Platonism and Intra-mathematical Explanation

Sam Baron*

Abstract

I introduce an argument for Platonism based on intra-mathematical explanation: the explanation of one mathematical fact by another. The argument is important for two reasons. First, if the argument succeeds then it provides a basis for Platonism that does not proceed via standard indispensability considerations. Second, if the argument fails it can only do so for one of three reasons: either because there are no intra-mathematical explanations, or because not all explanations are backed by dependence relations, or because some form of noneism—the view according to which non-existent entities possess properties and stand in relations—is true. The argument thus forces a choice between nominalism without noneism, intra-mathematical explanation and a backing conception of explanation. You can have any two, but not all three.

Keywords: Explanation; Platonism; Nominalism; Intra-mathematical; Mathematics; Ontic

1. Introduction

Platonists and nominalists disagree over the status of mathematical entities. Platonists contend that abstract, mathematical entities exist, whereas nominalists deny that there are any such entities. A central argument in favour of Platonism is the indispensability argument. The indispensability argument moves from the claim that mathematics plays an indispensable role in our best scientific theories, to the conclusion that mathematical entities exist.

A central aspect of the debate between Platonists and nominalists is the notion of extra-mathematical explanation: the explanation of a physical fact by a mathematical fact. The existence of genuine extra-mathematical explanations within science is widely thought to place serious pressure on nominalism. Much of the discussion has thus come to focus on the enhanced indispensability argument (Baker 2009), which operates on the idea that mathematics is playing an indispensable explanatory role in science.[1]

In addition to the interest in extra-mathematical explanation, there is a growing interest in intra-mathematical explanation: the explanation of one mathematical fact by another (see D'Alessandro 2020a; Inglis and Mejía-Ramos 2021; Lange 2016; Mancosu 2001; Weber and Frans 2017; Zelcer 2013). However, apart from some early attempts by Steiner

*University of Melbourne, Australia.

[1] See Andersen (2016); Baron et al. (2017); Baker (2005); Colyvan (2010); Knowles (2021); Knowles and Saatsi (2021); Leng (2012); Liggins (2012); Melia (2000); Saatsi (2016); Yablo (2012).

(1978) to understand extra-mathematical explanation in terms of intra-mathematical explanation, there has been little discussion of the implications that intra-mathematical explanation might have for the debate over mathematical ontology.

In this paper I introduce an argument for Platonism based on intra-mathematical explanation. If successful, the argument presents a route to Platonism that does not proceed via standard indispensability considerations. The argument is, however, controversial. For there are ways of thinking about explanation—either within mathematics or just in general—that would halt the argument in various places. Such views about explanation have their costs (and I'll explain what these are as we go) but such costs may well be worth paying. In this respect, I suggest that the argument is no worse than the enhanced indispensability argument, which is also hostage to views about the nature of explanation, both within science and in general.

Ultimately, however, the philosophical importance of the argument outruns the argument's potential for success. For if the argument fails then it does so for one of three reasons. Either it fails because there are no intra-mathematical explanations, or because not all explanations are backed by dependence relations, or because some form of noneism—the view according to which non-existent entities possess properties and stand in relations—is true. The argument thus forces a choice between nominalism without noneism, intra-mathematical explanation and a backing conception of explanation. You can have any two, but not all three. I find rejecting nominalism to be the most promising way forward, but I leave it to the reader to decide where they stand.

2. The Argument

Explanations, I will assume, are relations between propositions. Each explanation is a triple $\langle \Gamma, \alpha, \mathrm{r} \rangle$ where Γ is a set of propositions that, together, constitute the explanans, α is a set of propositions constituting the explanandum and r is a relation between the propositions at issue. In an intra-mathematical explanation, Γ and α state only mathematical facts.

According to the backing conception of explanation, *all* genuine explanations correspond to objective relations of dependence. As Taylor (forthcoming) puts the point:

> Backing theorists typically hold that the relata of explanations are sentences or propositions, and that explanations give information about worldly, metaphysical relations of determination or dependence, of which causation and grounding are taken to be central examples.

In this representative statement of the view, 'worldly' just means 'a relation of dependence between parts of the world'. Thus, on the backing picture, for any explanation $\langle \Gamma, \alpha, \mathrm{r} \rangle$, there is some dependence relation $\mathrm{Rxy} = (\mathrm{y}$ depends on $\mathrm{x})$ joining parts of the world that the explanation tracks. These relations are *tracked* in the sense that a triple $\langle \Gamma, \alpha, \mathrm{r} \rangle$ must give information about parts of the world joined by a dependence relation in order to count as an explanation. A 'part' of the world is here understood in a liberal sense. A part of the world might be an entity, or specific object; it might be a complex entity, such as a state of affairs (for instance, a particular entity a's being F); it might be a 'fact' in the ontological sense, namely a truthmaker; or it might be a structure/system of relations. The precise nature of worldly parts is not important. What matters is that they are genuine aspects of reality.

The backing conception is not to be confused with an account of the r-relation in $\langle \Gamma, \alpha, \mathrm{r} \rangle$. The dependence relation R holds between parts of the world that are *not* propositions; whereas r holds only between propositions. Accounts of the r-relation include DN-style views, where the propositions stand in some consequence relation; counterfactual approaches, where the propositions stand in a relationship of counterfactual dependence and unificationist approaches, where the propositions must fit within an explanatory scheme. Triples joined by the r relation represent parts of the world joined by the R relation (the dependency relation), but the r relation and the R relation are distinct.

According to one, particularly stringent version of the backing conception of explanation, the relations that back explanations are all causal relations. This version of the backing conception, however, is not very plausible (Pincock 2015: 878). For there appear to be many explanations that aren't backed by causal relations. Consider, for instance, the explanation for why a diamond is hard. This explanation appeals to the molecular structure of the diamond. However, the diamond's molecular structure doesn't cause the diamond to be hard. Rather, it constitutes (and in this sense explains) the diamond's hardness. Another example, due to Nerlich (1979: 73–74), is the explanation of physical phenomena in terms of the structure of spacetime. Consider, for instance, a group of particles in a dust cloud moving inertially through curved spacetime. These particles will gradually move closer to one another, and so the shape of the cloud will change, despite the fact that there are no forces acting on the particles. As Nerlich argues, this explanation does not seem to be a causal explanation since, in the context of classical mechanics at least, causal action is mediated by forces. Rather, the explanation of the change is the spacetime curvature along the dust cloud's worldline.

