Philosophical Perspectives, 20, Metaphysics, 2006

A DEFINITION OF NECESSITY

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In the history of philosophy, especially its recent history, a number of definitions of necessity have been ventured. Most people, however, find these definitions either circular or subject to counterexamples. I will show that, given a broadly Fregean conception of properties, necessity does indeed have a noncircular counterexample-free definition.

The argument has the following surprising consequence: a proposition is necessary iff it has a proof using only definitions and standard logical principles (for short, 'true by definition and proof'). In other words, necessity coincides with strong analyticity (the property of being true by definition and proof).[1] This generates two telling philosophical puzzles. First, it seems to collide with the highly plausible thesis that there are synthetic necessities (principles of supervenience, transitivity, incompatibility, etc.). Second, it seems to validate Frege's strong logicist thesis, but it is widely believed that this thesis is refuted by Gödel's First Incompleteness Theorem. Although these puzzles have a straightforward resolution in terms of a neglected distinction between intrinsic and extrinsic analyticities, the indicated equivalence of necessity and truth by definition and proof does necessitate some redrawing of the received epistemological map.

Three preliminary points. First, I will assume that possibilism and Meinongianism are mistaken. The framework in which I will be working will be actualist: everything there is actually exists and, accordingly, there are no nonactual objects and no objects that fail to exist. Second, I will assume a commonsense conception of facts, which is characterized by the following necessary truth: there is such a fact as the fact that P iff P is true.[2] Third, those in doubt about any of my background assumptions may take my main conclusions conditionally: *if* these assumptions are correct, *then* so are the conclusions. Taken this way, the outcome is still surprising; after all, a great many advocates of the indicated ontology think that necessity must be taken as a primitive.

Here is the game plan. After discussing the broadly Fregean conception of properties and some accompanying notions (§1), I give the definition and establish its correctness (§2). I then defend the indicated conception of properties (§3) and respond to criticisms of the definition (§4). Finally, I show how to resolve the aforementioned puzzles (§5).

1. Properties

As I indicated, our definition of necessity will be formulated in the setting of a broadly Fregean conception of properties. By this, I mean a conception on which properties are logical objects in the sense that they are neither physical nor psychological entities but are instead members of a third category whose existence is independent of all physical and psychological contingencies. For this reason, properties exist necessarily.[3]

Not only are the arguments for the broadly Fregean conception very compelling (see §3), but the majority of actualists in metaphysics, philosophy of language, and intensional logic are, I believe, committed to such a view at least implicitly. Consider, for example, those actualists (e.g., Robert Stalnaker, Peter Forrest, and Scott Soames) who identify possible worlds with "maximally specific ways the world could have been" and who identify "ways" with properties the world could have had (or states the world could have been in).[4] On the usual understanding, all these world-properties exist in the actual world and, indeed, exist regardless of which world-property is instantiated.[5] (This is no mere accident, but rather is required for world-properties to play their intended role of backing all modal truths, as I will explain in §3.) If all world-properties exist regardless of which world-property is instantiated, they would seem to exist necessarily. And if this is true of these properties, uniformity evidently demands that the same be said of all other properties as well, thus leading to the broadly Fregean conception.[6] For parallel reasons, the analogous thing evidently holds for actualists who develop their modal metaphysics in terms of (i) states of affairs (e.g., Roderick Chisholm and Alvin Plantinga) or (ii) maximal sets of compossible propositions (e.g., Robert Adams and Nathan Salmon). I will return to all these points in §3.

We will be concerned with a certain special class of properties called *identity properties.* An identity property is a property that by nature identifies its unique instance. Specifically, F is an identity property iff_{def} for some x, F = the property of being identical to x. In symbols: F is a identity property iff_{def} $(\exists x)\ F = [v: v = x]$. (Since an entity's identity property is its individual essence, we could adopt the following definition: F is an *individual essence* of x iff_{def} F = the property of being identical to x.)

Identity properties belong to a more general class of *essential individuating properties* which are distinguished, not by the fact that they identify an actual thing, but rather by the possibility of their identifying something. Specifically, F is an essential individuating property iff it is possible that, for some x, F = the property of being identical to x. In symbols: F is an essential individuating property iff_{def} $\Diamond(\exists x)\ F = [v: v = x]$. On the broadly Fregean conception, a great many of these properties need not, and indeed do not, have instances; they are distinguished by the possibility of their being an identity property for an object.[7] As a class, essential individuating properties serve significant theoretical roles in logic and metaphysics, and this is an important part of the justification of the broadly Fregean conception. For example, essential individuating properties are the sort of property that would serve to distinguish between the qualitatively identical spheres in Max Black's famous hypothetical case.[8]

The algebraic approach to intensional logic provides a logically perspicuous way of characterizing identity properties.[9] This approach begins with truisms like the following. The proposition that P & Q is the result of conjoining P and Q—that is, it is the value of the operation of conjunction applied to P and Q as arguments. Symbolically, conj(P, Q). The singular proposition that Fx is the result of predicating F of x—that is, the result of applying the predication operation to F and x. Symbolically, pred(F, x). Similarly for singular properties: for example, the property of bearing relation R to x is the result of applying the predication operation to R and x. Symbolically, pred(R, x). Thus, the property of loving x is the result of predicating the loving relation of x. That is, the property of loving x = pred(loving, x). Analogously for the identity properties: the property of being identical to x is the result of predicating the identity relation of x. That is, the property of being identical to x = pred(identity, x).

Our focus here will be on identity properties (i.e., individual essences) that, not only do have instances, but must have instances. More specifically, we will be concerned with the identity properties of facts—fact-identity properties—that not only are instantiated, but must be instantiated. In other words, our focus will be on fact-identity properties (fact-essences) of necessarily existing facts.[10]

Let x = the fact that P. Then, on the algebraic approach, we have the following identities:

> The property of being identical to x = pred(identity, x) = pred(identity, the fact that P).[11]

In what follows, the third way of referring to the indicated fact-identity property will play a pivotal role. In this connection, I will proceed on the supposition that the fact-abstract 'the fact that P' is a definite description.[12] And I will assume that definite descriptions have *a* use that is weakly Russellian such that: 'There exists such a thing as the present King of France' is provable from 'The present King of France is bald'. This is the use that will be operative in what follows.

2. The Definition

The key to our definition will be the match-up between the existence conditions of properties (i.e., they exist necessarily) and the existence conditions of necessary facts (i.e., such facts exist necessarily).[13] Specifically, P is necessary iff the existence conditions of the fact that P are necessarily the same as the existence conditions of the identity property of the fact that P.[14] Of course, we need a way to capture this necessary match-up of existence conditions without this circular appeal to necessity on the right-hand side of this biconditional. As we will see, this is accomplished by invoking the notion of definition as follows. There is a necessary match-up of these existence conditions iff the indicated fact-identity property is definable in a certain canonical way. Thus, P is necessary iff the identity property of the fact that P has the indicated sort of canonical definition. The correlation between P's necessity and the existence of such a definition underwrites our desired result—namely, that P is necessary iff P is strongly analytic.

I will now show that this biconditional is counterexample-free. This will require three preliminary steps dealing with: narrow-scope definite descriptions and fact-abstracts; necessary coexistence of definiendum and definiens; and canonical definitions of the identity properties of necessary facts.

