# The Mathematical Roots of Semantic Analysis

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Abstract: Semantic analysis in early analytic philosophy belongs to a long tradition of adopting geometrical methodologies to the solution of philosophical problems. In particular, it adapts Descartes' development of formalization as a mechanism of analytic representation, for its application in natural language semantics. This article aims to trace the mathematical roots of Frege, Russel and Carnap's analytic method. Special attention is paid to the formal character of modern analysis introduced by Descartes. The goal is to identify the particular conception of "form" developed by the analytic tradition, from Descartes to early analytic philosophy, and to determine its relation to similar notions, like 'function' and 'syntax'. Finally, I focus on how Frege, Russell and Carnap's methods of semantic analysis fit the general characterization of formal analysis previously developed.

## Introduction

According to Max Fernández de Castro's "Three Methods of Semantic Analysis" (2003), Bertrand Russell's "On Denoting" (1905) defined the project of a philosophical semantic analysis on three basic puzzles that, according to the English philosopher, "any semantic theory must try to solve":

> These are, to know, the identity paradox (How can an identity statement be informative?), the use of singular terms without denotation in meaningful phrases (How can we say something true about Pegasus when it does not exist?) and the exception to the identity laws that emerge in

> certain contexts (How is that we can say something false of London that is true of the English capital?) [My translation]

The goal of this paper is to clarify exactly in what sense a project as the one Russell defined requires a semantic *analysis*. The thesis that I want to defend in this paper is that the semantic analysis that Frege, Russell and Carnap developed for framing these three semantic puzzles and their solutions are part of a long tradition of adopting geometrical analysis' methodological developments to solve philosophical problems. In particular, the analytic methods of these authors arise from the application of formalization as a mechanism of analytic representation to semantic problems.

To this end, I trace and reconstruct the historical development of the concept of analysis in modern occidental philosophy, up to the foundation of what has been known as analytic philosophy[1]. This reconstruction is strongly based on (Beaney, 2002) and (Barceló, 2003). However, in contrast to Beaney, my central interest is how analysis bridged mathematics and philosophy in the late 19th century and early 20th centuries. Also, in contrast with (Barceló 2003), instead of the formal character of modern logic, I am interested in better understanding Frege's, Russell's and Carnap's philosophical semantics.

In the first section I introduce the very useful distinction Michael Beaney has made (2002) among analysis' three modes: regressive, decompositional and transformative. Despite the obvious importance of each mode, I will concentrate in the later, for it is inside it, that Descartes' formalization of geometry (as well as Frege's, Russell's and Carnap's philosophical semantics) acquires greater significance. To further refine Beaney's characterization, I will focus in the formal character of modern transformative analysis. The aim is to distinguish the analytical concept of "form" from similar conceptions of the same concept, and from other kinds of geometrical representation. To this purpose, I present the Cartesian solution of the three or four lines problem. Hopefully, this will illustrate the extreme importance of formalization in modern analysis.

In the fourth section, I take a brief look at the history of the formal-analytical tradition of modern mathematics, before doing the same with philosophy. I complement this history with a study of the close relation between function, form and syntax emerging from this development. Finally, once the formal character of analysis is clearer, I focus on how Frege's, Russell's and Carnap's methods of semantic analysis fit in the characterization of transformative formal analysis developed in this paper.

## 1. Michael Beaney: How Analytic is Analytic Philosophy?

### 1.1. The Regressive Mode: Going Backwards

> It is neverhteless far from obvious what this renowed method of the ancient geometrs really was. One reason for this difficulty of understanding the method is the scarcity of ancient descriptions of the procesure of analysis. Another is the relative failure of these descriptions to do justice to the practice of analysis among ancient mathematicians.
>
> Jaako Hintikka and Unto Remes (1976, 253)

According to Michael Beaney (2002, 2003), throughout western modern philosophy, the notion of 'analysis' has manifested itself in three different conceptions or modes: regressive, decompositional and transformative. Modern analysis, on the other hand, originates in Cartesian geometric analysis, where Descartes synthesizes all the aforementioned modes of analysis: regression, decomposition and transformation.

Modern Thought inherited the regressive mode of analysis from ancient Greek Geometry (particularly, from Pappus' commentary on Euclides, still "the only extensive description of analytical method in the ancient mathematical literature" (Remes and Hintikka 1976, 253) [2]). In this mode – which more or less corresponds to what Hintikka and Remes call 'directional analysis' –, analyzing a problem "involves working back to the principles, premises, causes, etc., by means of which something can be derived or explained." (Beaney 2002, 55) Beaney calls this mode 'regressive', because of its inverse direction regarding its complementary method of synthesis. In (Beaney 2003), he goes on to say that this mode defines analysis in its broadest sense, allowing for "great variation in specific method" (Ibidem). In contrast, in (1976), Hintikka and Remes had argued that, even though "the old geometrical procedure of analysis was accompanied by a complementary synthesis in all its typical uses in Antiquity" (Hintikka and Remes 1976, 265), such synthesis did not always involve a re-working forwards of the steps obtained in analysis. Furthermore, they claim, ancient analysis did not always go 'downstream', that is, "against the direction of logical consequence" (Hintikka and Remes 1976, 262).

