platonism · Theoretical virtues

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1 Introduction

The explanatory indispensability argument (IA) holds that the usefulness of mathematics in scientific explanations justifies mathematical Platonism. The main idea of the explanatory IA is to link scientific realism with mathematical Platonism, via the principle of inference to the best explanation. According to this version of the IA, just as the scientific realist rationally believes in the existence of unobservable entities and processes that feature in our best scientific explanations, she should also believe in the existence of mathematical entities, because they also feature in our best scientific explanations.

This argument has received many criticisms, one of which is that theories are ontologically committed to a posit if the posit plays an explanatory role, but this is not the case of mathematics. As Joseph Melia has pointed out, when mathematics is used in scientific explanations its role is to *represent* the relevant explanatory features, but it does not play an explanatory role in itself. For example, if we say that 'F occurs because P is $\sqrt{2}$ meters long', even though we are mentioning the number $\sqrt{2}$ in the explanation, it is the measure of the physical object P, not the real number $\sqrt{2}$ by which we represent it, that does the real explanatory work (cf. Melia (2002), p. 76). Melia's challenge for defendants of the indispensability argument is to show situations where mathematics is explanatory in itself, beyond its mere representational role.

> Melia's challenge has given rise to the following modified version of the explanatory IA:
>
> P1: We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in science.
> P2: Mathematical objects play an indispensable explanatory role in science.
> C: Hence, we ought rationally to believe in the existence of mathematical objects (Baker 2009, p. 613).[1]

This has been called the Enhanced Indispensability Argument (EIA). The success of the EIA depends on whether there are scientific explanations where mathematics can be indeed indispensably explanatory in the sense required by premise P1, and on whether these are the *best* explanations of a given phenomenon. These have been called Mathematical Explanations of Physical Phenomena (MEPPs). If it is found that the role of mathematics in MEPPs is genuinely explanatory, and that these explanations are better than their alternatives, then mathematical Platonism would be justified because, as Alan Baker puts it, "the mathematical postulates would have virtues that the nominalist has already conceded carry ontological weight" (Baker 2005, p. 225).

Many alleged cases of MEPPs have been advanced in the literature. One of the most discussed is that of the north American cicadas, which life cycle is explained by a number theoretic theorem. In what follows, I will discuss the cicada case in detail and argue that this case cannot be used to justify mathematical Platonism. I then generalize my analysis to other MEPPs and show that these explanations do not support the conclusion of the EIA.

[1] A similar reconstruction of the original IA can be found in Colyvan (2001), p. 11.

2 The cicada case[2]

Periodical cicadas of the genus *magicicada* remain underground, in nymphal state, for either 13 or 17 years, and then emerge simultaneously for two weeks. The only place in the world where these insects can be found is the North American eastern side, where fifteen different broods have been identified. Broods with 13-year life cycles are located in the South, and broods with 17-year life cycles in the North.[3] There are many features of these insects' behavior that are puzzling and require explanation, like the long length of the cycles and the simultaneous emergence. With respect to the long length of the cycles, two factors may be involved. First, Yoshimura (1997) suggests that the colder conditions during the glacial period in the Pleistocene (roughly until 11,700 years ago) slowed the growth and development of the cicadas. In addition, Cox and Carlton (2003) suggest that cicadas may have evolved long life cycles in order to minimize the times they emerge, thus minimizing the risk of emerging during a particularly cold year. On the other hand, the synchronized emergence may be explained by two factors. First, synchronized emergence increases mating opportunities, which constitutes an evolutionary advantage. In addition, simultaneous emergence increases the chances of survival from predators. Predators have a limited eating capacity. Even at their fullest, they will not be capable of eating the whole population of prey if the number of prey is too large. By emerging all of them at the same time, the cicadas guarantee that part of the population will survive.

Now, the feature of these insects' behavior that has generated most discussion among philosophers is that the numbers that represent both southerners and northerners' life cycles are prime. Many subspecies of cicadas may have emerged from the Pleistocene, with a spectrum of life cycles ranging from 12 to 20 years—Yoshimura (1997) even suggests that the possible life cycles may have been in the [14–18] range in the North, and [12–15] in the South. Eventually, those with 13 and 17 years survived. Why have precisely those with prime numbered life cycles survived?

Hypothetically, once more information is available, scientists may eventually be able to narrow down these ranges until a full explanation of each cycle is provided. However, that explanation would make it look as if the fact that both cycles are represented by prime numbers is just a coincidence. On the contrary, the two explanations available in the relevant scientific literature assume that the prime numbered cycles are not a coincidence. Rather, these explanations postulate that the cycles have been selected because they are evolutionarily advantageous, in virtue of having a property

[2] My presentation of this case relies heavily on information from the website: magicidada.org and on John Matson's article on *Scientific American* (Matson 2013).

