The Decomposition of Thought

Nathan Bice
nmb2138@columbia.edu

June 3, 2023

Abstract

This paper defends an interpretation of Gottlob Frege's views on the structure of thought. I argue that Frege did not think that a thought has a *unique* decomposition into its component senses, but rather the same thought can be decomposed into senses in multiple, distinct ways. These multiple decompositions will often have distinct logical forms. I also argue against Michael Dummett and others that Frege thought that the sense of a predicate is a function from the sense of a name (or names) to a complete thought. I defend my Frege interpretation against a puzzle often discussed in the Frege literature; namely, that the Multiple Decompositions Thesis is incompatible with Frege's stated view that a thought is built up out of its component senses as parts. I provide textual evidence and argument that Frege thought of the part/whole relation in such a way that a whole can be analyzed into (or built up out of) parts in multiple, distinct ways, thus dissolving the puzzle. I conclude with discussion of every sort of example of multiple decompositions that can be found in Frege's work.

Contents

1 Introduction

Gottlob Frege famously characterized the notion of the *gedanke* or *thought* expressed by a complete declarative sentence as the sort of content relevant to determining whether that sentence is true or false. Here is one of many examples: "So one has to separate off from the content of a sentence the part that alone can be accepted as true or rejected as false. I call this part the thought expressed by the sentence." (1906c, PW pg. 197-8)[1]

Furthermore, when discussing the structure of thoughts, Frege helped himself to mereological language, often speaking of parts of a thought corresponding to the significant parts of a sentence expressing it. For example, when I utter the sentence 'John has brown hair', my utterance expresses the thought that John has brown hair, a thought which contains the sense of 'John' and the sense of 'has brown hair' as parts.

Here is one classic quotation from Frege on the matter: "How can [language] achieve so much? By virtue of the fact that thoughts have parts out of which they are built up. And these parts, these building blocks, correspond to groups of sounds, out of which the sentence expressing the thought is built up, so that the construction of the sentence out of parts of a sentence corresponds to the construction of a thought out of parts of a thought. And as we take a thought to be the sense of a sentence, so we may call a part of a thought the sense of that part of the sentence which corresponds to it." (1914, PW pg. 225)[2]

Since a thought is a more abstract entity than the full content of a complete declarative sentence (a content which may include elements irrelevant to the truth or falsity of the sentence), it is no surprise that Frege wanted distinct sentences to be capable of expressing the same thought. For example, on Frege's view the sentences 'John loves Mary' and 'Mary is loved by John' express the same thought; the transition from active to passive voice does not affect the thought expressed. Discussing a similar example, Frege says "Although in actual speech it can certainly be very important where the attention is directed and where the stress falls, it is of no concern to logic." (1897, PW pg. 141) Intuitively, the sentence 'John loves Mary' carries no more and no less information than the sentence 'Mary is loved by John'; any difference between the two is irrelevant to determining their (shared) truth-value.

Many Frege scholars have found this aspect of Frege's view to be in tension with his view that thoughts have parts corresponding to the significant parts of sentences expressing them. Returning to our simple example, how could 'John loves Mary' and 'Mary is loved by John' express the same thought, given that

---

[1] In this discussion I follow Frege in treating *content* as an intuitive notion from which the more technical notion of *thought* can be isolated.

[2] Frege explicitly calls such mereological language "figurative" in the following passage: "To be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly ever bothered by the hitches which occur from time to time." (1923-6, pg. 36/390) However, this passage should be taken with a grain of salt. This was one of Frege's last papers, and it is unclear whether Frege's earlier positions are really compatible with such a noncommittal account of the mereological structure of thoughts.

the thought expressed by the former sentence contains a part corresponding to 'loves' while the latter does not? Surely the thought expressed by the latter sentence instead contains a part corresponding to 'is loved by'. These parts cannot be the same, since the former denotes the "loves" relation while the latter denotes its obviously distinct inverse: the "is loved by" relation.[3]

There are plenty of other seemingly-problematic examples in Frege's writings, some of which involve multiple analyses of the same sentence. Here is one: "Now if the same proper name occurs in both consequent and antecedent, we may regard the hypothetical thought as singular if we think of it as being analysed into the complete part that corresponds to the proper name and the unsaturated part left over." (Frege 1906b, PW pg. 188) Thus, the thought expressed by the sentence 'If John loves Mary, then John is happy' can not only be analyzed into the two thoughts expressed by 'John loves Mary' and 'John is happy' connected by the sense corresponding to the material conditional; it can also be analyzed into the sense expressed by 'John' and the predicative sense corresponding to the remainder of the sentence (with both instances of 'John' removed: 'If ( ) loves Mary, then ( ) is happy'). Yet these analyses give distinct logical forms: the former corresponds to a material conditional between atomic sentences (e.g. '$L(j,m) \longrightarrow H(j)$') while the latter corresponds to an atomic sentence (e.g. '$K(j)$').

It follows that on Frege's view, the structure of a thought does not determine a unique logical form. However, when one constructs a formal language for *expressing* thoughts (in order to give a precise account of various inferential transitions between thoughts, for example), one may choose to construct one's language in such a way that it satisfies *unique readability*: for every well-formed formula of the language, there is exactly one procedure for generating that formula from its component parts (e.g. '$((p \wedge q) \wedge r)$' is uniquely generated by applying the operator '$\wedge$' to atomic sentences 'p' and 'q', followed by again applying '$\wedge$' to '$(p \wedge q)$' and 'r'). This gives a unique decomposition of any well-formed formula into its components. The point being, on Frege's conception of thought, the same thought can be expressed by logically distinct sentences and in distinct formal languages, even if each choice of expression has a unique decomposition.

