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ON THE IDENTIFICATION OF PROPERTIES AND PROPOSITIONAL FUNCTIONS

The theory of properties, relations, and propositions (PRPs) promises to play a significant foundational role in logic, philosophy, semantics, and psychology. There is now a growing recognition of this fact by researchers in these disciplines.[1] Nevertheless, formalizations of PRP theory run into some technical complexities in connection with the treatment of free and bound terms occurring in PRP-abstracts (see, e.g., Bealer, 1979, 1982, 1983; Zalta, 1980; Mönnich, 1983). These complexities can be largely avoided by adopting the thesis that properties and propositional functions are identical (see, e.g., Aczel, 1980 and 1987; Mönnich, 1983; Turner, 1987; Chierchia and Turner, 1988). The purpose of this paper is to present some reasons against this thesis despite its short-term technical advantages.[2]

Philosophically, the propositional-function thesis seems unacceptable on at least two counts. First, it is highly counterintuitive. How implausible that familiar sensible properties are functions – the color of this ink, the aroma of coffee, the shape of your hand, the special painfulness of a burn or itchiness of a mosquito bite. No function is a color, a smell, a shape, or a feeling. Or consider the fact that navy blue is darker than yellow and that it covers the surface of my pen. No function is darker than another function, and no function covers the surface of my pen. To assert otherwise seems to be a category mistake.[3]

In addition to intuitive objections to the propositional-function thesis, a second philosophical objection is that the thesis threatens the prospect of certain explanations in epistemology, phenomenology, and philosophical psychology. For example, how are we to explain what is going on in the interplay of sensation and cognition when a person sees that two objects have some sensible quality in common? Or how are we to explain why various shades of color can look so similar? At best, the identification of properties and propositional functions complicates our epistemology, phenomenology, and philosophical psychology in connection with these and kindred phenomena.

My aim, however, is not to dwell on such philosophical objections to the propositional-function thesis, serious as they are. Rather, my aim is to discuss four logical difficulties facing the thesis.

(1) The first problem derives from the fact that functions are usually

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treated extensionally; i.e., the following extensionality principle is adopted for all functions f and g: $(\forall x)(f(x) = g(x)) \rightarrow f = g$. But this is wrong. There always exist functions f and g such that, even though f and g yield the same values for the same actual arguments, they could yield different values for some possible arguments. That is, even though $(\forall x)(f(x) = g(x))$, nevertheless $\Diamond(\exists x)(f(x) \neq g(x))$. In this case, $f \neq g$. Hence, a counterexample to the extensionality principle. This problem could be overcome by treating functions intensionally. To do this, one would reject the original extensionality principle; in its place, one might adopt a modal extensionality principle: $(\Box(\forall x)(f(x) = g(x))) \rightarrow f = g$. This move, however, does complicate things.

(2) The second problem is far more serious. One of the main purposes of PRP theory is to provide a logical framework for treating the propositional attitudes – belief, decision, memory, and so forth. It is now widely recognized that the sort of propositions suited to serve as objects of the attitudes must be very fine-grained.[4] For example, if two properties, say, being an x such that x Fs $\neq$ being an x such that x Gs, then for every x, the proposition that x Fs $\neq$ the proposition that x Gs. The intuitive idea here is that, if these two properties differ in any respect, then it will always be possible in principle for some person to have some propositional attitude to the proposition that x Fs and to fail to have that attitude toward the proposition that x Gs. Differences between properties are always reflected in differences between propositions formed from those properties. The logico-linguistic evidence supporting this principle of the distinctness among propositions is overwhelming.

This principle of distinctness among propositions entails (by contraposition and quantifier interchange) the following fine-grained principle of identity for properties: If, for some x, the proposition that x Fs = the proposition that x Gs, then being an x such that x Fs = being an x such that x Gs. This and kindred principles of property identity are formalized in my first-order PRP theory T2.[5] These property-identity principles comprise pretty much the object-language statement of the fine-grained conception of synonymy that Alonzo Church calls synonymous isomorphism and that he believes is required for treating the propositional attitudes. Church arrived at his conception by tightening up Carnap's principle of intensional isomorphism in response to various counterexamples.[6] Church has tried with very mixed success to formalize his conception in a higher-order ramified intensional logic called Alternative (0).[7] One of the problems is that Church himself adopts a version of the propositional-function thesis.

