REANALYZING CHISHOLM PARADOX. STRUCTURAL INSIGHTS

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Abstract. In this paper I focus on the conditions that have to be met for Chisholm's Paradox (CP) to occur. My claim is that identity and structure are notions closely related to each other. I propose a discussion in which the minimal framework for CP is set, then analyze the paradox in terms of S5, and suggest that in order to capture the core of the paradox one should use a dynamic valuation function for the model. Identity appears, at this point, to be dependent upon a structuralist point of view.
Keywords: Chisholm Paradox, transworld identity, valuation function, de re/ de dicto *statements.*

In this paper I analyze the conditions that have to be met for Chisholm's Paradox to hold. My claim is that the manner in which we use the notion of identity is closely related to the one of structure. The approach consists in historical sections, in which the Quinean critique to modal logic and Chisholm's Paradox (CP) are presented, a section in which replies to them are taken into account and finally the 'deconstruction' of an S5 model in which CP holds. Conclusions and further work suggestions are to be found in the end of the paper.

I. Introduction

Modal Logic Problems, Quine. Properties and Essences

Quine's critique to modal logic (ML) may have two readings: a technical one, and the target is the intensionality it involves, and a metaphysical one, where the essences and properties are at stake.

The two readings are faces of the same coin, and this is the status and implications of *de re* statements. ML is obtained by adding to the standard propositional and or predicate first order logic the modal operators necessity and possibility, and the devices they entail, in order for the new formed formulas to have a meaning. Depending on the scope of the modal operator, the obtained statements are either *de dicto* (that is, the scope is an entire wff) or *de re* (and here, in the scope of the modal operator enters only a part of a formula). The difference is between saying, for example, that

(i) Necessarily, any object is made of matter and
(ii) Any object is necessarily made of matter.

While the first sentence says that the proposition (in Fregean sense) that all the objects are made of matter is necessary, the second one attaches necessity to a predicate – being made of matter; hence all objects are necessarily made of matter. And this tells us somenthing about the objects, and not about the sentence. Formalised, the above statements become:

(i.i) $\Box\ (\underline{\forall}x)Mx$
(ii.i) $(\forall x)\ \Box\ Mx$, where x is a variable and M the predicate "_ is made of matter".

The first case is a *de dicto* one, while de second a *de re*. In the light shed by Quine's analysis, the second type of sentence is problematic since it involves, on the one part, intensional and referentially opaque contexts and, on the other one, its evaluation commits us to essentialist tendencies. We will focus in this paper on the metaphysical part of Quine's critique, and let aside the technical one[1].

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[1]   For the details of this approach, see (Quine, 1953)

1.1. A Weak Analogy between ML and TL

Modal and temporal logic are considered to be similar, and they are treated, according to this, in similar manners. But modal logic (ML) is not as similar to temporal logic (TL) as we think of, Quine's argument goes. In "Worlds Away", the discussion reaches this point. Let us consider the monetary content of John's pocket, for example. The discussion is directed towards the notion of identity, central for the evaluation of the statements about the 'monetary content of John's pocket', to which we will refer here on to as an 'object'.

In TL's terms, another character of our story, Mark, will state something about the present state of the respective object. One week later he will refer again to the object. And he will also be able to add temporal operators as Past and Future, not only Present. In evaluating his statements, one will be required to be able to pick up the object referred to. And this ability will certainly be dependent upon the object's identity and some device that allows a decision on the respective identity, whose warrant can be temporal continuity. Any point from our time interval can be evaluated and the object can be pointed out at, as in the process of identification temporal continuity is an accepted criterion.

Mark also wants to talk about the object in ML's terms. The evaluator of his statements' truth-value will use the devices at hand, that is, the possible worlds apparatus. He will have to evaluate Mark's modal statements at all possible worlds in his model, and he will also need to have an identity criterion for the object. This should not be a very difficult problem, unless Mark's statements are *de re*, and in this situation, things about the object's nature[2] would be at stake. Since possible worlds are made up, mere stipulations, what reassures him that the objects he evaluates across worlds are identical to each other? Any device that warrants such a thing would be one that allows him to discriminate between essential and non-essential properties and this, in Quine's view, is not possible. Evaluating *de re* statements in ML will drive us into

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2   That is, pertaining to its essence.

the "jungle of aristotelian essentialism", to which he is a fervent opponent, and so these statements are illegitimate. The similar treatment of ML and TL is, thus, not justified, because of the divergencies occurring between their *de re* statements. ML without them, that would be a recognised logic by Quine.