Interestingly, Frege's own formal language, as developed in Volume I of the *Grundgesetze* (1893), does not satisfy unique readability. For example, the formula '$—f(a)$' can be read as the application of the horizontal to the result of applying the function-name '$f(\xi)$' to the object-name 'a', but it can also be read as the application of the horizontal to the result of applying the two-place function-name '$\varphi(\xi)$' to the function-name '$f(\xi)$' and the object-name 'a', where this two-place function-name refers to a function which maps a pair of arguments to the result of applying the first argument to the second. (ibid pg. 39)

Frege was quite explicit that the same thought can be analyzed in distinct

---

[3]I trust the reader familiar with Frege's "concept horse" problem can rewrite this sentence in their preferred style, if they so choose.

ways. For example, in the essay quoted above he says, "If several proper names occur in a sentence, the corresponding thought can be analysed into a complete and unsaturated part in different ways. The sense of each of these proper names can be set up as the complete part over against the rest of the thought as the unsaturated part." (1906b, PW pg. 192)[4]

But how could a thought have multiple analyses, given that a thought is built up out of its constituent senses as parts? Won't the structure of a thought be uniquely determined by its constituent senses and the way they are combined? Wouldn't any distinct analysis of a thought force either distinct senses as constituents or a distinct manner of combination? While it is true that, as I pointed out in footnote 2, near the end of his life Frege explicitly called his mereological treatment of thoughts "figurative", this doesn't give us a satisfactory answer to these questions. Frege treated thoughts as composed of senses, and hence similar questions arise however one understands such composition.

Many recent authors have attempted to *revise* Frege's account of the structure of thought in order to avoid this difficulty. As paradigm examples, see (Dummett 1981b), (Bell 1996), (Bermúdez 2001), (Penco 2003), (Künne 2007), (Textor 2009), and (Kemmerling 2011). I lack the space to discuss each author in detail and will instead focus on (Dummett 1981b), which instigated the contemporary discussion.[5]

I will argue that revision is unnecessary. I will first defend Geach's (1975) position that Frege was committed to using his own function/argument distinction to account for the structure of thought. I will then provide strong textual evidence that Frege generally thought of the part/whole distinction in such a way that a thought can not only be decomposed into parts in multiple ways, it can also be built up out of parts in multiple ways.

Hence, a proper appreciation of Frege's understanding of the part/whole relation will allow us to reconcile his remarks on multiple analyses of the same

---

[4]There are many additional examples. Prior to making his sense/reference distinction, Frege spoke of multiple analyses of judgeable content in (Letter to Marty 8/29/1882, PMC pg. 101) and (1884, pg. 74-5), among others. After making this distinction, Frege spoke of multiple analyses in (1892b, PW pg. 107-8), (1906b, PW pg. 187), and (1906c, PW pg. 201-2), among others.

[5]Note that Dummett's revision involves interpreting Frege as working with two distinct notions: *analysis* and *decomposition*. While Dummett may deny that this is a revision, the majority of the literature disagrees. I will argue that this distinction is unmotivated and have chosen to use the two expressions interchangeably.

Bermúdez is less committal regarding interpretation vs emendation. Since Bermúdez's position involves positing that the same sentence can express multiple thoughts, even while holding the context fixed, and each thought has a unique analysis, it is straightforwardly incompatible with Frege's oft-repeated claim that thoughts have multiple analyses.

Finally, Kemmerling claims that his doctrine, that thoughts are unstructured and do not consist of parts, is in fact an interpretation of the mature Frege. But if thoughts are unstructured, how could there be any intrinsic distinction between one thought and another? Furthermore, Kemmerling says that thoughts can still be decomposed into parts. But how could an unstructured entity be decomposable at all? Surely something must be decomposed *in virtue of* its structure. I take it as fundamental to Frege's view of thought that thoughts have compositional structure. If reader feels this is a merely terminological disagreement, they are welcome to interpret Kemmerling in a manner consistent with the thesis I defend below.

thought with his remarks on a thought being composed of senses as parts that correspond to the significant parts of a sentence expressing it. Thus the difficulty dissolves. I will conclude with reflections on how far Frege wanted multiple decompositions to extend by considering every quotation in which Frege discusses a relevantly distinct example.

While this paper is almost entirely a work of Frege interpretation, I believe that applying multiple decompositions to Fregean propositions in the way Frege intended can resolve thorny issues involving truth, indexicals, and the hierarchy of senses that have been extensively discussed in the contemporary literature on content.

In particular, Frege often says that any thought that p and the thought expressed by 'the thought that p is true' express the same thought.[6] Frege also says that the *indirect* sense which refers to the thought that p in a belief ascription is the sense expressed by the expression 'the thought that p' (1892a, pg. 37/166). These two positions can be made compatible by seeing any thought that p as decomposable into its indirect sense and the sense expressed by the truth predicate.[7] Regarding truth, it follows that the sense of the truth predicate refers to a function from thoughts to their truth-values.

I will argue in future work that this is in fact a tenable position on the hierarchy of senses which resolves various issues left unresolved by contemporary accounts.

Finally, Frege states in his (1918-19a) that the sentences 'Today it is raining' and 'Yesterday it was raining', uttered in the appropriate contexts, in fact express the same thought. Frege explicitly points to multiple decompositions as a way to make this view tenable.[8] I agree, and will argue in future work that this view leads to a defensible and interesting account of the content of indexicals.

Hence, I believe Frege's position has wider appeal beyond the Frege literature.

2   The Structure of Thought

In Chapter 15 of his *The Interpretation of Frege's Philosophy* (1981b), Michael Dummett argues that Frege held the following four theses:

A1. A thought may be analyzed in distinct ways.