To show that it is a mistake to identify properties and propositional functions, we will make use of two intuitively compelling syntactic principles. First, if the word 'that' and a noun phrase α and a primitive verb phrase (i.e., a primitive predicate) 'Fs' are concatenated, the result is a well-formed 'that'-clause 'that α Fs'. Second, if 'x' is a variable and 'Fs' is a primitive verb phrase 'being an x such that x Fs' is a well-formed gerund. And we use the following semantical principles:

(A) Any property can in principle be expressed by a primitive verb phrase 'Fs'.

(B) If 'Fs' is a primitive verb phrase, the gerund 'being an x such that x Fs' denotes the property expressed by 'Fs'.

(C) If the variable 'x' is assigned the individual x as its value and the primitive verb phrase 'Fs' expresses the property of being an x such that x Fs, then the 'that'-clause 'that x Fs' denotes the proposition that x Fs.[8]

(D) If 'Fs' and 'Gs' are primitive verb phrases, then the following holds:

If, for some x, that x Fs = that x Gs, then being an x such that x Fs = being an x such that x Gs.

(E) If a variable 'x' is assigned the individual x as its value and a primitive verb phrase 'Fs' expresses a propositional function f, then the 'that'-clause 'that x Fs' denotes the proposition that is the value of f applied to argument x, that is, $f(x)$.

(A)–(C) are intuitively compelling principles of informal semantics. It would seem unreasonable to abandon these basic principles just to save the propositional-function thesis. (D) presents the fine-grained principle of property identity discussed above. (E) is a fundamental principle of all standard propositional-function semantic theories. (E) is a conditional whose antecedent implies that the primitive verb phrase 'Fs' expresses a propositional function f. However, given that the primitive verb phrase 'Fs' expresses a property, the antecedent of (E) implies that the propositional function f and this property are identical. If, as I maintain, this is false, then the entire conditional (E) is true. So we are free to use (E) in our argument against the propositional-function thesis. With principles (A)–(E) in place, we can now disprove the propositional function thesis. Here is the proof.

Proof. Let g be a constant propositional function characterized by the equation: $(\forall u)(g(u) =$ the proposition that x flies), where x is some arbitrarily chosen item, say, the number nine. Then by the propositional-function thesis, it follows that g is a property. So by (A), a primitive verb phrase 'Gs' could express g. Let the variable 'x' be assigned x (i.e., the number nine) as its value. Then, by (E), the 'that'-clause 'that x Gs' denotes the proposition that x flies. By (C), the 'that'-clause 'that x flies' denotes the proposition that x flies. So 'That x Gs = that x flies' is true on the above assignment. Therefore, the sentence 'For some x, that x Gs = that x flies' is true. Thus, by (D), 'being an x such that x Gs = being an x such that x flies' is true. It follows that these two gerunds denote the same property. Consequently, by (B), the primitive verb phrases 'Gs' and 'flies' express the same property. Given the propositional-function thesis, this property is really a propositional function. But the propositional function expressed by 'Gs' is g, so it follows that 'flies' expresses g. Now select some individual y that actually flies, say, this hummingbird, and let the variable 'y' be assigned y as its value. Given that the primitive verb phrase 'flies' expresses the propositional function g, it follows by (E) that the 'that'-clause 'that y flies' denotes the proposition that is the value of g applied to the argument y, that is, $g(y)$. However, by g's characterizing equation, $g(y)$ = the proposition that x flies (where x = the number nine). Thus, the 'that'-clause 'that y flies' denotes the proposition that x flies (where x = the number nine). However, by (B), 'that y flies' also denotes the proposition that y flies (where y = this hummingbird). It follows that the proposition that y flies = the proposition that u flies (where y = this hummingbird $\neq 9 = x$). But this is absurd. The proposition that y flies is true; the proposition that x flies is false. (This hummingbird flies; the number nine does not.) Since the propositional-function thesis leads to this absurdity, it follows that it is false.[9]

