QUA-LIFICATION

Conferencia pronunciada por el doctor Allan Bäck
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Presentación
del Académico Correspondiente Dr. Ignacio A. Angelelli

Tengo el gusto de presentar al Dr. Allan Bäck, quien disertará acerca de la reduplicación en la historia de la lógica. Este fue el tema de la tesis doctoral de Allan, en el Departamento de Filosofía de la Universidad de Texas en Austin, hacia fines de los 1970. Tiempo después, la tesis se transformó en un libro de mucha envergadura. El Dr. Bäck, además de dominar los idiomas clásicos también ha logrado convertirse en un experto en textos árabes filosóficos. Ha abordado muchos otros temas en historia y filosofía de la lógica, publicando libros y numerosos artículos.

Dr. Bäck: tiene la palabra.

QUA-LIFICATION

Dr. Allan Bäck

Resumen

Considero en este estudio las proposiciones 'qua', en que un sujeto posee un atributo en un determinado respecto. Luego de indicar por qué estos enunciados tienen importancia central en teoría científica y filosófica, ofrezco un modelo formal general para ellos. Resumo los análisis de las proposiciones 'qua' dados en la tradición aristotélica de la reduplicación. Muestro cómo derivar estos tipos aristotélicos a partir del modelo más general.

Abstract

I consider here 'qua' propositions, statements that a subject has an attribute in a certain respect. After indicating why these statements have central importance in philosophical and scientific theory, I offer a general formal model for them. I summarize logical analyses of qua propositions given in the Aristotelian tradition of reduplication. I show how to derive these Aristotelian types from the more general model.

Reality has a hierarchical structure… with each level independent, to some degree, of the levels above and below, 'At each stage, entirely new laws, concepts, and generalizations are necessary, requiring inspiration and creativity to just as great a degree as in the previous one.'[1]

Perhaps I should explain why a theory of qua objects, of qualification, has importance in advance. In ordinary discourse, people often qualify what they say by appealing to one respect or another of what is being said. Philosophers too seek to qualify their statements, for the sake of greater precision of theory, or for the sake of avoiding the ignominious elenchus, or for the sake of pretentiousness or...

[1] John Horgan, *The Undiscovered Mind* (New York, 1999), p. 250, quoting Philip Anderson, "More is Different", *Science*, 1972.

Indeed some whole metaphysical systems seem to have been built on qua-lification talk: Father Parmenides, one hopes; certainly the Carvaka and the Nyaya and certain Buddhists; perhaps Spinoza with his modes and Hegel with his moments, not to mention Aristotle in his attempt to avoid Eleatic refutations. So it is not surprising that philosophers have given logical analyses of the structure of statements containing qualifications.

For example, an atom on an adjacent lower level is composed of protons, electrons and neutrons. These, being quantum particles, do not exist continuously (and perhaps may themselves be sequences of independently occurring quantum phases). These subatomic particles obey statistical laws having quantum uncertainty. An atom composed of these particles may be taken to exist continuously and to obey laws that are at any rate often stated as if there is no quantum uncertainty etc. Now we, including physicists, talk about such an atom as if it is a thing separate from the particles that compose it. Indeed, from the perspective of those particles it is accidental where they are and whether or not they constitute parts of that atom. Yet we still talk as if electrons as well as atoms are real. We also assert that we would be double-counting if we were to talk about the particles and the atom existing as totally separate things—rather like Ryle's example of counting the football players and then counting the football team.

In my jargon, the atom is taken as an object composed of certain aspects of the subatomic particles. Note that the atom itself has certain aspects when it is taken as part of a molecule, which in turn has aspects when it is taken as part of an amino acid… of a cell… of an apple…

Again, all this is but one dimension of aspect and object. We can also, qua sociologists, consider electrons as a cultural construct as given in a theory, itself done by human beings in the role of physicist etc. Here we have EXFOLIATION, the proliferation of aspects, in another type of discourse, another dimension. We can also consider an electron to be an inspiration for a painting or a comic book.

So I want a logic that handles how we –here the human and not the ideally rational or the philosophers' "we"– talk truly about the world. Such a theory of things and respects provides a logical model for this ontology. It also gives a general analysis of the language of respects, when we say that things have different respects, or speak of something in respect of this or in respect of that.