I will have more to say about the backing conception later on. For now, let us grant a backing conception according to which explanations can be backed by causal and non-causal relations. Together, the existence of genuine intra-mathematical explanations, and a backing conception of explanation yield the following argument in favour of Platonism.

1. There are intra-mathematical explanations.
2. All explanations are backed by dependence relations between parts of the world.
3. If there are intra-mathematical explanations and all explanations are backed by dependence relations between parts of the world, then mathematical entities exist.

   Therefore,
4. Mathematical entities exist.

The backing conception of explanation requires that all explanations represent dependence relations between parts of the world. However, dependence relations are existence entailing: they require the existence of their relata in order to obtain. Thus, accepting the relevant dependence relations carries with it an ontological commitment to the parts of the world that feature in the stated dependencies. Assuming then that there are genuine explanations in mathematics, those explanations must also be backed by dependence relations between parts of the world. Since *only* dependence relations between mathematical entities (broadly construed, to include objects and structures) appear capable of backing intra-mathematical explanations, it follows that mathematical explanations imply the existence of mathematical entities.

What are the dependence relations that back intra-mathematical explanations? There are at least three other options. First, one might adopt Steiner's (1978) account in terms of characteristic properties. A characteristic property is like an essential property of a mathematical object, though Steiner resists a straightforward reduction to essence. The basic idea, though, is that an intra-mathematical explanation is one in which a mathematical fact is explained by showing how a certain mathematical property involved in a given explanandum depends on a mathematical property in the explanans, one that is core to the nature of some mathematical object.

Second, one can appeal to abstract dependence relations. For Pincock (2015: 878–879), abstract dependence is a *sui generis* variety of non-causal dependence between mathematical objects. Pincock understands abstract dependence in terms of ontological dependence. The idea being that intra-mathematical explanation is a matter of showing how one mathematical object depends for its existence on another. This is analogous to the way in which the existence of a swarm of bees is ontologically dependent on each bee.

Third, one might appeal to grounding relations, which are metaphysical relations of relative fundamentality (Schaffer 2016). Grounding relations are supposed to be primitive dependence relations. A grasp of grounding is typically given by reference to paradigm examples. Examples include the dependence of a boundary on the object it bounds; the dependence of mental states on physical states; and the dependence of social entities on mental entities. One of the core examples of a grounding relation often cited is a mathematical one. Sets, it is claimed, are grounded in their members. This grounding relation connects sets to sets, but also sets to numbers, as when we say that the singleton $\{2\}$ is grounded in the number 2.

In short, then, there are many kinds of dependence that might back intra-mathematical explanations. I won't prejudge the issue of what the 'correct' account might be, and so will leave the exact nature of the dependence open. What matters, for present purposes, is that some dependence relation is implicated in a backing-style account of how intra-mathematical explanation works.

But, one might wonder, why appeal to a backing conception of explanation at all? For there is another argument one might offer for Platonism that avoids the need to invoke any particular conception of intra-mathematical explanation. The argument is just this:

1. There are intra-mathematical explanations.
2. If there are intra-mathematical explanations, then some mathematical claims are true.
3. If some mathematical claims are true, then mathematical entities exist.

   Therefore,

4. Mathematical entities exist.

Call this the simple argument from intra-mathematical explanation. The simple argument relies on the factivity of explanation: a set of claims Γ explains a statement α only if each member of Γ is true and α is true. Of course, many explanations invoke idealizations in the explanans: claims that are, strictly speaking, false. Still, few would deny that explanation is at least *weakly* factive, in this sense: a set of claims Γ explains a statement α only if α is true. Given a commitment to the weak factivity of explanation, accepting that there are any genuine intra-mathematical explanations forces one to accept that there are mathematical truths and, from there, one might argue, it is a very short step to the conclusion that mathematical objects exist.

An argument along these lines has been considered before. Leng (2005: 174, fn. 2) for instance, dismisses such an argument on the grounds that it is question-begging (see also Bangu (2008)):

> ... one might wonder why it is mathematical explanations of physical phenomena that get priority. For if there are [...] some genuine mathematical explanations of mathematical phenomena, then these explanations must also have true explanans. The reason that this argument can't be used is that, in the context of an argument for realism about mathematics, it is question-begging. For we also assume here that genuine explanations must have a true explanandum, and when the explanandum is mathematical, its truth will also be in question.

Leng's worry is that because intra-mathematical explanations presuppose the truth of mathematical claims, we cannot use such explanations to establish the truth of those claims. My argument avoids Leng's worry entirely. That's because my argument does not use the factivity of explanation to establish the existence of mathematical objects. Ontological commitment is forced via the way that explanation works, rather than via any link between truth and ontology. Accordingly, even if one rejects the link between truth and ontology, if one accepts that there are intra-mathematical explanations that are backed by dependence relations and that those relations imply the existence of their relata, then one is forced to accept the existence of mathematical objects.

In short, denying the link between truth and ontology leaves each of the core premises in my argument untouched. This is advantageous because there are plenty of nominalists who reject that link. Many so-called 'easy road' nominalists accept that mathematical statements are true, but nonetheless deny that there are any mathematical objects. This is clear for deflationary nominalists like Azzouni (2004), but other easy-roaders seem to hold a similar view (such as 'weaseling' or 'indexing' nominalists, see Melia (2000)). The simple argument doesn't work against easy road nominalism. My argument, however, does. Thus, not only does my argument avoid Leng's objection, it also applies to a wider range of nominalist positions.

The easy road nominalist might disagree with this assessment. Explanations, they might argue, are *only* ontologically committing insofar as they feature true propositions and true propositions have ontological implications. If that claim is right, then the road to ontology through explanation is no different to the road through truth. Thus, an easy road nominalist can avoid the ontological implications of intra-mathematical explanations after all, by rejecting the ontological implications of mathematical truth in the usual way.

The nominalist who takes this line rejects the picture of ontological commitment that underpins my argument, since that picture routes ontology through dependence not truth.

That being so, the nominalist must explain where I've gone wrong. Assuming that they believe there are genuine intra-mathematical explanations, and assuming they accept the backing conception of explanation (and thus that there are dependence relations between mathematical entities), they seem to have only one option: they must deny that the presence of dependence relations between entities implies the existence of those entities. Defending that claim is an added burden beyond simply rejecting the link between truth and ontology and so the easy road nominalist must work a bit harder to develop her view.