[3] Although the number of species of cicadas is a matter of dispute, most biologists agree that there are seven species (cf. Cooley, magicidada.org 2016). Three of them (septendecim, cassini, and septendecula) have 17-year life cycles and each of them has at least one 13-year life cycles counterpart (thus: tredecim, neotredecim, tredecassini, and tredecula). Differences in morphology, behavior and calling signals are clear between species of the same life cycle. However—and here is where the dispute begins – these differences are not so evident between a species and its counterpart with the alternative life cycle. It is for this reason that some biologists claim that these counterparts may both belong to the same species (the only difference between subspecies would be the life cycle length). This is still an open question in the relevant scientific literature.

in common, and that this property is somehow related to the fact that the numbers representing the cycles time-lengths are prime.

The first explanation, due to Goles and Markus (2001), is that preys with prime numbered life cycles will avoid encounters with predators more than those with non-prime numbered cycles.

> [A] prey with a 12-year cycle will meet–every time it appears–properly synchronized predators appearing every 1, 2, 3, 4, 6 or 12 years, whereas a mutant with a 13-year period has the advantage of being subject to fewer predators (2001, p. 33).

This would explain why, amongst the species with different life cycles that may have emerged from the Pleistocene, only those with 13 or 17 years passed on.

The second explanation, due to Cox and Carlton (2003), emphasizes the evolutionary benefit of not overlapping with subspecies with different life cycles. When two broods overlap in some regions there may be interbreeding between some species and their counterparts with alternative life cycle. For example, *septendecula* (*decula* with 17-year cycles), and *tredecula* (*decula* with 13-year cycles) belong to different broods. But if their broods coincide, these two subspecies will interbreed, giving rise to descendants with life cycles between 13 and 17. These descendants will not overlap with other nymphs belonging to their progenitors' species. This will make them lose the advantage of synchronized emergence. Having prime life cycles ensures that this unfortunate event happens only every 221 years.

Both explanations rely on the fact that the chosen life cycles minimize the possibilities of intersection (in one case intersection with predators, in the other case intersection with sub species of different life cycles). Specifically, as Alan Baker puts it, "[t]he mathematical link between primeness and minimizing the intersection of periods involves the notion of *lowest common multiple* (LCM)" (2005, p. 231).

> **Lemma 1** the lowest common multiple of m and n is maximal iff m and n are coprime.
> **Lemma 2** a number m is coprime with each number n $< 2m; n \neq m$ iff m is prime.

It seems that without mentioning these facts about prime numbers neither of those explanations of the length of the cycles would work. And although a complete explanation of the cycles length must include empirical information about ecological and biological facts, that cycles of 13 and 17 years minimize the possibilities of intersection requires a purely mathematical explanation, which conclusion are the two lemmas mentioned by Baker. Indeed, biologist Robert MacArthur have noticed that this "may be the only application of number theory to biology" (cited in May 1979, p. 347).

One important feature highlighted by this explanation is that it shows that, given the already mentioned biological and ecological constrains, the life cycles were likely to be those numbers, independently of more details of their actual evolutionary history. The number theoretical explanation highlights the modal strength of the outcome in this case. And in fact, the explanation shows that any periodical species is likely to evolve cycles that are described by prime numbers (more on this below).

3 The concept of primeness in the cicada case

One important thing to note in these explanations is the way they use the concept of primeness. Sometimes primeness refers to a property of time lengths, and sometimes it refers to a property of numbers. The relevant scientific literature does not make this philosophical distinction, and so one important question here is whether we should take scientists at face value and ignore the distinction, or whether we should differentiate between two uses of the word ‘prime’: as referring to a geometrical property of empirical time lengths, or to a mathematical property of natural numbers.[4] According to Juha Saatsi, for example, we should not take scientists at face value in this case (2011, p. 153). Whenever the word ‘prime’ is used, it is done with the purpose of picking out the relevant property of time-lengths. For Saatsi, the starting point is that “the life-cycle period of North-American cicada [is] exactly 13 or 17 years” (2011, p. 149). Numbers 13 and 17 are used to represent the fact that “both cicada life-cycles are intersection-minimizing periods” (2011, p. 153). In the same line, Davide Rizza has pointed out that in these explanations we are dealing with “properties and relations of time intervals corresponding to life-cycles… [which] can be studied non-numerically” (2011, p. 106). In that sense, the property ‘being prime’ is used “to describe empirical relations between life-cycles measured in years” (2011, p. 106). The idea, then, is that the concept of primeness is responsible for picking out the empirical property of being ‘intersection-minimizing periods’. On this view, once we assume that the cycles are evolutionarily advantageous, the explanandum is that the ‘life-cycles represented by prime numbers are evolutionarily advantageous’, and the explanation must show how this is so.

But on the other hand, Alan Baker argues that we should not contradict scientists on their use of the term ‘prime’. On Baker’s view, as we will see shortly, the life cycles are themselves (mathematically) prime:

> Even once biologists had good explanations for the long duration and periodicity of cicada life cycles, they remained puzzled about why these periods have the particular lengths they do. And there is good evidence, based on what they write and say, that this puzzlement only arose because of the fact that both of the known period lengths are prime (2009, p. 617).