Here I 1) present the results of my earlier study of Aristotelian theories of qua propositions, also known as "reduplicative propositions" 2) give a general, formal model for qua propositions, and 3) show how the types of qua propositions appearing in Aristotle's texts and distinguished in the Aristotelian tradition fall under that model.

Qua Propositions

The basic structure of a qualified simple assertion is that of a simpler predication with an additional qualification. Let me now generalize and talk of 'qua propositions'. A qua proposition is a simple(r) proposition with a qualifying phrase with a connective like '*qua*', 'in respect of', 'in virtue of the fact that' and its complement. 'Qua' is the general connective of which '*qua*' is one instance. The general form is: 'S qua M is P'.[2] In many sentences, the 'S' term signifies the thing; the 'M' term the "respect" [not strictly speaking; see below].

The Aristotelian tradition has marked off two main logical types of qua propositions, of form 'S is P qua M'. In the (strictly) *reduplicative* type, the respect introduced by the M term has a predicative relation to the original subject and predicate, and sets restrictions on that predication. The original subject S is preserved along with 'P' continuing to be asserted of it. In the *specificative* type, the respect introduced by the M term has another, "mereological" relation to the original subject and predicate, and changes the original predication, so as not necessarily to be true of the original subject, at least for many types of parts, but rather of its "part".

A central criterion for a qua proposition being reduplicative is the validity of the *secundum quid ad simpliciter* inference:

S qua M is P; therefore, S is P.

Moreover, since S is being asserted to be P in the respect that it is M, it is being asserted that S is M. Because the respect of M, a feature of M, is a certain complex of properties of S, the reason why S is P is that S has those properties, it is being asserted [in most cases: perhaps not for qua this M] that M is P. So the general, default form

[Reduplicative] Every S qua M is P iff Every S is M, and every M is P (or: to be an M is to be a P)

We can get at the syntax by giving truth conditions for reduplicative propositions. I list below the truth conditions that I have given

[2] I use this form for the Aristotelian qua proposition as it has material presuppositions. For the purely logical form I use 'Φ qua Ψ is K'.

elsewhere for these types of qua propositions and examples for them that would be accepted in the Aristotelian tradition:

(i) Every S is P qua M (reduplicative) if and only if:
$(x)((Sx \supset Mx) \& (Mx \supset Px))$ (12)[3]

The Aristotelian tradition had two main subtypes: one, which I call *the restrictive*, used in scientific demonstration, where the reason or cause for the predication, given by the M term, should be commensurately universal with the predicate:

(ii) Every S is P qua M (restrictive reduplicative) if and only if:
$(x)((Sx \supset Mx) \& (Mx \equiv Px))$ (20)

Another, which I call *the abstractive*, concerns the demarcation of the sciences: physics studies substances qua moving; metaphysics studies beings qua being:

(iii) Every S is P qua M (abstractive reduplicative) if and only if:
$(x)(Sx \supset (Mx \& Px))$ and 'P' is an M-type predicate (97)

Some examples:

Every isosceles triangle *qua* isosceles triangle has its interior angles equal to 180°. [true reduplicatively and abstractively; false restrictively]

Every isosceles triangle *qua* triangle has its interior angles equal to 180°. [true restrictively; reduplicatively and abstractively]

The Great Pyramid *qua* geometrical is a triangular pyramid. [true abstractively; false reduplicatively and restrictively]

Being *qua* mathematical is quantitative. [likewise]

The reduplicative qua phrase then explains why S is P. This explanation can be given a weaker or a stronger causal sense.

With specificative propositions, the *secundum quid ad simpliciter* inference, 'if S qua M is P, S is P', does not follow. To use Aristotle's example, which became the standard one, if the Ethiopian with respect to his teeth is white (the Ethiopian's teeth are white), it does not follow that the Ethiopian is white. Here 'P', what is predicated of the original subject S need not be predicated of S in the re-

[3] The numbers in parentheses are the numbers for these analyses in *On Reduplication*, Leiden, 1996.

spect specified (M). That is, the predicates of that respect of S need not be predicates of S. If we take the notion of part broadly, as is traditional though not too contemporary,[4] we can think of the respect M as being a part of S. Then the fallacy of composition and division can apply here: what is true of the part need not be true of the whole, and *vice versa*. So we can give the following analysis of an accidental qua proposition:

> [Specificative] Every S qua M is P iff S qua M is a part of S, and everything that is S qua M is P [not that every M is P, but every M of S is P, in a mereological sense of 'of'].