(a) Narrow-scope definite descriptions and fact-abstracts

In this subsection, I will make some observations about the logic of narrow-scope occurrences of definite descriptions (and fact-abstracts) in modal contexts. First, the following is an elementary logical truth: if it is contingent that there is such a thing as the most frequently mentioned number, it is contingent that there exists such a thing as the successor of the most frequently mentioned number. Schematically: if it is contingent that there exists such a thing as the ϕ, then it is contingent that there exists such a thing as f(the ϕ).[15] The contrapositive of this conditional is the following: if it is not contingent that there exists such a thing as f(the ϕ), it is not contingent that there exists such a thing as the ϕ.

We also have the following logical truth: if it is impossible that there exists such a thing as the ϕ, then it is impossible that there exists such a thing as f(the ϕ). The contrapositive of this is: if it is not impossible that there exists such a thing as f(the ϕ), it is not impossible that there exists such a thing as the ϕ.

From these two contrapostives it follows that: if it is neither contingent nor impossible that there exists such a thing as f(the ϕ), then it is neither contingent nor impossible that there exists such a thing as the ϕ. Given that something is necessary iff it is neither contingent nor impossible, this conditional implies that: if it is necessary that there exists such a thing as f(the ϕ), then it is necessary that there exist such a thing as the ϕ.

Now let 'the ϕ' be instantiated by 'the fact that P' and 'f( )', by 'pred(identity,)'. Then the last conditional yields:

> If it is necessary that there exists such a thing as pred(identity, the fact that P), then it is necessary that there exists such a thing as the fact that P.

Since the foregoing has been wholly general, this holds independently of P's modal status (necessary, contingent, or impossible).

(b) Necessary coexistence of definiendum and definiens

In this subsection, I make a similar point about narrow-scope occurrences of fact-abstracts in *definition* contexts.[16] We start with the following logical truth: if it is true by definition that $\alpha = \beta$ and it is necessary that there exists such a thing as α, then it is necessary that there exists such a thing as β. So, in particular: if it is true by definition that F = β and it is necessary that there exists such a thing as F, then it is necessary that there exists such a thing as β. According to the broadly Fregean conception of properties, for every property F, it is necessary that F exists. So on this conception, it follows that: if it is true by definition that F = β, then it is necessary that there exists such a thing as β. Now let 'pred(identity, the fact that P)' instantiate β. Then we have:

> If it is true by definition that F = pred(identity, the fact that P), then it is necessary that there exists such a thing as pred(identity, the fact that P).

Taken together, this and the previously indented conditional imply the following:

> If it is true by definition that F = pred(identity, the fact that P), then it is necessary that there is such as thing as the fact that P.

As before, this holds independently of whether P is necessary, contingent, or impossible.

Since the foregoing is wholly general, the last indented principle implies that: for all F, if it is true by definition that F = pred(identity, the fact that P), then it is necessary that there is such as thing as the fact that P.[17] Since 'F' occurs free only in the antecedent, the principle implies (by quantifier logic) that: if, for some F, it is true by definition that F = pred(identity, the fact that P), then it is necessary that there is such as thing as the fact that P. For brevity, I will abbreviate this principle as follows:

(1) If, for some F, F $=_{\mathrm{def}}$ pred(identity, fact-P), then it is necessary that fact-P exists.

In symbols,

> If ($\exists$F) F $=_{\mathrm{def}}$ pred(identity, fact-P), then $\Box$ fact-P exists.

Using this sort of formulation in what follows will make it clear that we are dealing with definitions of fact-identity properties themselves.

(c) Canonical definitions of the identity properties of necessary facts

What I want to show next is that the converse of (1) holds as well. The key to this will be that the identity properties of necessary facts always have definitions that are of a certain canonical form. To show the acceptability of such definitions, I will make use of the following principle:

(2) If it is necessary that there exists such a thing as the fact that P, then for some property F, it is necessary that F = pred(identity, the fact that P).

This follows directly from the broadly Fregean conception of properties plus the following three points. First, it is necessary that everything have an identity property. Second, fact-abstracts ('the fact that P') are rigid designators—just as proposition-abstracts ('the proposition that P') are rigid designators.[18] Third, the predication operation has a rigid extension: for all u, v, and w, if pred(u, v) = w, then necessarily, if u, v, and w exist, then pred(u, v) = w.

Now choose some fact, say, fact-P. Let F be fact-P's identity property. Then, as we have seen, the following is a correct algebraic analysis of F: F = pred(identity, fact-P). Suppose that it is necessary that fact-P exists. Then, this algebraic analysis provides a basis for an acceptable definition of F:

(3) F $=_{\text{def}}$ pred(identity, fact-P).

That this is an acceptable definition can be established by means of the following two steps. The first has to do with two standard modal requirements on definitions (the first of which we have already discussed): (i) if $\alpha =_{\text{def}} \beta$, then it is necessary that α and β coexist (i.e., it is necessary that α exists iff β exists); (ii) if $\alpha =_{\text{def}} \beta$, then it is necessary that, if α and β both exist, then $\alpha = \beta$. To see that (3) satisfies (i), recall that, by hypothesis, it is necessary that fact-P exists. So it follows by (2) that, for some F, it is necessary that F = pred(identity, fact-P). And, in our broadly Fregean setting, any given property F is such that, necessarily, F exists. Therefore, given that 'pred(identity, fact-P)' is a rigid designator, it follows that, necessarily, both F and pred(identity, fact-P) exist. So it is necessary that F and pred(identity, fact-P) coexist, as (i) requires. As for (ii), since by hypothesis it is necessary that fact-P exists, it follows by (2) that there exists an F such that: it is necessary that F = pred(identity, fact-P). From this it follows (by modal propositional logic) that: it is necessary that, if F and pred(identity, fact-P) both exist, then F = pred(identity, fact-P). Thus, (3) satisfies (ii).[19]

The second step has to do with the form of an acceptable definition. The point is that (3) is based on a logically perspicuous algebraic analysis of F and is formulated in canonical terms (e.g., fact-abstracts are our canonical idiom for referring to facts). As long as (i) and (ii) are satisfied, a logically perspicuous analysis of a property formulated in canonical terms provides a basis for an acceptable definition of the property if there is not a superior candidate definition and if there is not some unacceptable circularity. But there is no candidate definition that is superior to (3).[20] Nor does (3) involve any unacceptable circularity.[21] Hence, (3) counts as an acceptable definition of F.

So, on the assumption that it is necessary that fact-P exists, (3) is an acceptable definition. Thus, we have:

(4) If it is necessary that fact-P exists, then for some F, F $=_{\text{def}}$ pred(identity, fact-P).

(d) The definition of necessity and its correctness

We are just about ready to state our definition of necessity. Putting the last indented conditional together with (1), we get the following biconditional:

(5) It is necessary that fact-P exists iff, for some F, F $=_{\text{def}}$ pred(identity, fact-P).