According to Baker, then, the explanandum in the cicada case is ‘prime life cycles are evolutionarily advantageous’. The explanation consists in showing how the property of primeness provides the desired evolutionary advantage.

This distinction between different descriptions of the explanandum in the cicada case is particularly important for Baker. In the first case, the quantification over mathematical objects can be avoided; in the second case, however, the explanandum-claim ineliminable quantifies over mathematical objects. As Baker explains, when one describes the cycles as being 13 and 17 years respectively, one can express the same idea without referring to numbers by using first order logic with identity. For example, a claim such as ‘the number of F’s is 2’ can be paraphrased like this:

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4 The distinction between geometrical and mathematical properties can be found, among other places, in Melia (2002), p. 76.

$$\exists x \exists y\, (Fx \wedge Fy \wedge x \neq y \wedge \forall z\, (Fz \supset (z = x \vee z = y)))$$
(cf. Baker 2009, p. 619)

Evidently, an analogous paraphrase can be done for ‘the length (in years) of the life cycle of one cicada subspecies is 13’ and ‘the length (in years) of the life cycle of the other cicada subspecies is 17.’ However, Baker points out, ‘the number of F’s is prime’ cannot be paraphrased away like this. Since there are infinite ways for a number to be prime, the paraphrase would involve an infinite disjunction (‘X has life cycle length 2 or length 3 or length 5 or …’) (Baker 2009, p. 619). Given the fact that scientists do describe the explanandum in terms of primeness, and that there is no nominalist paraphrase of this notion, the particular parts of number theory that have been used in the cicada example are ineliminable, and for this reason, Baker argues, “the mathematics in the [explanation of the] cicada case is indispensable” (2009, p. 620).

Now, given that Baker’s goal is to support mathematical Platonism, describing the explanandum in this way is problematic. In an explanation, the explanandum must be true (otherwise, there would be nothing to be explained in the first place). If the explanandum can *only* be expressed mathematically, one would be already committed to the truth of the mathematical part of it. Sorin Bangu has recently stressed this point in criticizing Baker’s cicada case:

> [The explanandum of the cicada case assumes that] there is a mathematical object (specifically: a number) to which the property ‘is prime’ applies. Therefore, by taking the explanandum as being true… Baker assumes realism before he argues for it (2008, p. 18) (see also Bangu 2012, p. 157 and ss).

Bangu’s argument shows that if the explanandum in the cicada case is described as Baker does, then we cannot use this case to support mathematical Platonism.[5] Baker himself has acknowledged the strength of Bangu’s objection, but he argues that it can be avoided if we pay close attention to the way the explanation is actually laid out. According to Baker, the explanandum is indeed that the cycles are, respectively, 13 and 17 (a description that is acceptable to both Platonists and nominalists). But in order to provide a common explanation of the 13 and 17 year cycles, we must *tentatively* describe the cycles as prime. If this explanation turns out to be better than its alternatives, then the conclusion of the EIA would be supported: the cycles are themselves prime and the explanandum is indeed committed to mathematical objects. This justification, however, *would not have* been made in a circular way.

So, Baker compares this explanation with a hypothetical historico-ecological explanation that would track down all the details of the cicadas’ evolutionary history, and concludes that the explanation that appeals to primeness is better. First because it is more general. Specifically, it “predicts that other organisms with periodical cycles are also likely to have prime periods” (2009, p. 621). In addition, from the perspective

[5] However, rather than rejecting mathematical Platonism, Bangu presents the ‘banana game’ as a case that requires a MEPP, and which explanandum is not committed to mathematical objects. I believe the account that I will develop below accommodates his example as well, but since it involves probabilities, addressing this example will require a larger discussion than the one I present here.

of a historico-ecological explanation, the fact that both cycles are prime would be a coincidence. But the mathematical explanation is better precisely because it explains why it was *somehow necessary* for the cycles to end up being prime. Thus, Baker concludes that:

> [B]y inference to the best explanation, we ought to believe in the entities invoked in the number theoretic explanation, which includes abstract mathematical objects such as numbers. But once numbers are included in our ontology, we need no longer be tentative about [describing the cycles as prime] (Baker 2009, p. 621).

4 Overlapping minimization and p-primeness

In Baker's reconstruction of the cicada case, the property that the 13-year cycles and 17-year cycles have in common is the mathematical property of primeness. For Baker, without mentioning this property the explanation would lose explanatory force, because it would be less general and it would fail to provide the modal information about the likelihood of periodical species to develop periods that are (described by a number that is) prime. But is it true that the only way of providing this generality and modal information is by expressing the property the cycles have in common in terms of primeness?