This can be formalized, not too informatively as:

> (iv) Every S is P qua M (specificative) if and only if:
> (x) ((Mx & x ε_i S) $\supset$ Px), where 'x ε_i M' indicates a part-whole relation between x and M (46)

This Ethiopian (say, Haile) is white with respect to his teeth.

The Aristotelian tradition claims that all qua propositions that are true reduplicatively are true specificatively on some part-whole relation.

The semantics for reduplicative qua phrases presents no more difficulty than what is needed for analyzing usual predicative sentences, as the analyses given above suggest. The reduplicative qua complexes formed may seem to have a reference different than the original subject.

However specificative qua phrases immediately, by themselves, change the reference of the original subject. This becomes controversial only because some of the "parts" needed for the reference of specificative qua complexes are of types for which some do not want an ontological commitment: such parts do not seem realistic. 'Haile in respect of his teeth' might be acceptable; this just refers to his teeth, traditionally called an i*ntegral part*. Here that phrase might be naming something different from Haile, his teeth. But 'Haile' in respect of being mentioned in this sentence' seems a less robustly real part, needing to distinction in reality. Yet right now I am considering a logical formal model and so allow all sorts of parts.

[4] Peter Simons, *Parts*, Oxford, 1987; J. T. J. Srzednick and V. F. Rickey, eds., *Leśniewski's Systems: Ontology and Mereology*, The Hague, 1984.

A Formal Model of Thing and Respect

Starting with these analyses, we can construct a much more general formal model of qua propositions than what the Aristotelian tradition offers. Being able to do this does not *ipso facto* repudiate the Aristotelian tradition, once we recognize what it is doing. For its situation here resembles the one with modal operators. Aristotle has a conception of, say, necessity, that in modern terms concerns physical and not logical necessity. What he finds necessary concerns what is necessary in this world, and not in any possible world.[5] This is clear from his general discussions of necessity as well as from his examples of necessary propositions, like ‘it is necessary that a swan is an animal’. In contrast, in modern modal logic, this statement would not be considered necessary [unless we add on material meaning postulates à la Carnap]. Rather, formal tautologies like ‘$A \supset A$’ are logically necessary.

As I am developing this model with an ontological eye to the generation of respects from their subjects with a quasi-independence, I take as basic the specificative qua complexes. For a complex qua phrase (‘S qua M’), taken reduplicatively, need not refer to a subject different from that of the simple expression and so need not generate new items in the domain. Here though I want to be able to model the generation of respects as independent subjects from their base objects.

This model will also explain just in what ways reduplicative propositions are true specificatively. For, although the Aristotelian tradition makes this claim repeatedly, it does not explain in detail how. For there is no explanation of how the reduplicative syntax and semantics can reappear in the specificative ones, apart from the vague intuition that reduplicative propositions describe respects of the original subject, and so too do specificative propositions, and that the specificative respects are more inclusive than the reduplicative. But the differences in syntax and in semantics suggest that not everything true for reduplicative propositions and qua complexes will hold for the specificative ones, and conversely.

[5] The notion of possible worlds became prominent only later, due to the theological consideration of having to have a God free to choose in creating. Cf. Simo Knuuttila, *Modalities in Medieval Philosophy*, London, 1993; Allan Bäck, “Avicenna and Averroes: Modality and Theology,” in *Potentialität und Possibilität*, ed. T. Buchheim et al., Stuttgart, 2001.

