A Solution to Frege's Puzzle

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My view that the English sentence 'Hesperus is Phosphorus' could sometimes be used to raise an empirical issue while 'Hesperus is Hesperus' could not shows that I do not treat the *sentences* as completely interchangeable. Further, it indicates that the mode of fixing the reference is relevant to our epistemic attitude toward the sentences expressed. How this relates to the question what 'propositions' are expressed by these sentences, whether these 'propositions' are objects of knowledge and belief, and in general, how to treat names in epistemic contexts, are vexing questions. I have no 'official doctrine' concerning them, and in fact I am unsure that the apparatus of 'propositions' does not break down in this area.
    Saul Kripke, Preface, *Naming and Necessity*

My own view is that Frege's explanation, by way of ambiguity, of what appears to be the logically deviant behaviour of terms in intermediate contexts is so theoretically satisfying that if we have not yet discovered or satisfactorily grasped the peculiar intermediate objects in question, then we should simply continue looking.
    David Kaplan, "Quantifying In"

1. The Problem

This paper presents a new approach to a family of outstanding logical and semantical puzzles, the most famous example of which is Frege's puzzle.[1] The underlying thesis is that what is needed to solve these puzzles is an enriched theory of propositions and their logical form. We begin by setting forth the problem and desiderata for a successful solution. Then we briefly assess the four main theories of propositions — the possible-worlds theory, propositional-function theory, the propositional-complex theory, and the algebraic theory — and we indicate why the algebraic theory avoids shortcomings inherent in the other three. The algebraic theory, we then show, allows for hyper-fine-grained intensional distinctions based on differences in logical form. It also leads

to a new understanding of the theory of predication implicit in Frege's theory of senses. Finally, it allows us to incorporate, not only Platonic modes of presentation (i.e., traditional realist properties and relations), but also non-Platonic modes of presentation. Taken together, these features create an opening in logical space that might at last allow us to identify the elusive non-descriptive, non-metalinguistic propositions that are responsible for our family of puzzles. The paper closes with a suggestion of how this enriched theory of propositions might enable us to give purely semantic, as opposed to pragmatic, solutions to the puzzles.

Frege's puzzle is this: how can $\ulcorner a = b \urcorner$, if true, differ in meaning or cognitive value from $\ulcorner a = a \urcorner$? A correlative puzzle is this: why do co-referential proper names fail to be substitutable *salva veritate* in propositional-attitude sentences? Given that $\ulcorner \ulcorner S \urcorner$ means that $S \urcorner$ holds for relevant sentences $\ulcorner S \urcorner$, these two puzzles appear to be instances of an underlying puzzle about the reference of 'that'-clauses: how can $\ulcorner$ that $a = b \urcorner$ and $\ulcorner$ that $a = a \urcorner$ refer to different things if $\ulcorner a \urcorner$ and $\ulcorner b \urcorner$ refer to the same thing? Or, more generally: how can $\ulcorner$ that $S \urcorner$ and $\ulcorner$ that $S' \urcorner$ refer to different things if $\ulcorner S \urcorner$ and $\ulcorner S' \urcorner$ are referentially isomorphic?[2]

We are assuming: 'that'-clauses are singular terms; expressions like 'believe', 'mean', 'assert', 'know' are standard two-place predicates that take 'that'-clauses as arguments; expressions like 'is necessary', 'is true', and 'is possible' are one-place predicates that take 'that'-clauses as arguments.[3] Intuitively valid arguments like the following provide strong evidence for these assumptions:

> Moore believes everything that his utterances literally mean.
> Moore utters 'Goodness is a nonnatural property'.
> 'Goodness is a nonnatural property' means that goodness is a nonnatural property.

> ∴ Moore believes that goodness is a nonnatural property.

This argument may be represented along the following lines:

> $(\forall s)(\forall x)((Utters(moore, s) \;\&\; Means(s, x)) \rightarrow Believes(moore, x))$.
> Utters(moore, 'Goodness is a nonnatural property').
> Means('Goodness is a nonnatural property', that goodness is a nonnatural property).

> ∴ Believes(moore, that goodness is a nonnatural property).

A satisfactory theory should be able systematically to represent "mixed" examples like this (i.e., examples that "mix" singular and general statements that deal simultaneously with belief and meaning and examples that "mix" singular and general statements that deal simultaneously with, belief, meaning,

and modality).[4]

There are arguments to show that 'that'-clauses do not refer to sentences or other linguistic entities and that they do not refer to entities whose existence depends on the existence of the mind.[5] Our thesis is that 'that'-clauses refer to an altogether different category of entities. We will use the term 'proposition' for these entities. However, it should be emphasized that the expression 'proposition' is used here solely as a term of art. Our intention is to use it merely as a term for the *sui generis* entities that are denoted by 'that'-clauses and whose existence is language-independent and mind-independent; we eschew other associations that the term has acquired in the history of its use.

Thus, our goal is to find a theory of propositions that solves our family of puzzles while, at the same time, preserving the standard logical syntax that treats 'that'-clauses as singular terms and 'believes', 'means', 'asserts', and so forth as standard two-place predicates that take 'that'-clauses as arguments.

Frege's solution to our family of puzzles aims to do just this. However, as it stands, Frege's solution, which is based on his theoretical distinction between *sinn* and *bedeutung*, falters on two counts: the Donnellan-Kripke critique[6] and Mates' puzzle.[7]

Donnellan and Kripke have given compelling arguments that proper names do not have descriptive senses. However, if proper names do not have descriptive senses, what could the sense of a proper name be? How could co-denoting names $\ulcorner a \urcorner$ and $\ulcorner b \urcorner$ have different senses? How could we have epistemic access to such senses? No satisfactory answer to these questions appears to be forthcoming.

Mates' puzzle is more general. Suppose that $\ulcorner D \urcorner$ and $\ulcorner D' \urcorner$ are synonymous sentences. Then, for Frege $\ulcorner D \urcorner$ and $\ulcorner D' \urcorner$ have the same sense. Given the principle that synonymous expressions—expressions having the same sense—can be substituted *salva veritate*, then the sentences $\ulcorner$ Nobody doubts that whoever believes that D believes that D $\urcorner$ and $\ulcorner$ Nobody doubts that whoever believes that D believes that D' $\urcorner$ must have the same truth value. But they do not.[8] Some Fregeans have been tempted to try to solve puzzles of this general sort by holding that, even though D and D' have the same ordinary sense, they have different *indirect* senses.[9] The problem with this attempted solution is that the hypothesized indirect senses are a complete mystery. What on earth could they be? How do we have epistemic access to them? Again, no satisfactory answer appears to be forthcoming. Another response to Mates' puzzle is simply to deny that syntactically distinct sentences can ever literally mean the same thing.[10] A consequence of this view is that exact translations between syntactically disjoint languages are in principle impossible. This response appears to be too radical to be taken seriously.

In summary, our goal is to develop a theory of propositions that (1) accommodates the various substitutivity failures, (2) preserves the standard logical syntax, (3) posits only epistemically acceptable senses, (4) permits syntactically distinct synonyms and interlinguistic translation.[11]

2. Four Theories of Propositions

There are four main theories: (1) the possible-worlds theory, (2) the propositional-function theory, (3) the propositional-complex theory, (4) the algebraic theory. Our hypothesis is that only the last is satisfactory. To motivate this view, we will survey a few of the difficulties—some well-known, others novel—that confront the first three theories.[12] At the heart of these difficulties is the fact that these three theories are reductionistic: each attempts to reduce intensional entities of one kind or another to extensional entities—either sets or extensional functions.[13] Our view is that this extensional reductionism has hampered the solution to our family of substitutivity puzzles. We need a theory that treats intensional entities as irreducibly intensional.

According to the possible-worlds reduction, a proposition is either a set of possible worlds or a function from possible worlds to truth values, and properties are functions from possible worlds to sets of possible (often non-actual) objects. First, there are standard epistemological and metaphysical objections to a theory that is truly committed to the existence of things that are not actual; other things being equal, such commitment should be avoided. Second, the possible-worlds theory is intuitively implausible. Are familiar sensible properties (e.g., colors, shapes, aromas) really functions from possible worlds to sets of possible objects? When I am aware that I am in pain, is a set of possible worlds—or a function from possible worlds to truth values—really the object of my awareness?[14] On the face of it, this is incredible. Many philosophers seem to forget these sorts of straightforward intuitive considerations. Third, there is a famous technical difficulty: the possible-worlds theory implies that all necessarily equivalent propositions are identical; a plainly unacceptable consequence. Certain possible-worlds theorists have responded to this problem by holding that 'that'-clauses denote abstract trees or structured meanings whose elements are possible-worlds constructs.[15] This revisionary view is a special case of the propositional-complex theory, and its difficulties are much the same.[16] We will come to that theory in a moment. Finally, the algebraic theory which we will defend promises to provide a framework within which the data that prompt people to become possibilists can be handled in an actualist fashion.

According to the propositional-function theory, a property (or relation) is a function from objects to propositions, where propositions are taken to be primitive entities. For example, the property being red = (λx)(the proposition that x is red). For any given object x, the proposition that x is red = (λx)(the proposition that x is red)(x) = the result of applying the function (λx)(the proposition that x is red) to the argument x.[17] But are the familiar sensible properties (e.g., colors, shapes, aromas) really *functions*? How implausible. Given that there are straightforward, intuitive theories that permit one to take properties at face value and given that intuitions form the evidential basis for the theory of properties, relations, and propositions in the first place, it is hard to

see what could justify accepting the counterintuitive theory that properties are functions. Perhaps the mathematization of this part of philosophy tends to mask this elementary epistemological point.

Besides this sort of intuitive difficulty, there are several technical difficulties.[18] Here is an illustration involving properties of integers. Being even = being an x such that x is divisible by two, and being self-divisible = being an x such that x is divisible by x. (If someone does not accept these identities, plainly there could be other identities that would serve to make the same point.[19]) Then, given the propositional-function theory, we may derive the following identities:

that two is even = (λx)(that x is even)(two) = (λx)(that x is divisible by two)(two) = that two is divisible by two = (λx)(that x is divisible by x)(two) = (λx)(that x is self-divisible)(two) = that two is self-divisible.

However, that two is even and that two is self-divisible are plainly different propositions: certainly someone could be consciously and explicitly thinking the former while not consciously and explicitly thinking the latter. Indeed, someone who is thinking that two is even might never have employed the concept of self-divisibility.[20]

Mates' example creates a kindred difficulty. Because that D = that D', we may derive the following identities:

that whoever believes that D believes that D =
(λxy)(that whoever believes x believes y)(that D, that D) =
(λxy)(that whoever believes x believes y)(that D, that D') =
that whoever believes that D believes that D'.

However, nobody doubts that whoever believes that D believes that D. So nobody doubts that whoever believes that D believes that D'. But people evidently do doubt this.

We shall see that these two puzzles—the self-division puzzle and Mates' puzzle[21]—can be solved by adopting the principle that propositions are distinct if the 'that'-clauses that denote them have different *logical form*. This principle, we shall see, is easy to accommodate if we take properties at face-value, not as functions, but as primitive entities. The algebraic approach permits us to do this.

Incidentally, because properties are not propositional functions and because λ-abstracts $\ulcorner(\lambda v)$(that A)$\urcorner$ denote propositional functions, these λ-abstracts do not denote properties. Philosophers' persistent use of λ-abstracts for this purpose invites unnecessary confusion. Another notation is called for. A perspicuous solution is to use $\ulcorner[v_1...v_n: A]\urcorner$ where $n \geq 0$. Thus, whereas $\ulcorner\{v_1: A\}\urcorner$ denotes the set of things v_1 such that A, $\ulcorner[v_1: A]\urcorner$ denotes the property of being a v_1 such that A. Whereas $\ulcorner\{v_1...v_n: A\}\urcorner$ denotes the relation-in-extension holding among $v_1...v_n$ such that A, $\ulcorner[v_1...v_n: A]\urcorner$ denotes the relation-in-intension

holding among $v_1...v_n$ such that A. In the limiting case where n = 0, $\ulcorner[A]\urcorner$ denotes the proposition that A.[22]

We come next to the propositional-complex theory, which has been called the "tinker-toy theory of propositions." According to this theory, 'that'-clauses denote ordered sets (or sequences or abstract trees or mereological sums) whose elements are properties, relations, and/or individuals.[23] For example, 'that you are running' denotes the sequence <running, you>; 'that you are running and I am walking' denotes <conjunction, <running, you>, <walking, me>>; 'that you are not running' denotes <negation, <running, you>>; 'that someone is running' denotes <existential generalization, running>; and so forth.[24] As with the two previous theories, this theory collides with intuition. On the face of it, this theory is highly implausible. When I am aware that I am in pain, is an ordered set the object of my awareness? When I see that you are running, do I see an ordered set? How implausible.[25] Moreover, there is in principle no way to determine which ordered set I allegedly see. Is it <running, you>? Or is it <you, running>? The choice is utterly arbitrary. And this is only the tip of the iceberg. What could justify admitting such wholesale arbitrariness into a theory when a good alternative exists? It is appropriate to recall Frege's sage observation (in "Gedankengefüge"), "we really talk figuratively when we transfer the relation of whole and part to thoughts."

Another problem with the propositional-complex theory is its redundancy. To develop the theory *systematically*, one introduces a family of logical operations—conjunction, negation, existential generalization, and so forth. But what are these operations? How do they behave logically? When one formulates a *general* theory for these operations, what one gets is pretty much an algebraic theory. True, even in the setting of a general theory of these operations one still might insist on artificially identifying propositions with ordered sets (e.g., neg (p) = <neg, p>). But it is hard to see what motivation there would be — especially in view of the fact that one has already conceded that there exist irreducibly intensional entities (namely, properties and relations).[26]

A rather different kind of difficulty is a logical problem that arises in connection with quantifying-in. Consider the following intuitively true sentence:

> Every x is such that, necessarily, for every y, either it is possible that x = y or it is impossible that x = y.[27]

In symbols,

> (i) $(\forall x)\Box(\forall y)$(Possible [x = y] $\lor$ Impossible [x = y]).

By the propositional-complex theory, this is equivalent to:

> (ii) $(\forall x)\Box(\forall y)$(Possible <x, identity, y> $\lor$ Impossible <x, identity, y>).

The singular term '<x, identity, y>' must have either narrow scope or wide

scope. If it has narrow scope, (ii) would imply:

$(\forall x)\Box(\forall y)(\exists v)\ v = <x, \text{identity}, y>.$

However, by the principle that, necessarily, a set exists only if its elements exist, this implies:

$(\forall x)\Box(\exists v)\ v = x.$

That is, everything necessarily exists. A manifest falsehood. On the other hand, suppose that in (ii) the singular term '<x, identity, y>' has wide scope. Then, (ii) would imply that every x is such that, necessarily, for all y, there exists an actual set <x, identity, y>. That is:

$(\forall x)\Box(\forall y)(\exists \text{actual} v)\ v = <x, \text{identity}, y>.$

But, by the principle that, necessarily, a set is actual only if its elements are actual, this implies:

$\Box(\forall y)\ y$ is actual.

That is, necessarily, everything (including everything that could exist) is already actual. Another manifest falsehood. So either way, (ii) implies something false. But (ii) is the propositional-complex theorists' way of representing the true sentence (i). So the propositional-complex theory appears unable to handle intuitively true sentences like (i).[28] We shall see that the algebraic approach, by contrast, can easily handle such sentences.

This sketch indicates a few problems with the possible-worlds theory, the propositional-function theory, and the propositional-complex theory. At the heart of these problems is the fact that all of these theories are reductionistic: each attempts to reduce intensional entities of one kind or another to extensional entities—either extensional functions or sets. This extensional reductionism has obscured basic facts about properties, relations, and propositions that, we believe, hold the key to the outstanding substitutivity puzzles. The algebraic theory promises to redress this situation.