Moreover, it is important to be clear about where such a nominalist ends up. The nominalist seems forced to defend a view according to which mathematical entities stand in genuine dependence relations to one another despite the fact that they don't exist. Such views are available and I briefly consider them in §3 under the name of noneism (see Routley (2018)). However, it is unclear whether such a view will be attractive to a nominalist. A nominalist may well want to deny that non-existent entities stand in relations; particularly the kinds of relations that back explanations (relations like ontological dependence, grounding and causation). Still, even if all I have done is push the easy road nominalist toward noneism, that would be a notable result, given that noneism remains a minority view in metaphysics.

2.1. Intra-mathematical Explanation

I don't expect the argument presented above will be compelling yet. A case must be made in favour of each of the premises. That task will occupy me in the rest of this section. My goal, however, won't be to make anything like a decisive case for the argument. Rather, I will outline what I take to be the central points in favour of each premise and thus make a *prima facie* case for Platonism only. My primary aim in doing so, however, is not to defend Platonism (though I do have sympathies in that direction). The goal, rather, is to use the defense of Platonism to spark discussion around the argument and, in particular, to encourage nominalists to say a bit more about how they think of intra-mathematical explanation.

Let us start, then, with the claim that there are intra-mathematical explanations. The reason to accept this claim is simple: mathematicians take mathematical explanation seriously, and therefore so should we. This reasoning can be sharpened as an appeal to naturalism. According to Maddy (1992: 276) naturalism demands that philosophers take scientific practice seriously, in this sense: we should not reject any element of scientific practice on purely philosophical grounds. Naturalism, according to Maddy, is motivated by a rejection of first philosophy; the idea that we start with philosophy as the basis for our theorising about the world. Maddy takes the rejection of first philosophy to motivate a naturalistic attitude toward mathematics as well:

> Mathematics, after all, is an immensely successful enterprise in its own right, older, in fact, than experimental natural science. As such, it surely deserves a philosophical effort to understand it as practiced as a going concern ... a philosophical account of mathematics must not disregard the evidential relations of practice or recommend reforms on nonmathematical grounds. (Maddy 1992: 276)

When philosophers disregard mathematical practice on philosophical grounds, they are failing to do justice to mathematics as a discipline. Thus, in so far as mathematicians make appeals to explanation within mathematics, we philosophers have good reason to take these appeals, and thus intra-mathematical explanation, seriously.

Bracketing naturalism for a moment, the central question is whether mathematicians do in fact appeal to mathematical explanation in their practice. Support for the affirmative comes from a number of sources. First, Lange (2016: 233–234) and Hafner and Mancosu (2005) all provide textual evidence that mathematicians make explicit appeals to explanation in their work. As Lange puts it:

> Mathematical proofs that explain why some theorem holds were distinguished by ancient Greek mathematicians from proofs that merely show that some theorem holds ... and this distinction has been invoked in various ways throughout the history of mathematics. (Lange 2016: 234)

Another data point comes from a recent corpus-analysis of mathematical papers conducted by Mejía-Ramos *et al.* (2019). Mejía-Ramos *et al.* searched over five-thousand mathematics papers, focusing on words like 'explain', 'explanatory' and 'explainable'. Their data provides substantial evidence that mathematicians use explanation talk in their work quite often. A similar analysis was conducted by Reutlinger *et al.* (forthcoming) on the use of counterfactual language in mathematics. Since counterfactual language is often linked to explanatory relevance, their study provides modest evidence that mathematicians are thinking in explanatory terms. In a further analysis, Mejía-Ramos *et al.* (2021) asked mathematicians to gauge the explanatoriness of mathematical proofs. They found some striking agreement among mathematicians about which cases are explanatory that provides further evidence mathematicians are thinking in explanatory terms.

One might respond to this data with scepticism. It could be that when mathematicians use explanatory talk, they are actually referring to non-explanatory facts. One possibility, recognised by Lange, is that mathematicians are 'merely gesturing toward an aesthetically attractive quality that the proof possesses' (Lange 2016: 231). Another possibility is that mathematicians use explanation talk as a way of referring to cognitive salience. When a mathematician says that a proof is explanatory, all they mean is that it gives one a kind of 'aha!' moment. Another option, offered by Kuorikoski (2021), is that appeals to explanation in mathematics are to be understand as claims about inferential connections between broad patterns of reasoning, rather than explanatory connections between mathematical facts.

Lange dismisses this kind of worry on the grounds that:

> ...no such suspicion is seriously entertained with regard to *scientific* explanation, and we should demand some reason why mathematical explanation deserves to be regarded differently. (Lange 2016: 231)

Now, to my knowledge, no evidence has come to light that mathematicians use explanatory language to mean something like 'aesthetically pleasing' or 'cognitively salient'. So we don't have a reason to treat mathematics differently from science.

But I think we can take Lange's argument a bit further. A principle of charity prevents us from adopting certain interpretations of explanation talk in mathematics without evidence. For consider what one is doing when one claims that mathematicians are using explanation talk to refer to something other than explanation. One is, effectively, charging mathematicians with using explanatory language improperly. To see just how uncharitable the proposed interpretation can be, consider the following passage from fields medallist Timothy Gowers:

> [Some] branches of mathematics derive their appeal from an abundance of mysterious phenomena that demand explanation. These might be striking numerical coincidences suggesting a deep relationship between areas that appear on the surface to have nothing to do with each other, arguments which prove interesting results by brute force and therefore do not satisfactorily explain them. (Gowers manuscript)

Gowers' use of explanatory language here seems completely unambiguous. He is talking about explanation in the usual way. For him, mathematical explanation relieves mysteries by reducing brute facts. It unifies apparently disparate phenomena. It goes beyond merely showing that something is the case to tell us *why* something is the case. To interpret Gowers as saying merely that we get an 'aha!' moment when we look at certain proofs seems to ignore the substance of his claims.

I suppose that being uncharitable to mathematicians is not a sin of the highest order, but it does have the whiff of anti-naturalism about it. Recall Maddy's insistence that we should seek to understand mathematics as a successful enterprise in its own right, as a going concern. Re-interpreting Gowers' talk of explanation as about aesthetics or cognition seems to sit in tension with this naturalistic position.

The question, of course, is whether we should be naturalists about mathematics. The main reason to endorse a naturalistic attitude toward mathematics is its track-record. Mathematics, as Maddy notes, is an incredibly successful enterprise. Mathematics has given rise to countless discoveries, which in turn have had a vast impact beyond mathematics. Success in philosophy, one might argue, is comparatively rare, and much less wide-ranging in its implications. The historical success of mathematical methods, and their comparative dominance over philosophical methods, renders philosophical criticisms of mathematical methods suspect. If one wishes to criticise mathematical methodology, one should do so from within mathematics, not from without.