1.2. Chisholm's Paradox (CP)

The lack of a criterion for cross-world identity has received attention from the part of other philosophers as well. Among them, Roderick Chisholm, with his famous paradox pointing out at the old problem of knowing who. Chisholm's characters are Adam and Noah, and the paradox goes as follows: take the two of them and exchange their properties starting, for example, with the number of years they lived, until you finally reach a world in which they switched sides. In all this transformation process beware to accommodate all the objects they relate to; thus Adam's wife will be, in the second world, the wife of a man (Adam) that lived 949 years, instead of 950, and Noah's will be the wife of a man living for 931, not 930 years. In the final world, the husband of Adam's wife will live for 930 years, and the companion of Noah's wife for 950 and so on, for each property. All this process seems correct, but we end up with this strange situation: the role of Adam from the actual world will be occupied in the final world by a man having all the properties and name of Noah, and vice versa. Let us sum up: we began from the actual world where our characters have their own (and only them have those) properties and we operated slight changes in each other such that, for any intermediate world, the differences between the same 'assumed' object are less relevant than the similarities, so there is no sound argument in favor of declaring them distinct. Still, when we take a look at the big picture, we notice that those differences we considered not relevant, taken together, changed the situation a lot.
    Chisholm notes:

"It seems to me that even if Adam does have such essential properties, there is no procedure at all for finding out what they are. And it also seems to me that there is no way of finding out whether he does have any essential properties. Is there really a good reason, then, for supposing that he does?" (Chisholm, 1967, p. 85)

Later on, this problem is subsumed to that of knowing who – namely, that we do not have information to be able to determine which are the properties that uniquely characterize an object, person etc. In other worlds, we have no strong identity criterion to apply.

1.3. Replies

Up to this point, we have two examples, the monetary content of one's pocket, and CP, both illustrating a problem specific to *de re* modalities: the unability to warrant for transworld identity. Answering to this challenge would mean either to give up the concept of identity, or to give an identity criterion strong enough to deal with such kind of puzzling situations. Here, we are interested with a solution preserving identity.

1.3.1. Modal Set Theory

A solution employing the apparatus of modal logic has been provided in (Forbes, 1983). Introducing the notion of 'individual essence' – the set of essential properties[3] that uniquely characterize an object, Forbes gives a solution for what constitutes the essence of a set – it is essential for any set to have the members it has – by explicitly formulating these axioms:

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[3]   Also, there are some remarks about what essential properties are not: (a) consequences of de dicto truths, (b) necessary universal/general properties (self-identity, existence) or their cognates, (c) properties had in virtues of necessary truths concerning items of another categories.

(Falsehood Principle)

$$\Box(\forall x)\Box(\forall X)\Box(x\in X\rightarrow E(x)\&E(X))$$

(Membership Rigidity)

$$\Box(\forall x)\Box(\forall X)\Box(x\in X\rightarrow\Box(E(X)\rightarrow x\in X))$$

(Crossworld Extensionality)

$$\Box_1(\forall X)\Box_2(\forall Y)([\Box(\forall z)(A_1(z\in X)\leftrightarrow A_2(z\in Y))]\rightarrow X=Y),$$

where x, y, z are variables for objects, X and Y are names for sets and E is the predicate of existence.

(Set Rigidity)

$$\Box(\forall x)\Box(Sx\rightarrow\Box(E(x)\rightarrow Sx))$$

(Individuality Rigidity)

$$\Box(\forall x)\Box(Ix\rightarrow\Box(E(x)\rightarrow Ix))$$

Roughly, what they say is that: Falsehood – necessarily, for any object and necessarily, for any set, it is necessary that, if an object belongs to a set, then this implies the existence of the object and of the set). This axiom would put aside the possibility of empty sets.

Membership Rigidity says that it is necessary, for any object and for any set, that if an object belongs to a set, then it is necessary that the set's existence implies that the object in question belongs to that set. Finally, Crossworld Extensionality brings the benefits of indexing properties and worlds, and "specifies an indiscernibility condition whose holding across worlds is sufficient for transworld identity between the entities for which it holds"

(Forbes, 1983, p. 112) it is necessary, for any two sets existing at different worlds, that if they have the same members, the respective sets are the same. Set Rigidity's justification is that if being a set requires to have been formed at that world and, if sets are necessarily sets, then their existence is necessary. The Individuality Rigidity simply says that for any object, if it has an individual essence, then its existence implies the individual essence. There are more axioms to be added to the system, in order to be complete, but for our purposes, the above-mentioned one are sufficient. Also, with the necessary adjustments, Forbes continues the discussion on categories other than sets (artifacts[4] and beings[5]).