3. The Algebraic Approach

On the algebraic approach, no attempt is made to reduce properties, relations, and propositions. Intuitively obvious truths like the following are accepted at face value requiring no reductionistic explanation. The proposition that A & B is the conjunction of the proposition that A and the proposition that B. The proposition that not A is the negation of the proposition that A. The proposition that Fx is the result of predicating the property F-ness of x. The proposition that there exists an F is the result of existentially generalizing on the property F-ness. And so forth. Throughout much of the history of philosophy, at least until the advent of extensionalism, examples like these were

accepted as plain truths. They are no doubt the sort of thing Plato and Aristotle had in mind in their famous remarks to the effect that truths arise through a "weaving together" of universals. Such examples serve to impart a firm intuitive grasp of the indicated logical operations—conjunction, negation, singular predication, existential generalization, and so forth. The aim of the algebraic approach is simply to systematize the behavior of properties, relations, and propositions (conceived as irreducible entities) with respect to these logical operations. The idea is that our intuitive grasp of these operations can be codified by means of appropriate elementary rules. An intensional algebra is a structure that contains a domain of primitive entities—particulars, properties, relations, and propositions—together with a list of relevant logical operations satisfying such rules.

There is a direct line of development in algebraic logic from Boolean algebras, to transformation algebras, to polyadic and cylindric algebras, and finally to intensional algebras. A Boolean algebra is a structure <D, disj, conj, neg, F, T>.[29] D is a domain of entities which may be thought of as primitive and irreducible; disj and conj are binary operations which may be thought of as the logical operations of disjunction and conjunction, respectively. The operation neg is a unary operation which may be thought of as the logical operation of negation. F and T are distinguished elements of the domain which may be thought of as falsity and truth, respectively. The operations in a Boolean algebra must satisfy certain standard rules which may be thought of as codifying our intuitive understanding of the operations of disjunction, conjunction, and negation, respectively. Boolean algebras are extensional models of sentential logic: in the standard case D would be the set of truth values {F, T} and disj, conj, and neg would be the standard truth functions. Boolean algebras are also extensional models of certain artificial fragments of first-order predicate logic. Consider, for example, a fragment of the monadic predicate calculus in which every atomic formula contains the same variable (and in which there are no quantifiers or individual constants). The following Boolean algebra would be a standard model for this fragment: D would be the power set of some given non-empty set of objects; disj would be the set-theoretical operation of union; conj would be intersection; neg would be complementation; F would be the null set; and T would be D itself. (We usually think of Venn diagrams as pictorial representations of this sort of Boolean algebra.) Or consider a fragment of the n-adic predicate calculus in which every atomic formula consists of an n-ary predicate letter followed by n distinct variables always occurring in the same order (and in which there are no quantifiers and no individual constants). (E.g., when $n = 3$, we have molecular formulas like '((Fuvw ∨ Guvw) & ¬Huvw)'.) The following Boolean algebra would be a standard model for this fragment: D would be the power set of the nth Cartesian product of some non-empty set of objects; disj would be the union operation; conj would be intersection; neg would be complementation; F would be the null set; T would be D. To obtain an extensional model of first-order predicate calculus (without quantifiers and

without individual constants) in which the indicated restriction on the variables is dropped, one considers structures <D, disj, conj, neg, τ, F, T> that are like Boolean algebras except for containing a new element τ. τ is a set of auxiliary logical operations intended to be semantical counterparts of the syntactical operations of repeating the same variable one or more times within a given formula and of changing around the order of the variables within a given formula.[30] For example, τ might contain an operation conv that would map the relation-in-extension {xy: x loves y} to its converse {yx: x loves y}; and τ might contain the operation reflex that would map the relation-in-extension {xy: x loves y} to its reflexivization {x: x loves x}. To obtain an extensional model of the predicate calculus with quantifiers (but without individual constants), one considers structures <D, disj, conj, neg, exist, τ, F, T> that are like the previous structures except that they contain an additional operation, exist.[31] This operation is to be thought of as the logical operation of existential generalization. For example, it takes a binary relation-in-extension (e.g., {xy: x loves y}) to an appropriate unary relation-in-extension (e.g., {x: ($\exists$y) x loves y}). All the above algebraic ideas are standard nowadays.

How can we extend these ideas to represent intensionality? The answer is this. To obtain an *intensional* model for the predicate calculus (without individual constants), we consider closely related algebraic structures <D, K, disj, conj, neg, exist, τ, F, T>. The domain D is the union of denumerably many disjoint subdomains $D_{-1}, D_0, D_1, D_2, ..., D_n, ...$. The subdomain D_{-1} is to be thought of as being made up of extensional entities; D_0, propositions; D_1, properties; D_2, binary relations-in-intension; D_n, n-ary relations-in-intension. The elements of D are to be thought of as primitive, irreducible items. The new element K is a set of *possible extensionalization functions*. Each extensionalization function $H \in K$ assigns to the elements of D an appropriate extension as follows: for each proposition x (i.e., for each $x \in D_0$), H(x) = T or H(x) = F; for each property x (i.e., for each $x \in D_1$), H(x) is a subset of D; for each n-ary relation x (i.e., for each $x \in D_n$), H(x) is a subset of the nth Cartesian product of D; in the case of particulars x (i.e., $x \in D_{-1}$), we let H(x) = x. Among the possible extensionalization functions in K there is a distinguished function G which is to be thought of as the *actual* extensionalization function; it tells us the actual extension of the elements of D. We require that operations conj, neg, and so forth in an intensional algebra behave in the expected way with respect to each extensionalization function $H \in K$. For example, for all x and y in D_0, H(conj(x, y)) = T iff H(x) = T and H(y) = T. For all x in D_0, H(neg(x)) = T iff H(x) = F. And so forth. For ease of presentation we will hereafter write simply <D, K, τ> with the understanding that D and K are as indicated and τ is an ordered set of operations including, in order, disj, conj, neg, exist, and those in τ. No harm is done if τ contains further operations in addition to those indicated; we will permit this. Finally, for convenience, F will be identified with the null set and T, with the domain D. With these details in place we can say what it

takes for one of these algebras M = <D, K, τ> to be *intensional*: there are elements in some $D_i \subset D$, $i \geq 0$, that can have the same possible extension and nevertheless be distinct. That is, M is intensional iff, for some x and y in $D_i \subset D$, $i \geq 0$, and for some H $\in$ K, H(x) = H(y) and x $\neq$ y. For example, if x and y are in D_0, perhaps G(x) = G(y) = T but x $\neq$ y.

These intensional algebras yield intensional models of the predicate calculus (without individual constants). An intensional interpretation is a function I that maps i-ary predicate letters to i-ary relations-in-intension; that is, I($\ulcorner F^i \urcorner$) $\in D_i$. Relative to an intensional interpretation I and an intensional algebra M, it is easy to define an intensional valuation function V_{IM} that maps sentences of the predicate calculus (without individual constants) to relevant propositions in D. For example, V_{IM} ('$\neg(\exists x)Fx$') = neg(exist(I('F'))). A sentence $\ulcorner A \urcorner$ is true relative to I and M iff its actual extension = T. That is, Tr($\ulcorner A \urcorner$) iff G($V_{IM}(\ulcorner A \urcorner)$) = T.

So far, however, we have not indicated how intensional algebras can model the predicate calculus *with* individual constants. By 'individual constant' we mean variables with fixed assignments, Millian (or Russellian) proper names,[32] and intensional abstracts.[33] For example, suppose that we extend the notion of interpretation so that I assigns to each variable a value in M's domain D and to each Millian (or Russellian) proper name a nominatum in D. Then, we should like to be able to assign some proposition in D as the intensional value of open-sentences $\ulcorner Fx \urcorner$ relative to an interpretation I and an algebra M. Similarly, suppose that $\ulcorner a \urcorner$ is a Millian (or Russellian) proper name. We should like to be able to assign a propositional meaning to the sentence $\ulcorner Fa \urcorner$. Finally, suppose that the language is fitted-out with intensional abstracts. For example, if $\ulcorner A \urcorner$ is a sentence, $\ulcorner$ that A $\urcorner$ in our notation is represented by the singular term $\ulcorner [A] \urcorner$. We should like to be able to assign a proposition in D as the intensional value of a sentence like 'F[($\exists x)Gx$]'. This threefold problem is solved simply by restricting ourselves to intensional algebras M = <D,K,τ> in which τ contains an additional logical operation, namely, *singular predication*—$pred_s$, for short. The operation of singular predication behaves exactly as one would expect. For example, when singular predication is applied to a property and an item, the proposition that results is true iff the item is in the extension of the property. That is, for all x $\in D_1$ and y $\in$ D, H($pred_s$(x, y)) = T iff y $\in$ H(x), for all extensionalization functions H $\in$ K. Using singular predication, we can then assign appropriate intensional values to our three problem cases: V_{IM}('Fx') = $pred_s$(I('F'), I('x')); V_{IM}('Fa') = $pred_s$(I('F'), I('a')), and V_{IM}('F[($\exists x)Gx$]') = $pred_s$(I('F'), exist(I('G'))).[34] Because we are able in this way to represent intensional abstracts, our intensional algebras thus provide models of first-order *intensional* logic.

A word about method. Consider the analogy between the possible-worlds approach and the algebraic approach.[35] In the former case, one begins with the informal theory that there exist possible worlds some of which are populated with non-actual possibilia. To model possible worlds and their inhabitants, one

considers a family of set-theoretical structures that are designed to have relevant formal similarities to possible worlds and their inhabitants. By studying the behavior of these set-theoretical structures, one is able to learn various salient facts about (languages that are equipped to talk about) possible worlds and their inhabitants. In most cases, the set-theoretical structures are built up from arbitrary objects having nothing inherently to do with possible worlds and their inhabitants; for example, some of these structures are built up from natural numbers, real numbers, or pure sets (i.e., sets built up ultimately from the null set). However, if the theory of possible worlds is correct—that is, if there really are possible worlds and if some of them really have non-actual inhabitants— then among these set-theoretical structures there will be a "natural model," a structure built up from genuine possible worlds and genuine non-actual inhabitants.[36] In a natural model, "the set of possible worlds" will really be the set of possible worlds; "the function that assigns to each possible world the set of inhabitants of that world" will really be the function that assigns to each possible world the set of inhabitants of that world; likewise for "accessibility among worlds," "similarity among worlds," and so forth. The informal theory of possible worlds may be thought of as a (partial) theory of this natural model.

The situation with our intensional algebra is analogous. We begin with the informal theory that properties, relations, and propositions are genuine irreducible entities that bear fundamental logical relations to one another— conjunction, negation, existential generalization, singular predication, and so forth. This theory is intelligible in its own right, certainly as intelligible as the possible-worlds theory. (Examples—such as the plain truth that the proposition that A & B is the conjunction of the proposition that A and the proposition that B—give us a good intuitive grasp of the fundamental logical operations posited in the theory.) This theory has historical credentials reaching as far back as perhaps Plato and Aristotle. In one form or another, it was the prevalent informal view of propositions prior to the advent of extensionalism. To study properties, relations, and propositions (and to study languages equipped to talk about them), one defines a family of set-theoretical structures most of which are built up from arbitrary objects having nothing special to do with properties, relations, and propositions and fundamental logical operations on them. However, on the assumption that the informal theory is correct, then among these set-theoretical structures there will be a "natural model" built up from genuine properties, relations, and propositions and genuine fundamental logical operations defined over them (i.e., the genuine operations of negation, conjunction, existential generalization, singular predication, and so forth).[37] The informal theory may be thought as a (partial) theory of this natural model.

From a formal point of view, the set-theoretical structures that we consider in our study of languages with intensional abstraction are plainly as legitimate as those considered in possible-worlds study of languages with modal operators. The correctness of the algebraic approach comes down to the correctness of the

underlying informal theory. In view of the failure of the alternatives discussed in the previous section and in view of the intuitive accessibility and historical credentials of the relevant algebraic concepts, there are good reasons for accepting this informal theory and, in turn, the algebraic approach itself.

4. Actualism and Anti-existentialism

Actualism. Thus far we have not discussed how the contingent existence of particulars is to be represented by intensional algebras. A technically convenient way to do this is to require that the possible extensionalization functions $H \in K$ take as arguments not only the elements of D but D itself. When H is applied to D itself, the value is a subset of D: that is, $H(D) \subset D$. $H(D)$ is to be thought of as the set of things that exist relative to H. Now if we want to be actualists—and if we want our quantifiers $\ulcorner \exists \alpha \urcorner$ to be actualist quantifiers—we need only confine ourselves to intensional algebras that impose relevant restrictions on the logical operation of existential generalization (i.e., the logical operation, exist). For example, for all x in D_1, $H(exist(x)) = T$ iff, for some $y \in H(D)$, $y \in H(x)$). Illustration: $H(exist([y: Fy])) = T$ iff, for some $y \in H(D)$, $y \in H([y: Fy])$. That is, relative to H, the extension of the result of existentially generalizing on the property F-ness = T iff, for some y that exists relative to H, y is in the extension of F-ness. Since $[(\exists y)Fy] = exist([y: Fy])$, this tells us that, relative to H, the proposition $[(\exists y)Fy]$ is true iff, for some y that exists relative to H, y is in the extension of F-ness.[38]

Anti-existentialism. Stated loosely, existentialism is the doctrine that, necessarily, a proposition exists only if its "constituents" exist; anti-existentialism is the contrary doctrine.[39] If propositions were mereological sums, sets, sequences, or some other kind of extensional complex, existentialism would surely be right. But we have seen that such extensional reductionism is the wrong way to think of propositions, for it cannot handle intuitively true sentences like: Every x is such that, necessarily, for every y, either it is possible that x = y or it is impossible that x = y.

On the algebraic approach it is easy to accommodate anti-existentialism (this does not mean that the algebraic approach requires one to be an anti-existentialist). To illustrate the key idea, let us consider a somewhat simpler example: for all x, necessarily, it is logically possible that x = x.[40] In symbols, $(\forall x)\Box$Possible $[x = x]$. Anti-existentialists would infer from this that, for all x, necessarily, something is logically possible, namely, that x = x. In symbols, $(\forall x)\Box(\exists v)$(Possible v & v = $[x = x]$). How can there be situations in which the proposition that x = x exists if the contingent particular x does not exist in those situations? The algebraic answer would go as follows. The proposition that x = x is simply the result of predicating self-identity of x. That is, $[x = x] = pred_S([u: u = u], x)$. This proposition, like all propositions, is to be thought of as an

irreducible intensional entity. Although the operation of singular predication maps self-identity and x to this proposition, we are free to hold that the existence of this proposition does not entail the existence of x. After all, on the algebraic approach this proposition does not "contain" x in a set-theoretical or mereological sense. Consider an analogy. The father-of function maps me to my father. However, the existence of my father does not entail the existence of me; my father certainly does not "contain" me. The following pair of diagrams illustrate what is going on. Figure 1 represents the actual relationship between the proposition [x = x], the property self-identity, and x; here all three items actually exist. Figure 2 represents a possible but non-actual relationship between [x = x], self-identity, and x; although [x = x] and self-identity exist in this circumstance, x does not.[41]


Figure 1   Figure 2
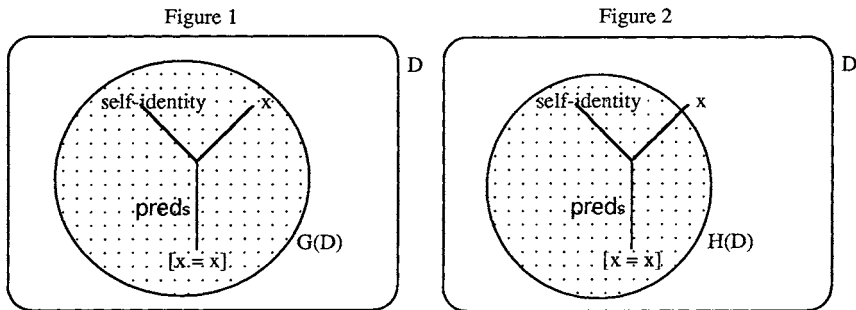
This account permits one to hold that, for all x, necessarily, something is logically possible, namely, that x = x. That is, $(\forall x)\Box(\exists v)$(Possible v & v = [x = x]).