A2. A thought is not built up out of its component [senses];[9] rather, the constituents of the thought are arrived at by analysis of it.

---

[6]One example is his (1892a, pg. 34/164). There are many others.

[7]It is controversial whether Frege thought of the truth predicate as expressing a sense. I believe he was at least *committed* to this, given his more fundamental commitments.

[8]Both claims are made in (1918-19a, pg. 64/358).

[9]Dummett actually writes 'concepts' rather than 'senses' here, claiming to follow Frege's *early* usage of 'concept' ('Begriff'). Dummett claims that early Frege used 'concept' in roughly the sense in which he would later use 'sense' ('Sinn'). (ibid pg. 261-2) As is well-known, after making his sense/reference distinction, Frege did not use the word 'concept' to denote a component of a thought, but rather the referent of a predicate. *Pace* Dummett, I do not think early Frege used 'concept' to in effect mean thought-constituent, since a judgeable content is

B1. The senses of the parts of a sentence are parts of the thought expressed by the whole.

B2. A thought is built up out of its constituents, which correspond, by and large, to the parts of the sentence expressing it. (ibid pg. 261)

As Dummett points out, the A theses seem to contradict the B theses. How can a thought be analyzed in distinct ways into distinct constituents if a thought is built up out of its constituents? How could a thought both be and not be built up out of its component senses? Could Frege have failed to notice this?

According to Dummett, the A theses and B theses correspond to two different relations of a whole thought to its parts: the A theses assume a relation much like that of a country that can be subdivided into regions in multiple ways for various purposes, with no unique best subdivision, while the B theses assume a relation much like that of a molecule to its atoms, with a unique analysis of that molecule into its constituent atoms. (ibid pg. 263-4)

Dummett mentions the beginnings of a possible solution: Peter Geach has defended the view that for Frege the structure of a thought is best analyzed using Frege's own argument/function distinction (Geach 1975, pg. 149). On such an interpretation, just as e.g. the number 16 is the value of the function $f(x) = x^2$ on the argument 4 and the value of the function $g(x) = 4^x$ on the argument 2, the same thought can be the value of multiple functions on multiple arguments. So, the thought that John loves Mary could be seen as the value of the function-sense expressed by 'loves Mary' with the object-sense expressed by 'John' as argument. But it could also be seen as the value of the function-sense expressed by 'John loves' with the object-sense expressed by 'Mary' as argument.

For Geach, this explains why Frege often spoke of one part of a thought needing to be *incomplete* or *unsaturated* in order for the parts of that thought to hold together.[10] Just as the referent of an incomplete expression is itself incomplete or unsaturated for Frege, the *sense* of an incomplete expression is incomplete as well. Given that Frege treats the referent of an incomplete expression as a function that must be saturated by an argument (or arguments) in order to result in a value, it is unsurprising that he treats the sense of an incomplete expression as incomplete in the same way: as a function that must be completed by an argument (or arguments) to result in a complete sense as value.

Dummett rejects this analysis of the structure of thought. Recall thesis B1: the senses of the parts of a sentence are parts of the thought expressed by the whole. Hence, given Geach's interpretation, a function-sense will be a part of its value along with its argument. For example, given that the sense of 'loves Mary' is a function from the sense of 'John' to the thought that John loves

---

partly a union of thought and truth-value (sense and reference), and have replaced 'concepts' by 'senses' in order to avoid confusion.

[10]Frege says this in multiple places. For example: "For not all the parts of a thought can be complete; at least one must be 'unsaturated', or predicative; otherwise they would not hold together." (1892c, pg. 205/193)

Mary, this function-sense, being the sense of 'loves Mary', is also a part of the thought that John loves Mary. This is quite different from the case of reference. Consider again the number 16. Few would find it plausible that the function $f(x) = x^2$ is a *part of* 16 in addition to mapping the number 4 to it. Functions over the natural numbers are not *parts* of those numbers.

Dummett notes this difference while primarily criticizing the interpretation for leaving us unable to say how the sense of a predicate contributes to the condition under which a sentence containing that predicate is true. (1981b, pg. 251-2) Recall that Frege often introduced thoughts as the sort of content of a complete declarative sentence relevant to determining whether that sentence is true or false. Hence, a thought fixes a truth-condition.[11] According to Dummett, if the sense of a predicate is a function from the sense of a name to a thought, then we can provide no account of how it contributes to fixing that truth-condition. The way the sense of a predicate contributes to fixing a truth-condition must be via determining a condition that objects either satisfy or fail to satisfy. This sense *just is* a way of determining such a condition, where that condition is a function from objects to truth-values. But if this sense is itself a function from object-senses to thoughts, then it isn't a way of determining such a condition. (1981b, pg. 270)

Dummett also complains that in order to understand such a function, we would already need to know what thoughts are and how to identify particular thoughts, so this account is of no help in explaining what thoughts are or how we grasp them. (ibid pg. 267ff) I see little reason to accept Dummett's insistence that to form a conception of a function, we must already know its range of possible values. Consider a function corresponding to a process of construction. I may have no idea what the output will be until after I have engaged in the process. I discover the output by engaging in the process relative to a particular input, and I conceive of the output as the result of that process. Frege's own geometric example meant to help explain the nature of a function, which we'll discuss below, can be viewed in this way.