I believe it is not. Let us assume that the life cycles are not prime. Rather, they possess a physical property that is responsible for their evolutionary advantage.[6]

Consider the definitions of the following empirical properties:

1) *Iteration of length L*: the resulting length of combining successive L's.

$$I^n(L) = L \oplus L \ldots \oplus\ L \text{ (n times)}$$

2) *Overlapping*: For any two objects with different lengths A and B respectively, at several points its iterations will have equal lengths:

$$I^n(A) = I^m(B);\ I^p(A) = I^q(B);\ \text{ etc. for some m, n, p, q, etc.}$$

3) *Overlapping minimization*: $I^m(L)$ and $I^n(L)$ will overlap at $I^n(I^m(L))$; if this is the first time they overlap, then they *minimize overlapping* with respect to each other.
4) *p-coprime*: If the only length of which *both* $I^m(L)$ and $I^n(L)$ are iterations is L, then $I^m(L)$ and $I^n(L)$ are *p-coprime* with respect to L.
5) *p-prime*: If $I^n(L) \neq I^p(I^m(L))$ for any m;p < n, then $I^n(L)$ is *p-prime* with respect to L

[6] There are many proposed nominalizations of the cicada case in the literature. What follows does not break new ground on this respect. However, I present my own nominalized version because it emphasizes how these explanatory facts depend on the extremely simple notions of combination and equality. The simplicity of these two notions will be crucial for defending my point below about these nominalizations providing generality and modal strength.

1-5 are geometrical properties of empirical lengths (including time lengths), not mathematical ones.[7] This is true despite the fact that, arguably, it would have been very hard (though not impossible) to discover these properties without mathematics. We can now express lemmas 1 and 2 of the number theoretical explanation in terms of these properties:

p-Lemma 1: p-coprime iterations minimize overlapping
p-Lemma 2: If $I^n(L)$ is p-prime, then it is co-prime with every $I^m(L) < I^2(I^n(L))$

The why-question in the cicada case is therefore 'why the $I^{13}(L)$ and $I^{17}(L)$ (L = one year) cycles are evolutionarily advantageous?'. The explanation is that they are p-prime, and that, by the two p-lemmas above, it follows that the cycles minimize overlapping, which is evolutionarily advantageous.

Mathematical lemmas 1 and 2 in the MEPP were used to pick out the physical relationships expressed by p-Lemmas 1 and 2, which can be cashed out in empirical terms using the basic notions of physical *combination* ($\oplus$) and *equality*, once the time lengths are properly idealized. In the cicada case, time is idealized as linear, and so the time lengths $I^{13}(L)$ and $I^{17}(L)$ (L = one year) are *p-prime* with respect to years, and that is why they have the *overlapping minimization* property, which is the relevant explanatory property in the cicada case.[8] It is the instantiation of this property that is responsible for the modal strength of the outcome (the fact that the cycles were going to be 13 and 17 independently of many aspects of the cicada's evolutionary history). And, just as in Baker's version, without mentioning this property it would appear that the life cycles are 13 and 17 by coincidence.

This is very plausible. We are trying to explain a scientific claim about insect species, and not an abstract mathematical claim. Do we need to appeal to the (mathematical) property of primeness to express the fact that the cicadas have p-primeness? In one way we do (I explore this below), but even in that case we would be able to paraphrase away such mathematical expression. The problem of the infinite series of disjunctions pointed out by Baker would disappear. Once we are in the domain of application of mathematics, there are constraints on the parts of mathematics that we need to use; so, we are not dealing with pure mathematics any longer. The 'p-primeness' property is a property of time, and the number of ways a life cycle may have this property is not infinite. As opposed to the claim about primeness, there is a finite number of cycles a species may adopt. Expressing this does not involve an infinite disjunction. It is difficult to see, for example, in what way it can be true that 'in principle' a living organism can have a life cycle of, say, 947 years.

The Platonist may protest, however, saying that this is exactly what we mean by 'in principle'. But we can set the number of a cicada cycle (in years) to some integer greater than the known lifetime of the universe (known to be around $13,82 \times 10^9$

[7] m, n, p and q represent natural numbers; but, as we saw, using these representations is unproblematic.

[8] I believe that once we express these lemmas in empirical terms it becomes crucial, for the overall explanation to work, to provide an *explanation of why these physical relations hold*. Moreover, I believe that this empirical explanation can be tracked down by an *appropriate* proof of the mathematical lemmas, suitable interpreted. I will not argue for this point here, since it is not relevant for my thesis; I am planning to do this in further work.

years).[9] In that case, we would have only a finite, although very large, disjunctive specification and we can nominalize it. The elements to perform such nominalization already exist.

Now, although it is possible to conceive an empirical version of the cicada case, I believe that this is indeed a MEPP. In the next section I present an account of MEPPs, and explain the sense in which the cicada case falls under this category.

