Chapter 1
Paraconsistency: Introduction

Koji Tanaka, Francesco Berto, Edwin Mares, and Francesco Paoli

1.1 Logic

It is a natural view that our intellectual activities should not result in positing contradictory theories or claims: we ought to keep our theories and claims as consistent as possible. The rationale for this comes from the venerable *Law of Non-Contradiction*, to be found already in Aristotle's *Metaphysics*, and which can be formulated by stating: for any truth-bearer A, it is impossible for both A and $\neg A$ to be true. *Dialetheism*, the view that some true truth-bearers have true negations, challenges this orthodoxy.[1] If some contradictions can be true, as dialetheists have argued, then it may well be rational to accept and assert them. For example, one may think that the naïve account of truth, based on the unrestricted T-schema: $\langle A \rangle$ is true

[1] Dialetheism itself has a venerable tradition in the history of Western philosophy: Heraclitus and other pre-Socratic philosophers were arguably dialetheists, for instance; and so were Hegel and Marx, who placed the obtaining and overcoming (*Aufhebung*) of contradictions at the core of their 'dialectical method'. For an introduction to dialetheism, see Berto and Priest (2008). A notable collection of essays on the Law of Non-Contradiction is Priest et al. (2004).

K. Tanaka (✉)
University of Auckland, Auckland, New Zealand
e-mail: k.tanaka@auckland.ac.nz

F. Berto
University of Aberdeen, Aberdeen, UK
e-mail: f.berto@abdn.ac.uk

E. Mares
Victoria University of Wellington, Wellington, New Zealand
e-mail: Edwin.Mares@vuw.ac.nz

F. Paoli
University of Cagliari, Cagliari, Italy
e-mail: paoli@unica.it

K. Tanaka et al. (eds.), *Paraconsistency: Logic and Applications*, Logic, Epistemology, and the Unity of Science 26, DOI 10.1007/978-94-007-4438-7_1,

if and only if A, should be accepted on a rational ground because of its virtues of adequacy to the data, simplicity, and explanatory power. However, the account is inconsistent, due to its delivering semantic paradoxes, such as the Liar.[2]

A dialetheist had better not be a classical logician. Classical logical consequence supports the principle often called *ex contradictione quodlibet* (ECQ): $\{A, \neg A\} \models B$ for any A and B. We are licensed by classical logic to infer anything whatsoever when we end up with a contradiction. To use a lively expression, classical logic is *explosive*: the truth of everything—a view often called *trivialism*—is classically entailed by the obtaining of a single contradiction; and trivialism is rationally unacceptable if anything is.[3]

A necessary condition for a logic to be *paraconsistent* is that its logical consequence relation, $\models$, is not explosive, invalidating ECQ. Although there is no general consensus on a definition of paraconsistent logic among researchers in the area, more often than not this necessary condition is taken to be a sufficient one too. Some logicians,[4] on the other hand, have argued that this 'negative' constraint should be supplemented by appropriate additional 'positive' properties. Be it as it may, since paraconsistent logics do not allow us to infer anything arbitrarily from a contradiction, their treatment of inconsistencies appears more sensible than the one in classical logic. But whereas a dialetheist should go paraconsistent, one does not need to accept that there are true contradictions to adopt a paraconsistent logic.[5] Dialetheism is a controversial view and many people find it counterintuitive. But, regardless of whether there are some true contradictions, it may be that in most cases when we find that we hold inconsistent beliefs or make inconsistent claims, we should revise them to be consistent.[6]

Whether or not there are *no* true contradictions, inconsistency is pervasive in our rational life. We often find that we have inconsistent beliefs or make inconsistent claims, and we are often subject to inconsistent information. Any philosopher who thinks that we may use a logic to make inferences from, determine commitments of, or otherwise logically examine the contents of people's beliefs, theories, or stories, should therefore think twice before being committed to explosion. For example, telling someone who has contradictory beliefs that they are committed to believing every proposition would be a very unproductive move in most debates, and do little more than merely pointing out that the person has inconsistent beliefs. Such considerations provide independent motivations for the development of paraconsistent logics: we need subtle, non-classical logical techniques to analyse the features of inconsistent theories and beliefs.