Two observations are in order. First, this argument does not show that propositional functions cannot be used in constructing *models* for intensional logic. In particular, one could construct artificial models such that every proposition in the model is the value of at most one propositional function in the model. In models like this the above sort of problem would not occur. Such models could be used for a variety of specific logical tasks. However, there is still a fatal problem. None of these artificial models is a *natural model* of propositional functions, for in a natural model there will always be more than one – indeed, there will always be infinitely many – propositional functions having the same proposition as one of its values. [For example, in a natural model containing the natural numbers there will be infinitely many propositional functions g_i, $i \geq 1$, that have as a value the proposition that $0 = 1$; functions like this might be characterized thus: $(\forall x)$ (if x is a natural number less than i, then $g_i(x)$ = the proposition that $0 = 1$; otherwise, $g_i(x)$ = the proposition that $x = x$).] But a condition of adequacy on any general semantical method is that the models it provides should include the natural model(s) for the entities it is supposed to model. Therefore, a general semantical method based on the above artificial models cannot be adequate.

The second observation concerns a weakening of the propositional-function thesis. We know that, for every proposition, there are infinitely many propositional functions having that proposition as a value. But given (A)–(E), we have shown that at most one of these propositional functions could possibly be a property. So it appears that a propositional-function theorist has no choice but to weaken the original propositional-function thesis in the following way. Originally, the thesis was that all and only properties are (unary) propositional functions. According to the weakened thesis, all properties would still be (unary) propositional functions; however, the converse would not hold. On the contrary, most (unary) propositional functions would not be properties. [On this weakened thesis, the propositional function f having the following defining equation might be an example of a propositional function that is really a property (i.e., the property of flying): $\Box(\forall x)(f(x) =$ the proposition that x flies). By contrast, the propositional functions g_i characterized above would not be genuine properties according to the weakened thesis.] True enough, this weakened propositional-function thesis avoids the difficulty given above. However, it does so at the price of making a mystery of the distinction between propositional functions that are supposed to be properties and those that are not. What is it about the propositional functions that are supposed to be properties that makes them special? The answer presumably is that these propositional functions are somehow more "natural" than others. The problem is that there evidently is no way to spell out clearly and precisely what this means without implicitly or explicitly using the logically prior idea of what it is to be a property (or related ideas not belonging to propositional-function theory as such). But if this is so, it would appear that despite its technical appeal, the weakened propositional-function thesis inevitably leaves something out: *it masks the true logical structure of the subject*. Conceptually, the right course is therefore to develop a theory of properties directly. Only such a theory can lay bare the true logical structure of PRPs. Propositional-function theory is at best artificial scaffolding.

(3) The third problem I wish to raise confronts even the weakened thesis that all (but not only) properties are unary propositional functions. As with the second problem, the third problem arises in the context of the logic for the propositional attitudes, which demands that very fine-grained distinctions be made among propositions. Let us introduce two new primitive verb phrases (primitive predicates) – 'rajneeshes' and

'fondalees' – by stipulating that:

(1) Being an x such that x rajneeshes = being an x such that x follows Rajneesh.
(2) Being an x such that x fondalees = being an x such that Jane Fonda follows x.[10]

(Jane Fonda seems to follow many people as time goes on; maybe it will be Rajneesh next.) According to the propositional-function thesis, the following identities hold:

(3) $(\lambda x)(x$ rajneeshes$)$ = being an x such that x rajneeshes.
(4) $(\lambda x)(x$ follows Rajneesh$)$ = being an x such that x follows Rajneesh.
(5) $(\lambda y)(y$ fondalees$)$ = being a y such that fondalees.
(6) $(\lambda y)($Jane Fonda follows $y)$ = being a y such that Jane Fonda follows y.

From (1), (3), and (4) it follows that:

$(\lambda x)(x$ rajneeshes$) = (\lambda x)(x$ follows Rajneesh$)$.

Apply each side of this identity to the argument Jane Fonda. The result is:

$(\lambda x)(x$ rajneeshes$)($Jane Fonda$) =$
$(\lambda x)(x$ follows Rajneesh$)($Jane Fonda$)$.