5 The cicada case as a MEPP

I have mentioned that there are other nominalized versions of the cicada case and other MEPPs in the literature. Rizza, for example, has a description of the cicada case in terms of congruence, juxtaposition, and iteration that is similar to mine. According to Rizza, what is distinctive about the cicada case is that the reasoning is 'essentially' mathematical, because mathematics plays a conceptual role in terms of expressing the appropriate physical concepts and drawing the relevant conclusions (2011, pp. 109–110). However, as Otávio Bueno has pointed out, these roles (expressive, inferential, and representational) are common to almost every application of mathematics (2016, p. 2592).[10] For this reason, I believe Rizza's view fails to describe the distinctive feature that makes explanations like the cicada case special. Specifically, Rizza does not explain what is it about a given physical explanandum that makes it suitable to be explained by a MEPP. This is what I aim to do in this section.

In order to explicate the distinctive feature of MEPPs I will introduce the notion of *optimal representation*. An optimal representation is a minimal model representation[11] that captures *all* the features that are relevant to explaining an empirical situation at a specified level of description.[12] What is a representational ideal for most minimal model representations is achieved by optimal representations. It is important to note that mathematical models are not 'optimal' in themselves. Optimality is a relative notion. A mathematical structure M is an optimal representation of a physical situation P only relative to explaining a specific feature p* of P. In that sense, M may be an optimal representation of P with respect to p*, but it may not be optimal with respect to another feature q* of P. It is also important to note that the mathematical model does not by itself tell us what is relevant in these cases; it is with our background knowledge about the overall situation, and its relation to the explanandum, that we choose a mathematical model M. In that sense, it is the physical situation that we want to explain that dictates whether or not we can represent it by an optimal representation, and consequentially, whether or not a MEPP would be appropriate.

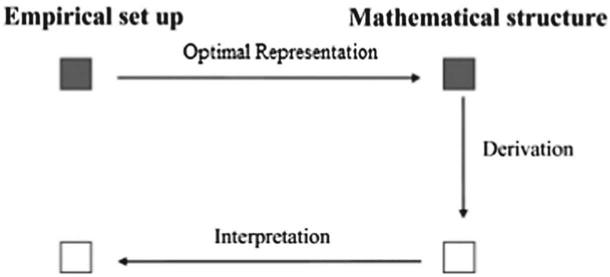
9 Three years ago the European Space Agency's Planck mission found evidence that the age of the universe might be around that number.

10 See also Bueno and Colyvan (2011), and Bueno and French (2012).

11 See Weisberg (2007) and (2013) for a detailed discussion on minimal models and the sense in which they differ from other kinds of models. However, Weisberg does not include the notion of optimal representation in his analysis.

12 Not to be confused with the notion of optimality models, which are models (mostly found in biology) that describe those traits that maximize fitness [see e.g., Orzack and Sober 2001; Rice 2015]. See also Baker (2016) for a description of the cicada case in terms of optimality models.

Now, assume that we know that a physical system P has property p*, and we represent this as a mathematical structure M that has property m*. Once the representation is in place we worry about why it is the case that M has m*; but M having m* can be perfectly understood in terms of P having p*. Schematically, this can be expressed as follows:[13]



My main point is that the distinguishing feature of MEPPs is that they rely on optimal representations; it is for this reason that the mathematics used in them seem to be indispensable: because the explanandum is mathematical. But if it is argued that the MEPP provides the only (or the best) explanation of P having p*, the reply would be, again, that as long as it is possible to establish a distinction between the mathematical explanandum and the physical explanandum, then the role of mathematics in the explanation would be merely representational. The confusion with MEPPs has been that the distinction between the empirical explanandum and its mathematical representation has not been clearly established in most of the cases discussed. In MEPPs, the mathematical explanation applies to a mathematical *representation* of P, and for that reason these explanations cannot be used to support mathematical Platonism, as proponents of the EIA hold.

Let us revisit the cicada case to see how this idea unfolds. Here the question that worries scientists is why life cycles represented by prime numbers have an evolutionarily desirable property. The mathematical representation which assumes that years are equal to each other captures all the relevant factors for the explanation of this feature of time. Representing years with the natural number system gives us an optimal mathematical model. This model captures all and only those features relevant to explaining how the 'overlapping minimization' property is evolutionarily advantageous. What is more, it is our background knowledge about the importance of natural cycles in explaining animal behavior that lead us to take years as the relevant measurement unit, and to idealize years as equal. Once this model is in place, we can 'forget' about the cicadas and focus on the prime number of years. The question 'why the $I^{13}(L)$ and $I^{17}(L)$ (L = one year) cycles are evolutionarily advantageous?' becomes 'why are prime periods evolutionary advantageous?' The (mathematical) explanation of this fact explains the empirical question about cicadas only insofar it shows us how the facts about time explain the life cycles. In other words, we use mathematics to represent time-lengths, and then we use the number-theoretic lemmas to understand why

[13] The scheme below is adapted from Bueno, Colyvan and French's presentation of the Inferential Conception (see Bueno and Colyvan 2011, p. 353; and Bueno and French 2012, p. 92.) However, here I emphasize that in the case of MEPPs the immersion step is done in terms of an optimal representation.