On the picture that emerges, traditional "logical atomism" is not quite right. In various possible situations there exist propositions (e.g., our proposition that x = x) that do not —as traditional atomism requires—have complete analyses exclusively in terms of: basic properties, basic relations, and particulars *that exist in those situations*. A "modal logical atomism" is nevertheless feasible. To illustrate, in a possible situation in which x does not exist, the proposition that x = x does not have the indicated sort of analysis; nevertheless, relative to such a possible situation, there are *other* possible situations (namely, any possible situation in which x exists) in which the proposition that x = x does have the indicated sort of analysis. Now this idea can be iterated to handle ever more complex examples (e.g., the example used earlier against the propositional-complex theory). In this way, propositions are not mystery entities requiring primitive Haecceities for their analysis: we can have a complete structural understanding of them, as atomists thought, but sometimes we must do so in a modal fashion. Our view is that by exploiting this idea we can handle all the data that prompt people to become possibilits.

Incidentally, the prospect of this kind of theory is important for an adequate presentist treatment of time. (Presentism is to eternalism as actualism is to possibilism.[42]) Specifically, it provides a logical framework in which we can explain the following sort of phenomenon: despite the fact that Socrates no longer exists, there nevertheless presently exist "singular" propositions about him, for example, the "singular" proposition that *he* does not now exist.

To show that the above proposal truly yields an actualist, anti-existentialist theory of propositions, one need only construct an intensional algebra that satisfies the indicated requirements. The easiest way to do this is to build a "syntactic" algebra. Consider a formal language like that described in section 2. Let this language have infinitely many primitive sentential letters F_i^0 ($i \geq 0$), infinitely many primitive predicate letters F_i^j ($i \geq 0$, $j \geq 1$), and a supply of names a_i ($i > 0$). The domain D of the algebra is to consist of these primitive expressions: D_{-1} is to consist of the names a_i; D_0, the sentential letters F_i^0; D_1, the 1-ary predicate letters F_i^1; and so forth. The remainder of the construction is straightforward.[43] This kind of syntactic algebra dramatizes the fact that the existence of the items in D_0 (i.e., the syntactically primitive sentential letters F_i^0 which are intended to model propositions) does not depend on the existence of any other items (e.g., the proper names $a_i \in D_{-1}$ which are intended to model contingent particulars). Likewise, the items in D_0 do not "contain" any other items in a set-theoretical or mereological sense. This of course is what real propositions are like.

Although propositions do not have constituents in a set-theoretical or mereological sense, a (non-set-theoretical, non-mereological) notion of "constituency" can be defined within the algebraic framework. In the next section we will characterize certain intensional algebras $<D, K, \mathcal{T}>$ that are *hyper-fine-grained*. In such an algebra, each element of D_0 has a unique "decomposition tree" that is determined by the (inverses of) the logical operations of $\mathcal{T}$. For example, the proposition [Fx] has the following unique decomposition tree:
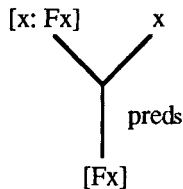
An item is a *constituent* of a proposition iff$_{def}$ the item appears somewhere (besides the initial node) in the proposition's decomposition tree. To say that an item is a constituent of a proposition thus does not imply that the item is a set-theoretical element of the proposition, nor does it imply that the item is a mereological part of the proposition. Propositions are the sort of thing that have neither set-theoretical elements nor mereological parts. As Frege tell us (in "Negation"), "the words 'made up of', 'consists of', 'component of', 'part' may

lead to our looking at it in the wrong way."

5. Propositions and Logical Form

It is widely agreed that substitutivity failures are traceable to differences in *form*, to differences in *content*, or to differences in both form and content. More specifically, substitutivity failures are traceable to differences in *logical form* of relevant 'that'-clauses; or they are traceable to differences in the *intensional content* of primitive constants occurring in relevant 'that'-clauses; or they are traceable to differences in both the logical form and intensional content.

The following is an example of a substitutivity failure that is traceable to a difference in the intensional content of constituent primitive predicates:

> All and only renates are cardiates.
> x believes that there are renates.
>
> ─────────────────────────────
>
> ∴ x believes that there are cardiates.

Here the two 'that'-clauses have the same logical form, but they contain primitive predicates that have different intensional content: 'renate' expresses the property of being a renate and 'cardiate' expresses the distinct property of being a cardiate.

The following is an example of a substitutivity failure traceable to a difference in logical form:

> $((\forall x)x = x \rightarrow (\forall x)x = x) \leftrightarrow (\forall x)(x = x \rightarrow x = x)$.
> x has just realized that $(\forall x)x = x \rightarrow (\forall x)x = x$.
>
> ─────────────────────────────
>
> ∴ x has just realized that $(\forall x)(x = x \rightarrow x = x)$.

Here the two 'that'-clauses contain the same primitive constants, but they have different logical forms.

A great many outstanding substitutivity failures can be (at least partially) explained by adopting the hypothesis that, whenever 'that'-clauses differ in logical form, the propositions denoted by them are different. To illustrate, consider the following symbolization of (the propositional version of) Mates' puzzle:

> $\neg(\exists x)$ x Doubts $[(\forall y)(y$ believes $[D] \rightarrow y$ Beleives $[D])]$
> $[D] = [D']$
>
> ─────────────────────────────
>
> ∴ $\neg(\exists x)$ x Doubts $[(\forall y)(y$ Beleives $[D] \rightarrow y$ Beleives $[D'])]$

Although the constituent intensional contents in the first premise and in the conclusion are alike, the two 'that'-clauses differ in logical form:

$$[(\forall y)(yB[1] \rightarrow yB[1])]$$

and

$$[(\forall y)(yB[1] \rightarrow yB[2])]$$

This fact, together with our hypothesis, implies that these 'that'-clauses denote different fine-grained propositions, and in turn, that substitutivity fails.[44] Likewise, the substitutivity failures in the self-division example (and in the fondalee/rajneesh example in note 19) can be explained in terms of differences in logical form. For two key steps in the argument—namely, [Even (two)] = [Divisible-by (two, two)] and [Self-divisible (two)] = [Divisible-by (two, two)]—can be rejected on the grounds that in each case the 'that'-clauses flanking '=' have different logical form: $[F^1(1)]$ and $[R^2(1,1)]$.

There exists a well-defined class of intensional algebras that capture the semantical analogue of this syntactical notion of logical form.[45] In terms of these hyper-fine-grained intensional algebras an associated notion of logical validity can be defined, and a hyper-fine-grained intensional logic can be formulated and proved complete.

There is a certain irony here. Frege's original puzzle about identity sentences is in a sense solved in this hyper-fine-grained setting. The reason is that the relevant 'that'-clauses —⌜that a = b⌝ and ⌜that a = a⌝ have different logical form: $[R^2(1,2)]$ and $[R^2(1,1)]$. Hence, that a = b and that a = a are different propositions. Therefore, because ⌜a = b⌝ means that a = b and ⌜a = a⌝ means that a = a, it follows that ⌜a = b⌝ and ⌜a = a⌝ mean something different. Thus, we have an answer (or, at least, a partial answer) to Frege's question of how, if true, ⌜a = b⌝ and ⌜a = a⌝ can mean something different.

This cannot be the complete answer, however. To see why, notice that there are very similar substitutivity puzzles that cannot be solved just by pointing to differences in logical form. For example, if names ⌜a⌝ and ⌜b⌝ are co-denoting, how can ⌜[a > b]⌝ mean something different from ⌜[b > a]⌝? (E.g., ⌜a⌝ might be 'Hesperus'; ⌜b⌝, 'Phosphorus'; ⌜>⌝, 'is brighter than'. Or ⌜a⌝ might be 'Cicero'; ⌜b⌝, 'Tully'; ⌜>⌝, 'is more eloquent than'.) After all, ⌜a > b⌝ means that a > b, and ⌜b > a⌝ means that b > a. However, the two 'that'-clauses ⌜that a > b⌝ and ⌜that b > a⌝ have the *same* logical form: $[R^2(1,2)]$. So the puzzle cannot be solved by pointing to a difference in logical form. Therefore, given the standard assumption (namely, that all substitutivity failures are traceable to differences in logical form or differences in intensional content of primitive constants or both), one would be forced to posit a difference in intensional content of the primitive constants ⌜a⌝ and ⌜b⌝. Or else one would be forced to hold that ⌜a > b⌝ and ⌜b > a⌝ do not, strictly and literally, mean something different. As we noted at the outset of the paper, the latter alternative is implausible on its face.[46] At the same time, the former alternative does not seem feasible in view of the arguments of Donnellan, Kripke, *et al.*: there do not appear to be distinct, epistemically

acceptable intensions that can serve as the respective senses of the co-denoting names ⌜a⌝ and ⌜b⌝. Because neither alternative appears acceptable, we must reject the standard assumption that all substitutivity failures are traceable to differences in logical form or differences in intensional content of primitive constants or both. How is this possible? This is the question we now must answer. (In giving our answer, we will find it convenient to discuss the following even simpler instance of the neo-Fregean puzzle: given that 'Cicero' and 'Tully' are co-denoting, how can 'Cicero is a person' and 'Tully is a person' mean something different?)

Before proceeding, notice that there seem to be neo-Fregean puzzles that involve predicates rather than proper names.[47] For example, suppose someone has just heard the predicate 'masticate' used for the first time. The person says, "Evidently, whoever masticates chews, but does whoever chew masticate?" Intuitively, this sentence means something different from the sentence 'Evidently, whoever chews masticates, but does whoever masticates chew?'. How is this possible given that 'chew' and 'masticate' mean the same thing? We cannot solve this problem by pointing to a difference in logical form, for the two sentences have the same logical form. Nor can we solve it by pointing to a difference in the intensional content of the constituent primitive expressions, for, as noted, 'chew' and 'masticate' mean the same thing. But, intuitively, the sentences really do mean something different. What could it be? (In discussing this question, we will also find it convenient to discuss the following variant: how can 'There exists something that chews and does not masticate' and 'There exists something that masticates and does not chew' mean something different?)

Our solution will depend on three further developments. The first concerns the kind of predication involved in certain descriptive propositions. The second concerns the distinction between Platonic and non-Platonic modes of presentation. The third concerns a variant of Russellian semantics.

6. Descriptions

There are four leading theories of definite descriptions: Frege's, Russell's, Evans's, and Prior's.[48]

(1) Frege.[49] On this theory ⌜the F⌝ is an ordinary singular term having a sense and often a reference. The term ⌜the F⌝ has the form ⌜(ιx)(Fx)⌝, where ⌜(ιx)⌝ is an unary operator that combines with a formula to yield a singular term.[50] If there is a unique item satisfying the predicate ⌜F⌝, the singular term ⌜the F⌝ refers to it; otherwise, ⌜the F⌝ has no reference. Truth conditions are as follows: (i) if ⌜the F⌝ has a reference, ⌜The F Gs⌝ is true (false) iff ⌜(∀x)(Fx → Gx)⌝ is true (false); (ii) otherwise, ⌜The F Gs⌝ is neither true nor false. Truth-value gaps are not essential to Frege's theory; to eliminate them, one need only revise clause (ii) as follows: (ii′) if ⌜the F⌝ has no reference, ⌜The F is G⌝ is false. In our subsequent remarks on Frege's theory, we will consider this revised theory for

simplicity of exposition.

(2) Russell.[51] On this theory ⌜the F⌝ is an incomplete symbol, meaningful only in the context of a complete sentence. Sentences containing definite descriptions are mere abbreviations for (or transformations from) sentences containing no descriptions. For example, ⌜The F Gs⌝ is an abbreviation for (transformation from) ⌜($\exists$x)Fx & ($\forall$x)($\forall$y)((Fx & Fy) → x = y) & ($\forall$x)(Fx → Gx)⌝.

(3) Evans.[52] On this theory 'the x' is treated as a binary quantifier ⌜[the x]⌝ that combines with a pair of formulas to yield a new formula. For example, ⌜The F Gs⌝ has the form ⌜[the x](Fx:Gx)⌝. The truth conditions are Russellian.

(4) Prior, et al.[53] On analogy with ⌜some F⌝ and ⌜every F⌝, ⌜the F⌝ is treated as a restricted quantifier ⌜[the x: Fx]⌝ that combines with a formula to yield a new formula. For example, ⌜The F Gs⌝ has the form ⌜[the x: Fx](Gx)⌝. The truth conditions are again Russellian.

Each of these four theories can easily be incorporated into our algebraic approach.[54] We will illustrate how to do this in the case of Frege's theory. Consider intensional algebras in which the set τ contains a unary operator *the* (akin to the Frege-Church operator ι) that takes properties to properties thus: for all properties u ∈ D_1, all H ∈ K, and all items w ∈ D, w ∈ H(*the*(u)) iff w ∈ H(D) & H(u) = {w}. The values of *the* are properties that may be thought of as "individual concepts." For example, *the*(F) may be thought of as the individual concept of being the F. Starting with the individual concept of being the F and the property of being G, how does one form the proposition that the F Gs? This proposition is not the result of a singular predication. When the operation of singular predication is applied to the property of being G and the property of being the F—i.e., pred$_s$([x:Gx], *the*([x:Fx]))—the result is the proposition that the property of being the F Gs. A very different proposition! The relation holding between the property of being G, the property of being the F, and the proposition that the F Gs is therefore not singular predication but rather a quite distinct kind of predication, which may be called *descriptive predication*--pred$_d$, for short. This relation of descriptive predication is implicit in Frege's informal theory of the senses: it is the relation holding between the sense of a predicate ⌜G⌝, the sense of a definite description ⌜the F⌝, and the sense of a sentence ⌜The F Gs⌝.[55] To represent Frege's theory of definite descriptions algebraically, we merely need to restrict ourselves to intensional algebras in which the set τ contains both *the* and pred$_d$, where pred$_d$ behaves thus: for all u,v ∈ D_1, and all H ∈ K, H(pred$_d$(u,v)) = T iff Ø ≠ H(v) ⊆ H(u).[56] So, for example, the proposition that the F Gs = pred$_d$([x: Gx], *the*([x: Fx])). This proposition is true relative to H ∈ K iff Ø ≠ H(*the*([x: Fx])) ⊆ H([x: Gx]). That is, relative to H, the proposition that the F Gs is true iff there exists something that is the unique element in the extension of the property of being F and the extension of the property of being F is included in the extension of the property of being G.

The operation of descriptive predication is implicit in Frege's theory. In what follows, we will make use of this aspect of Frege's theory; more

specifically, we will make use of intensional algebras in which the set τ contains the operation pred_d. In doing so, we do not commit ourselves to Frege's theory of definite descriptions. We could instead adopt something more in the spirit of Evans or of Prior. We pursue the Fregean option because it seems to be the most natural and because it has considerable historical interest.

Incidentally, the case of descriptions illustrates how the algebraic approach yields what may be called a *general semantics*. It does so in much the same way that the possible-worlds categorial method did except that it does not erroneously try to reduce properties, relations, and propositions to possible-worlds constructs and it is not hamstrung by implausible categorial restrictions. Davidsonian truth-conditional semantics is also a *general* semantics in a sense; however, it abandons the primary task of semantics, namely, the specification of what meaningful expressions *mean*.[57] (E.g., for English sentences $\ulcorner S \urcorner$, $\ulcorner S \urcorner$ means that S.) The fact that possible-worlds semantics and truth-conditional semantics are unsatisfactory in various ways does not mean that the valuable insights embodied in them cannot be preserved in the algebraic approach. Indeed, once relevant syntactic structures are uncovered and once the accompanying truth conditions are found, the rest is virtually automatic: one need only restrict oneself to intensional algebras in which the set τ contains corresponding logical operations whose behavior with respect to the extensionalization functions $H \in K$ match those truth conditions. The discussion in this section illustrates how this automatic adaptation works.

7. Non-Platonic Modes of Presentation

We have noted that the domain D in an intensional algebra partitions into denumerably many subdomains $D_{-1}, D_0, D_1, D_2, \ldots$ .We have been thinking of D_1 as being comprised of properties. Let us instead think of D_1 as being comprised of modes of presentation (*Arten des Gegebenseins*). Properties, which are purely Platonic entities, are one kind of mode of presentation, but they are not the only kind. There are also certain "constructed" entities that present objects to us. For example, pictures do. Certain socially constructed entities also fulfill this function. Prominent among these are linguistic entities. Indeed, linguistic entities provide the only access that most of us have to historical figures—for example, Cicero—and they are *public* entities which can be shared by whole communities. Intentional naming trees (or causal naming chains) are one kind of linguistic entity that serve this function. For example, the 'Cicero' historical naming tree provides us with access to Cicero. A closely related mode of access is our very practice of using 'Cicero' to name Cicero. Another is the name 'Cicero' itself. Of course, the name must be understood not as a mere phonological or orthographic type but as a fine-grained entity individuated by the social practice. (E.g., just as our practice of using 'Cicero' to name the Illinois town differs from our practice of using 'Cicero' to name the orator, so the town's name,

which is comparatively new, differs from the orator's name, which is much older.) Insofar as these linguistic entities (the tree, the practice, and the name) provide us with access to Cicero, they function as modes of presentation of Cicero.