Returning to Dummett's primary criticism, there are two separate questions here. The first is whether Geach's position on the senses of predicates was Frege's actual view. The second is whether this view is tenable. Regarding the first question, it is clear that Frege was at least *committed* to the senses of predicates being functions from the senses of names to thoughts. Recall that on Frege's view, when I assert the sentence 'Copernicus believed that the planetary orbits are circles', the expressions 'the planetary orbits' and 'are circles' refer to their customary senses, and the complete expression 'the planetary orbits are circles' refers to the thought customarily expressed by it.[12] Hence, the

---

[11] Consider the following famous passage from Volume I of the *Grundgesetze*: "Every such name of a truth-value *expresses* a sense, a *thought*. For owing to our stipulations, it is determined under which conditions it refers to the True. The sense of this name, the *thought*, is: that these conditions are fulfilled...Now, the simple or complex names of which the name of a truth-value consists contribute to expressing the thought, and this contribution of the individual name is its *sense*. If a name is part of the name of a truth-value, then the sense of the former name is part of the thought expressed by the latter." (1893, pg. 50-1)

[12] Cf. (1892a, pg. 28/159).

referent of this complete expression ought to be a function of the referents of 'the planetary orbits' and 'are circles', and since the former expression is complete while the latter is unsaturated, the latter must refer to a function on Frege's view, in particular, a function mapping the referent of the former to the complete thought.

In further support of this, note that in "Function and Concept" Frege says that an object is "anything that is not a function, so that an expression for it does not contain an empty place." (1891, pg. 18/147) Given that Frege's ontology divides completely into objects and functions, a function is therefore anything whose expression *does* contain an empty place.

Compare also his definition of function-names in Volume I of the *Grundgesetze.* (1893, pg. 43-4) Function-names are always unsaturated expressions, constructed via removing names from an expression in a way that leaves the resulting open places marked as fillable. Unsaturated expressions always denote functions; no other sorts of unsaturated expressions are allowed in the formal system.

Hence, the customary sense of 'are circles' not only *refers* to a function; it *is* a function from senses to thoughts. In this case, it maps the customary sense of 'the planetary orbits' to the complete thought expressed by 'the planetary orbits are circles', a thought which contains both senses as parts.[13]

In my view, much of the resistance to treating the sense of a predicate as a function from senses to thoughts while at the same time being a part of the thoughts it maps to comes from various prejudices concerning functions that have little to do with Frege's conception of them. In Frege's "Function and Concept" (1891, pg. 7/141), he uses the following analogy to explain the sense in which functions are unsaturated while objects are not: Consider a line divided by a single point. Now treat the line as divided into two segments, one which includes the point and one which doesn't. The segment which doesn't include the point is supposed to be analogous to a function while the other segment, being "complete in itself", is analogous to the object. Referring to the former, Frege says, "Only by completing it with this endpoint, or with a line that has two endpoints, do we get from it something entire." (loc. cit.) Extending Frege's analogy, what does this function map the other line-segment *to*? The whole line, of course. And yet at the same time, this function is a part of that line. More generally, it combines its arguments with itself in order to form a longer line-segment.

I think that this is a perfectly acceptable notion of "function", and find it plausible that this could very well be how Frege was thinking of the unsaturated

---

[13]I recently discovered a passage in Dummett in which he considered the related argument that, granted that objects alone cannot combine to form a thought, a complex proper name must contain a functional expression, and this functional expression must denote a function, in an opaque context such as that given by 'Plato believed that Socrates was wise', the referent of 'ξ was wise' must be a function in addition to being that expression's customary sense. (1981a, pg. 292) Dummett's response at the time was to make a radical modification to Frege's account of indirect reference by giving up the thesis that the referent of such a predicate in such an opaque context is its customary sense. (ibid pg. 294) I of course will argue that such a modification is unnecessary.

senses of predicates as both functions from senses to thoughts and parts of those thoughts.[14]

I am not saying that Frege would accept that the line-segment which doesn't include the point is *in fact* a function. I suspect that he would insist that line-segments are objects, as are analogous entities like half-open intervals, etc.[15] The point is that Frege used this as an *analogy* to help elucidate the unsaturatedness of functions, an analogy that immediately suggests a corresponding function/argument analysis to account for the unsaturatedness of the senses of predicates, and furthermore one *could* coherently see this line-segment as a function from line-segments to line-segments.[16]

Frege never explicitly asserted that the senses of predicates are themselves functions. My argument concerning indirect senses above demonstrates that he was *committed* to this position, given his more fundamental commitments.

This doesn't imply that *all* functions map their arguments to values which include those very functions and arguments as parts, which Frege rejected explicitly in his (1919, PW pg. 255), nor is this meant to somehow fully explain the nature of understanding a predicate. The point is merely that nothing in Frege's conception of functions implies that a predicative sense can't both be a part of its value and count as a function.[17] [18]

Furthermore, in response to Dummett, granting that the sense of a (first-

---

[14]In addition to Frege's geometric example, consider a formal syntax in which predicate symbols are functions from constant symbols and variables to atomic formulas. These functions will thus be components of their values, since their values are atomic formulas containing them. I trust the reader recognizes both this example and the previous geometric example as coherent, no matter how strange this conception of functions may seem. Haim Gaifman has suggested the possibility of such a formal syntax in unpublished work from 2006. I discuss set-theoretic objections in footnote 18 below.

[15]Perhaps considering half-open intervals of the real line makes the point even clearer. There is a natural sense in which the side of the interval which does not include an endpoint is "incomplete," approaching that point without ever reaching it, and would be "completed" by combining it with a disjoint closed interval which includes that point as an endpoint.

[16](Frege 1923-26) includes a nice discussion of unsaturated senses: "There can be no negation without something negated, and this is a thought. The unity of the whole comes about through the fact that the thought saturates the unsaturated part or, as we can also say, completes the part needing completion." (ibid pg. 37/390)

[17]In fact, prior to making his sense/reference distinction, Frege seemed to hold this view of conceptual contents carved into arguments and functions. Cf. his (1881, PW pg. 16) in combination with Section 9 of his (1879).