At the same time, as mentioned above, on the commonsense view of facts we have the following necessary truth: P is true iff fact-P exists. Hence, it follows (by two applications of the K axiom) that both sides may be necessitated: necessarily P is true iff necessarily fact-P exists. From this and (5), it follows that: necessarily P is true iff, for some F, F $=_{\text{def}}$ pred(identity, fact-P). Then, from this, we may infer that:

(6) P is necessary iff, for some F, F $=_{\text{def}}$ pred(identity, fact-P).[22]

That is, P is necessary iff there is a fact-identity property F such that F is, by definition, fact-P's identity property. Or, put more informally in terms of individual essence (as defined in §1): P is necessary iff fact-P's individual essence is definable in our standard way.

Of course, not just *any* entity is such that its existence is provable from a definition of its identity property (individual essence); this can be done, however, for entities having canonical descriptions that are at once rigid and necessarily nonvacuous. The remarkable thing about fact-abstracts is that, although they all have the first feature, they have the second feature (necessary nonvacuity) iff the proposition corresponding to the fact is necessary. For this reason, a fact-abstract may occur as it does in our canonical definition of the fact's identity property iff the proposition corresponding to the fact is necessary, thus assuring the correctness of (6).[23]

Since (6) provides counterexample-free necessary and sufficient conditions for P's being necessary, we could base a definition on it: P is necessary iff$_{\text{def}}$ for some fact-identity property F, F $=_{\text{def}}$ pred(identity, fact-P).

But we are now in position to give an alternate definition formulated in terms of strong analyticity (see note 1): Q is strongly analytic iff Q is true by definition and proof. More precisely: Q is strongly analytic iff Q has a proof from some set Γ of definitions.[24] From (6), we can prove the following:

P is necessary iff P is strongly analytic.

Proof. For the left-to-right direction, suppose that P is necessary. By (6), this implies that, for an appropriately chosen F, there is a correct definition: F $=_{\text{def}}$

pred(identity, fact-P). Our goal now is to show that P is strongly analytic—that is, that P has a proof using only definitions and standard logical principles.[25] The envisaged proof will make use of just one definition, namely, F $=_{\text{def}}$ pred(identity, fact-P). This is to be the first line of the proof. Next, using steps parallel to those in our earlier argument for (1),[26] we write down a proof of the nonmodal version of (1): for all properties G, if G $=_{\text{def}}$ pred(identity, fact-P), then fact-P exists. Instantiating on G, we get: if F $=_{\text{def}}$ pred(identity, fact-P), then fact-P exists. From this and the definition we took as a premise on the first line of the proof, we infer (by modus ponens) that fact-P exists. Finally, we infer P (by modus ponens) from this and the standard principle that, if fact-P exists, then P. Thus, there is a definition—F $=_{\text{def}}$ pred(identity, fact-P)—from which P is provable. So there is a set of definitions Γ such that Γ ⊢ P. Hence, by the definition of strong analyticity, P is strongly analytic.

For the right-to-left direction, suppose that P is strongly analytic. That is, for some set of definitions Γ, Γ ⊢ P. In the simplest case, Γ contains just one definition σ. We may suppose this without loss of generality.[27] Hence, $\sigma \vdash$ P. By conditionalization, $\vdash \sigma \rightarrow$ P. Since that which is provable is necessary, we may infer that $\Box$ ($\sigma \rightarrow$ P).[28] Now since definitions hold necessarily (e.g., if $\alpha =_{\text{def}} \beta$, then $\Box\, \alpha =_{\text{def}} \beta$) and σ is a definition, we may infer that $\Box\, \sigma$. Therefore, given that $\Box$ ($\sigma \rightarrow$ P) and $\Box\, \sigma$, it follows (by the K axiom) that $\Box$ P.

This proof, since it ensures that there are no counterexamples, provisionally justifies the following:

(7) P is necessary iff$_{\text{def}}$ P is strongly analytic.

3. Defense of the Broadly Fregean Conception

In a moment I will discuss the philosophical significance of the definition. Before doing so, however, I should say more on behalf of the broadly Fregean conception of properties and respond to some criticisms of the definition.

(a) Ersatz possibilia

The broadly Fregean conception yields a particularly neat actualist method for dealing with possible individuals, possible facts, and possible worlds. As I said earlier, the key idea underlying the definition was the perfect match-up between the existence conditions of necessary facts and their actualist identity properties. When I introduced the notion, I stipulated that F is an identity property iff, for some x, F = the property of being identical to x. And I said that identity properties belong to a more general class of *essential individuating properties* that are distinguished, not by the fact that they identify an actual thing, but rather by the possibility of their identifying something: F is an essential individuating property iff$_{\text{def}}$ $\Diamond(\exists x)$ F = [v: v = x]. Given such properties, instead of talking about possible but nonactual individuals, we may talk about associated actualist essential individuating properties. Likewise, instead of talking about possible but nonactual facts, we may talk about associated actualist fact-individuating properties. In particular, the possibilist idiom of possible worlds may give way to the idiom of possible maximal facts, and the latter idiom may in turn give way to talk about associated actualist maximal-fact individuating properties.[29] The point is that, by exploiting the broadly Fregean conception together with our framework of facts, we can reap the benefits of a possibilist ontology in a thoroughgoing actualist setting.

(b) From actualism to the broadly Fregean conception

As indicated in §1, I believe that a great many contemporary metaphysicians, philosophers of language, and intensional logicians are committed to the necessary existence of properties, at least implicitly. I will now explain why I take them to be committed to the necessary existence of essential individuating properties, in particular.

First, consider, as before, those actualists who would have us reject possible worlds in favor of "ways the world could be"—and who identify ways the world could be with properties the world could have. This sort of "world-property" would just be a lofty example of the sort of properties posited in the broadly Fregean conception. For example, amongst these world-properties, there would have to be world-properties F_w and F'_w corresponding to worlds w and w′ which differ from one another only in the identity of certain qualitatively identical individuals existing in them. For example, worlds w and w′ might be exactly alike except that some individual x exists in w whereas a distinct but qualitatively identical twin x′ exists in w′. On the present approach there would need to be world-properties that mark the nonqualitative distinction between x and x′. If there are such world-properties, however, uniformity plainly demands that there be analogous (broadly Fregean) essential individuating properties F and F′ that do nothing but distinguish x and x′ themselves. And given the existence of these broadly Fregean essential individuating properties, uniformity once again demands that we also accept the sort of fact-individuating properties upon which our definition turns.

Second, and for similar reasons, actualists who would have us replace possible worlds with maximal sets of compossible propositions (Adams, Salmon, etc.) are committed to such essential individuating properties as well. These actualists are committed to the necessary existence of propositions p and p′ that differ from one another only in the identity of certain qualitatively indistinguishable individuals x and x′ that they are about. If there are propositions like this, surely there are also associated essential individuating properties F and F′ like those just mentioned. Finally, actualists who embrace possible states of affairs (e.g., Chisholm and Plantinga) are committed to states of affairs s and s′ that differ from one another only in the identity of certain qualitatively indistinguishable individuals x and x′ that exist in them. If there are states of affairs like this, surely there are also the associated essential individuating properties F and F′.[30]

(c) "Parts" of properties?