> In one sense, analysis can even be said to proceed in either direction. The whole problem of the direction of analysis is also superficial in the sense that it is not connected with the heuristic usefulness of the method of analysis. (Hintikka and Remes 1976, 263)

However, it is important to identify two different senses in which Analysis is characterized by its 'inverse' direction. On the one sense, analysis goes in the inverse direction *with respect to Synthesis*. Thus, Synthesis just traces forward the steps Analysis laid out for it. On the second sense, Analysis works backwards *regarding the direction of logical consequence*. Under the otherwise reasonable assumption that what is sought is a deductive proof (such that Synthesis ought to follow the direction of logical consequence, from axioms, definitions and postulates to theorems), both senses become equivalent. However, such assumption stops being reasonable, once we leave such cases behind (cases which were not even paradigmatic in ancient geometry). First of all, in constructive cases (where what is sought is the construction of a figure) it makes little sense to talk about a logical direction among concomitants [*akóloytha*]. Second of all, in theoretical cases (where what is sought is a proof for a theorem), regressive analysis is, above all, a deductive hypothetical method, not an abductive one.[3] Thus, this is the most natural way to read Pappus remarks to the extent that proof is the reverse of analysis[4], and, later, Alexander of Aphrodisias claim that "analysis is the return from the end to the principles." (Gilbert 1960, 32 *apud.* Beaney 2003) Their directions are mutually inverse, not because only one of them follows the natural direction of logical consequence, but only because the starting point of one is the final point of the other. The conclusion of the synthetic proof is the hypothetical premise from which analysis starts. It is only in this sense that Analysis is said to work backwards.

Thus, instead of stressing the putative 'inverse' direction of analysis, it is better to characterize the regressive mode of analysis by its hypothetical and foundational dimension: (i) it starts with an assumption of already having what is sought and (ii) arrives to the principles, premises, causes, etc., by means of which something can be derived or explained.

Even though this article's main focus will be on the transformative mode, it is also worth mentioning that, just like the analytic method as a whole, the regressive mode of analysis has evolved in meaning since the time of Pappus. In (2002), Volker Peckhaus has started to trace this evolution. From a "very general view" of the regressive mode in analysis, he identifies three 'levels' on which the method works:

1. On the **practical** level, regressive analysis stands for the heuristics of research, i. e., the search for the necessary conditions to solve a given problem. On this level, which corresponds to Port Royal's understanding of 'analysis',[5] regressive analysis can also be identified with abduction.
2. On the **methodological** level, to analyze a set of statements is to set them in a logical order. Analogically, to analyze a statement is to find its place in such logical order. A paradigmatic example of this kind of analysis, according to Peckhaus, is Hilbert's axiomatic method.

3. Finally, on the **foundational** level, finally, the goal of regressive analysis is a justification for the starting points of deduction. Ancient Geometrical Analysis is the paradigmatic example of this kind of foundational analysis.

On the other hand, unlike Beaney, Peckhaus does not offer a satisfactory historical account of how regressive analysis came to occur on such different levels, and such different senses. This is not the occasion to further explore Veckhaus' hypothesis. Therefore, for the remaining of the current essay, I will restrict myself to the foundational understanding of regressive analysis.

1.2. The Decomposition and Transformative Modes

Bealey's second mode of analysis has deeper philosophical roots,[6] since it is a direct descendant of Plato's mature method of collection and division (as it appears in the *Phaedrus, Sophist, Politics* and *Philebus*), where concepts are analyzed – decomposed, that is – into other more general concepts.[7] A similar mode is observed in the Aristotelian method of definition through genus and specific difference.[8] So, for example, the concept of human being is decomposed in the concepts of 'animal' and 'political'. Even though the latter are extensionally broader than the original concept, this later contains them intensionally, its definition presupposes them. In this regards, Beaney writes:

> Understanding a classificatory hierarchy *extensionally,* that is, in terms of the classes of things denoted, the classes higher up [the more general ones] are clearly the larger, 'containing' the classes lower down as subclasses. . . *Intensionally*, however, the relationship of 'containment' has been seen as holding in the opposite direction. If someone understands the concept *human being*, at least in the strong sense of knowing its definition, then they must understand the [more general] concepts 'rational' and 'animal'. Working back up the hierarchy in 'analysis' (in the regressive sense) could then come to be identified with 'unpacking' or 'resolving' a concept into its

> 'constituent' concepts ('analysis' in the decompositional sense). (Beaney 2002, 69)

However, it is the third mode of analysis that that gives our modern conception its idiosyncratic sense. Bealey calls this mode, 'transformative' because it involves a *paraphrases* or change in the problem's representation.[9]

Even though there is a transformative element easily identifiable in Ancient Greek Geometrical Analysis,[10] it only acquires a special significance in modern analysis. It was Rene Descartes who, trying to reconstruct ancient analysis, developed the algebraic tools required to turn analysis from a figurative[11] into an eminently formal method.

Descartes' method of analysis aims to find the fundamental principles to build upon all knowledge –either geometrical or philosophical – in a synthetic fashion.[12] Descartes takes classical geometric analysis as methodological paradigm.[13] Given that, he could find just very little explicit information about this method in the available classic texts[14], its reconstruction is, indeed, the creation of a new analytic method.[15] The analitization of geometry that Descartes develops is the institution of a method of resolution that includes as much a change of representation as a method of regression[16] and decomposition[17]. In his *Geometry*, Descartes creates a new formal framework for the representation of geometrical problems. An essential element of this framework is the use of an algebraic language. As we will see in further detail, this change of notation is in itself a radical revolution in mathematics. Still, it is not all of the Cartesian method. It also involves an analysis by decomposition. However, such decomposition is completely dependant on the change of notation. Thus, Cartesian analysis synthesizes the three modes of analysis in a single method. From that moment on, the history of the concept of 'analysis' becomes a continuous dialog among these three modes. In fact, we can see its posterior history as a battle between the decompositional and formal-representational modes to capture the regressive-fundationist function of analysis.

In contrast, in (2003), Beaney attributes the origin of the hegemony of the decompositional mode in modern philosophical thought to Descartes. In particular, he traces it back to rule thirteen of the *Rules for the Direction of the Mind,* which states: "in order to perfectly understand a problem we must abstract from every superfluous conceptions, reduce it to its simplest terms and, by means of enumeration, divide it up into the smallest possible parts" (I, 51), and, later, to the second rule for his philosophical method presented in the *Discourse on Method*, where he instructs "to divide each of the difficulties I examined into as many parts as possible and as may be required in order to resolve them better." (I, 120)

Beaney stresses as an interesting fact that "Descartes' *Geometry* was first published together with the Discourse and advertised as an essay in the method laid out in the *Discourse*, for each part was responsible for the rise of a different mode of analysis on separate sides of the mathematics/philosophy divide that Descartes was trying to bridge. Thus, in early modern times, the decompositional account would become standard among philosophers, while the transformative mode revolutionized mathematics.