Then by (C) and (E) we have:

(7) The proposition that Jane Fonda rajneeshes = the proposition that Jane Fonda follows Rajneesh.

Similarly, from (2), (5), and (6), it follows that:

$(\lambda y)(y$ fondalees$) = (\lambda y)($Jane Fonda follows $y)$.

Apply each side of this identify to the argument Rajneesh. The result is:

$(\lambda y)(y$ fondalees$)($Rajneesh$) =$
$(\lambda y)($Jane Fonda follows $y)($Rajneesh$)$.

Then by (C) and (E), we have:

(8) The proposition that Rajneesh fondalees = the proposition that Jane Fonda follows Rajneesh.

From (7) and (8) it follows that:

> The proposition that Jane Fonda rajneeshes = the proposition that Rajneesh fondalees.

But this seems wrong. When a person consciously and explicitly thinks that Jane Fonda rajneeshes *must* that person be consciously and explicitly thinking that Rajneesh fondalees? It certainly does not seem so.[11] Thus, the weakened propositional-function thesis seems mistaken.

We have isolated another *prima facie* difficulty in the propositional-function thesis. It would be desirable to have a diagnosis of what has gone wrong. I will venture one, but I should emphasize that the problem should not be confused with the diagnosis (or with the technical apparatus used to state the diagnosis). Independently of the diagnosis, we have established that the propositional-function thesis leads to a *prima facie* problem.

According to the diagnosis, the objects of the propositional attitudes are so fine-grained that in the case of relational propositions the order in which relations are *predicated of* arguments is reflected in the identity of the propositions that are the outcome. For example, the relational property rajneeshing results from predicating the binary relation of following of Rajneesh, and in turn the relational proposition that Jane Fonda rajneeshes results from predicating this property of Jane Fonda. On the other hand, the relational property fondaleeing results from predicating the *inverse* of the binary relation of following of Jane Fonda, and in turn the relational proposition that Rajneesh fondalees results from predicating this property of Rajneesh. In symbols,

$$[\mathrm{R}j] = \mathrm{pred}(\mathrm{pred}([Fxy]_{xy}, r), j) \neq \mathrm{pred}(\mathrm{pred}([Fxy]_{yx}, j), r) = [\mathrm{F}r].$$

The reason that the propositional-function approach does not mark the distinction between these two propositions is that the *order* in which the corresponding propositional functions are applied to the arguments is not analogously reflected in the identity of the outcome:

$$(\lambda x)(\mathrm{R}x)(j) = (\lambda xy)(Fxy)(j)(r) = (\lambda yx)(Fxy)(r)(j) = (\lambda y)(\mathrm{F}y)(r).$$

Ironically, this and kindred phenomena are exactly the ones that make a propositional-function approach technically simpler than an approach that takes properties and relations as primitive entities not reducible to propositional functions. What the above example seems to show is that this very simplification blurs genuine distinctions among the type of propositions that figure in the logic for the propositional attitudes.[12]

I can think of two ways in which propositional-function theorists might try to regain the missing distinctions. First, they might try to regain them by introducing into their object theory a special primitive predicate for the application of a propositional function to an argument. For example, they might introduce the predicate 'App', where 'App(u, v)' is intended to mean the following: the result of applying propositional function u to argument v is a true proposition. Then, if these propositional-function theorists are willing to give up principle (E), they could claim:

(9) That Rj = that App($(\lambda x)(Fxr)$, j).

and

(10) That Fr = that App($(\lambda y)(Fjy)$, r).

Because:

That App($(\lambda x)(Fxr)$, j) $\neq$ that App($(\lambda y)(Fjy)$, r).

the missing distinction:

That $Rj \neq$ that Fr.

would be regained. This is to say:

That Jane Fonda rajneeshes $\neq$ that Rajneesh fondalees.