Our hypothesis will be that any of these three kinds of non-Platonic modes of presentation can be used in a solution to our puzzles.[58] Now observe that there is a natural one-one map from intentional naming trees onto conventional naming practices (the tree may be thought of as the practive "spread out in history"), and there is a natural one-one map from conventional naming practices onto the associated names. Because there exist these natural one-one maps, it will make little difference to us which kind is best — historical naming trees, conventional naming practices, or names themselves. For illustrative purposes, we will fill out the hypothesis first with naming practices playing the key role. Following that we will show how the hypothesis would work if we let names or naming trees play that role.

On the Kripke picture, a conventional naming practice typically consists of an initial act of baptism, with or without a baptized object actually present, together with an ongoing convention for using the name with the intention of referring to whatever it was that was referred to by previous uses of the name. Let us accept this picture, at least provisionally.[59]

The diagrams below are intended to represent three conventional naming practices:
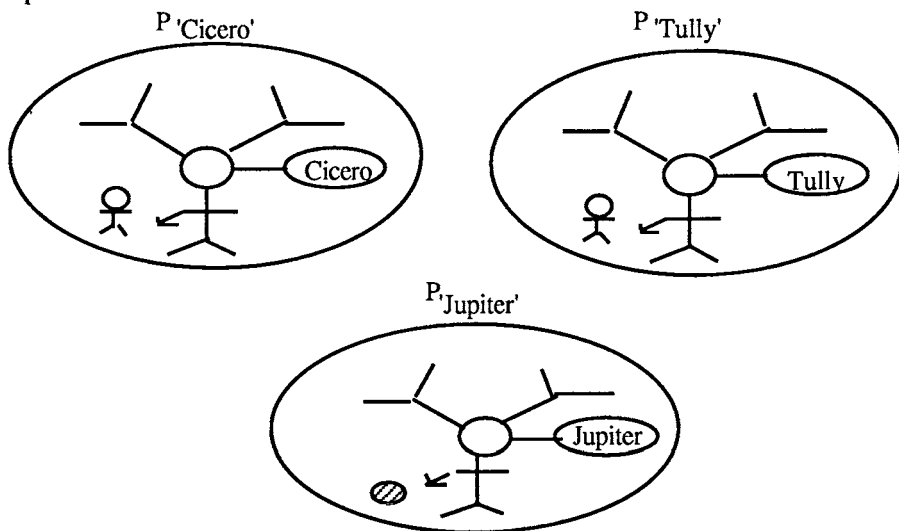
In $P_{\text{'Jupiter'}}$ no baptized object is there. By contrast, in $P_{\text{'Cicero'}}$ and $P_{\text{'Tully'}}$ there is a baptized object; indeed, the very same object. Nevertheless, $P_{\text{'Cicero'}}$ and $P_{\text{'Tully'}}$ are distinct. These two conventional naming practices present the object Cicero (= Tully) to us, yet they do so in different ways. (Similarly, $P_{\text{'Jupiter'}}$ *purports* to present an object to us; there just happens to be no object that is

really presented.) Insofar as our conventional naming practices present objects (or purport to present objects), they may be regarded as modes of presentation. Accordingly, even though these conventional naming practices are contingently existent, socially constructed entities, there are intensional algebras in which they are elements, not of D_{-1}, but instead of D_1. In such intensional algebras the extensionalization functions $H \in K$ would behave as one would expect: for all $H \in K$, $H(P_{\text{`Cicero'}}) = \{Cicero\} = \{Tully\} = H(P_{\text{`Tully'}})$; and $H(P_{\text{`Jupiter'}}) = \emptyset$. In these intensional algebras relevant logical operations would be defined over all modes of presentation, conventional naming practices as well as properties. So, for example, the operation of descriptive predication, pred_d, would be defined on the property of being a person and $(P_{\text{`Cicero'}})$: $\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})$. Likewise, descriptive predication would be defined on the property of being a person and $P_{\text{`Tully'}}$: $\text{pred}_d(\text{being a person}, P_{\text{`Tully'}})$. Given that we are dealing with hyper-fine-grained intensional algebras, and given that $P_{\text{`Cicero'}}$ and $P_{\text{`Tully'}}$ are distinct, these two propositions—$\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})$ and $\text{pred}_d(\text{being a person}, P_{\text{`Tully'}})$—are distinct. Given that our intensional algebras are actualist and anti-existentialist in character, there are possible circumstances in which the resulting propositions would exist even if our contingent, socially constructed conventional naming practices $P_{\text{`Cicero'}}$ and $P_{\text{`Tully'}}$ were not to exist. Now suppose that, for all $x \in D$, if $x \in G(\text{being a person})$, then, for all $H \in K$, $x \in H(\text{being a person})$. That is, if x belongs to the actual extension of being a person, then x belongs to every possible extension of being a person. Hence, every person is necessarily a person. Then, given that, for all $H \in K$, $H(P_{\text{`Cicero'}}) = \{Cicero\} = \{Tully\} = H(P_{\text{`Tully'}})$, it follows that, for all $H \in K$, $H(\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})) = T$ and $H(\text{pred}_d(\text{being a person}, P_{\text{`Tully'}})) = T$. That is, our two propositions $\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})$ and $\text{pred}_d(\text{being a person}, P_{\text{`Tully'}})$ are necessarily true. Hence, these two propositions have the same modal value that Kripke *et al.* would like to attribute to the proposition that Cicero is a person and the proposition that Tully is a person.[60] At the same time, our two propositions—$\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})$ and $\text{pred}_d(\text{being a person}, P_{\text{`Tully'}})$—are not descriptive propositions; that is, they are distinct from propositions expressed with the use of definite descriptions (with or without actuality operators). For example, $\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})$ is distinct from each of the following: the proposition that the thing presented by our conventional naming practice $P_{\text{`Cicero'}}$ is a person; the proposition that the thing presented by *this* conventional naming practice is a person; the proposition that the thing actually named 'Cicero' is a person; and so forth. Finally, these propositions—$\text{pred}_d(\text{being a person}, P_{\text{`Cicero'}})$ and $\text{pred}_d(\text{being a person}, P_{\text{`Tully'}})$—are not metalinguistic in any of the usual senses: First, given that our theory is anti-existentialist, these propositions are ontologically independent of the relevant linguistic entities $P_{\text{`Cicero'}}$ and $P_{\text{`Tully'}}$ insofar as it is possible for them to exist even if $P_{\text{`Cicero'}}$ and $P_{\text{`Tully'}}$ were not to exist.[61] These linguistic entities are certainly not *in* these propositions. Second, these propositions are

distinct from propositions expressed by sentences containing metalinguistic vocabulary. Third, when someone (e.g., a child or an ill-educated adult) is thinking one of these propositions, there is no evident need for the person to be employing any relevant notions from linguistic theory, e.g., the notion of a name or the notion of a conventional naming practice. These propositions are seamless; only in their logical analysis do the metalinguistic modes of presentation appear.[62]

We have been seeking a theory of propositions in which, for example, the proposition that Cicero is a person and the proposition that Tully is a person have the following features.[63] They should be distinct from each other. They should not be ontologically dependent on contingently existing things in the sense that it should be possible for them to exist even if relevant contingent things do not exist; contingently existing theings should not be *in* them. They should be necessarily true. They should not be descriptive (i.e., they should not be the sort of proposition expressed by sentences containing definite descriptions). Finally, they should not be metalinguistic in the usual senses just listed.[64] Propositions such as $pred_d$(being a person, P$_{\text{‘Cicero’}}$) and $pred_d$(being a person, P$_{\text{‘Tully’}}$) have all these features. Thus, they are promising candidates for the sort of propositions that have been eluding us.[65]

For our next candidate, consider names themselves. There is a notion of a name according to which Cicero, the famous orator, and Cicero, the town in Illinois, have the same name. However, there also seems to be a more fine-grained notion of a name according to which there are *two* names written and pronounced the same way, one naming the famous person and the other naming the town. Supposing this to be so, let ‘Cicero’$_{person}$ be the person's name, and let ‘Cicero’$_{town}$ be the town's name. Likewise, let ‘Tully’$_{person}$ be another one of the famous person's names in this fine-grained sense. In their way, both ‘Cicero’$_{person}$ and ‘Tully’$_{person}$ present the person Cicero to us; they may thus be regarded as modes of presentation. Accordingly, there are intensional algebras in which they are elements of D_1. In such intensional algebras the extensionalization functions $H \in K$ would behave as expected: H(‘Cicero’$_{person}$) = {Cicero} = {Tully} = H(‘Tully’$_{person}$). This suggests the following proposal: the proposition that Tully is a person = $pred_d$(being a person, ‘Tully’$_{person}$) and the proposition that Cicero is a person = $pred_d$(being a person, ‘Cicero’$_{person}$). The propositions $pred_d$(being a person, ‘Tully’$_{person}$) and $pred_d$(being a person, ‘Cicero’$_{person}$) are plausible candidates because they have all the features listed above that are possessed, respectively, by the proposition that Tully is a person and the proposition that Cicero is a person.

For our third candidate, consider the intentional naming tree associated with the name ‘Cicero’ and the intentional naming tree associated with the name ‘Tully’. (Hereafter T$_{\text{‘Cicero’}}$ and T$_{\text{‘Tully’}}$, respectively.) In an obvious way these intentional naming trees serve to present Cicero (= Tully) to us, and thus they may be regarded as modes of presentation. Accordingly, there are intensional

algebras in which $T_{\text{'Cicero'}}$ and $T_{\text{'Tully'}}$ are among the elements of D_1. In such intensional algebras the extensionalization functions $H \in K$ behave thus: $H(T_{\text{'Cicero'}}) = \{\text{Cicero}\} = \{\text{Tully}\} = H(T_{\text{'Tully'}})$. We thus have a third proposal: the proposition that Cicero is a person = $\text{pred}_d(\text{being a person}, T_{\text{'Cicero'}})$ and the proposition that Tully is a person = $\text{pred}_d(\text{being a person}, T_{\text{'Tully'}})$. These two propositions—$\text{pred}_d(\text{being a person}, T_{\text{'Cicero'}})$ and $\text{pred}_d(\text{being a person}, T_{\text{'Tully'}})$—have all the features listed above that are supposed to be possessed, respectively, by the propositions that Cicero is a person and that Tully is a person.

Besides the above three proposals there are several others that are based on other candidate types of non-Platonic modes of presentation.[66] It would be premature to declare any one of these proposals to be the best; rather, one should canvass the full range of proposals and simply let the data determine which one is best.[67] Nevertheless, because this general approach provides such a rich array of finely discriminated propositions, it is likely that it makes available enough propositions for a solution to the neo-Fregean puzzles. For the remainder of the paper we will assume that this thesis is correct.

Before bringing this section to a close, we will introduce a notational convention that will be useful in what follows. On each proposal we have considered there is a regular connection between expressions and associated non-Platonic modes of presentation. For example, between the expression 'Cicero' and our conventional linguistic practice $P_{\text{'Cicero'}}$, or between the expression 'Cicero' and the famous person's name 'Cicero'$_{\text{person}}$ (as opposed to the town's name 'Cicero'$_{\text{town}}$), or between the expression 'Cicero' and the associated intentional naming tree $T_{\text{'Cicero'}}$. Suppose that on the proposal that validates our thesis (just stated)—one of these three proposals or some further proposal—there is is some sort of a regular connection comparable to these three. We may then introduce the following notational convention: if e is an expression and m is the non-Platonic mode of presentation to which e bears the indicated regular connection, then m will be denoted by the expression that results from enclosing e in double quotation marks. So, for example, "Cicero" might be our conven-tional linguistic practice $P_{\text{'Cicero'}}$, the famous person's name 'Cicero'$_{\text{person}}$, the intentional naming tree $T_{\text{'Cicero'}}$, or perhaps some other non-Platonic mode of presentation, depending on which candidate proposal is correct.

We have been discussing non-Platonic modes of presentation that have regular connections with names. But there are also non-Platonic modes of presentation that have regular connections with predicates (e.g., our conventional linguistic practices of using a given predicate to express a relevant property or relation; intentional predicating trees; etc.) The above notational convention is also intended to apply to predicates. So, for example, "'chew'" and "'masticate'" are to denote relevant non-Platonic modes of presentation.

8. Semantics

> It does not belong in the garbage dump of informal pragmatics.
>
> Anil Gupta

We have devised a logical theory that provides a rich array of finely distinguished propositions—enough to serve as the objects of our attitudes and as the meanings of our utterances. Solutions to our family of puzzles should be close at hand. But will these solutions be semantic or pragmatic? For example, are meaning differences that we standardly attribute to ordinary sentences (e.g., 'Cicero is a person' and 'Tully is a person') semantical differences—i.e., differences in their literal meaning—or are they informal pragmatic differences determined by features of the context of utterance and Gricean rules?

An informal pragmatic solution to the puzzles is no doubt feasible.[68] However, there are two potential problems. First, it might entail that the literal meanings of 'Cicero is a person' and 'Tully is a person' are the same. Likewise for 'Cicero is more eloquent than Tully' and 'Tully is more eloquent than Cicero'. Second, an informal pragmatic solution cannot be made systematic. A semantical solution promises to be in the clear on these counts. Without committing ourselves to all the details, we will now sketch such a solution.

One proposal for specifying the semantics for primitive non-logical constants—names and predicates—is to treat the non-Platonic modes of presentation that we have been discussing as a new type of Fregean sense. Accordingly, one might venture the following:

> 'Cicero' expresses "Cicero".
> 'Tully' expresses "Tully".
> 'chew' expresses "chew".
> 'masticate' expresses "masticate".

And so forth. Using our algebraic techniques, one could then specify meanings for whole sentences in the obvious way. However, this proposal seems mistaken. First, consider predicates. Intuitively, 'chew' and 'masticate' mean *exactly* the same thing. Not so according to the present proposal. Indeed, this proposal would block the very possibility of distinct but synonymous predicates and, in turn, the very possibility of distinct but synonymous sentences. This would be a highly implausible outcome, incompatible with one of the desiderata stated at the outset of the paper. Second, in the case of names ('Cicero', 'Tully', etc.), this proposal also seems wrong: names, intuitively, do not seem to *express* anything at all; they just name. We should therefore look elsewhere.

Our proposal will be based on three ideas: (1) Russellian semantics, (2) a variant of the standard compositionality principle, (3) a relational (as opposed to a functional) approach to sentence meaning.

First, Russellian semantics.[69] This style of semantics is based on three theses. (1) Proper names merely name. They have no meaning above and beyond

what they name; in particular, they do not have Fregean senses. (2) One-place predicates express properties; two-place predicates express binary relations; and so forth. Predicates have no meaning other than the properties or relations they express; in particular, they do not have Millian denotations or Fregean references. (Certainly, the verb 'chews' does not *refer* to anything!) (3) Sentences express propositions. They have no meaning other than the propositions they express; in particular, they do not have Fregean references. (Surely the sentence 'Cicero chews' does not *refer* to anything!) Because names only name and because predicates and sentences only express, Russellians may offer a streamlined semantics based on a single underlying semantical relation, namely, meaning. Naming may then be treated as the restriction of this relation to names, and expressing may be treated as the restriction of this relation to predicates and sentences.[70] Accordingly, a Russellian semantics for primitive non-logical constants (names and predicates) in English may be written thus:

> ⌈a⌉ means a.
> ⌈F⌉ means F-ing.

For example:

> 'Cicero' means Cicero.
> 'Tully' means Tully.
> 'chew' means chewing.
> 'masticate' means masticating.