The idea of Cartesian method as a revolutionary change in scientific representation is already found in authors as diverse as Martin Heidegger (1977), Ernst Cassirer (1957), Michel Foucault (1970) and Jonathan Crary (1990), all of whom place it at the very origin of modern thought. Beaney's study, on the other hand, goes one step further by analyzing this modern notion into its decompositional, regressive, and transformative components. In other words, while the previous authors succeeded in identifying a transformative element in Cartesian analysis, they had not separated it from its regressive and decompositional elements.[18] Thus, they had failed to isolate the actual innovation that defined modern analytical method. As Beaney correctly points out, the distinction is essential to understand the truly innovate aspect of this kind of analysis. Both regression end decomposition had always been essential elements of analysis. It was the transformative element that was deeply transformed – no pun intended – by Descartes: from figurative into formal.[19]

The following section dives deeper into those waters. In order to make better sense of this revolution, I will differentiate between mere symbolic representation and full-fledged formalization, stressing the intimate relation between 'form' and 'function' in modern mathematical analysis. Later, this will allow me to contrast the figurative and formal transformations involved in classical and modern analysis respectively.

## 2. Formal Analysis[20]

It is easy to notice that the representational regime that operates in formal analysis is symbolic. Nevertheless, it is important to emphasize that the symbolic systems operating in (geometric, semantic or logical) formal analysis are not merely symbolic. At this point, it will be fruitful to appeal to the well-known distinction[21] between the 'syncopate' use of mathematical symbols in pre-Cartesian algebra and the 'formal' or 'analytic' use of modern mathematics. In ancient Arab Algebra and Western Cosistic, there were no proper variables as we know them. True, letters were used. However, they were no more than mnemothechnic devices or abbreviations of more complex expressions. Therefore, these primitive forms of algebra did not feature any means of expressing general calculations. Since its symbolic system[22] included only constants, it allowed only for particular calculations. Generality was expressed through particular cases that were used as examples or paradigms. It was not until Viète's work and his posterior refinement by Descartes'[23] that proper algebraic variables appeared in modern mathematics. Their introduction allowed two important advances in mathematics: the possibility to express general forms[24] – 'species', in Viète's terminology – and, even more importantly, the possibility to calculate with them. In this respect, Kline (1972) has written:

> Viète was completely conscious about that when he studied the quadratic general equation ax2 + bx + c = 0 (in our notation), he was studied **a whole class of expressions**. To distinguish between *numerous logistic*

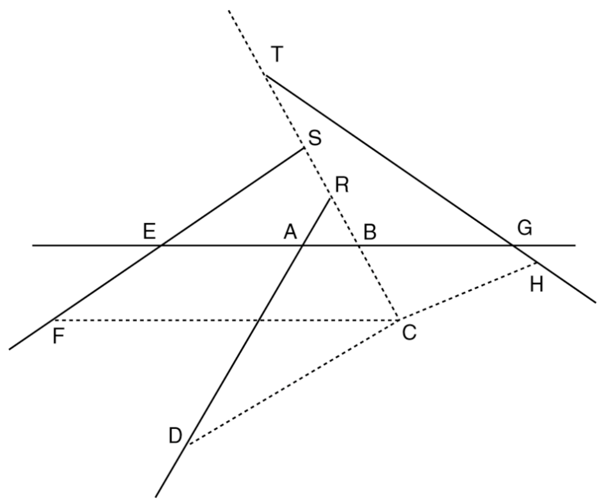
> and *specious logistic* in his *Isagoge*, Viète also distinguished between algebra and arithmetic. Algebra, the *specious logistic*, said, was **a method of the calculus with species or forms** of things. Arithmetic -the *numerous*- deals with numbers. So, in just one step, the algebra was converted in a study of general types of forms and equations, given that what it is done for the general case covers an infinite of special cases. (1972, 261-2)[25]

Thus, the central difference between modern and ancient algebra was that, through the use of variables, the former could abstract the common form of particular calculations and express it in a general formula. This new symbolic language allowed mathematicians to manipulate general forms in ways that were nearly impossible until then. It was then that mathematics' proper formal language was born. A formal language is not simply one that uses letters as symbols, but one that uses them *to calculate*. In this sense, modern algebra inaugurates the possibility of calculating with forms, instead of arithmetic quantities or geometrical magnitudes. Doors were opened to a new type of calculus, more abstract and general than arithmetic or geometry. It brought about a significant revolution in the development of mathematics, in particular, and scientific knowledge in general.

## 3. An Early Geometrical Example of Formal Analysis

Unlike philosophers, most mathematicians eventually recognized the value of Descartes' new method. His resolution of the famous 'three or four lines problem'[26] demonstrated its effectiveness in the mind of many modern mathematicians. It would be good, then, to take a closer look at this solution to illustrate the very important role the transformative way of analysis plays in this kind of analysis, so to understand the radical change that formalization represented in the development of mathematical analysis.

The problem is posed the following way.[27] Being *AB, AD, EF* and *GH* straight lines given as in the following figure:




Search all points *C* such that line segments *CB, CD, CF* and *CH,* drawn from *C* to the given four lines, satisfy the following condition: that the product of *CB* times *CD* is in a given proportion to the product of *CF* times *CH.* It is also asked whether such points are located inside a conic section – a circle, parabola, hyperbola, ellipse or similar –, or not.