Perhaps the greatest resistance to the broadly Fregean conception—and in particular to necessarily existing identity properties—may be traced to the naive idea that various properties literally (not just metaphorically) have *parts* in the mereological sense (or *elements* in the set-theoretic sense). According to this idea, there are no essential individuating properties beyond those that have *actual* instances because these actual instances would need to be parts (elements) of those identity properties. This, however, violates Frege's admonition in "Gedankengefüge" that "[W]e really talk figuratively when we transfer the relation of whole and part to thoughts [and other intensional entities]." Obviously, properties do not *literally* have parts. To speak of parts of properties is only a metaphor and a misleading one at that.

On the algebraic conception this naive picture is easily avoided. For example, the relation between an entity x and its identity property [v: v = x] is not that of part to whole; rather, [v: v = x] is simply the value of the predication operation applied to identity and x. Even though (for contingent x) the operation of predication maps the identity relation and the object x to this property, it does not follow that the existence of this property *entails* the existence of x. Consider an analogy: the father-of function maps me to my father. However, the existence of my father does not entail the existence of me; I am certainly not a part (or element) of my father.[31]

(d) Transmodal arguments for the broadly Fregean conception

Finally, the denial of the broadly Fregean conception is fraught with difficulties concerning *transmodal* truths.[32] For example, there could be an object x and there could be an object y such that it is impossible for x and y to coexist. That is, $\Diamond(\exists x)\Diamond(\exists y)\Box$ x and y do not coexist. This entails: there could be an object x and there could be an object y such that, necessarily, something (e.g., the number 1) has the property of being such that x and y do not coexist. That is, $\Diamond(\exists x)\Diamond(\exists y)\Box(\exists z)$ z has the property of being such that x and y do not coexist. I can see no way actualists can account for this modal truth without invoking the broadly Fregean conception. For example, on the Aristotelian in re view of properties, it is impossible for there to be a world in which the indicated property exists, for if there were such a world, this view implies that necessarily non-coexistent objects would coexist in that world. But if the indicated property exists in no possible world, it is simply impossible for it to exist.[33]

Taken together, these four considerations provide us with good reasons for accepting the broadly Fregean conception.

I close this section with a qualification (cf. note 22). Using considerations akin to the foregoing, we can, I believe, show that propositions conform to a broadly Fregean conception.[34] But suppose this is mistaken. In particular, suppose that there are necessarily true propositions P that have contingent entities as "constituents" and that, necessarily, if any of these entities do not exist, P does not exist.[35] Then, presumably, it would be necessary that, if any of these entities do not exist, fact-P does not exist either. In this case, the definition should be reformulated as follows. There is a straightforward procedure with which, for every proposition P having "constituents" that are not properties, relations, or propositions, one can specify an equivalent proposition P* "formed" from P but not having these entities as "constituents" and having their identity properties instead.

For example, if P is the proposition that Fx (where x is, say, some particular) and G = the property of being identical to x, then P* would be the proposition that, for some y, Gy & Fy. Given this, we may replace (6) with: P is necessary iff, for some fact-identity property F, F = pred(identity, fact-P*). And we may replace (7) with: P is necessary iff_{def} P* is strongly analytic.

4. Criticisms

In this section I will address some criticisms of the definition. I should note, however, that even if one or more of these worries were left standing, this would not affect my concluding discussion of the significance of our conclusion that all necessities are strongly analytic.

(1) Our definition relies exclusively on *logical* notions: identity, predication, property, proposition, fact, definition.[36] Some people, however, might reject out of hand any use of the notion of definition. But this would be just idle skepticism. The use of the notion of definition is commonplace in every discipline (as any descriptive survey would show). So we should feel free to employ it here absent some special reason to doubt its legitimacy. Most people are unconvinced by Quine's attack on the very possibility of definitions (i.e., nonstipulative definitions). One reason, I believe, is the robustness and intuitive cogency of the notion: it just seems incredible that there really could never be a correct definition of anything. The other reason is that Quine provides no serious argument for this radical nihilist view; and insofar as he provides any argument at all, it seems to be based on the mistaken supposition that good definitions require synonymy.

(2) Even granting this, however, someone might object to employing the notion of definition in a definition of necessity on the grounds that, since definitions have modal consequences (notably, schemas (i) and (ii) from above), the notion of definition should itself be counted as a modal notion, thereby rendering our definition circular. This worry, however, is unfounded. The mere fact that the notion of definition has modal consequences does not imply that the notion of definition is a modal notion: after all, the notion of being a tautology has modal consequences (if it is a tautology that P, it is necessary that P), but the notion of a tautology is clearly not a modal notion (it is definable by means of truth tables alone). Furthermore, the notion of definition is itself not definable in terms of necessity.[37] So our definition does not take us round in a circle. Further

support for this assessment comes from the fact that, historically, virtually no one thought that the definition of necessity as analyticity failed because it was circular.

(3) In a related vein, some people might object to our definition on the alleged grounds that someone who does not already understand the concept of necessity could not come to understand it just by reflecting on the definition. I do not find this at all obvious, but even if it were true, I do not see why it should threaten our definition. I know of no requirement that a correct definition of necessity be able to impart an understanding of modal concepts to people who lacked them (just as there is no requirement that a correct definition of experiencing red should be able impart genuine understanding to people who lack it).

(4) Someone might allege that the definition is unacceptably circular for a different reason: it relies on the modal features of certain entities—namely, the necessary existence of properties and of necessary facts. True, in *justifying* the definition, we made free use of the notion of necessity, which is the very notion being defined. But this way of proceeding is entirely standard, as the following analogy makes clear. Consider a standard way of defining the notions of finite and infinite set. We begin by inductively defining a certain set S: S is the smallest set such that (i) the null set belongs to S and (ii), for every set s belonging to S, the successor of s (i.e., the union of s and {s}) belongs to S. Now, by design, S has infinitely many elements, each of which is a set having finitely many elements. Consequently, we have the following definitions: a set is finite iff it is equinumerous with an element of S; and a set is infinite iff it, or one of its subsets, is equinumerous with S. These definitions are correct precisely because S's elements are finite and S is infinite—the very properties being defined. But the definitions are noncircular, for S is specified (vs. justified) without invoking the notions of finite set or infinite set. Our definition of necessity is wholly analogous in this respect.

(5) A final concern about our definition is that it is not *explanatory* in that it does not tell us what makes a necessary proposition necessary. This is curious criticism. For in the history of philosophy the definition of necessity as analyticity was never questioned on the grounds that it was not explanatory; quite the contrary, it was thought to be a particularly illuminating definition. The supposed problem with the definition was that it was subject to counterexamples (i.e., synthetic necessities). The proof at the close of §2, however, shows that it is not. (The discussion in the next section explains how we should think about the traditional candidate counterexamples.) But even if in its present form the proposed definition is not suitably explanatory, perhaps the following reformulated definition is.

Consider the analogy between truth and necessity. Just as P's truth is explained by the existence of the fact that P, P's necessity is explained by the *necessary* existence of the fact that P. True, but what explains the necessary existence of the fact that P? Perhaps the following does:

fact-P exists necessarily iff this fact's existence is strongly analytic.

More specifically, fact-P exists necessarily iff its existence is provable from a definition of its identity property (i.e., its individual essence). This does seem explanatory. This suggests reformulating the definition as follows:

P is necessary iff_{def} fact-P's existence is strongly analytic.