the cycles are evolutionarily advantageous. We see—within the mathematics—that this is because the cycles are prime; but then we must interpret back this result in empirical terms, thus discovering that it is the overlapping minimization property that explains the evolutionary advantage. The notion of mathematical primeness used to represent p-primeness has surplus structure that we get rid of once we interpret back these results in empirical terms.[14]

Crucial for this explanation to work is its appealing to the overlapping minimization property. For that reason, it is better than the hypothetical explanation that would track down the evolutionary history of each magicicada species. And, if we are scientific realists, we may follow the IBE principle and conclude that the overlapping minimization property is a real property of time-lengths. The main point is that we do not need to make the additional metaphysical assumption that the cycles themselves are mathematically prime in order to have an explanation that accounts for what the two cycles have in common. This explanation also shows that the fact that the two cycles are represented by prime numbers is *not* a coincidence.

The strategy of nominalizing alleged examples of MEPPs on a case by case basis has been called ‘piecemeal nominalism’,[15] as opposed to, on the one hand, hard road nominalism (e.g., Field 1980), which requires nominalizing the whole of science; and on the other, easy road nominalism [e.g., Bueno (2012); Azzouni 2004; Melia 2000, 2002], which focuses on the role mathematics plays in science. This is, I think, the correct way of describing my view. Although it does not require a full nominalization of all science, it does require, for every alleged case of MEPP, a clear distinction between the mathematical representation and the empirical set up. In order to do this, we must find a way of nominalizing the explanandum, which is what I did in the cicada case. This is not controversial; in fact, as Bangu has noted, establishing this distinction is also important for the Platonist, otherwise she would be begging the question.

6 Theoretical virtues

It could be argued that current scientific practice uses the notion of primeness in the mathematical sense (that is why they use number-theory in the explanation), and that this would be enough to establish the indispensability of mathematics in this case. This objection, however, has already been addressed in the literature; in fact, the answer to this objection is what motivated ‘Melia’s challenge’, which I mentioned above. There are many ordinary scientific explanations that use mathematics; and without using mathematics, they would not be as successful as they are. The same would happen in this case. It would be very hard to pick out the ‘p-primeness’ property without using mathematics; but from the moment we describe the role of mathematics as merely representational, the alleged indispensability of mathematics would be pragmatic. Mathematics would not be playing an explanatory role – it would be the property represented by the mathematics that would be playing this role. If the presence of

[14] This interpretation step is a crucial element of the Inferential Account (e.g., Bueno and Colyvan 2011, p. 357; Bueno and French 2012, p. 107 and ss).

[15] Cf. Baker (2016), p. 340.

mathematics to simplify calculations or to discover new physical properties was all that is needed to justify mathematical Platonism, the detour by the EIA would have been unnecessary. The original indispensability argument would have been enough. That would be a major drawback for defendants of the EIA.

In this section I will focus on other objections, based on whether MEPPs, in the way I have describe them, are the *best* explanations of the phenomena they are about. Rizza has pointed out that, in the cicada case, "the non-numerical explanation… does not suffer of any particular shortcomings with respect to Baker's numerical alternative" (2011, p. 112). I believe this is correct, and in fact I would go further and argue that this is the case for all MEPPs. Baker disagrees. He argues that that these nominalized resulting explanations are not the best explanations, because they do not share the theoretical virtues of the mathematical versions.[16] In particular, because they would "lack the generality of the original" [Baker (2016), p. 340], and because, by appealing to mathematical objects, Baker's version of this explanation uses facts that are modally stronger than empirical facts (Baker 2016, p. 16 fn. 22).

Below I examine five theoretical virtues (elegance, simplicity, unificatory power, generality, and modal strength) in which, allegedly, the mathematical version performs better than any nominalist version. I will show that this is not the case for any of these five virtues.

6.1 Elegance

Let us start with *elegance*. The idea is that an explanation is better than an alternative if it has greater aesthetical appeal. In the context of this debate, this is perhaps the most problematic notion of the five. First because intuitions are not clear regarding what it actually amounts to, but also because elegance, understood this way, seems to be reducible to simplicity and unificatory power (Colyvan 2001, p. 79 fn. 25)[17]. Below I will show that my version of the cicada case matches the mathematical version both in terms of simplicity and unificatory power. Therefore, it also matches it in terms of elegance.

6.2 Simplicity

One of the most cited theoretical virtues is *simplicity*: given two theories with the same empirical consequences, we must choose the one that is simpler, that is, that has less ontological commitments (2001, p. 78.) For Baker, using the notion of mathematical primeness makes for a simpler explanation. He argues (Baker (2016), p. 349) that, in the present context, what matters is simplicity with respect to our commitments to concrete posits, regardless of whether this entails overpopulating the world with abstract entities. According to him, even if this explanation would commit us to the whole world of mathematical entities, as long as it has the same *empirical* commitments than a given alternative, it will not be considered more complex than the said

[16] Thanks to an anonymous reviewer for raising this point

[17] See also Keas (forthcoming), Section 5.1.

alternative. The question of simplicity, then, comes down to whether the nominalized explanation has the same empirical posits as the mathematical explanation.