In his analysis of the problem above, Descartes starts by assuming that the condition is satisfied, that is, that such a *C* point exists. Up until here, the method follows closely Pappus definition of regressive analysis, according to which the first step is to assume that which is sought. However, the way Descartes represents this supposition is what differentiates his method from classical analysis. While, in classic analysis, point *C* is represent by a point in a geometric figure (similar to the one with which I have illustrated this problem), Descartes represents *C* by a pair of algebraic 'coordinates'. Given that *C* is determined by the length of segments *AB* and *BC*, given angle *ABC,* it can be modeled by an ordered pair $(x,y)$, where $x$ and $y$ correspond to the aforementioned lengths.

Risking to sound repetitive, let me stress again the revolutionary change of representation that Descartes performs here. To represent geometric hypotheses, classical figurative analysis could work with, at most, particular instances what was sought to be constructed or demonstrated in a general way (just like pre-formal algebra). This risked basing some posterior inference in the particularities of such instance, instead of the general specifications of the problem. The introduction of algebraic variables solved such problem. The use of variables permitted Descartes to represent his hypothesis in a formal, algebraic and universal way. In strict sense, the pair of Cartesian coordinates does not represent any particular point, but the general form of a point. In the current example, the introduction of algebraic variables allowed Descartes to represent, in a single view, all *C* points that constituted the general solution to the problem. This way, his analysis acquired the necessary formal and general character.

The next step is to show that all segments *CB*, *CD*, *CF* and *CH* are lineal functions of $x$ and $y$.[28] If so, the original condition of proportionality between $CB \cdot CD$ and $CF \cdot CH$ can be expressed by a quadratic expression with two variables. Each pair of coordinates $(x,y)$ satisfying the equation would represent each of the C points that are sought.

By representing the set of C points in a quadratic equation, one does not only algebraically represent the original geometric concept, but also 'formalizes' it. That way, it becomes possible to know the kind of conic section that such points stand on, attending merely to the equation's syntactic form. Geometrical information has thus been transformed into syntactic information.

I hope that this example clearly illustrates how the formal algebraic apparatus, as a mechanism of representation, let Descartes renovate geometrical analysis. Now, after fully understanding the role formalization plays in Cartesian analysis, can we finally see if formalization plays an analogous role in the semantic analysis of early analytic philosophy.

### 4. The Analytic Tradition in Mathematical Logic

Unfortunately, the importance of this new and powerful tool was not immediately recognized by all mathematicians of the time. On the contrary, for the next two hundred years, western mathematics lived in an intense inner fight between two different paradigms: the formal paradigm of algebraic analysis, and the constructive paradigm of geometry. The effects of this conflict are so broad and obvious that it is impossible to trace the historical developments of the following centuries without placing such controversy at the center. Therefore, it is easy to follow the development of formal analysis from France to England, and from there, under the guide of the Analytic Society, to early mathematical logic.[29]

The symbolic language of formal logic was developed in the algebraic-analytic tradition.[30] As such, it is not syncopated (it is not the abbreviated version of another non-formal language), but entirely formal. It does not only use algebraic formulae to express logical forms, but also features a calculus for their formal manipulation.[31] Both elements are essential for it to fulfill its analytic function.[32] Formalism and calculus are the two pillars upon which formal logic and semantic analysis are constructed. Both features distinguish analysis among the many representational regimes of modern science.

Furthermore, it is important not to mistake this mathematical notion of 'form' with the philosophical one. For philosophers, 'form' was characterized in opposition to 'matter'. It is temping to think that the formal character that the first mathematical logicians introduced is somehow related to the old philosophic notion. However, this is not so.[33] Early algebraic logicians were not in the business of isolating a certain logical form, by removing all non-logical matter. Instead, their method was establishing patterns of invariance between logical formulae. This is clear from the polemic between De Morgan and Mansel.[34] In his reply to (De Morgan, 1847), Mansel (1851) accused De Morgan of mismanaging the form/matter distinction. Nevertheless, it is clear that both thinkers were using the word 'form' in different senses: Mansel in the logic-Aristotelian sense and De Morgan in the algebraic-analytic one. In a first reaction to Mansel's criticisms, De Morgan tried to reconcile both notions. However, he soon realized that they were radically different notions. So he returned to his original opinion that the Aristotelian "metaphysical" notion was irrelevant to his own business of logical analysis (1847, 27).[35]

### 5. Form, Function and Syntax

Finally, it is also important to explain how this new kind of formal representation allows for a new sort of whole-part explanation in modern science. This, in turn, requires tracing in broad strokes the evolution of the notion of 'function' in mathematical analysis and its effect on the notion of 'form'.

The *Shorter Oxford English Dictionary*, defines the English term 'function' as "the special kind of activity proper to anything; the mode of action by which it fulfils its purpose." Even though this does not seem to be the way in which the term 'function' is currently used in mathematics (semantic or logic), this was the sense in which the word was introduced into the discipline, when Leibniz used it for the first time in his *Methodus Tangentium inverse, seu de Functionibus* (1673). There, Leibniz talks of function as a duty to be fulfilled, so that a line's function is identified with its role inside a figure. Similarly, in (1692) Leibniz talks about 'tangent', 'normal', etc. as a line's possible functions regarding a given curve.[36] It is Johann Bernoulli who transforms the Leibnizian notion into the more familiar conception of function as a correlation among quantities (even though he restricted it to analytically expressible correlations). Soon, Bernoulli's conception became standard.

Thus, the analytic notion of 'function' originally emerges as an attempt to mathematically capture the role a geometric object plays in the larger figure that contains it.[37] The analytic notion of 'function', therefore, aims at being a mathematical analogue of our ordinary notion of 'function'. Both are based on the whole/part relation. It makes sense to talk about the function of an object only in so far as it is part of another. In order for analysis to go beyond mere decomposition, it must be guided by the different functions that each part plays inside the analyzed whole. Thus, to functionally analyze a complex whole would involve decomposing it according to each part's function.