The reader may also find a separate discussion on CP in (Forbes, 1984), where two different approaches are examined. Forbes takes into account two formulations of the paradox, and then the two alternative modal theories that can be used to interpret them.

Let us suppose we have the predicate "_M_", with the meaning "_is constituted of exactly the sum of wood _", ⊚ a table h_1 the sum of wood the table is made of, for any h_i, h_{i+1}, sum of wood almost coextensive with h_i and h_n a sum of wood with nothing in common with h_1. Then, CP would take the form:

$$\Diamond\, M\alpha h_1 \qquad\qquad vs. \qquad\qquad \Diamond\, M\alpha h_1$$

$$\Box(\,M\alpha h_1 \rightarrow \Diamond\, M\alpha h_2) \qquad\qquad \Diamond\, M\alpha h_1 \rightarrow \Diamond\, M\alpha h_2$$

$$\begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array} \qquad\qquad \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array}$$

$$\Box(M\alpha h_{n-1} \rightarrow \Diamond\, M\alpha h_n) \qquad\qquad \Diamond M\alpha h_{n-1} \rightarrow \Diamond\, M\alpha h_n$$

$$\therefore \Diamond\, M\alpha h_n \qquad\qquad\qquad \therefore \Diamond\, M\alpha h_n$$

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[4] For a discussion on the importance of the category the object belongs to, see (Forbes, 1984).

[5] For beings, Krikpe's necessity of origin thesis is brought into attention (the corresponding moment is that of set formation), while concerning the artifacts, a logic of vagueness is developed.

The second formulation takes the form of Sorites, and Forbes argues for considering it as more suitable to express CP. Also, concerning the theory one assents to, candidates are counterpart theory and a system in which transitivity is rejected[6]. Forbes argues in favor of sustaining the counterpart theory; otherwise, best suited to express the logic of broad, or metaphysical necessity should be considered an S5-style system. This is what he is concerned with when building the modal set theory apparatus. Turning back to S5 then, how can you build a model in which the paradox arises?

2. S5 Models for CP

2.1. Discussing the Minimal Framework

A model for the minimal system required comprises a set of possible worlds W, among which we designate the actual one w* and a final world u, a non-empty domain of possible objects D, a valuation function V predicates to objects. In our case, the extra- and intra-world domain coincide. For the paradox to hold, the accessibility relation R has to be at least transitive, so that we have warrants that for any three consecutive worlds, if the first can 'see' an object in the second, and the second can see the same object in the third, then the first one also sees the object from the third. A supplementary condition is that the worlds' order is a strict one: once the first change has been done then it is necessary to take into account what happened at a previous world. If R is symmetric then, should it be also reflexive, W can comprise only two (possible) worlds, $w*$ and u: if $w*$ sees u and u sees $w*$, by transitivity, $w*$ sees $w*$. Still, CP would arise in a model with at least three possible worlds, and this is mainly because any change you operate while transitioning from a world to another one must be made accordingly to what happened at the immediate previous

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6   See Salmon's proposal and the Forbes discussion in the above mentioned (Forbes, 1984).

world and considering this then, CP does not hold if the relation is from w^* to u then to w^*, as the initial and final world would coincide. Symmetry or reflexivity aren't mandatory- there is no need to consider the possible worlds chain backwards and a world not seeing itself is no impediment for transitivity principle to hold. The minimal modal system in which CP arises, according to the observations made above, is the minimal system relying on Axiom 4 - that is K4. However, we will discuss CP in the S5 framework which includes among others K4 as a subsystem.

2.2. S5 – The Model

R is reflexive, symmetric and transitive. Each world can see itself, the worlds seen by a world see, at their turn, the worlds from where they are seen, and if any world sees some world seeing another one, then the first can see the latter one. Earlier it was mentioned that the worlds' appearance is not indifferent hence, form the multitude of sets of possible worlds that can be chained, by each step and world inclusion we eliminate more possible subsets, until we eventually reach the final one. The "selection" process being described, let us note some facts about the domains and the valuation function. Differences between worlds target properties, not objects; in definitive, maintaining the same domain could be a necessary condition, if we are to follow Chisholm's indications:

> "[...] We then arrive at our description of W^2 by accommodating our descriptions of the other entities of W^1 in such a way that these entities will be capable of inhabitating the same possible world as the revised Noah and the revised Adam. Both Noah and Adam, then, may be found in W^2 as well as in W^1." (Chisholm, 1967, p. 82)

The valuation function tells us, for each predicate in the model, which objects are satisfied by it, and at which possible worlds.