[2] See Priest (2005), Chap. 7.

[3] Trivialism finds, however, a recent, brilliant defence in Kabay (2010).

[4] See Béziau (2000).

[5] See Berto (2007) Chap. 5 and Priest and Tanaka (2009).

[6] Even dialetheists accept this. See, for example, Priest (2005) Chap. 8. For paraconsistent belief revision, see Mares (2002) and Tanaka (2005).

The history of paraconsistent logic has taught us that just taking classical logic and barring ECQ is not sufficient to produce an interesting non-explosive logic. In fact, a number of distinct logical techniques to invalidate ECQ have been proposed. As the interest in paraconsistent logic has grown, different people at different times and places have developed different non-explosive perspectives independently of each other. As a result, the development of paraconsistent logics has somewhat a regional flavour. This book is not a technical survey of the variety of paraconsistent logics[7]: it aims at illustrating their philosophical motivations, applications, and spin-offs. Since these logics are little known to non-specialists, though, in what follows we briefly summarise the most prominent logical strategies to achieve paraconsistency which feature in, or are presupposed by, the essays in this volume.

1.1.1 Discursive Logic

The first formal paraconsistent logic was developed in 1948 by the Polish logician Jaśkowski, in the form of *discussive* (or *discursive*) logic.[8] Jaśkowski's approach addressed situations involving distinct cognitive agents each putting forth her own beliefs, opinions, or reports on some event or other. Each participant's opinions may be self-consistent. However, the resultant discourse or set of data as a whole, taken as the sum of the assertions put forward by the participants, may be inconsistent.

Jaśkowski formalised this idea by modelling the inconsistent dialogical situation in a modal logic. For simplicity, Jaśkowski chose S5. We think of each participant's belief set (or set of opinions, assertions, etc.) as the set of sentences true at a world in a S5 model $\mathscr{M}$. Thus, a sentence A asserted by a participant in a discourse is interpreted as "It is possible that A" ($\Diamond A$). That is, a sentence A of discussive logic can be translated into a sentence $\Diamond A$ of S5. Then A holds in a discourse iff A is true at some world in $\mathscr{M}$. Since A may hold in one world but not in another, both A and $\neg A$ may hold in a discourse. In this volume, however, Marek Nasieniewski and Andrzej Pietruszczak show how Jaśkowksi's discussive logic can also be expressed via normal and regular modal logics weaker than S5 in their essay *On Modal Logics Defining Jaśkowski's D2-Consequence*.

1.1.2 Preservationism

In a discursive logic, a consequence relation can be thought of as defined over maximally consistent subsets of the premises. Given a set of premises, we can measure its degree of (in)consistency in terms of the number of its maximally consistent subsets.

[7] For surveys, besides Priest and Tanaka (2009), see Priest (2002) and Brown (2002).

[8] See Jaśkowski (1948).

For example, the level of $\{p, q\}$ is 1 since the maximally consistent subset is the set itself. The level of $\{p, \neg p\}$ is 2 since there are two maximally consistent subsets. If we define a consequence relation over some maximally consistent subset, then the relation can be thought of as preserving the level of consistent fragments. This is the approach which has come to be called *preservationism*. It was first developed by the Canadian logicians Ray Jennings and Peter Schotch.[9] In this volume, Bryson Brown's essay *Consequence as Preservation: Some Refinements* moves within this tradition, but proposes a more general view of the features a logical consequence relation can be seen as preserving.