Another reason to endorse naturalism about mathematics is a commitment to naturalism about science. For it is far from clear that there's a sharp distinction to be drawn between science and mathematics. Applied mathematics is clearly a part of science, and so views about science in general should apply to at least this branch of mathematics. But it is also doubtful that a clear distinction can be drawn between pure and applied mathematics. Mathematics moves seamlessly from the pure domain into the applied domain and back again as science progresses. The substantial traffic between the two domains would seem to scuttle the prospect of drawing any sharp lines.

Of course, there are many different shades of naturalism, some of which may not ratify an appeal to mathematical practice in the context of philosophy. But assuming that naturalism should extend to mathematics, the burden of proof does seem to lie with the sceptic to say why we shouldn't take mathematicians seriously when they talk about explanation.

2.2. Backing

This brings us to the second premise in the argument from intra-mathematical explanation: the claim that all explanations are backed by dependence relations between parts of the world. This strikes me as the most controversial premise. While I do think there is something to be said in favour of the backing conception (and I'll say it below), it's worth flagging that, from a nominalistic perspective, this is likely to be the chink in the argument's armour. Exactly how a nominalist should deal with the premise, however, is unclear, and warrants further discussion.

As noted, the backing conception allows that many different kinds of relations can play the backing role, not just causal relations. The backing conception so construed is advocated by a number of philosophers. For instance, Audi endorses a 'realist' account of explanation that 'requires determination relations to underlie explanations' (Audi 2012: 688, fn. 5). Similarly, Kim writes that:

> A realist about explanation believes that some objective relation between the events underlies, or grounds, the explanatory relation between their descriptions. (Kim 1994: 57)

Schaffer (2016) is another proponent of the backing conception, maintaining that explanations are generally backed by causal or grounding relations. Likewise Ruben maintains that 'the world is structured by various determinative or dependency relations' and 'it is the presence of these 'structural' determinative (and dependency) relations that makes explanation possible' Ruben (1990: 210). Other proponents of the backing conception include Pincock (2015: 865), Taylor (forthcoming), and Wilhelm (forthcoming: 1346).

I will defend the backing conception by outlining one argument in its favour, and by addressing three putative counterexamples to that view. The argument in favour of the backing conception that I consider is the one most likely to convince nominalists, but there are others (see, for instance, the extended defense of a backing conception offered by Strevens (2008)). The arguments in this section are by no means a complete defense of the backing conception. They are meant only to provide a preliminary case for backing and to reveal some of the costs for the nominalist if she gives it up.

The argument in favour of backing exploits a close association between the backing conception of explanation and the *ontic* theory of explanation. According to the ontic theory of explanation, explanation is a matter of showing how the explanandum depends on an ontic structure of mind-independent entities. The strictest version of the ontic theory is Salmon's (1984) causal account, according to which explanation involves showing how a fact was caused. There are, however, more liberal versions of the ontic theory, such as the one proposed by Saatsi, according to which 'not all explanatory dependence is causal' and where:

> Explanatory power derives from stating some relevant worldly facts: objective causal or mechanistic facts, or nomological facts, or statistical relevance relations, or symmetries, or whatever ontic structures can bear an objective relationship of explanatory relevance to the explanandum... (Saatsi 2016: 1052)

The liberalised ontic theory of explanation and the backing conception described above appear to be the same view. At the very least, the ontic theory described by Saatsi seems to imply the backing conception. For it is unclear what else 'an objective relationship of explanatory relevance' might be other than a dependence relation between parts of the world.

Whether the ontic theory just is the backing conception, or whether it merely implies it, the motivations behind the ontic theory of explanation carry over to the backing conception as well. One important reason for endorsing the ontic theory—also noted by Saatsi (2016: 1054)—is that it forges a powerful connection between explanation and ontology. If the ontic theory of explanation is true, then confirming an explanation should generally enforce some ontological commitment to whatever ontic structure underwrites the explanation at issue. The connection between explanation and ontology is particularly important for scientific realists, who often take explanation to be the royal road to ontology. Since scientific realism is common ground in the debate between mathematical Platonists and nominalists, this reason for endorsing the ontic theory of explanation and, with it, the backing conception, should move both parties.

In addition to this general reason to endorse the ontic theory of explanation, nominalists have a special reason to endorse that theory. Recall that the recent debate between Platonism and nominalism turns on extra-mathematical explanation. Platonists maintain that mathematical facts explain physical ones, and nominalists deny this. An important line of argument against Platonism in this context relies on the ontic theory of explanation. Saatsi (2016), for instance, argues against the existence of genuine extra-mathematical explanations on the grounds that mathematical entities do not generally enter into ontic relationships with physical objects (see also Knowles (2021) and Kuorikoski (2021) for similar arguments). If nominalists wish to uphold this kind of argument against Platonism, then they must uphold the ontic theory of explanation as well, and the backing conception along with it.

Now, one might object that the reasons to accept the ontic theory of explanation only motivate it as a theory of *scientific* explanation. It is, after all, scientific realists who are under pressure to preserve the connection between explanation and ontology; and it is explanation in the context of science that is at stake in the debate over extra-mathematical explanation. At best, then, the ontic theory of explanation can only motivate a backing conception of explanation in science. One might then argue that since intra-mathematical explanations are not scientific explanations, the ontic theory of explanation does not motivate a backing conception of these explanations.

However, as previously discussed, it is not clear that there is a sharp distinction between science and mathematics, particularly once we take applied mathematics into account and its permeable boundary with the pure domain. In order to prevent the ontic theory of explanation from generalising to intra-mathematical explanations (and thereby motivating backing in this case), a reason to treat scientific and mathematical explanation differently is required. It is not clear, however, what that reason might be.

Treating scientific explanation and mathematical explanation differently also incurs a further cost: it requires giving up explanation monism. Explanation monism is the view that, at some level, all explanations work the same way. If one accepts that scientific explanations work in an ontic fashion, while mathematical explanations do not, then it is difficult to see how monism about explanation can be sustained. Since explanation monism is a popular view in its own right (see, for instance, Held (2019); Nickel (2010); Reutlinger (2016); Wilhelm (forthcoming)) this may not be an entirely comfortable result. Of course, there are versions of monism that aren't ontic in nature (see, for instance, Reutlinger *et al.* (forthcoming)). The point, though, is that a nominalist who accepts an ontic approach to scientific explanation, can't easily withhold that approach from intra-mathematical explanation without giving up monism in some sense.