[18]The reader might still be bothered by such a conception of a function, since this function's values for appropriate arguments will contain the function itself as a part. This may seem somehow incompatible with the standard axioms of set theory (ZFC), since a set of ordered pairs can't include itself as an element of the second component of an ordered pair without violating the axiom of foundation. Let me make a few brief points in response: (1) There is no reason that one's formal representation of a function as a set of ordered pairs must also represent the mereological structure of the function's values; set theory and mereology are distinct enterprises. (2) Even if one wants one's formal representation of such a function to be such that its values are sets containing that function as an element, one can easily do so in a non-well-founded set theory, including ZFC itself without the axiom of foundation. (3) For Frege, a function is fundamentally an unsaturated entity while a set of ordered pairs is saturated (it is an object). Hence, such a formal representation doesn't respect Frege's conception of functions in any case.

level) predicate is a *way of thinking* of a condition that objects either satisfy or fail to satisfy (where that condition is a function from objects to truth-values), there is nothing to stop it from *also* being a function from object-senses to thoughts, as indeed it must be in order to remain consistent with Frege's views on the referents of expressions in indirect contexts, the referents of incomplete expressions, and the compositionality of reference (i.e. the referent of a whole expression is a function of the referents of its significant parts). One might even say that the sense of a (first-level) predicate is *fundamentally* a way of thinking of a condition on objects while adding that it is itself incomplete, much like its referent, and must be saturated by an object-sense to form a complete thought. The logical form of a predicative sense doesn't constrain its content to such an extent that it can't be a way of thinking of a function. Dummett has given us no reason to think otherwise, and hence we have no reason to reject the view Frege was clearly committed to.

Let me reiterate this point. We can completely agree with Dummett that the sense of a (first-level) predicate is fundamentally a *way of thinking* of a condition that objects either satisfy or fail to satisfy.[19] At the level of *form*, this strongly suggests that we make a structural distinction between senses denoting objects and senses denoting functions (and senses denoting second-level functions, etc.), which the function/argument analysis obviously provides. Taking the position that the senses of predicates are functions constrains the *logical form* of such senses; it puts few to no constraints on their content. We are free to adopt a wide variety of positions on the nature of predicative senses.

Doesn't this imply that the "concept horse" problem, discussed extensively in (Frege 1892c), also arises when attempting to refer to the senses of predicates? It does, but given that Frege explicitly claims that some senses are unsaturated, since otherwise a thought could not be unified, this problem arises on *any* acceptable interpretation. Frege could very well say that, strictly speaking, 'the sense of the predicate 'is a horse' ' doesn't refer to what we intend it to refer to, just as he does for concept-words. We can even agree with Dummett that the sense of a predicate is a way of thinking of a condition on objects while denying that such ways of thinking are themselves objects, even if we usually attempt to refer to them with complete expressions.

Dummett may have thought that the function/argument analysis is incompatible with the part/whole conception of thoughts (or any other conception of thoughts as having components), and furthermore this incompatibility forces an incompatibility between the function/argument analysis and Dummett's preferred conception of the senses of predicates. I argued extensively above that there is no incompatibility here.

Crucially, a thought can be a structured entity even if it can be analyzed in multiple ways. It simply has a more abstract structure than that given by any particular analysis. Compare the real line, which can be decomposed into intervals in infinitely many ways. Indeed, it is *in virtue of* its structure that such decomposition is possible.

---

[19]I take no stance on this position in this essay.

Heck and May (2011) raise a similar complaint to Dummett's: "Saying that thoughts are the values of sense-functions in no way explains how the composition of senses determines truth-conditions." (ibid pg. 147) It isn't meant to. This is in no way meant to completely explain the *content* of a thought, and in particular doesn't provide a complete explanation of the way in which a thought fixes a truth-condition, nor of the contribution the parts of a thought make to fixing that truth-condition.

Heck and May also complain that such a conception can't explain why 'The Morning Star is a planet' and 'The Evening Star is a planet' express distinct thoughts. (ibid pg. 147) While I won't address their complaint in detail here, it is important to recognize, as I argued above, that Frege held *both* the function/argument analysis of thought *and* a part/whole conception of thought: the predicative sense expressed by 'is a planet' is both a function from the sense expressed by 'The Morning Star' to the thought expressed by 'The Morning Star is a planet' and a part of that thought (along with its argument). This is why the sense of 'is a planet' can't be a function of an arbitrary argument to the thought that the Morning Star is a planet; it can only be a function of the sense of 'The Morning Star' to that thought, since that thought will contain both this function and its argument as parts.

3   Parts and Wholes

The function/argument analysis of thought is still insufficient to solve Dummett's claimed interpretive problem, however. As Dummett points out, even if this interpretive position could be made tenable (as I argued it could in the last section), it would only explain why Frege held the A theses, not the B theses, which seem to presuppose a different conception of the structure of thought that is manifestly incompatible with the A theses. (1981b, pg. 266)

In fact, for us the problem is worse, since I have argued above that Frege held (or, more carefully: was committed to) *both* the function/argument analysis of thought *and* a part/whole conception of thoughts, so that the function-sense expressed by 'loves Mary' is both a function from the sense of 'John' to the thought that John loves Mary and a part of that thought, thus leaving us seemingly unable to use the function/argument analysis to explain why the same thought can be analyzed in different ways. If such analysis can be made sense of, it is not simply on the model of distinct numerical functions mapping distinct arguments to the same value: as I stated above, the function $f(x) = x^2$ may map 4 to 16, but f is not a part of 16.