On this respect, however, I argue that either postulating mathematical primeness or p-primeness commit us to the same concrete posits. Mathematical primeness must be instantiated as a property of concrete time lengths in order to account for the observed cycles; the same happens with the property of p-primeness. In that respect, both explanations are equally simple. Baker has argued that the mathematical explanation is committed to less concrete posits, because it only requires predators of 2 or 3 year cycles (2016, p. 338). But the property of p-primeness would have exactly the same consequence; the only difference being that 2, 3, 13 and 17 would be p-prime; not mathematically prime.

6.3 Unificatory power

Another often cited theoretical virtue is *unificatory power*. Here the idea is to account for the maximum of *observed* phenomena with the fewer theoretical devices. According to Bangu, for example, quantification over mathematical objects brings the common factor that allows unifying otherwise disparate phenomena (2012, p. 172). In the cicada case, this would mean that by describing the cycles as mathematically prime we would be capable of providing a common explanation of the 13 and 17 cycles. However, this objection does not work against my version of the cicada case. By positing the concrete property of p-primeness we also provide a unified explanation of the two cycles. As we saw, the idea is to unify *observed* phenomena. On this respect both primeness and p-primeness can do the job—both explanations explain the observed 13 cycles and 17 cycles–, so both explanations are equally unifying.

Now, it may be argued that my piecemeal strategy is less unified than the Platonist strategy, in Philip Kitcher's sense (e.g., 1981). Piecemeal strategies deal with one MEPP at the time, whereas Platonism explicates what is common to all MEPPs. In that sense, the Platonist unifies all MEPPs under the same explanatory pattern. However, what my account shows is that all alleged cases of MEPPs can be understood in the same way: first, we must establish a clear distinction between the physical explanandum and its mathematical representation, and next we must show that the mathematical explanation is actually tracking down the relevant physical explanatory features. So, although it is true that I propose that we have to examine MEPPs one by one, the explanatory pattern used to make sense of these explanations is the same. So even at this level my proposal is as unified as the Platonist one.

6.4 Generality

As we have seen, Baker argues that the mathematical version is more general than the nominalist version. The generality objection runs as follows: postulating mathematical primeness accounts for a wider range of possible observations, and because of that, even if we nominalize the explanandum in the way proposed by piecemeal nominalists, the explanation that appeals to mathematical primeness is the best one (e.g., Baker 2009, p. 617; 2016, p. 340). But how much generality do we really need? As we saw, there are empirical constrains that limit the possible length of the cycles. For starters, they cannot be longer than the age of the universe. What is more, the overall explanation of the cicadas life cycle depends on the fact that during the Pleistocene the Earth temperature was extremely cold, and that that is why cicadas developed relatively long cycles. Therefore, it is a condition for the evolutionary explanation to work that the cycles are less than 11,700 years.

It may still be objected that the interesting thing about the explanation that appeals to primeness is that it would work in other planets older than ours, and that it is precisely this generality that would get lost if we do not use the notion of mathematical primeness. But, again, those planets would still be physical, just as ours. And however old these planets may be, their age would still be measurable in terms of finite cycles. Moreover, however long the cycles are, they cannot be infinitely long, because the very idea of an infinite life cycle does not make sense—if a time length does not have an upper bound, it would not make sense to call it 'a cycle'.

Now, the Platonist may still reply that the result is still less general than when we describe the explanandum as prime, even if we do not appeal to infinitely long cycles, because it would apply to more cycles than mere physical or realizable cycles. But I do not think this is going to be enough to justify the existence of those non-realizable cycles. Consider an example due to James Franklin (2014, p. 69). The statement 'all red things are colored' applies to all red things, and we can say that for anything, if it was red, then it would be colored. However, it does not seem right to justify the existence of infinite red things just because if there were infinite red things the statement 'all red things are colored' would be more general. In the same vein, it does not seem right to justify the existence of numbers just because if there were numbers, the statement 'n is prime' would be more general. If this were enough to justify the existence of numbers, the detour around the EIA would have been unnecessary (I take up this point below).

Also, it is important to note that what mine and similar nominalizations of the cicada case show is that the explanation relies on the simple notions of combination and equality, which are general enough properties. They apply to any set of objects once they are described in a certain way. This generality can be captured in modal terms. For example, if there are two objects of congruent lengths then the notion of equality would apply to them. But again, this by itself does not justify the existence of infinite objects that fit this description.