The remaining main task is then to define the meaning relation for whole sentences. It is here that we depart from Russell, for this task requires a variant of the principle of compositionality assumed by Russell (and Frege). This brings us to the second idea.

Russell and Frege assumed a very strong compositionality principle: sentence meaning is a function of the meanings of the primitive expressions contained within a sentence plus the logical form (or logical forms) of the sentence. Plausible though this principle may be, it yields unacceptable consequences within the setting of a Russellian semantics for names and predicates. For example, it would follow that there is no difference in meaning between 'Cicero is a person' and 'Tully is a person' (or between 'Cicero is more eloquent than Tully' and 'Tully is more eloquent than Cicero') and it would follow that there is no difference in meaning between 'There exists something that masticates and does not chew' and 'There exists something that chews and does not masticate'. But intuitively there are differences.[71] A more conservative compositionality principle is in order: sentence meaning is a function of (1) the primitive expressions contained within a sentence, (2) the (Russellian) meanings of those expressions, and (3) the logical form (or logical forms) of the sentence. In a moment we will apply this principle to some illustrative examples.

Our third idea is that meaning is a relation not a function. The phenomena

of lexical and syntactic ambiguity already show this. What is *the* meaning of 'The bank is flooded'? What is *the* meaning of 'Everyone loves someone'? These questions have no answer, for they carry a false presupposition, namely, that each of these sentences has only one meaning. In fact, they have more than one meaning. Meaning is a *relation* between a sentence and its several meanings. The job of semantics to identify these several meanings. It is a job of pragmatics to identify which (if any) of these several meanings is meant by a speaker in a context. Our thesis is that meaning is a relation even when the sentence in question is not lexically or syntactically ambiguous: such a sentence always means a cluster of closely related propositions. Let us build up to this thesis in four stages.[72]

First, 'Cicero is a person' and 'Tully is a person' mean something different; that is, each has a meaning (perhaps more than one) that that other does not have. What are they? A plausible (at least partial) answer is that 'Cicero is a person' means $pred_d$(being a person, "Cicero") and 'Tully is a person' means $pred_d$(being a person, "Tully"). Because these two propositions are different, the two sentences mean something different.

Second, 'There exists something that chews and does not masticate' and 'There exists something that masticates and does not chew' differ somehow in meaning; that is, each has a meaning (perhaps more than one) that the other does not have. (The interrogative forms dramatize the point: 'Does there exist something that masticates and does chew?' and 'Does there exist something that chews and does not masticate?' certainly seem to mean something different.) What could this difference in meaning be? A plausible answer (or, at least, partial answer) is that 'There exists something that chews and does not masticate' means exist(conj("chew", neg("masticate"))) and 'There exists something that masticates and does not chew' means exist(conj("masticate", neg("chew"))).[73] This proposal answers the question, for these two propositions are different on the hyper-fine-grained conception of propositions described earlier. Now if this proposal is right, then, by analogy, it would seem that another thing that 'Cicero is a person' means is $pred_d$("person", "Cicero").

Third, the two sentences '9 is a number' and 'ix is a number' intuitively have *a* meaning in common. What is it? We may assume that "9" ≠ "ix". If so, given our hyper-fine-grained theory of propositions, $pred_d$(being a number, "9") ≠ $pred_d$(being a number, "ix"). In this case, neither of these propositions is a plausible candidate for being a meaning shared by both '9 is a number' and 'ix is a number'. So what meaning do these sentences share? The only plausible answer is that the shared meaning is the singular proposition $pred_s$(being a number, nine)[74]. However, if $pred_s$(being a number, nine) is indeed one of the things that '9 is a number' means, then, by analogy, it would seem that $pred_s$(being a person, Cicero) is one of the things that 'Cicero is a person' means.

Fourth, if $pred_d$("person", "Cicero") is (by point two) one of the things 'Cicero is a person' means and $pred_s$(being a person, Cicero) is (by point three)

another thing that 'Cicero is a person' means, then, by analogy, it would seem that 'Cicero is a person' must also mean $pred_s$("person", Cicero).

Assembling these four points, we get the following conclusion. 'Cicero is a person' has a cluster of closely related meanings:

> $pred_s$(being a person, Cicero)
> $pred_s$("person", Cicero)
> $pred_d$(being a person, "Cicero")
> $pred_d$("person", "Cicero").

Notice that this cluster of meanings is determined in an obvious way by (1) the primitive expressions contained within the sentence, (2) their Russellian meanings, and (3) the logical forms of the sentence. This pattern generalizes: the cluster of meanings possessed by a sentence is a function of (1) the primitive expressions contained within the sentence, (2) their Russellian meanings, and (3) the logical forms of the sentence. (This is just the conservative compositionality principle stated earlier.) This style of semantics specifies the cluster of propositions that the sentence literally expresses. It is then the job of pragmatics to determine which of the propositions in this cluster is actually meant by a literal assertion of the sentence in a context.

Summing up, we have seen that, given a Russellian semantics for primitive expressions and given a satisfactory algebraic theory of logical form, a relational semantics for sentences looks feasible. In this semantics, proper names do not have Fregean senses, and predicates do not have Fregean references or Millian denotations. Nevertheless, a sentence like 'Cicero is a person' does have a meaning not shared with 'Tully is a person' and 'Tully is a person' has a meaning not shared with 'Cicero is a person'. We submit that this is what explains our initial intuition that these two sentences mean something different. The same thing goes for sentences like 'Cicero is more eloquent than Tully' and 'Tully is more eloquent than Cicero' and for sentences like 'There exists something that chews and does not masticate' and 'There exists something that masticates and does not chew'. At the same time, the propositions involved in this account have all the requisite features isolated in the course of our discussion: they are ontologically independent of the contingently existing things (in the sense that it is possible for them to exist when those contingently existing things do not exist); their modal values conform to those that Kripke *et al.* would assign; they are not the sort of proposition expressed by sentences containing definite descriptions, and they are not metalinguisitc in any of the standard senses. We thus appear to have the makings of a purely semantical solution to the neo-Fregean puzzles.

Notice that the proposed semantics allows for a significant amount of intralinguistic and interlinguistic translation. Consider *intra*linguistic translation. 'Something chews' and 'Something masticates' share a meaning—namely, exist(masticating). In many (perhaps most) contexts a literal assertion of one of

these sentences could be replaced by an assertion of the other without any change of meaning. Likewise, for infinitely many other pairs of English sentences. Next consider *inter*linguistic translation. Every English sentence has a meaning that is expressible in another language as long as the other language has primitive expressions with the same Russellian meanings as the primitive expressions in the original sentence and the other language has the same repertoire of logical forms as English. (The shared interlinguistic sentence meanings are those that arise by applying the relevant logical operations to the relevant Russellian word meanings.) In a great many contexts, these shared sentence meanings are the basis for exact interlinguistic translation.

However, English sentences have another kind of meaning, namely, the kind of meaning that arises by applying relevant logical operations to relevant *non-Platonic* modes of presentation. Typically, this kind of meaning cannot be expressed in a foreign language unless the translator engages in term-borrowing. For example, suppose someone were to make a literal assertion of 'There exists something that masticates and that does not chew'. The proposition meant would not be the trivial falsehood exist(conj(masticating, neg(masticating))); rather, it would be a proposition that arises by applying relevant logical operations to relevant non-Platonic modes of presentation—"masticate" or "chew" or both. Likewise for literal assertions of sentences like 'Cicero is more eloquent than Tully'. Typically, the proposition meant would not be the trivial falsehood $pred_s(pred_s(being$ more eloquent than, Cicero), Cicero); rather, it would be a proposition that arises by applying relevant logical operations to one or more non-Platonic modes of presentation—"Cicero" or "Tully" or both. Even if a foreign language has a pair of primitive predicates that express the same property that 'chew' and 'masticate' express (i.e., the property of masticating) and even if the foreign language has a pair of names that name the same person as 'Cicero' and 'Tully' (i.e., Cicero), the foreign language does not contain sentences that would express *exactly* the same propositions (unless, of course, the foreign language already contains the relevant primitive expressions themselves—'chew' and/or 'masticate'; 'Cicero' and/or 'Tully'). This assessment is borne out by the actual practice of translators. To capture one of these propositions exactly, a translator always borrows the relevant primitive expressions ('chew' and/or 'masticate; 'Cicero' and/or 'Tully') from the other language. Translators know that in those special cases this is the only way to get an exact translation.[75]

9. Three Applications

First, these remarks on translation put us in a position to propose a solution to Kripke's puzzle about Pierre's beliefs.[76] One of the meanings of 'Londres est jolie' is $pred_d(being$ pretty, "Londres"), and one of the meanings of 'London is pretty' is $pred_d(being$ pretty, "London"). Upon seeing a picture of a pretty-looking city labeled 'Londres', Pierre asserts 'Londres est jolie' with the

intention of speaking literally and sincerely. He means—and believes—$pred_d$(being pretty, "Londres"). Later, after living in an unattractive section of London, Pierre asserts 'London is not pretty' with the intention of speaking literally and sincerely. He means—and believes—neg($pred_d$(being pretty, "London")).[77] The latter proposition does not contradict the one Pierre meant—and believed—earlier because $pred_d$(being pretty, "London") ≠ $pred_d$(being pretty, "Londres"). This is so because "London" ≠ "Londres". The puzzle arises because Kripke mistakenly claims that 'Londres est jolie', as Pierre meant it, has as an exact translation 'London is pretty'. Kripke is right that the two sentences do share *a* meaning, namely, $pred_s$(being pretty, London). However, 'Londres est jolie' has another meaning not possessed by 'London is pretty', namely, $pred_d$(being pretty, "Londres"). This is what Pierre stated and believed. Since 'London is pretty' does not have this meaning, 'Londres est jolie', as Pierre meant it, does not translate into 'London is pretty'. Indeed, as Pierre meant it, 'Londres est jolie' does not have any *exact* English translation. To obtain an exact English translation, a translator would borrow the French name 'Londres'. Accordingly, as Pierre meant it, 'Londres est jolie' would be translated as 'Londres is pretty', which, as we have seen, means $pred_d$(being pretty, "Londres"). It is no coincidence that in discussion of Kripke's puzzle philosophers usually use 'Londres is pretty' to report Pierre's original belief.[78]

Second, consider an English speaker who is familiar with the name 'Phosphorus' but not 'Hesperus'. Suppose that by pure chance the person makes the stipulation that 'Hesperus' is hereafter to be another name for Phosphorus. By an adaptation of Kripke's meter-stick argument, Kripke would be committed to holding that the person would know something *a priori*. But what is it? Would the person know *a priori* that Hesperus = Phosphorus? That is, would the person know the often discussed necessity (about which Kripke puzzles in the quotation at the outset of the present paper)? If so, Kripke's famous doctrine that this necessity is essentially *a posteriori* would collapse. Our theory solves this problem, for the relevant non-Platonic modes of presentation are different: the new naming tree initiated by the person is different from the very old naming tree $T_{\text{'Hesperus'}}$; the person's newly instituted practice is different from our standing practice $P_{\text{'Hesperus'}}$; and the person's newly introduced name is different from our standing name. Accordingly, descriptive predications involving the new non-Platonic mode of presentation (the new tree, the new practice, the new name) result in propositions that are different from those which result from descriptive predications involving instead our standing non-Platonic mode of presentation "Hesperus". Even though the former proposition can be known *a priori*, the latter cannot. I believe that something this like this is required to solve the problem and, more generally, to reconcile Kripke's scientific essentialism with the sort of *a priori* knowledge that is associated with stipulative definitions.

Third, the foregoing ideas also provide the makings of a treatment of

demonstratives. Suppose that I see an object x directly in front of me and simultaneously see the same object x (without realizing that it is the same) through a complicated lens set-up on my left. Suppose that, while glancing straight ahead, I sincerely assert 'This is a pencil' with an intention of speaking literally. Intuitively, I would mean—and believe—something different from what I would mean—and believe—if, while glancing to the left, I sincerely assert 'That is a pencil'. What is the difference?[79] Our logical theory provides a range of promising answers. The simplest is this. When I assert 'This is a pencil', I mean—and believe—$pred_d$(being a pencil, "this"), and when I assert 'That is a pencil', I mean—and believe—$pred_d$(being a pencil, "that"). The idea is that "this" and "that" are limiting cases of the sorts of non-Platonic modes of presentation we have been discussing: for example, perhaps "this" = my act of referring to x by uttering 'this' on the indicated occasion, and perhaps "that" = my act of referring to x by uttering 'that' on the indicated occasion. In this case "this" ≠ "that", and therefore, $pred_d$(being a pencil, "this") ≠ $pred_d$(being a pencil, "that"). Perhaps this is the intensional distinction we are seeking.

There are other promising proposals in the same vein. However, in order to get to the philosophical point we wish to make, let us suppose that the above proposal is acceptable. How can we formulate a *semantics* for demonstratives and for sentences containing demonstratives? Several plausible formulations are feasible in our framework; at this stage we need take no stand on which one is best. The philosophical point is that among the alternatives is the view that Russell espoused in "Lectures on Logical Atomism."[80] According to this view, demonstratives are radically ambiguous expressions. When we strip Russell's view of its notorious privatism, we get something like the following: for any candidate object of reference x, 'this' strictly and literally means x. Likewise for 'that'. (Besides these purely semantical statements, a complete theory of demonstratives would also include "rules of use"—instructions for how and when to use a demonstrative.) Sentence meaning is then described as one might expect. For example, for each candidate object of reference x and each act y of referring to x by uttering 'this', the sentence 'This is a pencil' means each proposition in the following cluster: $pred_d$(being a pencil, y); $pred_d$("pencil", y); $pred_s$(being a pencil, x); $pred_s$("pencil", x).[81] Of course, as in all other cases, it is the job of pragmatics to determine which of these propositions is actually meant by a literal utterance of the sentence in a given context.

An advantage of a Russellian semantics for demonstratives is its unity: all sentences, including sentences containing demonstratives, have specifiable literal meanings; the sorts of things they mean are the same as the sorts of things that speakers believe when speakers sincerely assert them with the intention of speaking literally, and what they mean are bearers of such properties as truth and falsity, and necessity, contingency, and possibility.[82] However, it must be recognized that these ideas have been highly programmatic. Plainly there is a rich array of phenomena that need to be examined carefully (for example,

pronoun anaphora descending from an initial use of a demonstrative, etc.). Until this is done, it would be premature to suggest any one treatment as the best.

10. Conclusion

The foregoing discussion supports the view that, despite recent doubts,[83] the theory of properties, relations, and propositions still provides the most promising framework for a unified treatment of sentence meaning, mental content, truth, and modality. Certain cautionary remarks might be helpful, however.

(1) The proposed theory posits non-Platonic as well as Platonic modes of presentation. We know what it takes for something to be a Platonic mode of presentation (i.e., it is necessary and sufficient that it be either a property or a relation). What does it take for something to be a non-Platonic mode of presentation? Three points are in order. First, although it would be attractive theoretically to have a general analysis of the notion of mode of presentation (i.e., jointly necessary and sufficient conditions), there is no *requirement* that we have such an analysis. It is enough that we have a (relatively clear) grasp of what does and does not count as a mode of presentation in the intended sense. Surely we do. Second, if our algebraic theory is correct, then we *can* give a general analysis: a mode of presentation is any item over which relevant fundamental logical operations are well-defined. (E.g., u is a mode of presentation iff, for some v, $pred_s(u,v)$ is well-defined.) A mode of presentation is then non-Platonic iff it is not Platonic. Third, although this analysis is formally correct, there remains a question: by virtue of what do non-Platonic modes of presentation serve to present their respective objects? As before, there is no *requirement* to have an answer, but suppose that someone were to venture one. That answer would no doubt invoke intentional states. Would this pose a threat of circularity? There is no special reason to think so. First, if the propositions involved in these intentional states are ones whose constituents are all Platonic modes of presentation (and perhaps individuals), there would plainly be no circularity. I can see no reason to think that such propositions would not suffice for the contemplated answer. Second, among propositions of this sort are ones that are definable by means of diagonalization techniques. As is well known, such techniques can often eliminate a vicious circle even when there is *prima facie* appearance of one.[84]

(2) The proposed theory of propositions provides a framework for dealing in a unified fashion with sentence meaning, mental content, truth, and modality. The theory does not provide a specification of the conditions (necessary and/or sufficient) required for someone's having a given proposition as a mental content.[85] That is a separate project; whether or not it can be carried out makes no difference to the success of our theory. We certainly are not committed to the existence of specifiable necessary and sufficient conditions here.