In this respect, formal analysis aims to offer a new representation of the analyzed object, such that the location of (the representation of) the parts in (the representation of) the whole correspond to their role – i.e., their function – in it. Thereby, the form of an analytic representation captures all its element's functions. The disposition of the parts inside the representation – its syntax – must reflect their different functions in such a way that parts with similar functions occupy similar places. The goal is that the function of every element can be seen directly in the syntax of its representation. The function must be obtainable directly from the representation by simple decomposition.

During the XIX century and beginnings of last century[38], thinkers as De Morgan, Boole and Frege started introducing into philosophy analytic notions like 'function' and 'form'. As is well known, Frege's method for identifiyng functions is based on the identification of variables and invariable elements in formal representations. Thus, functions (and arguments) were seen as parts of a structured whole (the value of the argument to such function). During this long historical period, the notion of 'function' was introduced in one of two ways: (1) as the invariant element in a system of transformations, or (2) as an incomplete element in need of completion. In the former, the distinction between function and argument became the distinction between a variable element (the argument), and an element that remains constant during such variation (the function).[39]

This way, the notion of function is fully determined by patterns of substitutability (inside of an analytic formal representation). Different elements have the same function (as part of a whole), if their representations are interchangeable inside the formal representation of that whole. If substituting one for the other, the represented object changes, their function is different. Thereby, one may evaluate the success of an analysis through the substitution patterns codified in its formal representation. We know that an object has not been correctly analyzed if, by substituting (the representation of) one of its parts with (the representation of) another of the same function, we obtain (the representation of) a different object, or if substituting (the representation of) parts with different function does not affect (the representation of) the analyzed object.[40]

Finally, in this kind of analysis, the different representations resulting from the assignment of different values to the same function are said o be of the same 'form'. In other words, distinct values of the same function (under different assignments of arguments) share the same form. So, for example, one can indistinctly talk of a formula as the disjunction of other two, or as being of the form $A \vee B$. Thus, in analysis, the notions of 'form' and 'function' are so intimately joined that one can be easily obtained from the other.[41] In this sense, the representation's form tell us not just what are the constitutive parts of an object (formal analysis is not mere decomposition) but also their function. 'Syntax' becomes 'form' at the moment it captures the function of each part inside the represented object.

In conclusion, to analyze – in this transformative-formal sense – is to find the true form of an object: to represent it in such a way that the syntax of its representation directly reflects the different functions each part plays in its whole.

## 6. The Analytic Tradition in Philosophy

> To analyze is to reformulate,
> -to translate in better words.
>
> J.O. Urmson (1967, 295)

Even though Cartesian Analysis almost immediately ignited a radical revolution in the field of mathematics, the eventual success of his methodological proposal in philosophy was a long time coming. In contrast with what happened in mathematics,[42] the decompositional mode of conceptual analysis continued to be the philosophical paradigm for many years after Descartes. This conception of analysis is clearly present in Hobbes (*De Corpore* VI §§1-2), Locke (Essay, II, xxii, 9) and Leibniz (*De Arte Cobinatoria* 23 a-b). Nevertheless, it did not reach its pinnacle until Kant's work, whose distinction between synthetic and analytic judgments is clearly based on a decompositional vision of analysis.[43] By the XIXth century, the philosophical discussion around the synthetic and analytical methods was completely displaced by the study of the analytic/synthetic distinction. According to Beaney, most post-Kantian philosophers could be easily divided in two camps: those who accepted the weak Kantian notion of analyticity and those who, like G. Frege and B. Russell, tried to recuperate the complex Cartesian conception of analysis.[44] In the process, these later thinkers laid the foundations of analytic philosophy as it is still known today.

Beaney does not exaggerate when he says that "what Descartes and Fermat did for analytic geometry, Frege and Russell did for analytic philosophy." (Beaney 2002, 67) The method of logico-conceptual analysis they founded, resulted in a philosophical revolution comparable with the analytic revolution in mathematics. Their analytic method for philosophy reintegrated regression and decomposition (which was already present in philosophical analysis) with formal representation. Just like Descartes and his geometry, the true major contribution of early analytics philosophy was the introduction of formal representation back into the analytic method. Behind Frege's logic and Russell's philosophy, lays the idea that once properly – that is, formally – represented, the problems of both disciplines would become evident in their solutions and foundations. The transformation element involved in both of these methodological revolutions, in mathematics and philosophy, is formalization. Through the introduction of fomalization, analytic philosophy aimed at bringing logical and philosophical analysis up to date with mathematical analysis.

Furthermore, the logico-philosophical method of analytic philosophy remains regressive, in so far as it is also complemented by a synthetic method, where the solution is founded and reconstructed from the ultimate elements that constitute its formal decomposition. Consequently, the three steps of Cartesian analysis (corresponding to the three modes of analysis) are reproduced in the analytic method of analytic philosophy:

1. Formalization of the problem (analysis as transformation)
2. Decomposition or resolution of the formalized problem (analysis as decomposition)
3. Foundation of the problem's solution in the basic components obtained from the previous formalization and decomposition (analysis in the regressive sense).[45]

Thus, the early-analytic philosophical method synthesized the three modes of analysis in a similar way than Cartesian analysis had done for mathematical analysis three centuries before.

### 7. Frege, Carnap and Russell

Once clarified the sense of 'analysis' contained in our notion of 'semantic analysis', we can finally answer our original question: *¿In what sense did a semantic project as the one embarked by the early analytic philosophers require some form of analysis?* If we follow Fernández de Castro's diagnosis (2002), the puzzles that defined Frege, Russell and Carnap's semantic agenda are all essentially problems of (syntactic) substitution and (semantic) function. They all arise from a mismatch between syntactic form and semantic function. And, as seen before, these are problems of the sort that have defined modern analysis, even before arising as philosophical method early in the XX[th] Century. It was appropriate, therefore, for Frege, Carnap and Russell to look at formalization for the key to solving these puzzles. It had already proved to be a very fruitful analytic tool in geometry, and it was reasonable to expect similar success from its application in semantics.