6.5 Modal strength

Finally, it may be argued that the kind of *modal* information provided by mathematical properties is stronger than any provided by empirical properties (Baker 2016, p. 16 fn. 22; Lyon 2011, pp. 9–10).[18] Thus, the fact that primeness minimizes intersection between numbers is mathematically necessary. This necessity would be stronger than the fact that p-primeness, a property of time lengths, minimizes intersection

[18] Lange (2013) also defends that MEPPs point to mathematical necessities, but he thinks this is independent from the IA.

between biological species. Therefore, appealing to these 'stronger necessities' would provide a better explanation than appealing to the weaker physical necessities. This interpretation presupposes the existence of mathematical necessities in the physical world. In that sense, the explanation that appeals to mathematical primeness would highlight the fact that it is mathematically necessary that prime-numbered cycles minimize intersection with other cycles. This would be true in *any* world where the idea of cycles makes sense, so this necessity would be stronger than physical necessity.

The problem with this suggestion is that whether or not this justifies mathematical Platonism is independent of the Indispensability Argument. Let me illustrate this with a non-mathematical example. 'All red things are colored' is necessarily true, it applies to things in the real world, and it would still be true in any world where red is a color. So even though this is a fact about the physical world, it is necessarily true in a way stronger than physical necessity: even if the laws of nature were different, it would be true that 'all red things are colored' (Franklin 2014, p. 69). But again, the fact that 'all red things are colored' is necessarily true does not by itself prove that the property 'red' exists independently of its realizations. And the fact that 'in every world where red is a color, red things would be colored' does not by itself prove that those other worlds actually exist. In the same way, maybe some relations between time lengths are modally stronger than other physical relations;[19] but whether or not the existence of relations of necessity stronger than physical necessity shows that there exist abstract objects is a deep metaphysical question, which answer seems to be independent of considerations about the usefulness of mathematics in science, which is what the EIA is ultimately about.

Let us see how this works in the cicada case. I have shown that the empirical explanation of the life cycles length relies on the extremely simple notions of combination and equality of physical lengths. If we describe physical objects in a way such that combination and equality apply, then every physical object that fits such definition would be subject to these relations of combination and equality, which in turn entails that any physical object represented by 13 would not overlap with iterations of other lengths represented by 26 or less, apart from those represented by 1; similarly, it entails that any physical object represented by 17 would not overlap with iterations of other lengths represented by 34 or less, apart from those represented by 1. This will be *necessarily* true. But the crucial point is this: even if this relationship turns out to be that of mathematical necessity, it would be as surprising as the relationship of necessity between, say, the fact that if there were two objects of the same length, and a third object of the double length of the first object, then a *combination* of the lengths of the first two would be *equal* to the length of the third. If this fact were enough to support mathematical Platonism, then the detour by the EIA would have been unnecessary. Again, I do not want to say that these are not mathematical relationships of necessity

[19] Perhaps geometrical relations are physical and yet stronger than mere nomologically necessary. I cannot go into more details about this at this point. However, I hope my overall strategy is clear: the existence of geometrical relations does not by itself support mathematical Platonism.

in the world;[20] only that whether or not there are is independent of naturalism and the usefulness of mathematics in science, which is what the EIA is about. In other worlds, the usefulness of mathematics in science has nothing to do with this metaphysical claim.[21]

7 Conclusion

The defining feature of MEPPs is that they rely on optimal representations. These are cases where researchers are interested in special features of a physical situation, such that all that is needed to explain these features can be included in the mathematical representation. In these explanations, the role of mathematics, although perhaps pragmatically indispensable—in the sense that it would have been very unlikely that researchers would have discovered the relevant explanatory factors without using mathematics—, is ultimately representational. For that reason, these cases do not overcome Melia's challenge, and so they do not support the conclusion of the EIA.

Acknowledgements I would like to thank Paul Humphreys for his comments on several versions of this paper. The final version also benefited from helpful discussions with Otávio Bueno, James Cargile, and Juan Durán, and the comments of two anonymous reviewers.

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[20] I believe there are not, but I do not think this is relevant for my overall case against the EIA supporting Platonism.

[21] This point is perhaps clearer in the bridges of Königsberg case, which is another example of an alleged MEPP that is widely discussed in the EIA debate. The impossibility of performing a trip that crosses all the seven bridges of 18^{th} Königsberg without retracing one's steps is explained by a graph-theoretical theorem. For me, what is crucial for understanding this case is that Leonard Euler's proof of this theorem relied on the extremely simple fact that every time one crosses a bridge, two pieces of land are involved, the starting point and the ending point. It is often said that it is a matter of *mathematical* necessity that the trip over the bridges is impossible. This necessity would be both stronger than mere physical necessity, and about a physical system (cf. Lange 2013; Lyon 2011). As I said, I prefer not to enter this debate here. My point is that if the impossibility of performing an Eulerian trip over the bridges is described as a matter of mathematical necessity, then the fact that 'every time that I cross a bridge two pieces of land are involved' would also be mathematically necessary. But if *this* justifies mathematical Platonism, such argument does not need the complications of the EIA, or for that matter, of issues pertaining the applicability of mathematics in science. I will address this point in further work.

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