More specifically, P is necessary iff_{def} fact-P's existence is provable from a definition of its identity property.[38]

For the purposes of the next section, we need not judge whether this definition counts as explanatory in the above sense (or, indeed, whether a satisfactory definition must meet this requirement). Nor, for that matter, need we judge whether any of the other four criticisms is correct. All that will be required is that we have established that P is necessary iff P is strongly analytic. Given the legitimacy of broadly Fregean conception, I take it that we have.

5. Import

It is surprising that every necessary truth should have an easy proof from definitions and standard logical principles. This conclusion gives rise to two puzzles, whose solution will help to elucidate the import of the definition.[39]

(1) *Naming and Necessity* led us to redraw the traditional epistemological map: two categories previously thought to be empty—the necessary a posteriori and the contingent a priori—are now widely believed to be nonempty and, indeed, to have very important instances. These developments, however, did not call into question the traditional view that there are synthetic (i.e., nonanalytic) necessities: for example, various supervenience principles, transitivity principles, and incompatibility principles. This view is intuitively very plausible, yet our definition implies that it is mistaken. Our first puzzle is to resolve this apparent conflict.

(2) In the *Grundgesetze* Frege sought to show that the truths of arithmetic were analytic—that they held by logic plus definition. This thesis has a strong and weak reading, yielding a strong logicist thesis and a weak logicist thesis. On the weak (i.e., semantical) reading, the thesis is that every truth of arithmetic is weakly analytic—that is, a *logical consequence* of a set of premises containing only definitions and standard logical axioms. On the strong (proof-theoretic) reading, the thesis is that every truth of arithmetic is *strongly* analytic—that is, *provable* using as premises only definitions and standard logical axioms. Most philosophers of mathematics recognize that the weak logicist thesis holds (waiving worries about the definability of standard arithmetic notions, e.g., Benacerraf-style worries), for in a *higher-order* setting the truths of arithmetic are weakly analytic. Gödel's First Incompleteness Theorem does *not* refute this weak logicist thesis. It is, however, nearly universally believed that Gödel's Theorem does refute the *strong* logicist thesis—that every truth of arithmetic is provable using only definitions and standard logical axioms. Our result, however, shows that this

pretty much universally held view is mistaken: since every truth of arithmetic is a necessary truth, every truth of arithmetic is provable using only definitions and standard logical axioms; and, therefore, Gödel's Theorem does not refute the strong thesis. Our second puzzle is to explain where this widely held opinion goes wrong and to identify the true thing that underlies this opinion.

(a) Puzzle 1: Intrinsic and Extrinsic Analyticity

It certainly seems that many necessary facts provide information that "no definition could extract." But we have shown that, for every necessary fact (e.g., concerning supervenience, transitivity, incompatibility), its existence is provable using only a definition and standard logical principles. How can this be?

The result turns on the definitions of certain properties. But these properties are not themselves "constituents" of the necessary facts at issue. That is, the result depends upon definitions of properties that are, not themselves "intrinsic" to the necessities at issue, but rather "extrinsic" to those necessities. P is *intrinsically* analytic iff P is provable from definitions of constituents of P. P is *extrinsically* analytic iff P is provable from definitions of nonconstituents of P. This is a distinction that was (to my knowledge) overlooked in the traditional picture. Traditionally, analyticity was defined as we have defined it—true by definition or true by definition plus logic (proof)—or else it was defined in a way that was believed to be equivalent to this. At the same time, all the traditional examples of analytic necessities were intrinsic analyticities, and all the traditional examples of synthetic necessities (principles of supervenience, transitivity, incompatibility, etc.) were nonintrinsic analyticities. What we have shown, however, is that all necessities are extrinsically analytic—whether or not they are intrinsically analytic. For, specifically, all necessities are provable from definitions of the identity properties of the necessary facts to which they correspond.

Given the intrinsic/extrinsic distinction, there might be a temptation to think that the coincidence of extrinsic analyticity and necessity is just a logical fluke—a mere "chance necessity," as Aristotle would say. But this would be a mistake. We saw in the preceding paragraph that the "extrinsic" properties on whose definitions our result rides are hardly related to the associated necessary facts by "chance necessity." On the contrary, as we have seen, the definitions that guarantee the existence of necessary facts are definitions of the individual essences of those facts, namely, their identity properties. By their very nature, necessary facts have individual essences like this; contingent facts do not.

The fact that all necessary truths are true by definition and proof does not mean that necessary truths are any easier to discover than we originally thought. The reason is that, to know that there is a property F such that, by definition, F is the result of predicating identity of the fact that P, one must first know that it is necessary that there is a fact that P, and, to know this, one must already know that P is necessary—which in turn requires prior knowledge that P is true. At least, this is the order of discovery and justification available to beings with minds like ours.

Consider an example. According to Kripke's epistemology, the following is the path by which we come to know that necessarily water = H_2O. Using an a priori argument (e.g., Kripke's), one comes to know that, if water = H_2O, then necessarily water = H_2O. This (or related) a priori knowledge is then combined with the a posteriori knowledge that water = H_2O (or related a posteriori knowledge), thus yielding (by easy inference) the a posteriori knowledge that necessarily water = H_2O. Now consider how we come to know that, for some F, F $=_{\text{def}}$ pred(identity, the fact that water = H_2O). The path of discovery and justification is parallel to that just described but with an additional cycle. Using an a priori argument (like that provided in §2), one comes to know that, for all P, if P is necessary, then for some F, F $=_{\text{def}}$ pred(identity, the fact that P). This (or related) a priori knowledge is then combined with the a posteriori modal knowledge that necessarily water = H_2O (or some related a posteriori knowledge), thus yielding (by easy inference) the a posteriori knowledge that, for some F, F $=_{\text{def}}$ pred(identity, the fact that P). Of course, to acquire the requisite a posteriori modal knowledge that necessarily water = H_2O, the original Kripkean path (by way of a priori philosophical knowledge and a posteriori chemical knowledge) needs to be followed as well. Thus, just as the knowledge that necessarily water = H_2O requires a combination of a priori and a posteriori knowledge, so does the knowledge that, for some F, F $=_{\text{def}}$ pred(identity, the fact that water = H_2O). It follows (for the relevant property F) that it cannot be known purely a priori that F $=_{\text{def}}$ pred(identity, the fact that water = H_2O). In other words, the very methodology used to establish that some necessities are a posteriori also leads to the conclusion that some definitions are a posteriori.[40]

(b) Puzzle 2: Logicism, Extrinsic Analyticity, and Gödel's Incompleteness Theorem

The foregoing helps to solve our second puzzle. Let P be an arbitrary arithmetic truth. Since P is necessary, there is a property F $=_{\text{def}}$ pred(identity, fact-P). From this definition and standard logical principles, P is provable. By generalization on P, the strong logicist thesis follows: every arithmetic truth has a proof whose only premises are definitions and standard logical principles. So assuming no slips have been made, the strong logicist thesis must be consistent with Gödel's Theorem.