1.1.3 Adaptive Logics

One may think that we should treat a sentence or a theory as consistently as possible. However, once we encounter a contradiction in reasoning, we should adapt to the situation. *Adaptive logics*, developed by Diderik Batens and his collaborators in Belgium, are logics that 'adapt' themselves to the (in)consistency of a set of premises available at the time of application of inference rules. As new information becomes available expanding the premise set, consequences inferred previously may have to be withdrawn. However, as our reasoning proceeds from a premise set, we may encounter a situation where we infer a consequence provided that no abnormality, in particular no contradiction, obtains at some stage of the reasoning process. If we are forced to infer a contradiction at a later stage, our reasoning has to adapt itself so that an application of the previously used inference rules is withdrawn. Adaptive logics model the dynamics of our reasoning as it may encounter contradictions in its temporal development.[10] In this volume, Diderik Batens' essay *New Arguments for Adaptive Logics* presents four new arguments vindicating the utility of the adaptive approach.

1.1.4 Logics of Formal Inconsistency

The approaches to paraconsistency we have referred to so far retain as much classical machinery as possible (many paraconsistent logicians believe that the full inferential power of classical logic ought to be retained as much as possible, insofar as we find ourselves in consistent contexts). One way to make this aim explicit is to extend the expressive power of our logic by encoding the metatheoretical notions of consistency and inconsistency in the object language. The *Logics of Formal Inconsistency* (*LFIs*) are a family of paraconsistent logics which constitute consistent fragments of classical logic, yet reject explosion where a contradiction

[9]See for instance Schotch and Jennings (1980).

[10]For a general overview of adaptive logics, see Batens (2001).

is present. The investigation of this family of logics was initiated by the Brazilian logician Newton da Costa. An effect of encoding consistency and inconsistency as object language operators on sentences is that we can explicitly separate inconsistency from triviality. With a language rich enough to express consistency and inconsistency, we can study inconsistent theories without assuming that they are necessarily trivial, but at the same time admitting that *some* inconsistencies are so bad that they can trivialize a theory, whereas others are not. This makes it explicit that the presence of a contradiction is a separate issue from the non-trivial nature of paraconsistent inferences.

Prominent among the LFIs are the so-called *positive-plus systems*, which bear this name because they are paraconsistent logics whose negation-free fragment is just positive intuitionistic logic. The paraconsistent features of these systems are obtained by placing on top of the orthodox positive logic a profoundly modified treatment of negation, which turns out to be non-truth-functional: at least one of A and $\neg A$ has to be true, but given that A is true, $\neg A$ may be true or may be false. As a consequence, whereas Excluded Middle, $A \vee \neg A$, is logically valid, the Law of Non-Contradiction in the form of $\neg(A \wedge \neg A)$ is not. The negation of positive-plus systems displays some notable dualities with respect to intuitionistic negation.[11] In this volume, Walter Carnielli and Marcelo Coniglio provide a defense of the LFI approach and its epistemic viability in their essay *On Discourses Addressed by Infidel Logicians*.

1.1.5 Many-Valued Logics

In the standard semantics for classical logic there are exactly two truth values, namely true, 1 and false, 0. Many-valued logics allow more than two truth values. Not all many-valued logics are paraconsistent. Perhaps the most famous—Kleene's and Łukasiewicz's three-valued logics—are explosive. These logics admit, besides truth and falsity, a third value, say $\frac{1}{2}$, which can be thought of as *indeterminate*, or *neither true nor false*.

A many-valued paraconsistent logic typically allows inconsistent values to be designated, i.e., preserved in valid inferences (many-valued approaches to paraconsistency were first proposed by the Argentinian logician Florencio Asenjo[12]). The simplest strategy is to use three values. Suppose we start with the classical set of truth values, $\{1, 0\}$, and consider its power set, i.e., the set of all its subsets, minus the empty set, $\varnothing$: $\mathscr{P}\{1, 0\} - \varnothing = \{\{1\}, \{0\}, \{1, 0\}\}$. The three remaining items can be read as $\{1\}$ = true (only), $\{0\}$ = false (only), which can function as in classical logic, and $\{1, 0\}$ = *both true and false*, which, naturally enough, is a fixed point for negation: if A is both true and false, $\neg A$ is as well. Both $\{1\}$ and $\{1, 0\}$ are

[11] A classic paper in this tradition is Da Costa (1974).