(3) In this century, Russell's Principle of Acquaintance is the most famous attempt to specify a substantive necessary condition for having a given mental content. According to Russell's Principle, a proposition can be someone's mental content only if it is analyzable into constituents with which the person is acquainted. For decades, however, it has been recognized that the phenomena of quantifying-in, singular propositions, and *de re* thought refute Russell's Principle. Indeed, it was recognized long ago that these phenomena establish a form of "externalism"; after all, it is plain that a necessary condition for having ordinary *de re* thoughts is that relevant "external" conditions must be met.

What about *de dicto* thoughts? Arguments by Donnellan, Kripke, Putnam, and Burge provide reasons to question Russell's Principle of Acquaintance for *de dicto* thoughts. However, on their own, these arguments are not *absolutely* conclusive, for Donnellan, Kripke, *et al.* do not provide an analysis of the relevant *de dicto* propositions to determine conclusively that they are truly "externalist" in character. (Recall the quotation from Kripke at the outset of the paper.) To the extent that this is accomplished by the above theory of propositions, this gap in the refutation of Russell's Principle is closed.

These set-backs for Russell's Principle do not, however, show that all broadly Russellian theories of mental content are mistaken. Neo-Russellians are free to hold that at least some principle of the following form is true: x believes p iff for some q, x believes q, q is analyzable into constituents with which x is acquainted, and p stands in a certain logical-social-causal relation R to q. (Alternatively, neo-Russellians could advocate a *series* of such principles depending on the category of proposition under consideration. Moreover, they could replace the biconditional 'iff' with a mere conditional 'only if'. They have no need to suppose that there are non-circular jointly necessary and sufficient conditions. Necessary conditions are all that matter for the point we are about to make.) This sort of principle does not imply that the propositions that are the objects of most of our everyday beliefs are analyzable into acquaintables, nor does it purport to analyze the belief relation. It is simply a synthetic (*vs.* analytic) principle mapping the class of all beliefs into a privileged subclass that are analyzable into acquaintables. The compelling arguments of Donnellan, Kripke, Putnam, Perry, and Burge are entirely consistent with the truth of this sort of principle. Depending on the nature of the indicated logical-social-causal relation—more specifically, depending on which sorts of propositions are in the range of this relation—much of the traditional epistemology and metaphysics implicit in Russell's philosophy could still be right. Despite the impressive recent progress in philosophical logic and philosophy of language, as of today most of these fundamental philosophical issues are still open.

Notes

1. I presented many of these ideas in three courses: the University of Padua in spring 1989, the Second European Summer School in Logic, Linguistics, and

2. Many people advocate a pragmatic solution according to which, if $\ulcorner a = b \urcorner$ is true, $\ulcorner Fa \urcorner$ has the same literal meaning and cognitive value as $\ulcorner Fb \urcorner$. This solution was developed in section 39, "Pragmatics," in my book *Quality and Concept* (Oxford: Oxford University Press, 1982) although I refrained from officially advocating it. More recently, Nathan Salmon (*Frege's Puzzle*, Atascadero, CA: Ridgeview, 1992; originally published by MIT Press in 1986.) has adopted this as his official solution. However, nearly everyone (including advocates) admits that it is initially unintuitive.

3. These three assumptions are defended in detail in G. Bealer and U. Mönnich, "Property Theories," (*Handbook of Philosophical Logic*, volume 4, Dordrecht: Kluwer, 1989, pp. 133-251). For example, it is explained there why positing a three-place belief relation leads to an unacceptable treatment of iterated belief sentences.

4. The following is another example of this type: for all speakers x, all propositions $y_1, ..., y_n$, and all English sentences S, if x sincerely asserts S with the intention of speaking literally and if, for all propositions z, S means z only if $z = y_1$ or ... or $z = y_n$, then x believes y_1 or ... or x believes y_n. Certainly this is how sincerely meant literal assertion is tied to belief. Here is a further example:

> If there is a necessity which Kripke knows *a posteriori*, Kripke will utter a sentence which means it.
> That Hesperus = Phosphorus is a necessity which Kripke knows *a posteriori*.

∴ Kripke will utter a sentence which means that Hesperus = Phosphorus.

The pragmatic theory in my section 39, *Quality and Concept, op. cit.* and in Nathan Salmon's *Frege's Puzzle, op. cit.*, have difficulty providing a systematic treatment of such "mixed" cases. Further, as I understand them, many other recent theories have difficulty with examples like these. I have in mind the theories proposed by David Lewis ("Attitudes *De Dicto* and *De Se*," *The Philosophical Review* 87, 1979, pp. 513-43); Roderick Chisholm (*The First Person: An Essay on Reference and Intentionality*, Minneapolis: University of Minnesota Press, 1981); Mark Crimmins and John Perry ("The Prince and the Phone Booth: Reporting Puzzling Beliefs," *The Journal of Philosophy* 86, 1989, pp. 685-711); and Graeme Forbes ("The Indispensability of *Sinn*," *The Philosophical Review* 99, 1990, pp. 535-564). However, this is not the place to elaborate on this point.

5. See, e.g., Alonzo Church ("On Carnap's Analysis of Assertion and Belief," *Analysis* 10, 1950, pp. 97-99) and George Bealer ("Universals," *The Journal of Philosophy*, 90, 1993, pp. 5-32).

6. Keith Donnellan ("Proper Names and Identifying Descriptions," *Synthese* 21, 1970, pp. 335-338) and Saul Kripke ("Naming and Necessity," in D. Davidson and G. Harman, eds., *Semantics of Natural Language*, Dordrecht: Reidel, 1972, pp. 253-355 and pp. 763-9; reprinted in book form as *Naming and Necessity*, Cambridge, Mass.: Harvard University Press, 1980). Aggravated versions of the problems in Frege's theory arise in connection with co-referential demonstratives (see John Perry, "Frege on Demonstratives," *The Philosophical*

Review 86, 1977, pp. 474-97) and in connection with co-referential names in interlinguistic translation (see Saul Kripke, "A Puzzle About Belief," in *Meaning and Use, Papers Presented at the Second Jerusalem Philosophical Encounter*, ed., A. Margalit, Dordrecht: Reidel, 1979, pp. 239-83). In recent history the rejection of Frege's theory of ordinary proper names and the return to a Millian theory was first advocated by Ruth Barcan Marcus, "Modalities and Intensional Languages," *Synthese* 13, 1961, pp. 303-322. See also W.V. Quine, Saul Kripke, Dagfinn Follesdol, and Ruth Barcan Marcus, "Discussion: On the Paper of Ruth B. Marcus," *Synthese* 14, 1962, pp. 132-143.

7. Benson Mates, "Synonymity," *University of California Publications in Philosophy* 25, 1950, pp. 201-26; reprinted in L. Linsky, ed., *Semantics and the Philosophy of Language*, Urbana: University of Illinois Press, 1952.

8. Mates' puzzle is about synonymous sentences (sentences having the same sense). There is a corresponding puzzle about the identity of propositions, namely, why is the following an invalid argument:

That D = that D′
Nobody believes that whoever believes that D believes that D.

∴ Nobody believes that whoever believes that D believes that D′.

E.g., D might be 'someone chews' and D′ might be 'someone masticates'.

9. See Alonzo Church, "Review of M. White and M. Black," *The Journal of Symbolic Logic* 11, 1946, pp. 132-3.

10. See Nelson Goodman, "On Likeness of Meaning," (original citation); reprinted in L. Linsky, *op. cit.*, pp. 67-74.

11. Such a theory is needed to accommodate Kripke's scientific essentialist doctrine that 'Hesperus = Phosphorus' expresses an essentially *a posteriori* necessity (see section 9 below). And such a theory is needed in order to explain the oblique uses of expressions such as 'arthritis', 'contract', 'sofa' that Tyler Burge discusses in "Individualism and the Mental," *Midwest Studies in Philosophy* 4, 1979, pp. 73-122. Burge emphasizes (e.g., pp. 86 f., 96 f.) that it is the oblique use of such expressions that is at work in his arguments against the individualist conception of mental content (see note 64 below).

12. For a more complete survey, see Bealer and Mönnich, *op. cit.*

13. Functions f and g are extensional if $(\forall x)(f(x) = g(x)) \rightarrow f = g$.

14. A possible-worlds theorist might reply that this objection is an instance of the so-called fallacy of incomplete analysis. However, this reply is theoretically weak, for it requires that we not take our intuitions at face value. By contrast, the algebraic theory permits us to take our intuitions at face value.

Someone might doubt the latter claim on behalf of the algebraic theory: is it not odd to say that, when I am aware that I am in pain, I am aware of a proposition? But this is not to take seriously our admonition that 'proposition' is being used here merely as a term of art. According to the algebraic theory, 'that'-clauses denote a *primitive* category of mind-independent, language-independent entities. We have a firm intuitive grasp of these entities, and they can be characterized in terms of fundamental logical relations that hold among them and in terms of the distinctive roles they play in logic, linguistics, psychology, and philosophy. They certainly are not sets or functions.

As a terminological convenience, we will also use the expression 'property' as a term of art: it will apply to any 1-ary intensions without regard to the distinction between "natural" properties and "Cambridge" properties. Likewise, for 'n-ary relation' and n-ary intensions.

15. David Lewis ("General Semantics," in D. Davidson and G. Harman, eds., *Semantics of Natural Language*, Dordrecht: Reidel, 1972, pp. 169-218) and Max Cresswell (*Structured Meanings*, Cambridge, Mass.: MIT Press, 1985).

16. The revisionary view also has some special problems of its own. See Bealer and

Mönnich, *op. cit.* For example, notice that the proposition that something is necessary is necessary. In symbols: Necessary [(∃x) Necessary x]. Given this elementary modal fact, the revisionary theory has the consequence that there exist non-well-founded sets: for example, {x: Necessary x} ∈ ... ∈ {x: Necessary x}. Surely the mere fact that it is necessary that something is necessary does not justify rejecting classical well-founded set theory. A safer, more conservative response is simply to seek an alternative to the possible-worlds theory.

For other objections to the possible-worlds theory, see: Robert M. Adams, "Theories of Actuality," *Nous* 8, 1974, pp. 211-231. Michael Jubien, "Problems with Possible Worlds," in *Philosophical Analysis*, D.F. Austin, ed., Dordrecht: Kluwer, 1988, pp. 299-322.

17. Except for the matter of type distinctions, this is the sort of propositional-function theory Russell had in mind in "Mathematical Logic as Based on the Theory of Types" (*American Journal of Mathematics* 30, 1908, pp. 222-62; reprinted in *Logic and Knowledge*, ed., R. C. Marsh, 1956) and in *Principia Mathematica* (with A. N. Whitehead, Cambridge: Cambridge University Press, 1910). Type-free versions of this sort of theory have been developed by Peter Aczel ("Frege Structures and the Notions of Proposition, Truth and Set," in J. Barwise, H.J. Keister and K. Kunen, eds., *The Kleene Symposium*, Amsterdam: North Holland Publishing, 1980, pp. 31-59) and "Algebraic Semantics for Intensional Logics, I," *Properties, Types, and Meaning: Volume I: Foundational Issues*, Dordrecht: Kluwer Academic Publishers, 1989, pp. 17-46); Uwe Mönnich ("Toward a Calculus of Concepts as a Semantical Metalanguage," R. Bäuerle, C. Schwarze, and A. von Stechow, eds., *Meaning, Use, and Interpretation of Language*, Berlin: W. de Gruyter, 1983, pp. 342-60); Raymond Turner ("A Theory of Properties," *The Journal of Symbolic Logic* 52, 1987, pp. 455-72); and Raymond Turner and Gennaro Chierchia ("Semantics and Property Theory," in *Linguistics and Philosophy* 11 1988, pp. 261-302). In their original theory of propositions (what they call 'situations') John Perry and Jon Barwise adopted a complicated propositional-complex theory (*Situations and Attitudes*, Cambridge, Mass.: MIT Press, 1983); recently, however, Barwise has dropped the latter in favor of a propositional-function theory based on Aczel's theory. Alonzo Church advocates a non-Russellian propositional-function theory. According to this theory, the sense of a predicate, e.g., 'is red', is a function from individual concepts (i.e., senses of proper names of individuals) to propositions. See his "A Formulation of the Logic of Sense and Denotation," in P. Henle, H. M. Kallen, and S. K. Langer, eds., *Structure, Method, and Meaning: Essays in Honor of Henry M. Scheffer*, New York: Chelsea Publications, 1951, pp. 3-24. However, this theory runs into problems analogous to those that confront the Russellian propositional-function theory. See Bealer "On the Identification of Properties and Propositional Functions," *Linguistics and Philosophy* 12, 1989, pp. 1-14. Moreover, as it stands, Church's theory is unable to handle the phenomenon of quantifying-in. This deficiency could be overcome by incorporating a propositional-complex theory of propositions into Church's propositional-function theory of predicate senses. Unfortunately, the resulting theory would fall prey to all the difficulties facing the propositional-complex theory as well as all the other problems facing the propositional-function theory.

18. See "On the Identification of Properties and Propositional Functions," *ibid.*

19. For example, one could just stipulatively define new primitive predicates 'eves' and 'sdivides' so that 'x eves' is synonymous to 'x is divisible by two' and 'x sdivides' is synonymous to 'x is divisible by x'. In this case, being an x such that x eves = being an x such that x is divisible by two, and being an x such that x sdivides = being an x such that x is divisible by x. The rest of the argument would then go through *mutatis mutandis*: the propositional-function theory would yield the consequence that the proposition that two eves = the proposition that two sdivides. But, intuitively, someone could be consciously and explicitly thinking the former proposition without consciously and explicitly thinking

the latter. This strategy resembles that used in the fondalee/rajneesh example (cf., "On the Identification of Properties and Propositional Functions," *ibid*.) To fondalee = to be something y such that Jane Fonda follows y; to rajneesh = to be something x such that x follows Rajneesh; therefore, given the propositional-function theory, the proposition that Rajneesh fondalees = the proposition that Jane Fonda rajneeshes. But this identity evidently does not hold: surely someone could consciously and explicitly think that the former while not consciously and explicitly thinking the latter.

20. The occurrence of the name 'two' in this example is not the source of the difficulty. If 'two' were replaced throughout by a fresh free variable that has the number two as a fixed assignment, there would still be a problem.

21. And the fondalee/rajneesh puzzle. Incidentally, there are "synonymously isomorphic" sentences ⌈D⌉ And ⌈D′⌉ such that ⌈the proposition that D ≠ the proposition that D′⌉ is intuitively true. (E.g., 'the proposition that there exists something that chews and does not masticate ≠ the proposition that there exists something that masticates and does not chew'.) Our eventual theory provides for such propositions, thus making it possible to block associated instances of Mates' puzzle at the very first step. This, however, would be of no help to the propositional-function theory, for our way of treating the two propositions — that D and that D′ — is incompatible with a propositional-function theory. See note 73.

22. For more on this sort of notation see my "Theories of Properties, Relations, and Propositions," *The Journal of Philosophy* 76, 1979, pp. 643-648, and *Quality and Concept*, Oxford: Oxford University Press, 1982.

23. Russell discusses such a theory in *The Principles of Mathematics*, New York: Norton 1903. Versions of it can also be found in David Lewis, (*op. cit.*); Stephen Schiffer ("The Basis of Reference," *Erkenntnis*, 13, 1978, pp. 171-206; and *Remnants of Meaning*, Cambridge, Mass.: MIT Press, 1987); David Kaplan ("Dthat," *Syntax and Semantics 9: Pragmatics*, ed. P. Cole, New York: Academic, 1978); Jon Barwise and John Perry (*op. cit.*); Max Cresswell (*op. cit.*); Nathan Salmon (*op. cit.*); Scott Soames ("Direct Reference, Propositional Attitudes and Semantic Content," *Philosophical Topics* 15, 1987 pp. 47-87); Joseph Almog ("Logic and the World," in J. Almog, J. Perry, H. Wettstein, eds., *Themes from Kaplan*, New York: Oxford University Press, 1989, pp. 43-66); and Mark Richard (*Propositional Attitudes: An Essay on Thoughts and How We Ascribe Them*, New York: Cambridge University Press, 1990).