The generation of respects in the formal model has the following stages:

Form PREDICATES from propositions. Singular terms can be copulated: ‘s’ becomes ‘being s’.[6] Any individual term in a sentence can be taken as the argument; with the rest of the sentence then is the predicate. Pull these predicates out of the sentence so as to make them available for combination. [IZZING]

Then apply the subordination function to each predicate so as to get the predicates under which a predicate falls and those predicates falling under it. This can be done formally or materially, with the introduction of meaning postulates and stipulations of fact. [UO (*unterordnen*)]

Intersect them. [The QUA operator]

Optional: add back in those predicates under which a predicate falls and those predicates falling under it. [UO+ or UO–]

Take the resulting complex as a thing in its own right. [THINGING]

Generation of Respects

Let us start with a simple qua proposition of form ‘Φ qua Ψ is K’. ‘Φ qua Ψ’ is the qua complex. When taken specificatively, it signifies “a respect of that Φ”, which is a different thing (which will become a different item in the domain once thinged) than ‘Φ’. I now show how to generate such respects from the qua proposition. Although ‘Φ qua Ψ’, taken specificatively, will have a fundamental symmetrical structure of the intersection of izzed Φ and izzed Ψ, we shall still be able to get to asymmetrical respects in ordinary discourse by distinguishing ‘Φ considered with
respect to Ψ’ and ‘Ψ considered with respect to Φ’.

So begin with izzing ‘Φ’ and ‘Ψ’. Izzed Φ and izzed Ψ will each be correlated with a set of “predicates” as just defined above. Let these sets be named ‘Φ’ and ‘Ψ’ [when there is no confusion in doing so]. Then perform the qua operation: ‘Φ qua Ψ’. I shall represent this as ‘ΦΨ’ [which will become useful when I use matrices and group theory instead of set theory]. This gives the basic structure from which respects of different sorts are generated.

[6] Quine in *Word and Object* has predication serve a similar function: ‘is’ transforms a singular term into something that can be attributed.

So far we have generated the sets of predicates for Φ and for Ψ with the izzing. The qua operator produces the intersection of those sets so as to generate a set of predicates in each of which predicates of Φ and Ψ appear. 'ΦΨ' itself, once thinged, is a respect.

This basic, symmetrical structure can be modified in various ways in order to generate the various sorts of respects, or specificative qua complexes, that have been distinguished according to the Aristotelian theories. Most of these sorts are asymmetrical, as with the respect "the Ethiopian with respect to his teeth".[7]

To get these various sorts we need also to define 'Σ considered with respect to M' and 'M considered with respect to Σ'. These will be respectively: 'UO^{-}(ΦΨ) qua Φ' and likewise 'UO^{-}(ΦΨ) qua Ψ'. For 'UO^{-}(ΦΨ) with respect to Φ', take 'ΦΨ', the intersection of Φ and Ψ, i.e., Φ qua Ψ. For each predicate π_i in ΦΨ, take $UO^{-}(\pi_i)$. This amounts to the *possible* instantiations for π_i, namely all predicates falling under π_i with a higher degree of saturation than π_i, until we get completely saturated predicates like 'Φα'.

To get the *actual* instantiations for π_i with respect to Φ, intersect $UO^{-}(\pi_i)$ with Φ—again a qua operation: $[UO^{-}(\pi_i)]$ Φ. Then take the union of all those sets. Let this be symbolized as: Φ_Ψ, as in effect we have here all the predicates of Φ in which Ψ appears.

'UO^{-}(ΦΨ) with respect to Ψ' can be defined likewise.

Using Φ_Ψ and Ψ_Φ we can define the various respects signified by qua complexes and give a typology for the various types distinguished in the Aristotelian tradition. (I indicate parenthetically how Φ^Ψ and Ψ_Φ could be taken as basic instead of Φ and Ψ.)

Reduplicative Propositions

If we return to the Aristotelian theory of reduplication, we can see how this model accommodates it. To recap, the salient points for the reduplicative propositions are: 1) the particular truth conditions for the different types distinguished 2) the general claim that the *secundum quid ad simpliciter* inference holds 3) the *dictum* that in reduplicative syllogisms the qua phrase forms part of the predicate and 4) the qua phrase does not make the subject term change its reference 5) some account of why reduplicative propositions imply their specificative counterparts.

[7] I assume as an ontological fact that the world has fundamental asymmetry. Thus in the interpretation I break the symmetry along Aristotelian lines.