[12] See Asenjo (1966).

designated, the idea being that a designated value must have *some* truth, 1, in it. ECQ is invalidated by having a propositional parameter p which is both true and false; then $\neg p$ is both true and false as well, and the inference to a q which is false (only) does not preserve the designated values. This is the approach of the paraconsistent logic LP (the *Logic of Paradox*) developed by Graham Priest.[13]

If one lets $\varnothing$ play the role of a fourth (and non-designated) value, to be read as *neither true nor false*, which behaves in an appropriate way, one obtains *Belnap's four valued logic* and, in particular, its linguistic fragment FDE (*First Degree Entailment*), a basic *relevant* logic.[14] In this volume, innovative informational models for FDE are proposed by R.E. Jennings and Yue Chen's essay *Articular Models for First Degree Entailment*.

1.1.6 Relevant Logics

Relevant (or *relevance*) *logics* are perhaps the most developed and discussed among paraconsistent logics. The approaches to paraconsistency we have mentioned above target ECQ on the basis of the pervasive presence of inconsistencies in our inferential practices. One may think, though, that ECQ is just one of a set of inferences that are problematic for a more general reason, having to do with the lack of *relevance* between the premises and the conclusion. $(A \wedge \neg A) \rightarrow B$, an 'object-language' counterpart of ECQ, is called, not accidentally, a 'paradox' of the (material or strict) conditional even within classical logic. The problem with such entailments as 'If it is both raining and not raining, then the moon is made of green cheese' is that rain (even inconsistent rain!) seems to have little to do with the material constitution of the moon. Other paradoxes of the conditional, such as $A \rightarrow (B \vee \neg B)$ ('If the moon is made of green cheese, then either it is raining or not'), and $A \rightarrow (B \rightarrow B)$ ('If all instances of the Law of Identity fail, then (if it is raining, then it is raining)') are also taken in this approach as 'fallacies of relevance', due to the lack of a connection between antecedents and consequents.

Relevant logics were pioneered by the American logicians Anderson and Belnap, in order to provide accounts of conditionality free from such fallacies.[15] Anderson and Belnap motivated the development of relevant logics using natural deduction systems; yet they developed a family of relevant logics in axiomatic systems. As research on relevance proceeded and was carried out also in Australia, more focus was given to semantics and model theory. The mainstream approach consists in developing worlds semantics including, besides ordinary possible worlds, also so-called *non-normal* or *impossible* worlds, to be thought of, roughly, as worlds

[13] See Priest (1979).

[14] For Belnap's logic, see Belnap (1977). The interpretation of the truth values of FDE in terms of sets of classical truth values has been suggested by Dunn (1976).

[15] See Anderson and Belnap (1975) and Anderson et al. (1992).

where the truth conditions of logical operators are non-classical. The main semantic tool to obtain a relevant conditional consists in specifying its truth conditions in terms of a three-place accessibility relation on worlds, due to the logicians Richard Routley and Robert Meyer. By accessing worlds which are locally inconsistent or incomplete, one can also invalidate $(A \wedge \neg A) \rightarrow B$ and $A \rightarrow (B \vee \neg B)$.[16]

The core of the philosophical debate on these models is what intuitive sense one is to give them. In this volume, Koji Tanaka's essay *Making Sense of Paraconsistency* addresses the issue in a general setting, turning tables around and challenging the classical logician to make intuitive sense of ECQ, while Ed Mares' *Information, Negation, and Paraconsistency* proposes an informational interpretation that, in a sense, dispenses with possible and impossible worlds altogether, in favour of situations interpreted *à la* Barwise and Perry. In his *Assertion, Denial and Non-Classical Theories*, a notable exponent of the relevantist tradition like Greg Restall provides innovative insights to paraconsistency by considering what he calls 'bitheories'—formal theories based on assertion and denial operators. The expressive powers of bitheories allow them to abstract away from much logical vocabulary whose meaning is controversial in the debate between classical and non-classical logicians.