Dummett himself attempts to solve the interpretive problem by making a distinction between *analysis* and *decomposition*, arguing that thoughts have a unique *analysis* but can be *decomposed in* multiple ways. Dummett argues that although Frege never says so explicitly, he was clearly working with such a distinction, and the apparent contradiction in his remarks can be resolved by interpreting the B-theses in terms of analysis and the A-theses in terms of decomposition. (ibid pg. 271) For Dummett, analysis reveals the ultimate

constituents of the thought (and how they are combined), while decomposition reveals a pattern that is shared by multiple thoughts that can be exploited for the purposes of inference.[20]

This attempt at a resolution has been roundly criticized by Frege scholars. Many point out that it seems to have little to no textual support.[21] If Frege really distinguished analysis from decomposition, he never seemed to say so. It would be preferable to find a resolution that corresponds to what Frege actually says.

There is another way of solving this interpretive problem. Note that Dummett assumes that when Frege spoke of a thought as being built out of parts, Frege was implying that there is a *unique* analysis of that thought into its component parts. That is, Dummett assumes that Frege thought of the part/whole distinction as being such that a whole can only be built up out of parts in a unique way. But (1) there is ample textual evidence that Frege thought of wholes as being analyzable into parts in multiple, distinct ways. (2) Frege's discussion of thoughts being built up out of their parts does not imply that thoughts are the result of a temporal process, since for Frege thoughts are timeless, abstract entities. Furthermore, Frege's own examples provide cases in which a whole is not only analyzable in distinct ways; it can be built up out of parts in distinct ways as well.

Once we recognize these interpretive points, there is no need to postulate an analysis/decomposition distinction, for Frege's treatment of thoughts as built up out of parts in no way contradicts his treatment of thoughts as having multiple analyses. Given that Frege thought of wholes more generally as having multiple decompositions into parts and multiple ways of being built up, Frege can quite consistently speak of a thought as built up out of parts and as having multiple decompositions.[22]

---

[20]To see what Dummett has in mind here, consider his example: 'If anyone killed Brutus, then he was an honorable man.' From this sentence one can infer 'If Brutus killed Brutus, then Brutus was an honorable man.' This inference is plausibly explained in virtue of the shared pattern 'If ( ) killed Brutus, then ( ) was an honorable man' present in both sentences. But this pattern will not correspond to a constituent of the thought expressed by the latter sentence on Dummett's view, since the latter thought is uniquely analyzed into the sense corresponding to the material conditional and the two thoughts expressed by 'Brutus killed Brutus' and 'Brutus was an honorable man', each having a unique analysis of their own. Recognizing such a pattern in a thought that is not a constituent of it corresponds to decomposing it in a particular way. (ibid pg. 273ff)

[21]Two examples are Currie (1985, pg. 286) and Garavaso (1991, pg. 201). There are various other objections to Dummett's interpretation that we needn't discuss here, since we will see that Dummett's distinction between analysis and decomposition is unmotivated.

[22]Levine (2002, pg. 202ff) made a similar point against Dummett (although in terms of multiple decompositions instead of multiple ways of being built up), although unfortunately it has been overlooked by many Frege scholars. *Pace* Levine and in fairness to Dummett, in Dummett's early work on this matter (e.g. his (1981b), as we've been discussing) he mentions the possibility of a part/whole relation in which the whole can be decomposed into parts in multiple ways (I pointed this out above). His example is that of a country that can be divided into component regions in different ways depending on one's purposes in doing so. (ibid pg. 264) The essential issue is Dummett's insistence that a thought can only be *built up out of* parts in a unique way, thus making the B-theses nontrivially incompatible with the A-theses. So Levine's objection strictly speaking misses the mark.

Let's first consider the textual evidence for (1). Here are five quotations which demonstrate that Frege thought of wholes as divisible into parts in a variety of nonequivalent ways, the first four of which occurred in the context of Frege distinguishing extensions of concepts from aggregates (a distinction which, he argued, the set theorists failed to appreciate):

What Mr. Schröder calls 'inclusion' or 'subsumption' is here, properly speaking, nothing but the part–whole relation, extended in such a way that every whole is to be treated as a part of itself. From the point of view we are now adopting, we do not need the words 'individual' and 'single thing'. Divisibility can be imagined as going on *ad infinitum*...and we have no need at all to assume there are parts insusceptible of further division; so perhaps it is better at this stage not to talk about elements at all. (1895, pg. 434-435/211)

[I]f we are given a whole, it is not yet determined what we are to envisage as its parts. As parts of a regiment I can regard the battalions, the companies or the individual soldiers, and as parts of a sand pile, the grains of sand or the silicon and oxygen atoms. On the other hand, if we are given a class, it is determined what objects are members of it. (Letter to Russell 7/28/1902, PMC pg. 140)

It is determined by a concept which objects fall under it; it is not determined by an aggregate what should count as its parts, whether, for example, in a regiment they are the individual soldiers, the companies, or the battalions; or whether, in a chair, they are the atoms, the molecules, or the artificially joined pieces of wood. (1903, pg. 150)

The aggregate is composed of its parts. Whereas the extension of a concept is not composed of the objects that belong to it...Now of course it can happen that all objects which belong to the extension of a concept are at the same time parts of an aggregate and what is more in such a way that the whole being of the aggregate is completely exhausted by them...A grain of sand is an aggregate. And it can be that the extension of the concept *silicic acid molecule contained in this grain of sand* apparently coincides with the aggregate which we call this grain of sand. But we could just as well let the extension of the concept *atom contained in this grain of sand* coincide with our aggregate. But in that case the two extensions of concepts would coincide, which is impossible. From which it follows that neither of the two extensions of concepts coincides with the aggregate, for if one of them were to do so, then the other could with equal right be said to do so. (1906a, PW pg. 183)

We must notice, however, that one and the same thought can be split up in different ways and so can be seen as put together out of parts in different ways. (1906c, PW pg. 201-2)

The first four quotations establish that a whole can be decomposed into parts in multiple ways on Frege's view. The final quotation establishes that a thought can be *put together* out of parts in multiple ways. Indeed, this is already suggested by the earlier quotations. Take the third example. Constructing a chair by arranging its atoms is a different procedure than constructing it by arranging its component pieces of wood.