As already seen, the substitution and assignment of functions to parts of a whole are the defining elements of formal analysis. In the case of semantic analysis, propositions are the objects of analysis, and their parts are analyzed by their function (role) in determining such semantic unit. Problems arise when the surface grammatical syntax of the statement that expresses the proposition does not capture the different semantic elements and their functions that constitute such proposition. In other words, problems arise when the object of semantic analysis – the proposition – is represented by a structured element – the sentence – , whose structure – its surface grammatical syntax – does not reflect its semantic form, that is, does not tell us much about the different components of the proposition and they way they are composed together.

From the analytic standpoint, there is a mismatch between syntactic and semantic form, if semantic elements of the same semantic function – reference, in this case – cannot be mutually replaced inside the statement, without significantly affecting its semantic content. Thus, it is required to reveal the true semantic form lying underneath the superficial grammatical form in order to leave the truly semantic functions of its parts evident. The aim is to rescue the substitution of parts with the same semantic function. The means lay in formal semantic analysis.

In this way, it is clear to see how the different semantic analysis proposed by Frege, Russell and Carnap correspond to different changes in formal semantic representation. Frege proposes to change the representational regime of natural language for a first formal representation (to substitute the grammatical syntax of the natural language for the formal syntax of an artificial symbolic language). By running into the aforementioned puzzles of substitution, Russell recognizes fissures in Frege's formalization and analysis and, through his theory of definite descriptions, proposes a new formal representation. Analogously, by differentiating the intensional from the extensional functions of a term, Carnap once again appeals to patterns of substitution. Fernández de Castro (2003) writes:

We denote "$\alpha\equiv\beta$"to

$\alpha=\beta$ if $\alpha$ and $\beta$ are individual constants, names, or definite descriptions

$(\forall x)(\alpha x=\beta x)$ if $\alpha$ and $\beta$ are predicates

$(\alpha\leftrightarrow\beta)$ if $\alpha$ and $\beta$ are statements

So, we will say that $\alpha$ and $\beta$ are equivalents if "$\alpha=\beta$" is true, and that $\alpha$ and $\beta$ are equivalents-L if "$\alpha=\beta$" is true-L. As always, from an equivalent relation is possible to define an object for each of the elements of the correspondent partition. By this we can say that two designators (in other words, individual constants, predicates or statements) have same extension if they are equivalents and intension if are equivalent-L.

For Carnap, a referential term's intension is determined by the patterns of substitution in L-contexts, while its extension is determined by patterns of substitution in other contexts. This way, extension and intension are just different possible semantic functions.[46]

It has been mentioned that an object has not been correctly formalized if substituting (the representation of) one of its parts with (the representation of) another one of the same function, we obtain (the representation of) a new and different object, or if the substitution of (the representation of) functionally different objects does not change the (represented) analyzed object.[47] Now, when facing puzzles of this kind, our reaction could be either to look for a new form of representation or conclude that objects that we originally believed to have the same function, actually have different. In this case, Russell's response to the puzzles of semantic substitution was of the first kind, he looked for a new way of formalizing the puzzling cases. Frege and Carnap, in contrast, combined both strategies: they introduced both new formalisms and new semantic distinctions: between sense and reference

(Frege), and between extension and intension (Carnap). In every case the goal was to save the patterns of substitution determined in the semantic form, either by changing the formal representation or by incorporating the results of analysis to the semantic theory. In every case, it is clear that, despite being different methods, they share a common idea of what is the purpose and what the available means for semantic analysis.

### 8. Formalization and Ideal Language

Before finishing I would like to clarify a couple of issues regarding the role formalization plays inside these authors' philosophical analysis. First of all, the importance given to the transformative mode in their brand of analysis must not be confused with the linguistic turn in philosophy, at least not in the sense popularized by Michael Dummett (1978, 1993). As Ray Monk (1996) has made clear, Bertrand Russell never gives language the fundamental role that presumably defines this 'linguistic turn'. It is equally doubtful that, as Dummett holds (1978, 1993), Frege's philosophy emerges mostly from his philosophy of language. Besides, as I emphasize in this paper, their interest in formalization stems from their search for a proper way of representing philosophical problems in order to perform analysis on them. In other words, they are interested in formalization only in so far as they find it to be a useful method of representation for philosophical analysis. Thus, formalization is neither the goal nor the object of analysis. Their putative 'pursuit of a perfect language' must be so understood. Just as Descartes' formalization did not aim at substituting synthetic geometric language, so Frege, Russell and Carnap's formal systems did not pretended to substitute natural language. They simply wanted to make a tool that would facilitate philosophic analysis.[48]

I do not want to suggest that these philosophers advocated formalization as the only proper way to do philosophy. It is clear that the complex philosophic thought of these three figures cannot be reduced to its formal contributions, not even if restricted to their semantic theories. After all, their methods of philosophical analysis contain both regressive and decompositional elements. Besides, their philosophical work went far beyond that of analysis, either formal or otherwise. In this respect, I want to appropriate G.E. Moore's words, whose response to Josh Wisdom stated:

> It is not true that I have said, thought or implicated that analysis was the only appropriate duty to philosophy! ... I could not even imply that. (1942 675-676, quote by Ayer 1971 179-80)

By concentrating on the three semantic riddles identified by Russell, and the different responses given by Frege, Russell and Carnap, it has become evident that there is a certain 'analytic' way of identifying and representing semantic problems. The basic problem at the root of these three riddles –how is it possible for linguistic elements of the same grammatical type to differ in their semantic function – is an especially adequate problem to be solved through formalization. On the other hand, I also believe to have demonstrated how such problems and the three solutions proposed by Frege, Russell and Carnap belong to the analytic methodological tradition restored by Descartes. Of course that I think that my conclusions could extend beyond these three problems and authors, and that formalization plays a more important role in the thought of Frege, Russell and Carnap that the one presented here. Besides these semantic examples, other important philosophical distinctions also emerged from problems of substitution similar to those considered here and, clearly, other philosophers have shown methodological features related to the analytic tradition. However, for now, I must leave these issues unexplored, and leave such exploration for further occasions.