Relevant logics belong to the family of *substructural* logics, which, besides rules of inference for the logical operators, have structural rules allowing one to operate on the structure of the premises and conclusions.[17] In this volume, the topic is addressed by Francesco Paoli's *A Paraconsistent and Substructural Conditional Logic* via a formal system providing an innovative approach to *ceteris paribus* conditionals. Patrick Allo's work, *Noisy vs. Merely Equivocal Logics*, connects substructural logics to ambiguities of logical connectives that are overlooked within classical logic, in order to shed new light on the issue of rivalry between logics.

1.2 Applications

We claimed that the main motivation for paraconsistency, apart from dialetheism, is the need to model, and account for, non-trivial inferences from inconsistent theories, data bases, and belief sets. It is therefore no surprise that paraconsistency has many applications, given how pervasive these phenomena can be. They can manifest themselves in ordinary life reasoning (a paraconsistent approach to commonsensical inference is proposed in this volume by Michael Anderson, Walid Gomaa, John Grant and Bon Perlis, in their essay *An Approach to Human-Level Commonsense Reasoning*). But they also show up in more theoretical contexts. Working scientists can and have worked productively with inconsistent theories

[16]For a general introduction to relevant logics, see Mares (2006) and, for a philosophical interpretation, Mares (2004). On non-normal or impossible worlds, see Berto (2009).

[17]On substructural logics, see Restall (2000) and Paoli (2002).

(which they could not do if they merely inferred that, then, everything is true according to such theories).[18] Readers of fiction understand and appreciate stories that are inconsistent, and at times not accidentally (because of authorial inaccuracy), but essentially so.[19] Similarly, we may have real moral dilemmas, in which we have inconsistent obligations; and we do have inconsistent legal codes. Other examples of inconsistent but intuitively non-trivial information and theories traditionally suggested are: quantum mechanical phenomena on the micro-scale; predicates with over-determined criteria of application; the intuitive metaphysics of change and becoming.[20] The relation between quantum mechanics and paraconsistency is addressed in this volume by Ross Brady and Andrea Meinander's essay, *Distribution in the Logic of Meaning Containment and in Quantum Mechanics*.

We have singled out two paradigmatic (sets of) cases for closer, albeit still rapid, inspection: the role of paraconsistency in the philosophy of mathematics, and its application to the modeling of vagueness in natural language. Many of the papers in the second part of this volume can be located within these two areas.

1.2.1 Philosophy of Mathematics

Historically speaking, paraconsistency comes into the philosophy of mathematics via the celebrated paradoxes of naïve set theory, such as Russell's (the set of non-self-membered sets does and does not belong to itself) and Cantor's (the set of all sets is, via Cantor's Theorem, and of course is not, larger than itself). There are various axiomatised set theories, such as ZF-ZFC or VNB, that are free from these paradoxes; it is well-known, though, that they all introduce more or less *ad hoc* limitations to the unrestricted Comprehension Principle for sets, stating that any well-formed condition, $A[x]$, delivers a set of all and only the items satisfying $A[x]$. Also given Gödel's Incompleteness Theorem, a consistent theory capable of representing basic arithmetical truths cannot represent its own consistency proof. And since theories of sets like ZFC can represent such truths, they cannot therefore represent their own consistency proofs. In fact, the situation is worse: ZFC can formalize *all* of standard mathematics; therefore, a consistency proof for ZFC, not being representable in ZFC by Gödel's result, would be, in some sense, beyond

[18]For instance, Bohr's atomic theory assumed that energy comes in discrete quanta, and also assumed Maxwell electromagnetic equations to make predictions on atomic behaviour. The two assumptions are inconsistent, but the theory was quite successful—and, more importantly, nobody would find intuitively acceptable that the theory entails that everything is true. On this story, see Brown (1993).

[19]For instance, Priest (1997a) is a story centred on an inconsistent box which is both empty and not empty; the contradiction is only true in the fiction, of course, but if we bracketed the inconsistency we would miss the whole point of the narration. And intuitively, not everything happens in the story.