Importantly, Frege was quite explicit that discussion of building up a thought from parts is *metaphorical*; he says so explicitly in one of the very papers that Dummett cites as evidence for an inconsistency: "Compound Thoughts". Frege says, "By filling the gaps [in the unsaturated expression] with expressions of thoughts, we form the expression of a compound thought of the second kind. But we really should not talk of the compound thought as originating this way, for it is a thought and thoughts have no origin." (1923-26, pg. 40/394). The point being, thoughts are timeless entities and shouldn't be thought of as the result of a temporal process. That is merely a helpful metaphor and should not be taken so literally. While it is true that thoughts *have* parts, it is not literally true that thoughts are *built up out of* parts.

Does Frege ever assert the opposite? Wolfgang Künne (2007) claims that at several points Frege endorsed the thesis that if a sentence expresses a particular thought, then any sense which is a part of that thought is expressed by at least one part of that sentence. He provides one explicit quotation in support of this: "As the thought is the sense of the whole sentence, so a part of the thought is the sense of part of the sentence." (Frege 1906b, PW pg. 192)

But this quote is ambiguous: Frege does not explicitly assert that *any* part of the thought is the sense of part of the sentence. He can naturally be read as asserting that any part of the sentence expressing a sense is such that that sense is part of the thought expressed. Furthermore, if Frege *was* endorsing this thesis, he would be contradicting himself in this very same essay. Earlier in the essay (ibid pg. 187) he speaks of singular thoughts, but then asserts that, strictly speaking, the same thought can be analyzed in ways that lead to distinct logical forms, and so shouldn't be called e.g. singular independently of a particular analysis. So even if we accept Künne's interpretation, we can add that Frege would say that, strictly speaking, this quotation does not express his official position.[23]

Hence, the apparent inconsistency in Frege dissolves. Frege can perfectly well speak of a thought as built up out of parts while denying that a thought has a unique analysis into its components, for each distinct analysis of the thought will reveal distinct parts of the thought that combine into the whole.[24] Consider again Frege's example of a line divided by a point. One can see this point as

---

[23]Similar remarks apply to the other citations Künne makes, e.g. (Frege 1914, PW pg. 243), which in my opinion has no reasonable interpretation under which it could be read as endorsing this thesis.

[24]Returning to Dummett's four theses, we can accept all four once we remove the insistence in thesis A2 that a thought is not built up out of its component senses, which is in no way implied by the latter half of the thesis: the constituents of a thought being arrived at by analysis of it. One should instead say that a thought can be built up out of constituents in multiple ways.

dividing the line into two parts, one which contains the point and one which doesn't. And yet, if we divided the very same line at another point, we would have a distinct pair of parts that combine to form the very same line. These parts can further be divided into parts *ad infinitum*. Crucially, one should *not* think of the line as unstructured. Indeed, it is in virtue of the structure of the line that it is capable of being divided into parts in this way.[25]

Frege held an analogous view of the structure of thoughts. To return to our simple example, the thought that John loves Mary can be seen as composed of the function-sense expressed by '( ) loves Mary' and the object-sense expressed by 'John', but it can also be seen as composed of the function-sense expressed by 'John loves ( )' and the object-sense expressed by 'Mary'. Furthermore, it can be seen as composed of the function-sense expressed by '( ) is loved by John' and the object-sense 'Mary', and in various other ways as well. Each way of decomposing it corresponds to a particular analysis of the whole thought into senses as parts.

What consequences does this have for the commonly-expressed intuitive criterion of difference for thoughts: namely, two sentences express distinct thoughts if and only if it is possible to reject one while accepting the other?[26]

First of all, this already assumes that we are discussing *rational, linguistically competent* agents. Such an agent must understand both sentences and (for example) must not endorse contradictions. Understanding both sentences requires grasping the senses expressed by the expressions making up these sentences. But if this agent is also a *rational* agent and grasps the relevant senses, then they arguably *will* be able to recognize that the two sentences either *do* or *do not* suggest multiple analyses of the same thought, at least so long as which thought is expressed is not dependent on features of the external context.[27]

4 Consequences

As we have seen, Gottlob Frege held that a thought does not have a unique decomposition into parts, but rather the same thought can be decomposed into parts in various ways. We have seen two primary examples of this: (1) the transition from a sentence in active voice to the equivalent sentence in passive voice does not affect the thought expressed, and (2) any sentence containing names[28] expresses a thought that can be decomposed into a complete sense corresponding to one of those names and a predicative sense corresponding to the

---

[25]As any elementary topology textbook makes clear, the real line has quite a bit of topological structure that distinguishes it from other topological spaces. Analogously, a thought can have quite a bit of structure and yet still be decomposable in multiple ways.

[26]A related yet more nuanced version of this criterion is found in e.g. (Frege 1906c, PW pg. 197).

[27]As is well-known, this criterion seems straightforwardly incompatible with various other claims made by Frege. I think this criterion has serious problems on its own in any case. My point is merely that it is compatible with multiple decompositions.