Gödel's Theorem does, however, imply that there are arithmetic truths that are not provable from definitions of standard arithmetic properties (natural number, successor, less than, addition, multiplication) plus standard logical axioms. Thus, Gödel's Theorem implies that there are truths of arithmetic that are not *intrinsically analytic*. But it is simply silent about whether there are any truths of arithmetic that fail to be *extrinsically* analytic. Since our result depends upon definitions of properties "extrinsic" to arithmetic truths—namely, definitions of fact-identity properties associated with those truths—there is no collision with Gödel's Theorem.[41]

We have been dealing with the provability of *individual* arithmetic truths (i.e., arithmetic truths considered one at a time). We now turn to the question of whether there are any *systems* of axioms in which *all* truths of arithmetic are provable. Gödel showed that there are not—specifically, that there can be no consistent system that has a recursive set of premises (axioms) from which all the truths of arithmetic are provable.[41] Now when we adjoin a definition to a theory, let us think of the definition as just one more premise (axiom) of the theory. What our result shows, then, is that every arithmetic truth is provable from a consistent set of premises consisting of a standard (and, hence, recursive) set of logical axioms plus a set of definitions of the form: F $=_{\text{def}}$ pred(identity, the fact that P). This does not contradict Gödel's Theorem. On the contrary, taken together, Gödel's Theorem and this result imply that each such set of definitions is nonrecursive.

This outcome is not really surprising, for, as we saw at the close of the previous subsection, the fact that all necessities are true by definition and proof does not improve our epistemic situation with respect to them. In the present case, this is manifest in the familiar idea that beings with minds like ours are unable to determine the membership of nonrecursive sets. With regard to the nonrecursive set in question, in order to decide whether or not a candidate definition of the indicated form is included in this set, a being with a mind like ours would typically already have to know whether P is true (and, indeed, whether P is necessary). In other words, in order to decide the membership of the set of definitions in question, one must first be in a position to decide the truths of arithmetic.

In summary, the fact that all necessities are true by definition and proof does necessitate some redrawing of the received epistemological map in terms of the notions of intrinsic and extrinsic analyticity and the distinction between recursive sets of definitions and sets of definitions simpliciter. Of course, the primary import of the discussion as a whole lies in its definition of necessity: if it is correct, modal notions are definable in nonmodal terms.[42]

Notes

1. I will be employing the standard inclusive use of 'definition' which applies to definitions of properties as well as definitions of concepts. I use 'strong analyticity' as shorthand for 'true by definition and proof', where 'definition' is understood in this inclusive way. I should also note that by 'logical principles' I will mean pure, nonindexical logical principles (cf. note 24).
2. This biconditional is necessary on the reductive view that facts are either obtaining states of affairs or true propositions. On this view, the biconditional in the text

would be equivalent to 'There is such a thing as the obtaining state of affairs that P iff P is true' or 'There is such a thing as the true proposition that P iff P is true'. The biconditional is also necessary on the nonreductive view that facts are primitive entities. According to this view, contingent facts are like events in that their existence is contingent; necessary facts exist necessarily, and there simply can be no such things as "impossible facts." I believe there are good reasons to accept the nonreductive view (cf. note 29). I also think readers will find the definition more illuminating when considered in the context of the nonreductive view (cf. §4.5).

3. That is, for all properties F, it is necessary that F exists. In fact, it will be enough for our purposes that identity properties of necessary facts exist necessarily.
4. For example, Scott Soames tells us (p. 23, *Beyond Rigidity: The Unfinished Semantic Agenda of Naming and Necessity*, New York: Oxford University Press, 2002):

   As Kripke understands the notion, a possible world is not another universe; rather, it is a way the universe could have been. Following him, I take a possible world to be a maximally complete property that the universe could have had (instantiated). The actual world is also such a property; it is a maximally complete property that the universe does have. To say that a proposition p is true in (or at, or with respect to, or according to) a possible world w is to say that p would have been true if w had obtained . . . Ordinary sentences containing modal notions . . . are systematically connected with truth-conditionally equivalent sentences that talk about possible worlds, but since possible worlds themselves are defined as properties the universe could have had, there is no attempt to provide a reductive analysis of ordinary modal notions in terms of nonmodal notions.

   If correct, the definition below shows that there does exist a noncircular "analysis of ordinary modal notions in terms of nonmodal notions." Thanks to Marc Moffett for pointing this out.
5. This is how most people have understood Robert Stalnaker's classic statement of the view in "Possible Worlds," *Noûs* 10, 1976: 65–75. But in a recent unpublished paper "On What There Isn't (But Might Have Been)," he proposes a "ways" view intended to have weaker commitments, although in my opinion this view runs into difficulties (see below §3.2–3.4, especially notes 31–2). In any case, he does accept that all world-properties exist in the actual world and that all of them are uninstantiated except for one, the actual world-property.
6. This is not to say that these further properties cannot be somehow reduced to (or explained in terms of) set-theoretical constructs over world-properties. Whether this is so is another question (though I am dubious). The present question is whether these further properties exist necessarily; evidently they would according to the envisaged reduction (explanation).
7. Some might worry that in order for there to exist a property of being identical to x, x itself must exist, and so if x exists contingently, the property of being identical to x also exists contingently. In §3 I show why this is mistaken. For now, suffice it to say that, if correct, this worry would apply equally to the actualist states of affairs discussed two paragraphs above.
8. Another application comes from actualist philosophy of religion: essential individuating properties are the sort of property that would enable one to explain how, prior to their creation, God could consider what contingent entities to create.

9. See, e.g., my "Theories of Properties, Relations, and Propositions" (*Journal of Philosophy* 76, 1979: 634–48).
10. An even simpler definition than the one proposed below—one which invokes neither the broadly Fregean conception nor fact-identity properties—is available to those who think that contingently existing entities (e.g., contingent facts) do not have definitions (see note 23). Incidentally, if you prefer, you may take the location 'facts exist' as shorthand for 'there are facts' and 'the fact that P exists' as shorthand for 'there is such a thing as the fact that P'.
11. It is crucial to distinguish between identity properties and descriptive properties that are contingently equivalent to them. For example, if x is the most frequently mentioned fact, the property of being identical to x and the descriptive property of being the most frequently mentioned fact are contingently equivalent but distinct. The algebraic approach helps to bring out the difference: an algebraic analysis of the descriptive property involves the property of being a fact that is more frequently mentioned than every other fact whereas an algebraic analysis of the identity property would instead involve x itself. Thus, the property of being identical to x = pred(identity, x) = pred(identity, the most frequently mentioned fact) $\neq$ the property of being identical to the most frequently mentioned fact.

    Let the most frequently mentioned fact = the fact that P. Then the property of being identical to the most frequently mentioned fact $\neq$ the property of being identical to the fact that P, for these properties are only contingently equivalent. Let x = the fact that P. Are the property of being identical to x and the property of being identical to the fact that P distinct properties? This depends on the granularity of properties—a topic on which I take no stand.
12. Instead of supposing that 'the fact that P' is a definite description, we could shift our discussion from fact-abstracts to the associated canonical definite description 'the fact to which P corresponds' (or an associated algebraic analysis of that fact). Alternatively, we could simply assume directly that fact-abstracts have weakly Russellian uses.
13. A more circumscribed match-up would serve our purposes equally well; see the closing paragraph of the next section.
14. I am using 'P' as a variable whose intended values are propositions and whose substituends are sentences. Alternatively, 'P' may be taken as a propositional variable whose substituends are 'that'-clauses (or related singular terms; cf. note 12); in this case, the sentential uses of 'P' in the text should be replaced with the formula 'P is true' (or related formula; cf. note 12). Following any one of these alternatives consistently ought to yield a sound argument in this section.
15. A note about conditionals. I am going to proceed as if they are material conditionals. But nothing turns on this. My use of 'If ϕ then ψ' may be replaced with 'not both ϕ and not ψ'; analogously for 'iff'. And 'Fx iff_{def} ϕ(x)' may be replaced with 'the property of being F $=_{\text{def}}$ the property of being ϕ'. (Note that here and certain other places I take the liberty of using single quotation marks where corner quotation marks are strictly speaking required.)