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[1] For considerations of space, I concentrate in two historical processes that I consider essential for the development of philosophical analysis: the emergence of modern algebra in early 17$^{th}$ century, and the birth of analytic philosophy at the end of the 19$^{th}$ century. Specially, I am interested in the intersection between these two disciplines. Thus, I hope that my work sheds some new light on the complex dialogue between modern mathematics and philosophy.

[2] Einerson Benedict (1936, 36) points out that the mathematical source of the term 'analysis' has been already recognized at least, since (Blancanus 1615), the comments of Waitz to the Aristotelic *Organon* translation (1844-46, I 366) and (Solmsen 1929).

[3]. For more on the relation between Analysis and Abduction see Aliseda (2005)

[4]. On Pappus' *Mathematical Collection*, composed around 300 AD.

[5]. Peckhaus quotes the following passage from the section 'On Method' of Arnauld and Nicole's *La Logique ou l'art de Penser*: "Hence there are two kinds of method, one for discovering the truth, which is known as analysis, or the method of resolution, and which can also be called the method of discovery. The other is for making the truth understood by others once it is found. This is known as synthesis, or the method of composition, and can also be called the method of instruction" (1996, 233)

[6] Nevertheless, it is reasonable to assume that ancient Greek geometry had a strong influence on both Plato and Aristotle. Cf. Beaney (2003) Therefore, all three modes of analysis have strong mathematical roots. Cf. Benedict (1936, 36-39).

[7]. Furthermore, Beaney (2003) finds in the decomposotional mode another bridge between the formal and the analytical. In his interpretation, Plato's method of dihairesis lays the basic groundwork not only of the decompositional mode of conceptual analysis, but also of its formal dimension. Although Plato did not use the term 'analysis'– his word for 'division' was 'dihairesis' – its goal was finding of the appropriate 'forms' and, subsequently, laying down synthetic definitions.

[8] Aristotle follows a decompositional kind of analysis in his analysis of figures. (*An. Pr.*I32, 42-10) See Benedict (1936,39)

[9] It is very important not to confuse the use of the term 'representation' in contemporary philosophy of science, and in the philosophy of mind and language. In this paper I restrict my use of the term to the first sense.

[10] Beaney quotes Hankel 1874, 137-50 and Heath (1921) I, 140-2) It is especially clear in Aristotle, whose *Analytics* show very sophisticated syntactic methods of transforming a syllogism's structure.

[11]. I will say about figurative analysis just enough to contrast it with formal analysis. For a more detailed view of this kind of transformative analysis, see Panza (*forthcoming*)

[12] It is important to remember that Descartes' *Geometry* was originally published alongside the *Discourse of Method* as a sample application of such method.

[13] Descartes points out the similarities between his method and classic geometrical analysis in (1965) VII 424, 444-5, (1985, 1984, 1992) I 18-19, II 5, 111. Cf. (Flage 1999, 3) François Viète, who was first to introduce variables to geometrical analysis, was of the same opinion.

[14] Descartes accuses the classics geometers of hiding their method of analysis in (1965) X 336, (1985, 1984, 1992) I 19 and (1965) VII 157, (1985, 1984, 1992) II 111.

[15] Even though it maintains a strong continuity with Pappus' method. Compare Pappus' definition with Descartes' in his *Geometry* (1965 VI 372)

[16] In the preface to the French edition of the *Principles* (1965 IXB 5, 1985, 1984, 1992 I181), Descartes describes his method of analysis as the search for 'first causes'. See (Flage 1999, 1, 14)

[17] See (Flage 1999 32-43)

[18] Another important difference between these authors' interpretations, and Beaney's (and mine) is the strong emphasis they place on 'order' in Cartesian analysis. True, Descartes stresses the importance of order in passages like (1965) X 379, 451, VI 21, VII 155, (1985, 1984, 1992) I 64, 121, II 110. See (Flage 1999, 38-43). However, a closer reading of these passages shows that order is not important for analysis, but for (mathematical) induction. Descartes himself recognizes this in (1965) X 388-9, (1985, 1984, 1992) I 25-6.

[19]. That is why this paper focuses so much on formal representational. Despite having correctly pointed out the importance of formal symbolism in modern analysis, Beaney does nothing to characterize it or contrast it with similar representational regimes in the history of analysis. Helena Pycior, on the other hand, besides signaling that "the analytic art of the moderns was not the same as that of the ancients. For Oughtred as for Viète, the analytic art was inextricably linked to the symbolic style" (1997, 45) gives a more detailed account of such style and its importance for the development of (British) algebra. I must admit that her analysis goes farther than mine at least in one direction. She traces the importance of formal symbolism as visual representation within the British empiricist tradition. For the British, it was essential that there be an intuitive component to analysis, in order to compete with geometry as a

foundation for mathematics. Viète's formal symbolism allowed this by "involving the human eye as well as reason in the mathematical process." (Pycior 1997, 45)

[20] The following section is an abbreviate version of the history of formal analysis in (Barceló 2003)

[21]. In the history of mathematics literature.

[22]. My use of "symbolic system" here is clearly anachronistic. It simply refers to the set of symbols involved in arithmetic calculation.

[23]. With important contributions from Harriot, Girard, Oughtred and Hudde. See Kline (1972) 259-63.

[24] In modern mathematics, talk of 'generality' must not be understood in the same inductive sense it has outside of mathematics. Instead, every formal statement is mathematically 'general', in so far as it is a general schema for expressions or calculations of the same form. Thus, it would be justified to say that, in mathematics, one does not generalize, but formalizes.

[25]. For Kline, Viète's introduction of variables was "the most significant change in the character of algebra" during the XVI and XVII centuries (Kline 1972, 261) Mahoney (1971, 372) goes as far as stating Viète's work as giving birth to 'algebra', as distinct from the previous mere 'algebraic approach'. How ironic, then, that Viète himself preferred the term 'analysis' over the Arab term 'algebra'. Cf. (Pycior 1997, 31).