24. It should be emphasized that on this theory sequences are not being used to "represent" propositions; they are actually *identified* with them. If someone wished, instead, to use sequences merely to "represent" propositions, that person would still owe us an account of what propositions *are*. That is the question under discussion in the text.

25. For a reply and counter-reply, see note 14.

26. The propositional-complex theory also provides an implausible treatment of "complex-properties" (e.g., conjunctive properties such as the property of being rich and famous). On the one hand, such properties could, on analogy with propositional complexes, be treated as ordered sets. E.g., being rich and famous might be identified with <conjunction, being rich, being famous>. But how implausible that some properties (e.g., being rich, being famous, etc.) are primitive *sui generis* entities whereas other properties (e.g., being rich and famous) are sets. On the other hand, complex properties could be treated algebraically, as we advocate. But in this case it would be altogether *ad hoc* not to treat propositions algebraically as well. For example, suppose that the complex property of loving someone is treated algebraically: [x: (∃y) x loves y] = the result of existentially generalizing on [xy: x loves y]. Then, it would be *ad hoc* not to extend the algebraic treatment to the proposition that someone loves someone: [(∃x)(∃y) x loves y] = the result of existentially generalizing on [x: (∃y) x loves y] = the result of existentially generalizing on the result of existentially generalizing on [xy: x loves y]). The moral is that there is no way

for the propositional-complex theory to provide a smooth *unified* theory.

27. Our argument will go through if *mutatis mutandis* 'x = y' is replaced by 'if x and y exist, x = y'.

28. The above style of argument is examined in detail in my paper "Universals," *op. cit.* Incidentally, another difficulty with the propositional-complex approach concerns *self-constituency*. For example, sometimes when you look in a mirror, perhaps you see u, where u = that you see u. If so, the propositional-complex theory must revolutionize set theory simply to allow for this possibility: u = \<seeing, you, u\>. See Jon Barwise, "Three Views of Common Knowledge," in M. Vardi, ed., *Theoretical Aspects of Reasoning about Knowledge, II*. Los Altos, CA: Morgan Kaufmann, 1988, pp. 365-80; and Peter Aczel, *Non-Well-Founded Sets*, Stanford: CSLI Publications, 1988. The algebraic approach allows us to accommodate this possibility smoothly without touching standard well-founded set theory.

29. It is more common to write: \<D, +, •, -, 0, 1\>. The notation in the text should be more perspicuous for present purposes.

30. These structures \<D, disj, conj, neg, τ, F, T\> are closely related to Halmos's transformation algebras (p. 27f., *Algebraic Logic*, New York: Chelsea Publishing, 1962). The differences are immaterial philosophically. For related ideas, see W. V. O. Quine, "Variables Explained Away" (*Proceedings of the American Philosophical Society* 104, 1960, pp. 343-47; reprinted in Quine, *Selected Logic Papers*).

31. These structures are closely related to, but not identical with, cylindric algebras (L. Henkin, D. Monk, A. Tarski, *Cylindric Algebras, Part I*, Amsterdam: North Holland Publishing, 1971) and polyadic algebras (Halmos, *op. cit.*). For other approaches to algebraic models for the predicate calculus, see W. V. O. Quine, *op. cit.*, and William Craig, *Logic In Algebraic Form*, Amsterdam: North-Holland, 1974.

32. By 'Millian (or Russellian) proper name' we mean a syntactically simple singular term that is not a variable and that has a rigid denotation and no connotation or sense.

33. An intensional abstract is a 'that'-clause or a gerundive (or infinitive) phrase. That is, a proposition abstract, a property abstract, or a relation abstract. In our notation intensional abstracts have the form $[v_1...v_n: A]$, where $n \geq 0$.

34. This solution to the algebraic representation of individual constants is given in my "Theories of Properties, Relations, and Propositions," *The Journal of Philosophy* 76, 1979, pp. 643-48. Much the same solution is adopted by Terence Parsons, *Nonexistent Objects*, New Haven: Yale University Press, 1980; Edward Zalta, *Intensional Logic and the Metaphysics of Intentionality*, Cambridge, MA: MIT Press, 1988; Peter Aczel "Algebraic Semantics for Intensional Logics I," *op. cit.*; and Chris Menzel, "The Proper Treatment of Preication in Fine-grained Intensional Logic," this volume. This solution seems inevitable once one gives up the propositional-function theory and the propositional-complex theory.

35. Much the same points could be made if instead we were to consider the analogy between the Tarski-style extensional semantics and our approach.

36. Because of cardinality limitations, there might not be a unique natural model that contains *all* possible worlds and *all* non-actual possibilia.

37. As was the case with the natural possible-worlds model, cardinality limitations might also prevent there being a unique natural algebraic model containing *all* properties, relations, and propositions. Moreover, according to actualism, the only genuine particulars in the natural model are actually existing particulars; therefore, in order to accommodate actualism, certain adjustments in our models are needed. Specifically, certain "singular" properties are to serve as ersatz "non-actual particulars." See note 38.

38. It might be objected that, although our quantifiers ⌜α⌝ are actualist, our models are not, for their domains contain particulars that do not actually exist. The reply is that the "non-actual particulars" in the domains of our models are ersatz

entities—actually existing entities playing the role of "non-actual particulars." This can be accomplished as follows: a "singular" property x is to serve as an ersatz "non-actual particular" iff, although there actually exists no particular y such that x = the property of being identical to y, it is possible that there exists a particular y such that x = the property of being identical to y.

39. For more on the existentialism/anti-existentialism dispute, see the following. Arthur Prior, *Time and Modality*, Oxford: Oxford University Press, 1957. Kit Fine, "Postscript," in Arthur Prior and Kit Fine, *Worlds, Times and Selves*, Amherst: University of Massachusetts Press, 1977, pp. 116 ff. Robert Merrihew Adams, "Actualism and Thisness," *Synthese* 49, 1981, pp. 3-41. Alvin Plantinga, "On Existentialism," *Philosophical Studies* 44, 1983, pp. 1-20. See also John Pollock's "Plantinga on Possible Worlds" (pp. 121-144) and Kit Fine's "Plantinga on the Reduction of Possibilist Discourse" (pp. 145-186) — and Alvin Plantinga's replies—in *Alvin Plantinga, Profiles*, J. Tomberlin and P. van Inwagen, eds., Dordrecht: D. Reidel, 1985. The existentialism/anti-existentialism problem drove Arthur Prior to a radical reformulation of his modal logic (see, for example, "Tense Logic for Non-Permanent Existents," in *Papers on Time and Tense*, Oxford: Oxford University Press, 1957). The same type of problem has also driven Alan McMichael ("A New Actualist Modal Semantics," *Journal of Philosophical Logic* 12, 1983, pp. 73-99) and Christopher Menzel ("The True Modal Logic," *The Journal of Philosophical Logic* 20, 1991, pp. 331-374), and Harry Deutsch ("A Logic for Contingent Beings," manuscript) to advocate a re-working of the semantics and axiomatic formulation of modal logic. The algebraic approach provides a far easier solution to the problem. Moreover, it yields as an immediate corollary an actualist solution to the problem of contingent existents and iterated modalities posed by Alan McMichael ("A Problem for Actualism about Possible Worlds," *The Philosophical Review* 92, 1983, pp. 49-66), a solution that does not require us to posit necessarily unanalyzable individual essences ("Haecceities"). Alvin Plantinga's theory (cf., "Actualism and Possible Worlds," *Theoria* 42, 1976, pp. 139-160), by contrast, is usually understood as positing necessarily unanalyzable individual essences.

40. In these remarks we could replace 'x = x' with 'if x exists, x = x'. Furthermore, our discussion would carry over *mutatis mutandis* to the example we discussed in section 2 in connection with the propositional-complex theory: $(\forall x)\Box(\forall y)(\text{Possible } [x = y] \lor \text{Impossible } [x = y])$.

41. Is the proposition that x = x true in all possible circumstances? Suppose that it is not. In this case, we would require that for all properties v (i.e., for all $v \in D_1$) and for all $H \in K$, $H(v) \subseteq H(D)$. In particular, $H([x: x = x]) = \{x \in H(D): x = x\} = H(D)$, for all $H \in K$.

Suppose, however, that the proposition that x = x is true in all possible circumstances. (This supposition seems right to me.) Then, we omit this requirement that, for all properties v and all $H \in K$, $H(v) \subseteq H(D)$, and we require that $H([x: x = x]) = \{x \in D: x = x\} = D$. These adjustments allow for possible extensionalization functions H such that the H-extension of the property of being self-identical contains items that do not exist relative to H. That is, possible extensionalization functions $H \in K$ such that $H([x: x \text{ exists}]) = H(D) \subset H([x: x = x]) = D$. More generally, we allow there to be properties $v \in D_1$ and functions $H \in K$ such that $H(v)$ is not a subset of $H(D)$. This is the kind of formal semantical theory most actualists have been seeking.

Incidentally, certain actualists might want to take advantage of this opening in "logical space" to explain the apparent truth of atomic sentences concerning non-existent entities. Such actualists might hold that, e.g., 'Pegasus flies' is literally true, and they might try to explain its truth by holding that Pegasus $\in$ G(flying). This is consistent with holding that Pegasus $\notin$ G(D) and therefore, that '$\neg(\exists x)x = \text{Pegasus}$' is literally true. However, one should not accept this account of 'Pegasus flies' unless certain difficult epistemological problems can be solved. In any event, the theory presented later in the present paper provides

a perfectly acceptable account for the truth and meaning of negative existentials like '$\neg(\exists x)x = \text{Pegasus}$'. No positive proposal will be made in this paper regarding sentences like 'Pegasus flies'.

42. See Mark Hinchliff (*A Defense of Presentism*, Ph.D. dissertation, Princeton University, 1988) for an illuminating discussion of these issues.

43. The rest of the construction can be done along the following lines. (For other constructions, see my "General and Hyper-fine-grained Intensional Logic," *Noûs*, 1994, forthcoming.) Choose any set τ of 1-1, disjoint, well-founded functions <disj, conj, neg, ...> on D satisfying the appropriate requirements on degree. For example, conj(F, G) is defined iff F and G are predicate letters of the same degree; likewise, conj(F, G) = H only if H is a predicate letter of the same degree as F and G. It remains to characterize the set K of extensionalization functions. Let K = {G, G'}, where G and G' are defined as follows. G(D) = D. G'(D) = D~D_{-1}. G and G' are otherwise the same. (If we wish G and G' to differ from each other in accordance with the caveat discussed in note 41, we should then restrict G'(F_k^j) to G'(D) in the fashion indicated in that note.) It remains to define G for the elements of D. First, for the elements a_i in D_{-1}, G(a_i) = a_i. The remaining elements of D are predicate letters F_k^j, j ≥ 0. If F_k^j is not the value of any function in τ, then G(F_k^j) = Ø. Otherwise, G(F_k^j) is determined in the obvious way by the algebraic rules that characterize the behavior of the functions in τ with respect to G. For example, suppose F_k^j = neg(F_i^j) and G(F_i^j) = X; then by the rule for neg, G(F_k^j) = D^j~X. Illustration: if F_i^j is not the value of any function in τ, then G(F_i^j) = Ø and so G(F_k^j) = D^j~Ø = D^j.

44. A similar solution to Mates' puzzle was suggested by Hilary Putnam, "Synonymity and the Analysis of Belief Sentences," *Analysis* 14, 1954, pp. 114-22.

45. In these intensional algebras the set τ contains only operations that are 1-1 and whose ranges are disjoint, and τ is closed under an operation of intensional composition •. These intensional algebras are thus 4-tuples <D, K, τ, •>. τ is closed under • in the sense that, if $\tau_1,...,\tau_n \in \tau$ and $\tau_0 \cdot (\tau_1,...,\tau_n)$ is defined, then $\tau_0 \bullet (\tau_1,...,\tau_n)$ is defined and $\tau_0 \bullet (\tau_1,...,\tau_n) \in \tau$; and if $\tau_0 \bullet (\tau_1,...,\tau_n)(\vec{v})$ is defined, so is $\tau_0 \bullet (\tau_1,...,\tau_n)(\vec{v})$. Here • is ordinary functional composition and $\vec{v} \in \underset{i}{\times}_{\geq 1} D^i$. We require that, when defined, H($\tau_0 \bullet (\tau_1,...,\tau_n)(\vec{v})$ = H($\tau_0 \bullet (\tau_1(\vec{v}),...,\tau_n(\vec{v}))$) for all H ∈ K. Thus, although in hyper-fine-grained intensional algebras H($\tau_0 \bullet (\tau_1,...,\tau_n)(\vec{v})$ and H($\tau_0(\tau_1(\vec{v}),..., \tau_n(\vec{v}))$) are distinct intensions (this follows from the fact that operations in τ have disjoint ranges), they must always have the same extension and so in this sense are well-behaved. In my "General and Hyper-fine-grained Intensional Logic," (*Noûs*, 1994, forthcoming) these hyper-fine-grained intensional algebras are fully specified, examples are constructed, validity is defined, and a sound and complete logic is formulated.

46. This alternative was developed but not officially adopted in "Pragmatics," in *Quality and Concept*. Nathan Salmon (*op. cit.*) has since embraced this solution officially.

47. There are also neo-Fregean puzzles involving common names. For example, although, 'filbert' and 'hazelnut' are co-denoting, 'Filbert is sweeter than hazelnut' and 'Hazelnut is sweeter than filbert' intuitively mean something different. Similarly, although 'puma' and 'cougar' are co-denoting, 'Cougars are native to North America but pumas are not' and 'Pumas are native to North America but cougars are not' intuitively mean something different. Likewise, 'Greeks are more famous than Hellenes' and 'Hellenes are more famous than Greeks' intuitively mean something different as do 'Schmogs are furrier than dogs' and 'Dogs are furrier than schmogs' (see pp. 57 ff.., Stephen Schiffer, *Remnants of Meaning, op.cit.*).

48. The points in this section about definite descriptions carry over *mutatis*

mutandis to number-neutral descriptions (such as 'Whoever shot Kennedy' as it occurs in the sentence 'Whoever shot Kennedy is crazy'.) According to Stephen Neale, this sort of number-neutral description is the key to a successful traditional treatment of donkey sentences. See Neale, "Descriptive Pronouns and Donkey Anaphora," *The Journal of Philosophy* 87, 1990, pp. 113-50, and Neale, *Descriptions*, Cambridge, MA: MIT Press, 1990. Like most other treatments of these sentences, Neale's treatment provides only truth conditions; it does not identify the *propositions* expressed by such sentences. This remaining task can be accomplished by adapting in the obvious way the techniques sketched in the present section. See note 56 below.

49. "On Sense and Reference" and "Begriffschrift: a Formalized Language of Pure Thought Modelled upon the Language of Arithmetic," both in *Translations from the Philosophical Writings of Gottlob Frege*, P. Geach and M. Black, trs., Oxford: Blackwell, 1952. First appearing in German as "Über Sinn und Bedeutung," *Zietschrift für Philosophie und philosophische Kritik* 100, 1892, pp. 22-50; and *Begriffschrift: eine der arithmetischen nachgelbildete Formelsprache des reinen Denkens*, Halle: L. Nebert, 1879. See also Peter Strawson, "On Referring," *Mind* 59, 1950, pp. 320-344, and Alonzo Church, "A Formulation of the Logic of Sense and Denotation," *op. cit.*

50. In Alonzo Church's language (*op. cit.*) ⌐the f⌐ has the form ⌐ι((λx)fx)⌐, where 'ι' is an operator that combines with a propositional-function term to form an individual term.

51. "On Denoting," *Mind* 14, (1905), pp. 479-93; and *Principia Mathematica, op. cit.*

52. Gareth Evans, "Pronouns, Quantifiers, and Relative Clauses (I)," *Canadian Journal of Philosophy* 7, 1977, pp. 467-536, and "Pronouns, Quantifiers, and Relative Clauses (II)," *Canadian Journal of Philosophy* 7, 1977, pp. 777-97.