In the model, the features of the base thing Φ captured by the qua complex, to form a predicate of the subject, comes from the izzed but not thinged 'Φ qua Ψ', here expressed as the conjunction of all predicates generated. As discussed, the predicates are generated by taking all true wffs in which 'Φ' and 'Ψ' appear, replacing the term for 'Φ' with a free variable, and making a conjunction of them (or some of them, if there are further restrictions as with the restrictive sort). Let this conjunction be represented by 'Qx'. In effect 'Qx' is 'x is Φ qua Ψ'.

Given the truth of the reduplicative singular proposition, 'φ qua Ψ is K', we can say: Qφ. Now certainly: Kφ. Likewise for particular and universal reduplicative propositions we can get 'every/some Φ is K'. So the *secundum quid ad simpliciter* inference holds—for logical affirmations but not for all denials. In particular take a predicate P* that the subject S falls under but not qua M. Then S qua M is not P*, S is P*, but it is not the case that S is not P*. As above, it is better to read the qua proposition thus: S is P not qua M'.

Now also: $(x)(\Phi x \supset Qx)$; $(x)(Qx \supset \Phi\Psi x)$. That is, if we izz the reduplicative complex so as to make it a predicate function, we get for every x if x is Φ, then x is ΦΨ', i.e., x is Φ qua Ψ.

Thus far we have no respects but only predicates. To get the respects these reduplicative predicates ('ΦΨ') must be thinged.

General reduplicative respects

The specificative respect generated from the general reduplicative qua complex is: thinged $(\Phi \cap \Psi)$ (thinged $\Phi_\Psi \cap \Psi_\Phi$)—i.e., the intersection of the sets of Φ and Ψ, once thinged. Here 'Φ' has all the (first-order) predicates ['π_i's'] of ΦΨ ('Φ qua Ψ'), and so the *secundum quid ad simpliciter* inference holds for the simple atomic wffs using the first-order predicates of ΦΨ (containing no negations) and the individual constant signifying the respect.

For instance, the respect signified by 'Socrates qua rational' leaves out Socrates' being snub-nosed. However, the *secundum quid ad simpliciter* inference does not hold for 'Socrates qua rational is not snub-nosed; therefore Socrates is not snub-nosed'. (Actually it is better to analyze the scope of the negation thus: Socrates is snub-nosed not qua rational. In this case we do not consider the occurrence of 'qua rational' to generate a respect but to be merely reduplicative. If we do take it to generate a respect, still it is not the case that the respect signified by ‘Socrates qua rational’ is snub-nosed.)

Restrictive reduplicative respects

The specificative respect generated from the restrictive reduplicative qua complex is: thinged $(\Phi \cap \Psi) \cap K$ (thinged $[(\Phi_\Psi \cap \Psi_\Phi) \cap K]$), i.e.:

> the intersection of the sets of Φ_Ψ and Ψ_Φ and K. once thinged.

Here ‘Φ’ has all the predicates [‘π_i’s’] of ‘$(\Phi$ qua $\Psi) \cap K$’, and so the *secundum quid ad simpliciter* inference holds for the simple atomic wffs using the first-order predicates of $\Phi\Psi$ (containing no negations) and the individual constant signifying the respect. This one has a double qua operation. So you might say that this is the purest feature or respect as well as Aristotle’s favorite.

Abstractive reduplicative respects

The specificative respect generated from the abstractive reduplicative qua complex is: thinged Φ_Ψ (thinged $[((\Phi_\Psi \cap \Psi_\Phi) \cap UO^{-} \Phi_\Psi)]$), which amounts to: the union of the intersection of the sets of Φ_Ψ and Ψ_Φ, with the possible or actual instantiations of Φ added in, once thinged. Here ‘Φ’ has all the predicates of ‘Φ qua Ψ’, and so the *secundum quid ad simpliciter* inference holds for the simple atomic wffs using the first-order predicates of $\Phi\Psi$ (containing no negations) and the individual constant signifying the respect.

Specificative Propositions

To recap, the salient points for the specificative propositions are: 1) some way to account for different part-whole relations 2) the general claim that the *secundum quid ad simpliciter* inference does not hold 3) the *dictum* that in specificative qua complexes the qua phrase forms part of the subject and 4) the qua phrase makes the subject term change its reference.