However, this way out of the problem is not acceptable. Not only does it involve giving up principle (E), which is the central principle of standard propositional-function semantic theories, but also it is manifestly mistaken on its face. For (9) and (10) are plainly false: a person could consciously and explicitly think that Jane Fonda rajneeshes without consciously and explicitly thinking that the result of applying the propositional function $(\lambda y)(Fxr)$ to Jane Fonda is true. Indeed, someone could think the former proposition and not even have the concept of applying a function to an argument! So this way of trying to solve the problem must be abandoned.

The second way in which propositional-function theorists might try to regain the missing distinctions is by holding that these distinctions are *pragmatic* not semantic. That is, they could hold that strictly and literally:

The proposition that Jane Fonda rajneeshes = the proposition that Rajneesh fondalees.

is true. But in conversation when you say:

I think that Jane Fonda rajneeshes.

what you would mean differs from what you would mean when you say:

I think that Rajneesh fondalees.

This pragmatic difference can then be explained by means of Gricean rules of conversation without calling into question the above strict and literal identity. So goes the pragmatic solution.[13]

However, this kind of pragmatic solution creates a special problem for propositional-function theorists. It can be shown (Bealer and Mönnich, 1989) that such pragmatic solutions require an *independent* solution to the *paradox of analysis*.[14] Specifically, such pragmatic solutions require positing a distinction between *analyzed intensions* and *unanalyzed intensions*. That is, there must be *two* types of intension, analyzed and unanalyzed. Given this, propositional-function theorists who advocate the pragmatic solution are faced with two grave problems. First, they are forced to decide which type of 0-ary intension – analyzed or unanalyzed – are to be the values of propositional functions. Inevitably, the choice will be utterly arbitrary. Second, they must work out a theory of the *other* type of 0-ary intension (i.e., the type of 0-ary intension not chosen to be values of propositional functions). Presumably, some further logical machinery besides that provided by the propositional-function theory will be needed for this purpose, and the use of this further logical machinery will lead propositional-function theorists to a disunified general theory of PRPs. On the algebraic approach to PRPs (developed in Bealer 1979, 1982, and 1983), both of these defects – the arbitrariness and the disunity – are avoided.

(4) I have just indicated that the existence of two types of PRP – analyzed and unanalyzed – creates grave problems for propositional-function theorists who would try to save their theory by pragmatic maneuvers. I now want to show that these grave problems are quite general: they arise as long as there is more than one type of PRP. Suppose for a moment that I am wrong about the need to introduce a distinction between analyzed and unanalyzed intensions to solve the paradox of analysis. No matter, there are also compelling intuitive and theoretical reasons for positing a distinction between fine-grained and coarse-grained PRPs.[15] This distinction is enough to produce the same sorts of special problems for the propositional-function theory noted in the previous paragraph. I will spell this out more fully.

Certainly both fine-grained and coarse-grained intensions exist. Consider an example. Intuitively, the thought that the glass is half empty is different from the necessarily equivalent thought that the glass is half full. (Thoughts are fine-grained 0-ary intensions.) Not so for the conditions (situations, states of affairs) to which thoughts correspond in the world. (Conditions are coarse-grained 0-ary intensions.) Intuitively, the glass's being half empty is the *same* condition (situation, state of affairs) in the world as the glass's being half full. It is just the physical condition that you are observing right there in front of you.[16]

Now the existence of more than one type of 0-ary intension gives rise to the following question. If properties are treated as propositional functions, are the values of these functions thoughts, or are they conditions (situations, states of affairs)? Are the values of these functions to be identified with fine-grained or coarse grained 0-ary intensions? Two observations.

First, the answer seems utterly arbitrary. If properties are identified with propositional functions, what *grounds* are there for thinking that their values are coarse-grained rather than fine-grained or fine-grained rather than coarse-grained? No answer seems available. The propositional-function theory requires the assumption of an arbitrary dogma on this point.