Why believe monism? Providing a full answer to this question lies beyond the scope of this paper. For now it is enough to note that monism appears to be better than the alternative, namely: explanation pluralism. The pluralist denies that there is any commonality to the way that explanations work. The pluralist is thus committed to two or more quite distinct families of explanatory notions, and so her view is more complex and less unified. As Reutlinger *et al.* (forthcoming) put it, the pluralist's view is 'piecemeal' and thus 'less general'. The pluralist also struggles to explain the common features of explanations. More generally, she struggles to say what all explanations share in common such that they all count as explanations. This is not to say that pluralism is unworkable, of course, only that it seems to be at a disadvantage (one might go so far as to say that 'monism is the default option in the theory of explanation' (Held 2019: 447).

In sum, then, giving up the ontic theory incurs two costs for the nominalist: (i) it weakens the link between explanation and ontology and (ii) it strips the nominalist of an important objection against the enhanced indispensability argument. If the nominalist tries to hold onto an ontic conception of scientific explanation while denying the same for intra-mathematical explanation, then she must give up explanation monism as well.

I anticipate two objections to my argument in favour of the backing conception of explanation. First, I have assumed that the backing conception, in its most plausible form, is not the view that all explanations are backed by causal relations. I have similarly assumed that the ontic theory of explanation, in its most plausible form, is not the thesis that the representation of causal structure is necessary and sufficient for explanation. I have made these assumptions on the grounds that there appear to be compelling instances of non-causal explanation.

But what happens if one simply doubles-down on a causal backing conception or a causal ontic theory? Then, one might argue, my argument for Platonism fails, since it is implausible to suppose that mathematical objects enter into causal relations with one another, and so mathematical explanations cannot be backed by such relations. This, however, is too quick. For some sufficiently broad notions of causation (such as those defended by Wilson (2018) and Zardini (2019)) it is possible to say that intra-mathematical explanations are causal as well. So further argument would be needed to rule out causation of a very general sort between mathematical entities. Moreover, if one accepts a purely causal version of the ontic theory for scientific explanation, and yet one continues to believe in the existence of intra-mathematical explanations (and believes they can't be causal), one is still forced to give up explanation monism. Thus at least one of the costs already identified for the liberal ontic and backing conceptions applies equally well to more stringent, causal versions.

This brings me to the second objection. By appealing to the ontic theory of explanation, my argument in favour of Platonism becomes hostage to broader views about the nature of explanation. Because of this, one might contend, my argument is at best a weak argument for Platonism. But the argument is not so different from the enhanced indispensability argument in this respect. I have already noted that critics of the enhanced indispensability argument lose a core objection if they give up an ontic conception of scientific explanation. But, by the same token, a proponent of the enhanced indispensability argument seems compelled to *reject* the ontic conception of scientific explanation in order to run her argument (on pain of falling prey to the nominalist's objection). Both my argument and the enhanced indispensability argument must therefore take a stand on the ontic theory of explanation at some point.

It is notable that the enhanced indispensability argument and the argument from intra-mathematical explanation fall on different sides of the ontic/non-ontic divide. This is potentially advantageous for Platonism. For either the ontic theory of explanation is correct for all explanations or it isn't. If it is, then the enhanced indispensability argument likely fails, but the argument from intra-mathematical explanation succeeds. If, by contrast, the ontic theory of explanation is not correct for all explanations then the argument from intra-mathematical explanation is harder to defend, but the enhanced indispensability argument is rendered more plausible, since it need not be the case that extra-mathematical explanations are ontic (though a link between a non-ontic theory of explanation and ontology must still be rebuilt).

Thus, rather than weakening my argument, basing my case for the backing conception on the ontic theory of explanation strengthens and widens the case for Platonism.

As noted, there are three putative counterexamples to the backing conception in the literature. Thus, in order to complete my defense of the backing conception, I will respond to each case. The first two cases are particularly important to consider as they focus on intra-mathematical explanation. I will argue that none of the cases presented so far rules out the backing conception, and so none provide a counterexample to that view.

Here's the first case. In a recent paper, Lange (2019) offers examples of intra-mathematical explanation that, he argues, are not backed by grounding. Lange's arguments for this claim are convincing, and deserve a much more involved discussion than I can provide here. For now it is enough to note that even if Lange's arguments are successful, they do not yet rule out a backing conception of the examples he considers. This is so for two reasons. First, at best, Lange has shown that intra-mathematical explanations don't involve grounding relations. But, as discussed in §2, grounding is just one possible dependence relation between mathematical objects. So it is compatible with Lange's arguments that intra-mathematical explanations involve dependence relations of some other kind, and thus that a backing conception holds for these explanations after all.

Second, the notion of grounding that Lange uses is drawn from Fine (2012). But, as Schaffer (2016) has recently argued, Fine's notion of grounding is not a dependence relation between entities. As Schaffer puts the point, Fine's account 'regiments grounding as a sentential operator' (Schaffer 2016: 85). It therefore appears to be a relation between propositions. At best, then, Lange has shown that a specific notion of grounding is not implicated in some intra-mathematical explanations. It doesn't follow from this that there are no grounding relations that back those explanations. For there may yet be grounding relations in the sense advocated by Schaffer doing this work (as noted in §2 grounding for Schaffer is a dependence relation between parts of the world, analogous to causation).

The second challenge to the backing conception is offered by D'Alessandro (2020b). D'Alessandro argues that there are 'viewing-as' explanations within mathematics that may not fit the mould of the backing conception of explanation. These 'viewing-as' explanations involve viewing one mathematical object as another. So, for instance, one can view numbers as arrangements of dots. By doing so one can provide an explanatory proof of the fact that the sum of the first n odd natural numbers is always n^2 (see D'Alessandro 2020b for the details).

D'Alessandro calls the numbers that the dots represent the 'source' and the dot representation the 'target'. He argues, quite rightly, that there is no dependence relation between the source and the target that underwrites the viewing-as explanation. However, it is compatible with there being no dependence relation between the source and the target that there is nonetheless some dependence relation between the source objects at issue; a relation that the target objects manage to represent. Which is to say that it is compatible with D'Alessandro's arguments that numbers stand in dependence relations, and that these dependence relations back the dot-based explanation of why the sum of the first n odd natural numbers is always n^2.