[20]For an overview of applications of paraconsistency, see Priest and Routley (1989). Specifically on the metaphysics of change, see Priest (1987), Chaps. 11, 12 and 15.

standard mathematics (e.g., by including so-called large cardinal axioms whose epistemic status may be more problematic than that of the consistency of ZFC itself).

This landscape has motivated the development of paraconsistent theories of sets which retain the full Comprehension Principle of naïve set theory. This delivers inconsistent sets like Cantor's and Russell's, but the underlying non-explosive logic prevents the inconsistencies from trivializing the theory. Whereas consistency proofs are not at issue for such formal theories, there exist non-triviality proofs for paraconsistent set theories, and they are representable within the theories themselves.[21] Interesting new results in this tradition are provided in this volume by Zach Weber's essay, *Notes on Inconsistent Set Theory*.

Paraconsistent arithmetics have also been developed. The first such theory, the system of *relevant arithmetic* R#, had an underlying relevant logic and was proposed in the 1970s by Robert Meyer. Its most interesting feature is that it can be proved absolutely consistent (i.e. nontrivial) by finitary means. However, Friedman and Meyer somewhat downplayed the significance of this result by showing that there are (purely mathematical) theorems of classical Peano arithmetic that cannot be proved in R#. Classes of inconsistent arithmetical theories were later explored by Meyer and Chris Mortensen, and they proved capable of representing also algebraic structures like rings and fields. Their inconsistency and 'finitary' features allow them to escape from Church's undecidability result: they are, that is, provably decidable.[22] The topic of paraconsistent arithmetic is addressed in this volume by Chris Mortensen's essay, *Arithmetic Starred*, while Francesco Berto's *Wittgenstein on Incompleteness Makes Paraconsistent Sense* attempts to make sense of Wittgenstein's (in)famous remarks on Gödel's First Incompleteness Theorem by advocating a paraconsistent reading of Wittgenstein's deeply finitistic philosophy of mathematics.

Just as the issue of logical pluralism is turned on by the development of paraconsistent logic, the one of pluralism in the philosophy of mathematics is triggered by the development of paraconsistent and radically non-classical formal mathematical theories. In this volume, Michelle Friend's *Pluralism and 'Bad' Mathematical Theories* defends such a form of pluralism, in the light of paraconsistency as well as in that of Stewart Shapiro's structuralism.

1.2.2 Philosophy of Language: Vagueness

Natural language abounds in vague predicates, that is, predicates whose criteria of application admit of borderline cases. What must your age be in order for you to

---

[21] See Brady (1989) for a proof of the non-triviality of paraconsistent set theory, and Brady (2006) for a general account.

[22] See Meyer (1976), Friedman and Meyer (1992), Meyer and Mortensen (1984) and, for a general characterization, Priest (1997b) and Priest (2000).

be *old*? How much money must you make in a year to be *rich*? How many hairs must you lose to become *bald*? And so on. Vagueness causes notorious problems to classical logic, for the latter licenses paradoxical inferences, like the *Heap* (a form of the Sorites paradox—from the Greek *soros*, which means precisely 'heap'): one million grains of sand form a heap; if n grains of sand form a heap, then also $n - 1$ grains form a heap (what difference can one grain make?); apply the latter repeatedly, until you get that one single grain of sand forms a heap, which will not do.

In fact, with the exception of the so-called epistemic solutions, all the main approaches to vagueness, such as the ones based on many-valued logics, or supervaluations, already require some departure from classical logic, in the form of under-determinacy of reference, and/or the rejection of Bivalence: if a middle-aged man, m, is a borderline case with respect to the predicate 'is old', $O(x)$, then $O(m)$ may turn out to have an intermediate truth value between truth and falsity, or no truth value at all. But it may be conjectured that a borderline object like m, instead of satisfying neither a vague predicate nor its negation, satisfies them *both*: a middle-aged man, in some sense, can be correctly characterized both as being and as not being old. Similarly, in a borderline rainy day we may safely answer to the question whether it is raining with a 'Yes and no', and get away with it. If these phenomena have, as is usually claimed in this context, a *de re* reading, then actually inconsistent objects may be admitted, together with vague objects. To the satisfaction of the dialetheist, this would spread inconsistency all over the empirical world: if borderline cases can be inconsistent, inconsistent objects are everywhere, given how pervasive the phenomenon of vagueness notoriously is: teen-agers, borderline bald people, middle-age men, etc. Again, however, it is an open option for the paraconsistent logician to assume that the inconsistencies due to vague predicates and borderline objects are only *de dicto*: they may be due to merely semantic under- and/or over- determination of ordinary language predicates.