    I am assuming here that it is necessary that the function f is total (i.e., defined on everything). In what follows, 'f( )' will be instantiated by 'pred(identity,)'. This expression denotes a total function of indicated kind: since it is necessary that everything has an identity property, it is necessary that this function is defined on everything.

16. I am speaking here of objectival definitions, (see the following note).
17. Since the occurrence of 'F' in the definition in the antecedent is a wide-scope externally quantified (de re) occurrence, the indicated definition is an objectival (vs. linguistic or conceptual) definition. But, on the intended reading, 'pred(identity, the fact that P)' occurs with narrow scope within the intensional operator 'It is true by definition that' (and so occurs de dicto). This combination of wide and narrow scope (de re and de dicto) is commonplace in objectival definitions (as well as in many other familiar contexts). Of course, all these points also hold for 'F $=_{\text{def}}$ pred(identity, the fact that P).'
18. α is rigid iff the following two principles hold (for all x): first, if $\alpha =$ x then, necessarily, if x exists, $\alpha =$ x; second, if $\alpha =$ x then, necessarily, if α exists, $\alpha =$ x. (Some people invoke only the first clause, but that formulation is open to counterexamples.)
19. One might wish to add the following further requirement: (iii) if $\alpha =_{\text{def}} \beta$, then β specifies what α is essentially (its essence). But (3) satisfies this requirement as well: what F is essentially is the result of predicating identity of the fact that P.
20. This is not to say that there is no other way to define F. (I assume here that a property can have more than one correct definition. E.g., circularity may be defined in terms of equidistance from a common point and also in terms of having arcs all with equal curvature.) One candidate comes to mind. Let x be fact-P. Then the following might also be acceptable: F $=_{\text{def}}$ pred(identity, x). In this formula, the occurrence of 'x' has wide scope and as such is a de re occurrence—whereas the occurrence of 'fact-P' in (3) has narrow scope and as such is a de dicto occurrence. My point in the text is that this (or any other alternative) is not *superior* to (3). (Of course, even if there were a superior definition, that would not automatically disqualify (3) as a correct definition.)
21. This point is straightforward if F is a familiar sort of identity property that is not involved in P. If there are nonstandard identity properties F that are involved in P, then a circle would be involved. But since P is something fixed at the outset, plausibly this is not an unacceptable circle. At any rate, (3) provides a correct analysis of F (even if a circle is involved). If there is any doubt on this score that (3) qualifies as a definition it may be replaced with the following: it is true by analysis that F = pred(identity, fact-P).
22. This inference makes use of two equivalences: P is necessary iff P is necessarily true; and P is necessarily true iff it is necessary that P is true. The first is obvious. The second holds if the broadly Fregean conception applies to propositions as well as properties (for a defense, see the papers of mine mentioned in notes 31–2). If it does not, we may implement the more cautious strategy described at the close of §3. Since the remainder of the argument in that setting would proceed pretty much as the argument in the text does, the argument in the text may continue without prejudice.
23. As indicated in note 10, a simpler definition of necessity would be possible if contingently existing entities—specifically, contingent facts—have no definitions. In this case, given that P is necessary iff the fact that P exists necessarily, we would be led to the following: P is necessary iff, for some fact x, it is true by definition that x = the fact that P. It seems credible to me, however, that at least some contingent facts do have definitions (for example, contingent facts denoted by fact-abstracts involving only expressions for the most fundamental universals).

Alternatively, suppose the definition contemplated in note 20 is a correct definition of F, and suppose Alvin Plantinga's doctrine of "serious actualism" holds (see his "On Existentialism," *Philosophical Studies* 44, 1983: 1–20). Then the following biconditional would hold (and thus would underwrite an associated definition of necessity): P is necessary iff, for some x identical to fact-P, there is an F such that, F $=_{\text{def}}$ pred(identity, x). This should be of interest to serious actualists. This biconditional fails, however, if serious actualism fails. The definitional strategy in the text is designed to be neutral with respect serious actualism.

24. You may take the notion of proof to be just an informal notion. If, however, you are concerned that this notion is covertly modal (cf. §4.2), the following (manifestly nonmodal) syntactic explication of 'proof' eliminates the problem: derivation in a standard first- or second-order logical framework supplemented with '$=_{\text{def}}$', fact-abstracts, property variables, proposition variables, and standard principles for these devices. For example, the following nonmodal versions of principles (i) and (ii): (i′) if $\alpha =_{\text{def}} \beta$, then α and β coexist; (ii′) if $\alpha =_{\text{def}} \beta$, then if α and β both exist, then $\alpha = \beta$. Another example is the following standard nonmodal principle from the logic of facts: if fact-P exists, then P. Whether the background logical framework is first-order or second-order depends on how one chooses to understand proposition variables (see note 14). There are to be no proper names in this framework so that 'Socrates exists' and the like are (rightly) not provable; relatedly, in a proof a premise may contain no free variables beyond property and proposition variables. Notice that alleged contingent logical truths (e.g., those exploiting 'actual') are not provable in this framework.
25. In compliance with the concern voiced in the previous note, the envisaged proof will be manifestly nonmodal.
26. To ensure that these steps are modality-free, we would, for example, use the nonmodal principle (i′) from note 24 in place of (i).
27. This is so on the assumption that proofs have finitely many premises—as they do, e.g., in the framework described in note 24.
28. See note 14.
29. Analogously, the idiom of states of affairs may give way to the idiom of facts and fact-individuating properties: talk of actual states of affairs may give way to talk of actual facts, and talk of possible but nonactual states of affairs may give way to talk of possible but nonactual facts, which in turn gives way to talk of associated actualist fact-individuating properties. This approach is optimally developed in the setting of the nonreductive view of facts (see note 2), according to which contingent facts are like events in that their existence is contingent. (As I understand them, this is how David Armstrong and Michael Tooley think of states of affairs.

Here is one argument for the nonreductive view of facts. Obviously, there is such a thing as the fact that someone is coming into being iff there is such a thing as the event of someone's coming into being. So (for any u and v) if u is the indicated fact and v the associated event, it follows that: u exists iff v exists. But, intuitively, this biconditional is not just an accidental truth; it is necessary that u exists iff v exists. Now virtually everyone agrees that events exist contingently; for example, it is contingent whether v exists. From these two propositions, it follows that it is contingent whether u exists. (See Kit Fine for another line of defense of the nonreductive view, "First-Order Modal Theories III–Facts," *Synthese* 53: 43–122.)