D'Alessandro does come close to considering this possibility, when he writes:

> Perhaps the explanantia in the viewing-as cases are really facts about *the source objects themselves*. On this picture, facts about the target objects don't directly contribute anything to the relevant explanations. They may serve some sort of purpose—making an explanation more obvious, or easier to state, or something like that—but they aren't essential ... The problem with this story is that it's not at all clear how to subtract the target objects from the explanations in the above cases. (D'Alessandro 2020b: 781)

The view that I am proposing—that there are dependence relations between the source objects (numbers)—is rejected on the grounds that such relations would make the dot representation dispensable. However, the fact that there is a dependence relation between the source objects doesn't imply that the dot representation is dispensable. The dot representation may be an indispensable way of capturing the dependence relation between numbers that ultimately backs the explanation.

This is a familiar point from the debate surrounding extra-mathematical explanation. As Saatsi (2016) has argued, while mathematics may be indispensable to a scientific explanation of a physical phenomenon, the dependence relation that backs the explanation may be a relation between physical entities only, and need not involve any of the mathematical entities in the representation. Similarly, it is an open possibility that in the cases D'Alessandro considers, the dots are indispensable to an intra-mathematical explanation of a mathematical phenomenon, while the dependence relation that backs the explanation connects mathematical objects only, and does not involve dots in any way.

What are the dependence relations between numbers being represented by the dot explanation? It is difficult to say without an account of what numbers are, and thus how they might depend upon one another. But here is a very tentative suggestion. Suppose that numbers are sets on the Zermelo-Frankel model. Thus, $0 = \varnothing; 1 = \{\varnothing\}; 2 = \{\varnothing, \{\varnothing\}\}$ and so on. Then numbers literally constitute one another in so far as each number is in the next (and all subsequent numbers). What the dot representation may be doing is representing these constitution relations. This seems plausible since the way the dot explanation works is arguably via constitution: each odd number corresponds to an array of dots, and when the dot arrays for the first n odd natural numbers are combined they always constitute a dot array that corresponds to n^2.

As I said this is just a tentative suggestion. The main point is that D'Alessandro hasn't completely ruled out a backing interpretation of the examples he considers, and so viewing-as explanations are not yet a counterexample to the backing conception of explanation. To properly rule backing out, he would need to show that there cannot be any dependence relations between numbers that back the dot explanations he considers. Doing that, however, would likely require settling on a metaphysics of numbers first, and then arguing that such a metaphysics does not support any dependence relations between numbers that might back viewing-as explanations.

The third and final challenge to the backing conception comes from highly inaccurate models. According to Bokulich (2011), inaccurate models can play a genuine explanatory role in science. But, one might argue, it is hard to see how an inaccurate model could be backed by any genuine dependence relation. However, I don't see why this should be so. An explanation can be distorted in so far as it may not represent the world with complete accuracy. Despite this, it can still manage to capture a real dependence relation between parts of the world.

This, I take it, is what's happening with Bohr's model of the atom. The model itself is inaccurate, but it is not completely wrong. It still manages to capture a real dependence relation between atoms in some cases. Indeed, as Bokulich (2011: 43) notes, Bohr's model manages to capture some real structure involving the emission spectrum of hydrogen despite being largely inaccurate. As with the previous two cases, then, inaccurate models are compatible with the backing conception of explanation and so do not yet provide a counterexample to that view.

2.3. Mathematical Entities

I turn now to the third and final premise of my argument from intra-mathematical explanation. Recall the premise:

> If there are intra-mathematical explanations and all explanations are backed by dependence relations between parts of the world, then mathematical entities exist.

This premise relies on two assumptions. First, it assumes that dependence relations are existence entailing. That is, if y depends on x via some dependence relation Rxy, then x and y should both exist. Second, it assumes that only dependence relations between mathematical entities are fit to back intra-mathematical explanations.

The first assumption appears to be built into the backing conception of explanation. The relations that back explanations are supposed to be dependence relations between parts of the world. Assuming that the world only has parts that exist (and doesn't have any non-existent parts), then whatever the dependence relations join must be real.

One might respond, however, that there are clear counterexamples to the idea that dependence is ontologically committing. For instance, it seems possible that XYZ causes cancer in humans. But it doesn't follow from this that XYZ exists. So not all dependence relations are ontologically committing. In fact, I believe we should conclude from this that XYZ exists, just not in the actual world. Dependence relations are ontologically committing even between possible objects, and so an analogous argument to the one for intra-mathematical explanation can be run for modal realism as well.

However, the success of my argument does not rely on the imagined case for modal realism. For it is possible to modify the claim that dependence relations are existence entailing to avoid potential counterexamples from possibilia. The modification involves limiting the modal scope of the claim to this: if y *actually* depends on x via some dependence relation Rxy, then x and y should both exist. Since the kind of dependence at stake in my argument from intra-mathematical explanation is actual dependence between mathematical entities, this weaker claim is enough to establish the third premise. Moreover, the weaker claim remains untouched by merely possible cases of dependence.

Conceivably, however, one might reject even this weaker dependence claim. Noneists believe that non-existent objects can possess properties and stand in relations. Indeed, some non-existent objects can even stand in causal relations: a clear case of a dependence relation (see Routley (2018: 35)). If one is a noneist, then one will reject any general implication from dependence to existence. Since I have no argument to offer against noneism, I concede that a nominalist can use noneism as a basis for resisting the third premise of my argument. By the same token, it is difficult to see how a commitment to some form of noneism can be avoided if a nominalist denies that dependence relations are (actually) existence entailing. For it seems she must thereby accept that entities that don't exist can nonetheless stand in relations. It may come as a surprise that noneism is the price of nominalism.

This brings us to the claim that *only* dependence relations between mathematical entities can back intra-mathematical explanations. The reason to believe this is simple: it is unclear what else could do the relevant work. An intra-mathematical explanation just is an explanation of one mathematical fact by another. Assuming that the explanans and the explanandum only quantify over mathematical entities—as should be the case in an intra-mathematical explanation—it follows that the only entities relevant to the explanation and thus available to stand in a dependence relation are mathematical in nature. Of course, one could try to deny that mathematical claims involve quantification over mathematical entities. But, as Benacerraf (1973: 662) argues, any such view adds an ugly disunity into one's general semantic picture of language, and likely into the theory of truth itself.

3. Nominalist Responses

This concludes my preliminary defense of the argument from intra-mathematical explanation. In the process of defending the argument, I have already identified three potential responses available to the nominalist: she can either reject the claim that there are any intra-mathematical explanations, reject the backing conception of explanation or endorse noneism.