Whatever one's attitude on this issue is, given the obvious dualities between Excluded Middle, $A \vee \neg A$, and the Law of Bivalence, $T\langle A\rangle \vee T\langle \neg A\rangle$ (with T the relevant truth predicate), on the one side, and the Law of Non-Contradiction in 'syntactic' ($\neg(A \wedge \neg A)$) and 'semantic' ($\neg(T\langle A\rangle \wedge T\langle \neg A\rangle)$) formulations on the other, it has not been too difficult for authors in the paraconsistent tradition to envisage a 'sub-valuational' paraconsistent semantic approach, dual to the supervaluational strategy.[23] However, it is not uncontroversial that super- and sub-valuational approaches are the right paraconsistent way to address the phenomena at issue. In this volume, David Ripley's essay, *Sorting out the Sorites*, proposes an alternative paraconsistent strategy, based on Priest's logic LP.

In fact, also the connections between the paradoxes of self-reference (taken by dialetheists, as we have claimed, as a decisive motivation for their view) and the paradoxes of vagueness may be quite tighter than expected. In this volume, Graham Priest's essay *Vague Inclosures* shows how the Sorites can fit into Priest's general

[23] Sub-valuational semantics have been proposed by Hyde (1997) and Varzi (1997).

'Inclosure Schema' for the paradoxes of self-reference. Dominic Hyde's *Are the Sorites and Liar Paradox of a Kind?* also addresses the issue of the structural similarities and differences between the two kinds of paradox, finding their common source in the under-determinacy of the relevant predicates in a paraconsistent setting.

References

Anderson, A.R., and N.D. Belnap. 1975. *Entailment: The logic of relevance and Necessity*, vol. 1. Princeton: Princeton University Press.

Anderson, A.R., N.D. Belnap, and J.M. Dunn. 1992. *Entailment: The logic of relevance and necessity*, vol. 2. Princeton: Princeton University Press.

Asenjo, F.G. 1966. A calculus of antinomies. *Notre Dame Journal of Formal Logic* 7: 103–105.

Batens, D. 2001. A General characterization of adaptive logics. *Logique et Analyse* 173–175: 45–68.

Belnap, N.D. 1977. How a computer should think. In *Contemporary aspects of philosophy*, ed. G. Ryle, 30–55. Boston: Oriel Press.

Berto, F. 2007. *How to sell a contradiction: The logic and metaphysics of inconsistency*. London: College Publications.

Berto, F. 2009. Impossible worlds. In *The stanford encyclopedia of philosophy*, Fall 2009th ed, ed. E.N. Zalta. Stanford: Stanford University.

Berto, F., and G. Priest. 2008. Dialetheism. In *The stanford ncyclopedia of philosophy*, Summer 2010th ed, ed. E.N. Zalta. Stanford: Stanford University.

Béziau, J.-Y. 2000. What is paraconsistent logic? In *Frontiers in paraconsistent logic*, ed. D. Batens, 95–112. London: Wiley.

Brady, R. 1989. The nontriviality of dialectical set theory. In *Paraconsistent logic: Essays on the inconsistent*, ed. G. Priest, R. Routley, and J. Norman, 437–471. München: Philosophia Verlag.

Brady, R. 2006. *Universal logic*. Stanford: CSLI Publications.