Suppose, however, that this inevitable arbitrariness is swallowed (as it should not be) and that the values of propositional functions are arbitrarily identified with one of the two types of 0-ary intensions. How is one to develop a theory of the *other* type of intension? This job will require some new kind of logical machinery, machinery not used in the original propositional-function approach. My second observation is this. This new logical machinery is likely to be very much like that used in the algebraic approach to intensional entities, which is the main competitor to the propositional-function approach (see Bealer, 1979, 1982, and 1983). If so, what is gained by not using an algebraic approach to both types of intension from the start? Furthermore, whatever the new kind of logical machinery is like, it certainly must go beyond that needed by the original propositional-function approach. For this reason, the resulting propositional-function theory inevitably fails to provide a *unified* treatment of both types of intension. One type of intension will be treated one way (i.e., by means of the original propositional-function machinery); the other type will be treated some other way (i.e., by means of the additional logical machinery). (Perhaps someone will try to identify coarse-grained intensions with equivalence classes of necessarily equivalent fine-grained intensions. But how unintuitive and unnatural! Not to mention the risk of logical paradox courted by such equivalence classes.) The algebraic approach, by contrast, provides a unified treatment of all types of intension; there is no ad hoc disanalogy in the way different types of intension are treated.

Conclusion

In addition to the various philosophical problems cited at the outset, we have isolated four significant logical problems confronting theories that identify properties and unary propositional functions. By similar arguments, we can also show that there are four analogous problems confronting theories that identify n-ary relations and n-ary propositional functions ($n \geqslant 2$). In view of these conclusions, we should not be tempted by the short term technical simplifications imparted by propositional-function theories. Properties cannot be reduced to unary functions (or other such entities made prominent by mathematics). Likewise, n-ary relations cannot be reduced to n-ary propositional functions. Properties and relations must be taken at face value as primitive, logically fundamental entities. By saying this, I am not suggesting that propositional functions do not exist. (Indeed, on the most economical theory of propositional functions, a unary propositional function is just a univocal[17] binary relation-in-intension; a binary propositional function is just a univocal ternary relation-in-intension, and so forth.) The point is this. Once properties and relations are taken as irreducible entities, propositional functions will cease to play the pivotal role that they have played in earlier formulations of intensional logic motivated by mathematics. A correct formulation of intensional logic treats properties and relations directly, and propositional functions are treated, not as something special, but as just one more kind of relation.

Notes

1 For example, this fact was assumed as the point of departure for the 1986 University of Massachusetts, Amherst, Conference on Property Theory.

2 Some of the reasons I will give against the propositional-function thesis were reasons that originally guided me toward the algebraic approach in Bealer (1979, 1982, and 1983). Others, however, have occured to me since then.

3 A propositional-function theorist might reply that this argument is an instance of the "fallacy of incomplete analysis." However, this reply is theoretically weak, for it forces the propositional-function theorist to hold that our intuitions here cannot be taken at face value. But other things being equal, a theory is superior if it can take relevant intuitions at face value. Our theory that properties are not propositional functions permits us to do just this.

4 It is important to stress that fine-grained intensions are not needed just for treating the propositional attitudes. They are also needed for treating various purely logical matters such as logical truth and analyticity for propositions. Many philosophers and cognitive scientists overlook this important point. For further discussion see Section 1 of G. Bealer (1986).

5 See Bealer (1979), Chapter 2 (1982), and (1983).

6 See Church (1954).

7 See Church (1951 and 1974); see also Anderson (1980).

8 This principle is Russellian in flavor. Fregeans would wish to modify it slightly. However, when the appropriate Fregean principles are substituted, we still can make much the same argument against the propositional-function thesis.

9 C. Anthony Anderson (1986) has independently given a somewhat similar argument in connection with a comparison of Russellian and Fregean higher-order intensional logics.

10 More colloquially, being someone who rajneeshes = being someone who follows Rajneesh, and being someone who fondalees = being someone whom Jane Fonda follows. Or,

to rajneesh = to be someone who follows Rajneesh, and to fondalee = to be someone whom Jane Fonda follows. In the symbolism of *Quality and Concept*, $[\mathrm{R}x]_x = [Fxr]_x$ and $[\mathrm{F}y]_y = [Fjy]_y$.

[11] This argument is given in a broadly Russellian setting in which the relevant functions may be applied to individuals, for example, to Jane Fonda and Rajneesh. But much the same argument can be given in a Fregean setting in which the relevant propositional functions are instead applied to individual concepts, for example, to the individual concept of (being) Jane Fonda and the individual concept of (being) Rajneesh.