One might argue, however, that there's a fourth-style of response that I haven't considered. This fourth option allows the nominalist to accept that there are intra-mathematical explanations and to accept a backing conception of explanation, and yet deny that intra-mathematical explanations carry ontological implications. Here's the idea. Consider a fiction, like *Lord of the Rings*. One might think that there are explanations within such a fiction. For example, why does Frodo make the long trip to Mordor? Because he wants to destroy the one ring and free Middle Earth. Why does Aragorn ascend the throne of Gondor? Because he is the last living descendent of Isildur. These explanations are not ontologically committing. The fact that Frodo makes the long trip to Mordor to destroy the one ring does not imply the existence of Frodo, the ring and Middle Earth. Aragorn's ancestry gives us no reason to believe that he exists.

The nominalist can thus argue that explanation within mathematics is just like explanation within any fiction: genuine, but not metaphysically potent. One way to develop this idea is to say that claims about dependence between mathematical entities, while true, do not force an ontological commitment to those entities (a view reminiscent of the easy road nominalist positions discussed above). This, one might argue, allows the nominalist to take full account of mathematical practice. She is thus not placed in the invidious position of telling mathematicians that they are wrong about their own field. The nominalist can thereby dispose of the argument from intra-mathematical explanation in a manner that leaves her naturalistic credentials intact.

However, this nominalist approach is not a distinct option from the ones already considered. That's because a nominalist who maintains that claims about dependence are true but not ontologically committing seems to be committed to a form of noneism. In the fictional case, such a nominalist must accept that the causal relations that underwrite Aragorn's ancestry genuinely hold, despite the fact that Aragorn doesn't exist. Similarly, in the mathematical case, she must accept that mathematical entities depend on one another in the manner needed to satisfy the backing conception even though such entities don't exist. In both cases, then, she maintains that non-existent entities stand in relations.

It is also worth noting that the motivation for the current view—namely a particular interpretation of explanation in fiction—is not the only interpretation available. An alternative view would be to simply deny that fictions ever contain explanations, precisely because of one's commitment to a backing conception of explanation. What then of the apparent intuition that there are genuine explanations in fictions? This intuition can be addressed by leaning on a distinction between *fictional explanations*, on the one hand, and *explanations in fictions*, on the other. A fictional explanation, is an explanation that is not, strictly speaking, a genuine explanation, but would be a genuine explanation, were the fiction real. Explanations in fictions, by contrast, are genuine explanations involving fictional entities.

Given this distinction, the apparent existence of explanations in fictions can be explained away as follows. When thinking about fictions, we tend to automatically engage in an act of pretence: we pretend as if the fiction is real for a time (say, by imagining a world in which Frodo travels to Mordor to destroy the one ring). It is this automatic act of pretence that gives rise to the intuition that there are genuine explanations in fictions. If we pretend as if *Lord of the Rings* is real, it does seem right to say that there are genuine explanations of Frodo's behaviour. Once we throw off the pretence, however, it is much less clear that the intuition about explanation remains. If we really see the fiction for what it is—something that does not correspond to reality—the sense that there are genuine explanations about fictional entities appears much less stable. The game of pretend can make it all to easy to confuse fictional explanations with genuine explanations in fictions.

In general, then, I am inclined to reject the claim that there are genuine explanations in fictions, and regard the intuitions in this direction as mistaken. Since we can explain these intuitions away, and since they seem to be the sole reason to believe that fictions contain genuine explanations, I think we can dispose of the fictional case entirely. Now, a nominalist might turn this all around and claim that, within mathematics as well, we are simply engaging in a game of pretend. The intuition that there are genuine explanations in mathematics is based on a similar failure to properly emerge from the fiction. But, again, this is a view one has to take of mathematical practice, and it's not at all clear that mathematicians are pretending as if there are mathematical explanations. Pretence is much more plausible as a take on how we engage with explanation in fiction than it is as a take on how mathematicians engage with explanation in mathematics.

4. Conclusion

What are we to make of intra-mathematical explanation, metaphysically speaking? One can draw a stronger conclusion and a weaker conclusion. The stronger conclusion is that we should be Platonists. We should be Platonists because we should accept that there are intra-mathematical explanations, such explanations are backed by dependence relations between parts of the world because all explanations are, and only relations between existing mathematical entities can do the backing. The weaker conclusion is just that there is a tension between the following three claims:

1. Nominalism is true.
2. All explanations are backed by dependence relations between parts of the world.
3. There are genuine intra-mathematical explanations.

Only a noneist can accept all three claims. Everyone else can accept any two of these claims, but not all three. This reveals three stable positions in addition to noneism. The first is the Platonist view just mentioned. The second is *non-backing nominalism*. The non-backing nominalist accepts that there are genuine intra-mathematical explanations, but denies that all explanations are backed by dependence relations between parts of the world. The third position is *sceptical nominalism*. The sceptical nominalist accepts that all explanations are backed by dependence relations between entities but denies that there are any genuine intra-mathematical explanations.

I believe we should just be Platonists. Barring that, non-backing nominalism seems like the best bet. But, as discussed, there are good reasons to accept a backing conception of explanation. No matter how the argument is addressed, something precious to someone has to go. Nominalism, the backing conception of explanation and the existence of genuine intra-mathematical explanations all have their advocates, and are all under threat. In this way, the philosophical interest of the argument reaches well beyond its capacity to establish Platonism.[2]

References

Andersen, H. (2016) 'Complements, Not Competitors: Causal And Mathematical Explanations', *British Journal for the Philosophy of Science*, 69: 484–508.

Audi, P. (2012) 'Grounding: Toward A Theory Of The "In-Virtue-Of" Relation', *The Journal of Philosophy*, 109: 685–711.

[2] I am grateful to three anonymous referees for their extremely useful comments which greatly improved the paper. Work on this paper was partly supported by the Australian Research Council through a Discovery Early Career Award (DE180100414).

Azzouni, J. (2004) *Deflating Existential Consequence: A Case for Nominalism*. Oxford: Oxford University Press.

Baker, A. (2005) 'Are There Genuine Mathematical Explanations of Physical Phenomena?' *Mind*, 114: 223–38.

Baker, A. (2009) 'Mathematical Explanation In Science.' *British Journal for the Philosophy of Science*, 60: 611–33.

Bangu, S. (2008) 'Inference To The Best Explanation And Mathematical Realism', *Synthese*, 160: 13–20.