What are the conditions we have to impose on this function in order to obtain the equilibrium between the properties/predicate modifications and the other objects?

Consider two objects, a and n, and the sets of properties they have, {F, G, H}, respectively {K, L, M}. If we are to build a CP-similar context, then V must provide a mechanism allowing us to do the following moves, from:

$[Fa]_{W*} = 1$, $[Fn]_{W*} = 0$; ... .

$[Ka]_{W*} = 0$, $[Kn]_{W*} = 1$; ... ; to

$[Fa]_{W1} = 0$, $[Fn]_{W1} = 1$, $[Ga]_{W1} = 1$, ... ;

$[Ka]_{W1} = 1$, $[Kn]_{W1} = 0$, ...;

and so on and keeping track on them such that all the relevant cognate properties satisfy their objects adequately, maintaining thus the internal coherence Chisholm refers to in the paragraph cited above. In other words: can we picture a dynamic valuation function for our model?

If we are to offer an affirmative answer to this question, then it may be necessary to divide our work in two parts: one focused on the assignments for the actual world, and another one derived from the first, that keeps track on the crossworld modifications, provided a first property shift/ change has been done.

The valuation function would link the predicates and objects upon the following model:

Shift & Change: for any possible world from the set W, except for the actual one, there must exist a previous world such that the differences between them consist in the (unique) change of the object (understood as one of the objects CP applies to) satisfied by some predicate (specification of the property modified) and the accommodation of the objects entering in relations (and hence of the cognate predicates) such that the relations remain the same.

We will start by depicting the predicate that changes about one object for some other one. Once this has been set, we can start populate the new world with objects. The predicates satisfying them, if they are cognates of the predicate(s) that were modified, would apply to them if and only if the objects mirror the relations they initially engaged into.

Consequences, Conclusions and Further Work

An observation that can be drawn from the detailed specification of this valuation function is that, for the big picture, retainment of the world's structure is necessary. By 'structure' we mean, here, n-place relations (n >1), and they can be woven in spite of not being able to determine any of the objects' essential properties. A direct consequence of this is that, should it enter in the 'right' relations, any object can occupy any position in the structure. Hence, it is plausible that by the end of a chain of possible worlds we legitimately infer identities of recognized non-identicals.

Analyzing a possible S5 model in which CP holds did not offer a solution to it; there are well-known replies and answers given to this challenge. The relevancy of this approach resides, in my opinion, in the emphasis on the conditions that enable CP and in the fact that, in the end, it seems that our general notion of identity is dependent on a structuralist point of view.

In a series of articles on the notion of essence, Kit Fine argues that the standard account of this notion in terms of necessity is misleading (Fine, 1994), and he gives an alternative in which essence is a primitive, and necessity is explained by appeal to it. A very interesting mark of his approach is that an essentialist statement has the following form: *A* (the statement) is true in virtue of the nature of the objects that *F,* where *F* is the predicate of *A*, and the class of "objects that *F*" is formed using the devices of abstraction. The result, for the metaphysician, is a web of relations and dependencies of objects and properties of objects upon some others, and this is how we enter in the realm of grounding theories.

CP's above analysis is relevant, in this respect, for more reasons; it shows that CP belongs to a family of problems pertaining to the notion of structure and problems of modal logic have common roots with problems from other disciplines. Structural realists, for example, seek for a metaphysical framework for their perspective and one of the most famous proposals is ontic structural realism (OSR), which asserts that the world has an objective modal structure represented by the theoretical part of our best scientifical theories (Berenstein & Ladyman, 2012). I consider that the Finean grounding theories are a good candidate to fulfill the structural realists' demands, and they are also compatible with OSR. Recent work in this direction has been made by Shamik Dasgupta who, in one of his 2014 articles discusses on plural grounds and structural realism. His alleged case is about mass quantities, and he proposes a relativist perspective, in which plural grounds support a holist-style approach.[7] Accommodating grounding theories and structural realism is a recent orientation, so it is a challenge for anyone considering it to prove its coherence and plausibility, so projects questioning the compatibility of these interdisciplinary theories would prove to useful.

Acknowledgements

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[7]   I find this a strong point in favor of adopting the Finean accounts on essence and grounds. Should we translate Dasgupta's proposal into a suggestion for solving CP, it could take the following form: our objects cannot be identified or defined by means of separate relations (which is what we do when build the paradox by shifting, changing and modifying properties), but as the result of the reunion of all the relations they engage into. For the long term, we find ourselves in a pragmatist sphere.

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