Brown, B. 1993. Old quantum theory: A paraconsistent approach. *Proceedings of the Philosophy of Science Association* 2: 397–441.

Brown, B. 2002. On paraconsistency. In *A companion to philosophical logic*, ed. D. Jacquette, 628–650. Oxford: Blackwell.

Da Costa, N.C.A. 1974. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic* 15: 497–510.

Dunn, J.M. 1976. Intuitive semantics for first-degree entailments and coupled trees. *Philosophical Studies* 29: 149–168.

Friedman, H., and R.K. Meyer. 1992. Whither relevant arithmetic? *Journal of Symbolic Logic* 57: 824–831.

Jaśkowski, S. 1948. Rachunek zdań dla systemów dedukcyjnych sprzecznych. *Studia Societatis Scientiarun Torunesis (Sectio A)* 1(5): 55–77. (trans) Propositional calculus for contradictory deductive systems, Studia Logica, 24(1969): 143–157.

Hyde, D. 1997. From heaps and gaps to heaps of gluts. *Mind* 106: 641–660.

Kabay, P. 2010. *On the plenitude of truth: A defense of trivialism*. Saarbrücken: Lambert Academic Publishing.

Mares, E.D. 2002. A paraconsistent theory of belief revision. *Erkenntnis* 56: 229–224.

Mares, E.D. 2004. *Relevant logic: A philosophical interpretation*. Cambridge: Cambridge University Press.

Mares, E.D. 2006. Relevance logic. In *The stanford encyclopedia of philosophy*, Spring 2009th ed, ed. E.N. Zalta. Stanford: Stanford University.

Meyer, R.K. 1976. Relevant arithmetic. *Bulletin of the Section of Logic of the Polish Academy of Sciences* 5: 133–137.
Meyer, R.K., and C. Mortensen. 1984. Inconsistent models for relevant arithmetics. *The Journal of Symbolic Logic* 49: 917–929.
Paoli, F. 2002. *Substructural logics: A primer*. Dordrecht: Kluwer.
Priest, G. 1979. Logic of paradox. *Journal of Philosophical Logic* 8: 219–241.
Priest, G. (1987). In *Contradiction. A study of the transconsistent*. Dordrecht: Martinus Nijhoff. 2nd and expanded edition, Oxford: Oxford University Press, 2006.
Priest, G. 1997a. Sylvan's box: A short story and ten morals. *Notre Dame Journal of Formal Logic* 38: 573–582.
Priest, G. 1997b. Inconsistent models for arithmetic: I, finite models. *The Journal of Philosophical Logic* 26: 223–235.
Priest, G. 2000. Inconsistent models for arithmetic: II, the general case. *The Journal of Symbolic Logic* 65: 1519–1529.
Priest, G. 2002. Paraconsistent logic. In *Handbook of philosophical logic*, vol. 6, ed. D. Gabbay and F. Guenthner. Dordrecht: Kluwer.
Priest, G. 2005. *Doubt truth to be a liar*. Oxford: Oxford University Press.
Priest, G., J. Beall, and B. Armour-Garb (eds.). 2004. *The law of non contradiction*. Oxford: Oxford University Press.
Priest, G., and R. Routley. 1989. Applications of paraconsistent logic. In *Paraconsistent logic: Essays on the inconsistent*, ed. G. Priest, R. Routley, and J. Norman, 367–393. München: Philosophia Verlag.
Priest, G., and K. Tanaka. 2009. Paraconsistent logic. In *The stanford encyclopedia of philosophy*, Summer, 2009th ed, ed. E.N. Zalta. Stanford: Stanford University.
Restall, G. 2000. *An Introduction to substructural logics*. London: Routledge.
Schotch, P.K., and R.E. Jennings. 1980. Inference and necessity. *Journal of Philosophical Logic* 9: 327–340.
Tanaka, K. 2005. The AGM theory and inconsistent belief change. *Logique et Analyse* 48: 113–150.
Varzi, A. 1997. Inconsistency without contradiction. *Notre Dame Journal of Formal Logic* 38: 621–639.