Some might wonder whether, in the Russellian setting, the problem might not turn on the use of names in intensional contexts. This is not so, for the entire argument can be given using externally quantifiable free variables instead of names. Where u = Jane Fonda and v = Rajneesh,

$$[\mathrm{R}x]_x = [Fxv]_x.$$
$$(\lambda x)(\mathrm{R}x) = [\mathrm{R}x]_x.$$
$$(\lambda x)(Fxv) = [Fxv]_x.$$

Therefore,

$$(\lambda x)(\mathrm{R}x) = (\lambda x)(Fxv).$$

So:

$$(\lambda x)(\mathrm{R}x)(u) = (\lambda x)(Fxv)(u).$$

Hence, by (C) and (E),

$$[\mathrm{R}u] = [Fuv].$$

Similarly,

$$[\mathrm{F}y]_y = [Fuy]_y.$$
$$(\lambda y)(\mathrm{F}y) = [\mathrm{F}y]_y.$$
$$(\lambda y)(Fuy) = [Fuy]_y.$$

Therefore,

$$(\lambda y)(\mathrm{F}y) = (\lambda y)(Fuy).$$

So:

$$(\lambda y)(\mathrm{F}y)(v) = (\lambda y)(Fuy)(v).$$

Hence, by (C) and (E),

$$[\mathrm{F}v] = [Fuv].$$

Combining these two results, we get:

$$[\mathrm{R}u] = [\mathrm{F}v].$$

That is,

The proposition that u rajneeshes = the proposition that v fondalees.

However, it seems possible that someone could be consciously and explicitly thinking that u rajneeshes while not consciously and explicitly thinking that v fondalees.

[12] If we wish a PRP theory that makes even finer-grained intensional distinctions (for example, so that $[\mathrm{R}j] \neq [Fjr]$), we should adopt the apparatus introduced in chapter 3 of *Quality and Concept* for resolving the paradox of analysis. Indeed, I now think that a full diagnosis of the problem cannot be given until we first confront the paradox of analysis head-on.

Incidentally, some people have suggested that the fondalee/rajneeshe example is a counterexample to the principle of β-conversion from λ-calculus. If this were right, so much the worse for λ-calculus. However, β-conversion does not seem to be what the example calls into question. How could it? On the standard interpretation of λ-calculus '$f(x)$' denotes the value that results when function f is applied to argument x, where f is assigned to 'f' and x is assigned to 'x'; and '$(\lambda x)(f(x))(x)$' denotes the value that results where a certain function is applied to argument x, where that function is one that yields $f(x)$ as its value when it is applied to argument x. Accordingly, given an assignment to 'f' and 'x', '$f(x)$' and '$(\lambda x)(f(x))(x)$' cannot fail to denote the same thing. And this generalizes. Therefore, β-conversion seems unassailable. So the counterexample does not call into question β-conversion; rather, it calls into question the identification of properties with propositional functions. There is nothing wrong with λ-calculus. The point is that λ-calculus cannot be *applied* as a theory of properties, for properties and propositional functions behave differently.

[13] See Section 39 'Pragmatics' (Bealer, 1982) for a discussion of how a pragmatic solution could be developed along these lines.

[14] See Section 11 'Mates's Puzzle, the Paradox of Analysis, and the Need for Fine-grained Intensional Distinctions', (Bealer and Mönnich, 1989) for a full presentation of the argument.

[15] We need not take a stand here on the relationship between unanalyzed intensions and coarse-grained intensions. For the purposes of the present argument, it is enough that there should be intuitive and/or theoretical grounds (independent of the paradox of analysis) for positing the existence of coarse-grained intensions. Such grounds are spelled out more in some detail in Chapters 8–10 of *Quality and Concept* (especially Section 40). See also (Lewis, 1983) and Section 1 of (Bealer, 1986).

[16] This dual theory of thoughts and conditions is developed in Chapters 8 and 9 of *Quality and Concept*.

[17] A binary relation r is univocal *iff* for all x, y, z, if x, y stand in relation r and x, z stand in relation r, then $y = z$.

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