In the model, the features of the subject captured by the qua complex to form a respect “that”, a respect strictly speaking, independent of the subject, comes from the izzed and thinged ‘Φ qua Ψ’, here expressed as the conjunction of all predicates generated. So, for all qua com-plexes taken to signify respects, the qua phrase changes the reference of the original subject: even the traditional “reduplicative” qua propositions taken specificatively. In addition, with the traditional specificative qua propositions, we izz ‘Φ qua Ψ’ “from the side of the predicate”, i.e., starting from ‘Ψ’.

Consider the set of properties of the Ethiopian and the set of properties of the teeth.[8] The idea is that here ‘qua teeth’ will restrict the properties of the Ethiopian to his teeth part and its properties. We do this by considering only those statements in which ‘the Ethiopian’ and ‘his teeth’ appear. By making the respect on the side of ‘his teeth’ we can shift the reference so as to get something having properties different from those belonging to the Ethiopian ‘e’. For ‘e’ has predicates like ‘(∃t)(Ee & Tt & Het & Wt)’, and the teeth ‘d’ has ones like ‘(∃x)(Ex & Td & Hxd & Wd)’, which differ.

The restrictive respect will have only those predicates of the reduplicative respect that are coextensive with the predicate of the original qua proposition—in this example, ‘white’. The respect then is limited to the teeth predicates having to do with its color at or above the level of generality of ‘white’: e.g., ‘being white’, ‘being colored’, ‘appearing on a surface’ etc.

The abstractive respect has the predicates common to the Ethiopian e and his teeth t, like ‘Ex & Ty & Hxy & Wy’, as well as those particular predicates and instantiations of it belonging to the Ethiopian e, like ‘(∃t)(Ee & Tt & Het & Wt)’.

The specificative respect thinging all the teeth predicates/properties of the Ethiopian will have the common predicates, like ‘Ex & Ty & Hxy & Wy’, as well as predicates proper to these teeth and different from those of its subject, the Ethiopian, like ‘(∃x)(Ex & Td & Hxd & Wd)’.

Conclusions

In sum, on this analysis: 1) the *secundum quid ad simpliciter* inference does not hold for the specificative respect even for all the

[8] Ordinary language may have to be regimented or translated into a part-whole language to make all the details clear. But taking ‘predicate’ in my sense enables us to bypass this issue, for the statements explaining part-whole relations are in the domain and we can then makes those statements into ‘predicates’ by izzing them.

positive attributes. If we require some overlap of predicates between a thing and its respect, a specificative respect will have some predicates of the thing. Still it will always have some predicates that its base does not have [at least predicates that are not respectable ('being thought up or stated by me') or are of higher order ('being its respect')]. Different part-whole relations can be accommodated by the predicates' being generated from dif-ferent sets of propositions about 'Φ' and 'Ψ'. 2) The qua phrase now modifies the subject term. 3) It changes the reference of the subject term.

4) We can make partial sense of the claim that reduplicative propositions hold specificatively. First, there is the notion that what is izzed and not thinged is a constituent of the same thing izzed and thinged. Second, as we have seen, there is a way to generate specificative respects using the reduplicative analyses. Beyond this, as the syntax and semantics for the reduplicative and the specificative types differ so much, we should not expect too much agreement: in particular that a qua proposition taken reduplicatively is true specificatively. For think of the *secundum quid ad simpliciter* inference.

These results agree with the main claims made about the relationship between specificative and reduplicative propositions by the Aristotelian tradition: that the *secundum quid ad simpliciter* inference fails for the specificative but holds for the reduplicative, and that there is a (vague!) sense in which what is true reduplicatively is true specificatively. That sense has to be vague. For, strictly speaking, a reduplicative proposition still does not have the logical form of a specificative proposition. For propositions of the two types differ in their semantics especially. To give the reduplicative a common ground with the specificity, we must suppose the same semantics and take the reduplicative propositions to generate respects too. Even then, strictly speaking, the *secundum quid ad simpliciter* inference fails, in the case of reduplicative propositions taken so as to generate respects, for predicates that are not logically affirmative.

Thus I have presented a general model for qua propositions. It remains to show how useful it can be.