Baron, S., Colyvan, M. and D. Ripley (2017) 'How Mathematics Can Make A Difference', *Philosophers' Imprint*, 17: 1–19.

Benacerraf, P. (1973) 'Mathematical Truth', *The Journal of Philosophy*, 70: 661–79.

Bokulich, A. (2011) 'How Scientific Models Can Explain', *Synthese*, 180: 33–45.

Colyvan, M. (2010) 'There Is No Easy Road To Nominalism', *Mind*, 119: 285–306.

D'Alessandro, W. (2020a) 'Mathematical Explanation Beyond Explanatory Proof', *British Journal for the Philosophy of Science*, 71: 581–603.

D'Alessandro, W. (2020b) 'Viewing-as Explanations And Ontic Dependence', *Philosophical Studies*, 177: 769–92.

Fine, K. (2012) 'A Guide To Ground', in F. Correia and B. Schneider (eds.) *Metaphysical Grounding: Understanding The Structure Of Reality*, 37–80. Cambridge: Cambridge University Press.

Gowers, W. T. (manuscript), "The Two Cultures Of Mathematics." *https://www.dpmms.cam.ac.uk/ wtg10/2cultures.pdf [Accessed May 2021]*.

Hafner, J. and P. Mancosu (2005) 'The Varieties Of Mathematical Explanation', in K. F. Jørgensen and P. Mancosu (eds.) *Visualization, Explanation And Reasoning Styles In Mathematics*, 215–250. Dordrecht: Springer.

Held, C. (2019) 'Towards a Monist Theory of Explanation', *Journal for General Philosophy of Science*, 50: 447–75.

Inglis, M. and J. P. Mejía-Ramos (2021) 'Functional Explanation In Mathematics', *Synthese*, 198: 6369–92.

Kim, J. (1994) 'Explanatory Knowledge And Metaphysical Dependence', *Philosophical Issues*, 5: 51–69.

Knowles, R. (2021) 'Platonic Relations And Mathematical Explanations', *Philosophical Quarterly*, 71: 623–44.

Knowles, R. and J. Saatsi (2021) 'Mathematics And Explanatory Generality: Nothing But Cognitive Salience', *Erkenntnis*, 86: 1119–37.

Kuorikoski, J. (2021) 'There Are No Mathematical Explanations', *Philosophy of Science*, 88: 189–212.

Lange, M. (2016) *Because Without Cause*. Oxford: Oxford University Press.

Lange, M. (2019) 'Ground And Explanation In Mathematics', *Philosophers' Imprint*, 19: 1–18.

Leng, M. (2005) 'Mathematical Explanation', in C. Cellucci and D. Gillies (eds.) *Mathematical Reasoning, Heuristics and the Development of Mathematics*, 169–189, London: King's College Publications.

Leng, M. (2012) 'Taking It Easy: A Response To Colyvan', *Mind*, 121: 983–95.

Liggins, D. (2012) 'Weaseling And The Content Of Science', *Mind*, 121: 997–1005.

Maddy, P. (1992) 'Indispensability And Practice', *Journal of Philosophy*, 89: 275–89.

Mancosu, P. (2001) 'Mathematical Explanation: Problems And Prospects', *Topoi*, 20: 97–117.

Mejía-Ramos, J. P., Alcock, L., Lew, K., Rago, P., Sangwin, C., and M. Inglis (2019) 'Using Corpus Linguistics To Investigate Mathematical Explanation', in E. Fischer and M. Curtis (eds.) *Methodological Advances in Experimental Philosophy*, 239–264. Great Britain: Bloomsbury Academic.

Mejía-Ramos, J. P., Evans, T., Rittberg, C., and M. Inglis (2021) 'Mathematicians' Assessments Of The Explanatory Value Of Proofs', *Axiomathes*, 31: 575–99.

Melia, J. (2000) 'Weaseling Away The Indispensability Argument', *Mind*, 109: 455–79.

Nerlich, G. (1979) 'What Can Geometry Explain?' *British Journal for the Philosophy of Science*, 30: 69–83.

Nickel, B. (2010) 'How General Do Theories Of Explanation Need To Be?', *Noûs*, 44: 305–28.

Pincock, C. (2015) 'Abstract Explanations In Science', *British Journal for the Philosophy of Science*, 66: 857–82.

Reutlinger, A. (2016) 'Is There A Monist Theory Of Causal And Noncausal Explanations? The Counterfactual Theory Of Scientific Explanation', *Philosophy of Science*, 83: 733–45.

Reutlinger, A., Colyvan, M. and K. Krzyżanowska (forthcoming) 'The Prospects For A Monist Theory Of Non-causal Explanation In Science And Mathematics', *Erkenntnis*.

Routley, R. (2018) *Exploring Meinong's Jungle And Beyond: The Sylvan Jungle*, Volume 1, 1st Edition, Eckert, M. (ed). Switzerland: Springer.

Ruben, D. H. (1990) *Explaining Explanation*. London and New York: Routledge.

Saatsi, J. (2016) 'On The "Indispensable Explanatory Role" Of Mathematics', *Mind*, 125: 1045–70.

Salmon, W. (1984) *Scientific Explanation And The Causal Structure Of The World*. Princeton: Princeton University Press.

Schaffer, J. (2016) 'Grounding In The Image Of Causation', *Philosophical Studies*, 173: 49–100.

Steiner, M. (1978) 'Mathematical Explanation', *Philosophical Studies*, 34: 131–51.

Strevens, M. (2008) *Depth: An Account Of Scientific Explanation*. Cambridge, Massachusetts: Harvard University Press.

Taylor, E. (forthcoming) 'Backing Without Realism', *Erkenntnis*.

Weber, E. and J. Frans (2017) 'Is Mathematics A Domain For Philosophers Of Explanation?' *Journal for General Philosophy of Science*, 48: 125–42.

Wilhelm, I. (2021) 'Explanatory Priority Monism', *Philosophical Studies* 178/4: 1339–59.

Wilson, A. (2018) 'Metaphysical Causation', *Noûs*, 52: 723–51.

Yablo, S. (2012) 'Explanation, Extrapolation And Existence', *Mind*, 121: 1007–29.

Zardini, E. (2019) 'Instability And Contraction: Méditations Hégéliennes I', *Journal of Philosophical Logic*, 48: 155–88.

Zelcer, M. (2013) 'Against Mathematical Explanation', *Journal for General Philosophy of Science*, 44: 173–92.