53. Arthur Prior, "Is the Concept of Referential Opacity Really Necessary?" *Acta Philosophica Fennica* 16, 1963, pp. 189-98. Paul Grice, "Vacuous Names," in Davidson and Hintikka, eds., *Words and Objections*, Dordrecht: Reidel, 1969, pp. 118-45. Richard Sharvy, "Things," *The Monist*, 53, 1969, pp. 488-504. Richard Montague, "The Proper Treatment of Quantification in English," in Hintikka, Moravcsik, and Suppes, eds., *Approaches to Natural Language: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics*, Dordrecht: Reidel, 1973, pp. 221-42.

54. This is trivial in the case of Russell's theory. For discussion, see pp. 161-66, *Quality and Concept*. To incorporate Evans's theory, we simply restrict ourselves to intensional algebras in which the set τ contains a binary operator the that takes pairs u,v of properties to propositions, where for all $H \in K$, $H(\text{the}(u,v)) = T$ iff, for some $w \in H(D)$, $H(u) = \{w\}$ & $H(u) \subseteq H(v)$. See p. 204f., Bealer and Mönnich, "Property Theories," *op. cit.* To incorporate Prior's theory, we simply restrict ourselves to those intensional algebras in which the set τ contains a unary operator The that takes properties to properties of properties: for all properties u and v and all $H \in K$, $v \in H(\text{The}(u))$ iff, for some $w \in H(D)$, $H(u) = \{w\}$ & $H(u) \subseteq H(v)$. Then, the proposition that the F Gs = $\text{pred}_s(\text{The}([x: Fx]), [x: Gx])$. So for all $H \in K$, $H(\text{pred}_s(\text{The}([x: Fx]), [x: Gx])) = T$ iff, for some $w \in H(D)$, $H([x: Fx]) = \{w\}$ & $H([x: Fx]) \subseteq H([x: Gx])$.

55. In his formulation of Frege's theory (p. 50, "A Formulation of the Logic of Sense and Denotation," *op. cit.*) Church adopts the thesis that the sense of a predicate of individuals is a function whose arguments are individual concepts and whose values are propositions (i.e., that the sense of a predicate of individuals is a kind of propositional function.) Accordingly, in Church's system the relation of descriptive predication collapses into a special case of the relation of application of function to argument. However, the propositional-function thesis unnecessarily exposes Frege's informal theory of senses to the flaws noted earlier in the propositional-function theory; moreover, it generates certain internal technical difficulties as well. When the propositional-function thesis is removed from Frege's theory, one gets the picture presented in the text.

Evidently, this is also the picture one would get if one were to attempt to formalize Graeme Forbes' operation ∧ for combining senses that is introduced informally in "The Indispensability of *Sinn*" (p. 548 ff., *op. cit.*).

56. The operation of descriptive predication may also be used to represent other sorts of descriptive propositions within a Fregean setting. For example, consider one of Neale's number-neutral descriptive propositions: the proposition that whoever shot Kennedy is crazy. Within a Fregean setting this proposition may be represented thus: $pred_d([x: Cx], whe([x: Sx]))$, where whe is Neale's number-neutral description operator. This operator behaves as follows: for all $u \in D_1$ and all $H \in K$, if $H(u) \subseteq H(D)$, $H(whe(u)) = H(u)$; otherwise $H(whe(u)) = \emptyset$. If number-neutral descriptions were treated *a la* Russell, Evans, or Prior, the corresponding algebraic representations would be analogous.

57. Ironically, the only reason offered in Davidson's "Truth and Meaning," (*Synthese* 17, 1967, pp. 304-23) against the latter kind of semantics is that it is undermined by Mates' puzzle. However, a suitably fine-grained theory of propositions forestalls this objection.

58. Someone might wonder whether there is something circular about invoking the indicated non-Platonic modes of presentation. In the closing section we will indicate why this worry is unfounded.

59. For some ways in which we might wish to sharpen Kripke's picture, see, e.g., p. 62 f., Dennis Stampe, "Toward A Causal Theory of Linguistic Representation," *Midwest Studies in Philosophy II*, 1977, pp. 42-63.

60. Likewise, $pred_d(pred_d(identity, P_{'Tully'}), P_{'Cicero'})$ has the same modal value (i.e., necessity) that Kripke *et al.* attribute to the proposition that Cicero = Tully.

61. E.g., there are $H \in K$ such that $pred_d(being\ a\ person, P_{'Cicero'}) \in H(D)$ and $P_{'Cicero'} \notin H(D)$.

62. Nevertheless, a proposition like $pred_d(being\ a\ person, P_{'Cicero'})$ is metalinguistic in a hitherto unnoticed sense: it is the value that results when a certain logical operation (i.e., $pred_d$) is applied to a socially constructed linguistic entity. In this way, we obtain a type of proposition that is "metalinguistic without being metalinguistic."

63. Our intention here is that these two propositions are those responsible for the salient difference in meaning between 'Cicero is a person' and 'Tully is a person'. At this stage we are not supposing that these two sentences *literally* express these propositions. That semantical question comes later.

64. This requirement is insisted upon by Tyler Burge (pp. 127 ff., "Belief and Synonymy," *The Journal of Philosophy* 75, 1978, pp. 119-39, and p. 97, "Individualism and the Mental," *op. cit.*) and by Stephen Schiffer (pp. 67 ff., *Remnants of Meaning, op. cit.*).

65. Non-Platonic modes of presentation could be incorporated into a propositional-complex theory. (See, e.g., Mark Richard, *op. cit.*) However, as we have seen, there are independent reasons not to adopt the propositional-complex theory. These reasons are only magnified by a propositional-complex theory that uses non-Platonic modes of presentation to solve Frege's puzzle. For example, consider the proposition that Cicero is a person (i.e., the elusive proposition that is distinct from the proposition that Tully is a person). On the envisaged propositional-complex theory, this proposition would be identical to the ordered set <being a person, $P_{'Cicero'}$>. But there are possible situations in which Cicero exists and in which this ordered set does not exist, namely, situations in which Cicero exists and the linguistic practice $P_{'Cicero'}$ does not exist. Therefore, on the envisaged propositional-complex theory, there would be possible situations in which Cicero exists and the proposition that Cicero is a person does not exist! This is only the tip of the iceberg.

66. For example, the "discourse objects" that, according to Hans Kamp, occur in a "discourse representation structure." See Kamp, "A Theory of Truth and Semantic Interpretation," in Groenendijk, Janssen, and Stokhof, eds., *Formal Methods in the Study of Language*, vol. I., Amsterdam. See also Henk Zeevat, "A Treatment of Belief Sentences in Discourse Representation Theory," in *Studies in Discourse*

Representation Theory and the Theory of Generalized Quantifiers, D. de Jongh, J. Groenendijk, and M. Stokhof, eds., Dordrecht: Foris, 1987, pp. 202-229. Another candidate type of non-Platonic mode of presentation is the sort of "name dossier" posited by Graeme Forbes, *op. cit*. A third candidate type are the "notions" and "ideas" posited by Mark Crimmins and John Perry, *op. cit*. Our algebraic theory of propositions is consistent with the thesis that these three types of theoretical posits exist, that they are truly modes of presentation, and that they play genuine roles in human cognition. Indeed, the algebraic approach provides a way (the only way, I suspect) to incorporate these non-Platonic entities into an acceptable anti-existentialist theory of propositions. These three proposals, however, might not yield satisfactory solutions to the neo-Fregean puzzles. The reason is that each of the three candidate types of non-Platonic modes of presentation is "private" in the sense that two individuals seldom, if ever, implement exactly the same one. Accordingly, the prospect of genuine public meaning—propositions that are asserted and believed by one person and truly understood and accepted by another (e.g., the proposition that Cicero is more eloquent than Tully)—is evidently ruled out, at least as we understand these proposals.

67. A potential advantage of the linguistic entities we have discussed (trees, practices, and perhaps even names) is that, on one way of talking about them, they can be said to merge and/or to divide over the course of their own histories. It is plausible that there will be data that call for non-Platonic modes of presentation that exhibit just this sort of behavior. In this connection, the criteria of individuation for these linguistic entities might (desirably) turn out to exhibit certain indeterminacies that correspond to associated indeterminacies in the data.

68. As already noted, outlines of such a solution were sketched, but not officially adopted, in section 39, "Pragmatics," in *Quality and Concept, op. cit*. Nathan Salmon advocates this solution in *Frege's Puzzle, op. cit*.

69. This style of semantics is defended and developed in section 38, *Quality and Concept*.

70. I.e., ⌜a⌝ names a iff ⌜a⌝ is a name & ⌜a⌝ means a; ⌜F⌝ expresses F-ing iff ⌜F⌝ is a predicate and ⌜F⌝ means F-ing. This streamlined formulation of Russellian semantics is not crucial to the proposal we are about to make. We could instead employ two primitive semantical relations, naming and expressing: 'Cicero' names Cicero; 'Tully' names Tully; 'chew' expresses chewing; 'masticates' expresses masticating; etc. We will use the formulation in the text only because it is neat.

71. This compositionality principle yields the same consequences in the setting of a Fregean semantics for names and predicates once it is recognized that there is no difference in Fregean sense between 'Cicero' and 'Tully' or between 'chew' and 'masticate'.

72. Some readers might want to go along with only the first, or first and second stages.

73. In a formal presentation we should want to make minor adjustments to accord with the intensional algebra described in note 45. Incidentally, as we forecast in note 21, the propositional-function theory is incapable of handling the present example. The reason is that "chew" and "masticate" are not propositional functions; they are non-Platonic modes of presentation.

74. This answer seems inevitable given that "9" ≠ "ix". However, suppose that there is a notion of *numeral* according to which '9' and 'ix' are distinct orthographic presentations of the *same* numeral, i.e., the English name of the number nine. In this case, someone might hold that "9" = "ix". If this were right, $pred_d$(being a number, "9") and $pred_d$(being a number, "ix") would be the same proposition. In that case, perhaps this proposition would be meant by both '9 is a number' and 'ix is a number'. If so, there would be no need to hold, as we do in the text, that these sentences mean the singular proposition $pred_s$(being a number, nine). In turn, perhaps we could avoid holding that 'Cicero is a person' and 'Tully is a

person' both mean the singular proposition $pred_s$(being a person, Cicero). If so, adjustments would need to be made at various points in the coming pages. For the present we wish to remain neutral on the ultimate disposition of this issue.

75. Another practice that a translator might follow in these special cases is to adopt some merely conventional pairing of relevant primitive expressions in one language with relevant primitive expressions in the other language. For example, a translator might adopt a convention of always translating 'Hesperus' as 'the Morning Star' and 'Phosphorus' as 'the Evening Star' even though there is, strictly speaking, no semantical difference between 'Hesperus' and 'Phosphorus'. Accordingly, the translator might translate the Greek sentence 'Phosphoros phôteineros esti Hesperos' as 'The Morning Star is brighter than the Evening Star' and not as 'The Evening Star is brighter than the Morning Star'. Although the translator would be commended for having given a good translation, no translator would deem it a *perfectly exact* translation. To obtain an exact translation in these special cases, translators always simply borrow the relevant primitive expressions from the original text.

76. "A Puzzle About Belief," *op. cit.*

77. He might also mean — and believe — neg($pred_s$(being pretty, London)). This proposition does not contradict the one he stated and believed originally. After all, London ≠ "Londres"; moreover, on the hyper-fine-grained conception singular predications and descriptive predications are always distinct.

78. Kripke (*op. cit.*) poses a second puzzle. Peter has been party to a conversation about a pianist named 'Paderewski', and he has been party to another conversation about a polish Prime Minister named 'Paderewski'. Unbeknownst to Peter, the pianist and the Prime Minister are the same person. Besides what he has gleaned from the two conversations, Peter has no other relevant information about Paderewski. In the course of the first conversation Peter sincerely asserts 'Paderewski has musical talent'. In the course of the second conversation he sincerely asserts 'Paderewski does *not* have musical talent'. In both conversations, Peter intends to speak literally; we may suppose that he succeeds in doing so. What does Peter mean? What does he believe? It can be argued that on the first occasion Peter literally meant $pred_d$(having musical talent, "Paderewski") and on the second occasion he literally meant the contradictory proposition neg($pred_d$(having musical talent, "Paderewski")). Here is one argument. Suppose that someone else, Paul, knows that the prime minister = the pianist, and suppose that, during the two conversations to which Peter was party, Paul asserts our two sentences with the intention of speaking literally. (Of course, Paul would *not* be speaking sincerely on the second occasion; indeed, he would be lying.) Certainly, the two propositions meant by Paul would be contradictory. But, just as certainly, Paul would have meant the same things that Peter meant when he asserted the two sentences with the intention of speaking literally. Otherwise, linguistic communication would fail far more often than it in fact does! Now since Peter spoke sincerely on both occasions, he believed what he meant; it would follow, therefore, that Peter had contradictory beliefs. This assessment does not imply that Peter was irrational. Rather, it shows that a person's rationality is determined, not by *all* of the person's beliefs, but only by a certain privileged subset of them. Our theory of propositions promises to make it easier to identify what sorts of propositions these are.

It should be emphasized in strongest terms that our general approach is not wedded to the above assessment. Our general approach could in obvious ways be used to produce alternative treatments that would not attribute contradictory beliefs to Peter. But I am at present disinclined to advocate these alternative treatments, for they seem to me to focus on *auxiliary* beliefs that Peter must have had rather than on the two beliefs that Peter actually articulated when he sincerely asserted the relevant sentences with the intention of speaking literally.

79. The theory given in David Kaplan's "On the Logic of Demonstratives" (*Contemporary Studies in the Philosophy of Language*, ed. by P. French, T.

Uehling, and H. Wettstein; Minneapolis: University of Minnesota Press, 1979, pp. 401-413) does not, as it stands, provide a solution. Nor does the theory suggested in John Perry's "Frege on Demonstratives," (*op. cit.*) and in "The Problem of the Essential Indexical" (*Noûs* 13, 1979, pp. 3-21).

80. *Logic and Knowledge*, ed., R. C. Marsh, London: Allen and Unwin, 1956.

81. For the purpose of illustration we are still supposing that the proposal of the previous paragraph is acceptable. If it is not, the present proposal would need to be modified accordingly.

82. See note 4. This unity is evidently lost on Kaplan's theory, (*op. cit.*) and Perry's theory, (*op. cit.*), at least as these theories were formulated there. Perhaps, however, the insights of Kaplan and Perry could be incorporated into a more elaborate algebraic theory along the general lines we have been advocating.

83. E.g., the quotation by Saul Kripke at the outset and Stephen Schiffer, *op. cit.*

84. Stephen Schiffer (p. 257f., "The Mode-of-Presentation Problem," in *Propositional Attitudes*, Anderson, *et al.*, eds., 1990, Stanford: CSLI Publications, pp. 249-268) argues that the use of intentional states in a setting like this *must* be circular. His argument is fallacious, for it overlooks these two possibilities for avoiding circularity.

   Schiffer also tries to argue that propositions *cannot* do the theoretical work demanded of them. His argument fails, however, for he does not recognize that propositions act in two distinct roles. First, an essentially shared public role as the meanings of sentences and as the objects of beliefs that are expressible with such sentences. Second, an individualized, sometimes unshared (indeed, perhaps even private) psychological role. Both roles are crucial to a *full* account of our mental states. Some propositions are suited for both roles, but others are not. In the present paper, my focus is of course on the first role. (See note 78 above and section 39 in *Quality and Concept* for more on these two roles.)

85. Russell, for example, pursued this project; his Principle of Acquaintance offers a necessary condition for someone's having a given proposition as a mental content. See, e.g., p. 58, *The Problems of Philosophy*, Oxford University Press: Home University Library, 1912. Recently, Christopher Peacocke has undertaken this project in *Thoughts: An Essay on Content*, Oxford: Oxford University Press, 1986, and in "The Limits of Intelligibility: A Post-Verificationist Proposal," *The Philosophical Review* 97, 1988, pp. 463-496. Colin McGinn (*Mental Content*, Boston: Basil Blackwell, 1988) provides a valuable overview of the competing attempts at this project found in the (philosophy of) cognitive science literature.