30. The analogous point holds for those actualists who accept the existence of possible worlds, taken as primitive entities, but who reject the existence of nonactual particulars.
31. This is particularly compelling on the dominant theory of properties, namely, the coarse-grained theory of properties (according to which necessarily equivalent properties are identical). For example, there is not the slightest temptation to say that being an angle is part of the property of being a closed planed figure with three sides. But, on a coarse-grained conception, the latter property is identical to the property of being a closed plane figure with three angles. So it follows on the coarse-grained conception that being an angle is not part of the property of being a closed plane figure with three angles. Given this, parity would imply that no property has parts and, in particular, that an entity is not a part of its own identity property.

    The temptation to speak of parts of properties arises, if at all, only in a fine-grained setting. In this case the broadly Fregean conception would be committed to "anti-existentialism." For a defense of anti-existentialism, see my "A Solution to Frege's Puzzle" (*Philosophical Perspectives* 7, 1993:17–61) and "Universals" (*Journal of Philosophy* 90, 1993: 5–32). On this picture, fine-grained and coarse-grained properties have the same existence conditions; they differ only in that fine-grained properties have more discriminating identity conditions. Mutatis mutandis for relations and propositions.
32. See my "Universals" and "Propositions" (*Mind* 107, 1998: 1–32) for a discussion of transmodal truths. The following is a variation of the example discussed there: for all x, it is necessary that, for all y, the property of being identical to either x or is instantiated. For a reason much like that just given, the Aristotelian view cannot account for this intuitively obvious transmodal truth. For in a world in which x does not exist and in which there exists an object y that does not exist in the actual world, y would nevertheless have the property of being either x or y. But this cannot be so on the Aristotelian view; for, on that view, the indicated property exists neither in the actual world (because y might not exist in the actual world) nor in the envisaged world (because x does not exist there). Similar problems arise if in this example 'instantiated' is replaced with various other atomic predicates (e.g. 'unthinkable').
33. In "On What There Isn't," Stalnaker (who is perhaps the leading advocate of coarse-grained properties) proposes a possible-worlds view on which properties—including "ways the world might have been"—exist contingently. I believe this view runs into difficulties concerning the above transmodals. I also think it runs into difficulties concerning the "constancy of propositions." To put the point the way a possibilist would, Stalnaker's view permits distant worlds w relative to which our world does not exist but in which there could be beings whose mental lives are functionally just like ours. Given this, the proposition (i.e., set of worlds or set of world/truth-value pairs, on Stalnaker's view) expressed by, say, our English sentence 'Everything is self-identical' and the proposition expressed by our counterparts' sound-alike sentence would have to be different. But this seems counterintuitive: when functionally equivalent counterpart speakers in w utter their sentence 'Everything is self-identical', surely they assert the same proposition we assert when we utter our sentence 'Everthing is self-identical'. The same difficulty arises, mutatis mutandis, for properties.
34. For details, see the papers cited in note 31–2.

35. As Frege would remind us, "constituent" is a metaphor. It has the following neutral gloss: z is a "constituent" of P iff there is a way of analyzing P such that z occupies a node in the corresponding labeled analysis tree. This gloss allows for the theoretical possibility of a world in which z not exist (and, in turn, the labeled tree not exist) even though P does still exist. The labeled tree is an extensional entity and, hence, depends for its existence on the contingent entity occupying one of its nodes (just as sets depend on their members and mereological sums, their parts). By contrast, P is an intensional entity; its existence is not dependent on this contingent entity (just as the existence of properties is not dependent on their contingent instances). What is dependent on z is a representation of P which, happily, exists in the actual world, though not in all worlds. The supposition in the text just dismisses this theoretical possibility (to my mind, without good reason).
36. In saying that these are logical notions, I am using 'logical' in a sense not tied to this or that formal system, but rather in its core sense: the relevant abstract entities belong to a family of entities that together determine logical form. Logical truths are those whose logical form guarantees their truth: necessarily, every proposition of the same form is true. Necessity is itself a logical notion in this core sense. My objective has been to show that necessity has a noncircular, counterexample-free definition in terms of the restricted class of logical notions just listed in the text.
37. The reasons are akin to those showing that the notion of essence is not definable in terms of necessity and possibility (see Kit Fine's "Essence and Modality"). For example, although $\{2\} =_{\text{def}}$ the set whose unique element is the number 2, it is not the case that $2 =_{\text{def}}$ the number that is the unique element of $\{2\}$. Modal notions, however, are blind to this asymmetry: it is necessary both that $\{2\} =$ the set whose unique element is 2 and that $2 =$ the unique element of $\{2\}$.

    Incidentally, definition is prima facie a narrower notion than essence. For example, it is of the essence of circles to have a size, but it seems that there is no correct definition of circularity that identifies having size as a necessary condition.
38. This definition is explanatory only if the nonreductive view of facts holds; cf. note 29. (I find the definition to be most appealing if 'fact-P' is understood as shorthand for 'the fact to which P corresponds' or an associated algebraic analysis of that fact; cf. note 12.) A similar definition of necessity can be formulated using as a primitive the notion of essence rather than the notion of definition. Given the broadly Fregean conception of properties, the correctness of this essence-based definition can be demonstrated along lines similar to those used in §2. This definition would not succeed, however, without the broadly Fregean conception (for example, if this conception were replaced with an Aristotelian conception). Other things being equal, this is one more count in favor of the broadly Fregean conception.
39. Recall that I am employing the standard inclusive use of 'definition' which applies to definitions of properties as well as definitions of concepts, and I am using 'strong analyticity' for the property of being true by definition and proof. For scholarly concerns about ambiguities in the use of 'analytic' in the history of philosophy, see my "Analyticity," *Routedge Encyclopedia of Philosophy*, vol. 1, London: Routedge & Kegan Paul, 1998, pp. 234–9.

40. In particular, if there exist a posteriori necessities, there exist a posteriori extrinsic definitions of their fact-identity properties. Since these would be extrinsic scientific definitions, this adds credence to the idea that there exist intrinsic scientific definitions (e.g., water $=_{def}$ H_2O) though we are not committed to this.
41. Frege and other classical logicists *believed* that the truths of arithmetic were provable from nothing but definitions of the standard arithmetic notions and standard logical principles. This belief is equivalent to the thesis that every arithmetic truth is intrinsically analytic, and Gödel's Theorem shows that it is mistaken. But Gödel's Theorem does not show that the strong thesis (as usually *stated* by classical logicists) is mistaken. Note that all the truths of arithmetic are provable without the aid of any definitions of standard arithmetic properties; definitions of properties "extrinsic" to arithmetic suffice. Consequently, none of the familiar technical difficulties arise (Benacerraf-style worries about definitions of the arithmetic properties, paradox-prone abstraction principles, etc.).
42. It is understood that the rules of inference are all standard (modus ponens, etc.).
43. I appreciate the valuable audience comments I received during colloquia at Rutgers University, MIT, University of Colorado, University of Florida, and Yale University. I thank Susanne Bobzien and Geoffrey Pynn for very helpful scholarly discussions of Frege. Finally, I am grateful to David Barnett and especially Chad Carmichael and Dan Korman for their numerous insights and suggestions.