[26] According to Pappus, this problem had been discussed, but not solved, by Euclides and Apolonius.

[27] I take the reconstruction of the problem from (van der Waerden 1985, 74-5).

[28] Descartes achieves this through the algebraic calculation of the arithmetic relations between *AB*, *BC* and the aforementioned lines. Notice that, since the segments are represented in function of coordinates $x$ and $y$, these calculations are neither geometrical, nor arithmetical, but algebraic.

[29] See (Grattan-Guiness 2000, 14-74)

[30] It is not by chance that the first systems of mathematical logic were algebraic. More on the algebraic sources of modern logic can be found in (Kramer 1982)

[31] This double nature of logic –as language and calculus – is at least as old as Leibniz's '*characteristica universalis'* from (1666), which contained both a universal language (which he variously called '*lingua generalis'*, '*lingua universalis', 'lingua rationalis'* or '*lingua philosophica*'), and a '*calculus ratiocinator'*, as a general technique for the reduction of all reasoning into mere calculation

[32] See (Barceló 2003)

[33] For a competing view about this, see MacFarlane (2000)

[34] See (Grattan-Guiness 2000, 28-29)

[35] Unfortunately, more than half a century later of the discussion between Mansel and De Morgan the form/matter distinction returned to the logic vocabulary with the distinction between 'material' and 'formal implication' introduced by Russell. Grattan-Guiness (2000, 318) conjectures that Russell must have been influenced by De Morgan's effort to reconcile both (the philosophical and the mathematical) notions of 'form'.

[36] It is interesting to note that in this same work, Leibniz uses the term '*relatio*' to refer to what we now call a 'function', that is, a regular correlation among magnitudes. See Cabillón (2002).

[37] This way of conceiving mathematical functions was strongly criticized since the middle of the XIX[th] century and, by the middle of the XX[th] century, it had already been abandoned, thanks to the work of Dirichlet, Riemann, Hausdorff and others. Our modern vision of mathematical function still holds a weak relation with this old notion, even if it does not fully correspond to it. In the rest of the paper I will use the notion of 'function' in this primary primitive sense. See Kramer (1982) and Kleiner (1989) for a broader historical analysis.

[38] See Luizin (193?) 33

[39] The distinction is explained more or less like this: Take a complex formal representation – for example, an equation. Vary one of its elements (not necessarily simple), this is, substitute one of its parts for another of the same type such that the new representation is well formed. The part that remains constant in such variation represents the function of the element represented by the part that varies (its argument) in the analyzed whole (its value). In the later, start by eliminating one of the elements. The remaining part represents the function of the object represented by the part that is eliminated. This way, the function is not invariant, but incomplete.

Both treatments are very similar and it is enough to take substitution to be the process of eliminating an element and putting another in its place for them to become equivalent. I disagree with Sandra Lapinte (2002), who believes that the substitutional and compositional modes of analysis are completely independent. Unfortunately, in her paper from (2002), Lapinte does not give us an argument on behalf of this thesis (except to say that it "does not seem likely", p. 109)

[40] When faced with substitution-problems of this kind, two reactions are possible: either look for a new representation that avoids the problem or conclude that the objects believed to have the same function, actually had different ones. As we will see later, Russell's response to the problems of semantic substitutability is of the first kind, while Frege and Carnap combined both strategies.

[41] Cf. (Hart 1990, 203) It is also important to remember one of the major achievements of analytic geometry was the discovery that geometrical objects of similar geometric form, i.e. of similar shape, could be characterized by equations of similar syntactic form. In the aforementioned problem of the three lines, for example, the geometric shape of the conics can be easily identified directly from the syntactic form of the corresponding second-degree equation.

[42] With the caveats explained later in the paper.

[43] See Beaney (2002) and (2003)

[44] In the first camp, Beaney places Hegel, the Idealists and German Romantics, Bradley and the British Idealists and Bergson. On the other side, we can find thinkers like Bolzano, Frege and Russell, Moore, the first Wittgenstein and the Logical Positivists. He also recognizes that phenomenology and the hermeneutic tradition are not easily classifiable in this dichotomy.

[45] Just after commenting on Russell's philosophical method, Philip P. Weiner (1944 274-5) draws a continuous historical line from Plato to Carnap and Wittgenstein, including Plotinus, Aristotle, Neo-Platonism, Descartes, Spinoza, Leibniz, Locke, Berkeley, Hume and Russell, identifying regressive and decompositional elements in the analytic methods of all of these thinkers. As a matter of fact, in the section on "Analysis and Synthesis" of his post-humously published manuscript *Theory of Knowledge*, Russell explicitly defines analysis in decompositional terms: "Analysis may be defined as the discovery of the constituents and the manner of combination of a given complex" (ca.1913, 119) It is also clear that Russell's (and Wittgenstein's) logical atomism is intimately related to the decompositional and regressive methods of analysis (Cf. Tomassini 1994). The fact that, before Russell and Wittgenstein, Moore had also defined analysis in decompositional and regressive terms, motivated authors like A.J. Ayer (1971) to interpret this philosophic tradition's method of analysis as predominantly regressive and

decompositional, more than transformative. I am not interested in the primacy between different modes of analysis involved in analytic philosophy. What I hope to leave clear in this paper is that Frege, Russell and Carnap's methods of semantic analysis were not just regressive and decompositional, but also transformational.

[46] In this sense, Carnap clearly illustrates the formal character of his semantic analysis. However, he emphasizes the role of calculation rules in the determination of the semantic form of a proposition. That way, Carnap can distinguish between L-truths and other kinds of truths.

[47] It is important to notice that these patterns of substitution are a necessary – but not sufficient – condition for good analysis. Productivity and explaining power are other criteria to judge a successful analysis by.

[48] See Frege (1879), Russell (1959, 1985) and Carnap (1934, 1951).