[28]Here (following Frege) a *name* is understood broadly as any expression meant to denote an entity, and hence a definite description counts as a name on Frege's view. The reader unhappy with this terminology may prefer the less-natural expression 'entity-expression'.

remainder of the sentence. One could also decompose a thought with multiple names into complete senses corresponding to two or more names (or instances of the same name) and a predicative sense corresponding to a relational expression. Since sentences often contain multiple names or multiple instances of the same name, examples of type (2) will often suggest multiple decompositions of the same thought.

How far do examples in Frege of multiple decompositions extend? In the Introduction I showed that many examples of type (2) lead to multiple decompositions of the same thought that have distinct logical forms. In fact, Frege pointed out that distinct analyses may have distinct logical forms explicitly in (1906b, PW pg. 187), (1906c, PW pg. 201-2), and (Letter to Linke 8/24/1919, PMC pg. 98).

A more controversial example of multiple decompositions is Frege's famous recarving of the judgeable content that line a is parallel to line b into the judgeable content that the direction of line a is identical with the direction of line b, a transition meant to allow the thinker to acquire the concept of direction (1884, pg. 74-5). It is unclear whether Frege would have still accepted this type of example of multiple decompositions after he made his sense/reference distinction, since it is not mentioned in the *Grundgesetze.*

Frege *did*, however, bring up similar examples in a letter to Bertrand Russell of 7/28/1902, stating that so long as the relation in question is an equivalence relation (to use the modern expression), "this relation can be transformed into an equality (identity)..." (PMC pg. 141). Whether this counts as a case of multiple decompositions of the same thought depends upon what Frege meant by "can be transformed into" or "can be replaced by".

Here is another example where Frege may have changed his mind: in "Function and Concept", Frege speaks of the two sides of a particular case of Basic Law V as "express[ing] the same sense, but in a different way." (1891, pg. 10-11/143) However, there is no evidence in the *Grundgesetze* that Frege continued to think of Basic Law V in this way.

Another radical example is Frege's assertion that the sentences 'There is at least one square root of 4', 'The concept *square root of 4* is realized', and 'The number 4 has the property that there is something of which it is the square' all express the same thought. (1892c, pg. 199/107) Decomposing this thought in the manner suggested by the first sentence results in a logical form with an existential quantifier as its main connective, while the latter two suggested decompositions have distinct atomic logical forms, with the first including an object-sense referring to a proxy-object for the concept *square root of 4* [29] while the second includes an object-sense referring to the number 4.

Near the end of the same paper ("On Concept and Object"), Frege makes several remarks about the saturated/unsaturated distinction that suggest he thought of e.g. 'The number 2 is a prime number', 'The number 2 falls under

---

[29] Recall that for Frege the expression 'the concept *square root of 4*' actually denotes a proxy-object rather than a concept. This isn't the place to discuss the "concept horse" problem beyond noting this point and the resulting different decompositions of the expressed thought on Frege's view.

the concept *prime number*', 'The *falls under* relation holds between the number 2 and the concept *prime number*', 'The *holds between* relation holds among...', etc., as all expressing the same thought. (1892c, pg. 204-05/193) However, he does not say so explicitly, instead stating "It is thus easy for us to see that the difficulty arising from the 'unsaturatedness' of one part of the thought can indeed be shifted, but not avoided." (ibid pg. 205/193) If Frege meant that the unsaturatedness of one part of the thought can be shifted to another part of the same thought, then this would indeed count as a case of multiple decompositions of the same thought. Furthermore, such cases fit naturally with Frege's explicit examples discussed above.

In "Compound Thoughts" (1923-26) Frege identifies the thoughts expressed by several logically equivalent types of sentences, including 'A and B' and 'B and A' (ibid pg. 39/393), 'A and A' and 'A' (ibid pg. 39 fn. 5/393 fn. 21), 'not (not B)' and 'B' (ibid pg. 44/399), and 'not[(not A) and A]' and 'If A then A' (pg. 50/405). He even suggests in a letter to Husserl that so long as two sentences don't contain logically self-evident component parts, if both sentences follow logically from each other, they express the same thought. (Letter to Husserl 12/9/1906, PMC pg. 70)[30] However, since Frege seems to make incompatible claims outside of this letter, it is unclear whether it is appropriate to attribute it to him, even with respect to his later work.[31]

Indeed, some of these examples lead to a non-standard treatment of the part/whole relation. For example, if 'A and A' and 'A' always express the same thought, then it seems that every thought is a proper part of itself.[32] This may be one of the "hitches" regarding applying the part/whole relation to thoughts that Frege (1923-26) mentioned (see footnote 2). Note, however, given that this issue arises so long as thoughts contain components, whether understood in the sense of part/whole or otherwise, and given that thoughts containing components is essential to Frege's position, if the reader finds this unpalatable I recommend rejecting the position that these and similar logically equivalent sentences express the same thought.

As I pointed out in the Introduction, there are additional cases of multiple decompositions stemming from Frege's remarks on truth, indexicals, and (arguably) the hierarchy of senses. Arguing for these points in detail requires a

---

[30]Why the qualification? I suggest Frege was thinking of the following sort of case: compare the sentences 'John loves Mary and either Jim has brown hair or Jim does not have brown hair' and 'John loves Mary'. Although the two sentences are logically equivalent, the former contains additional content not found in the latter, and hence expresses a distinct thought. The thought expressed by the former must include an object-sense referring to Jim, while the latter thought contains no such component. Hence, logical equivalence is not a sufficient condition for thought identity.

[31]One relevant quote which puts further pressure on this view comes from Frege's essay "Negation" (1918-19b): "Thus the two thoughts: A, and the negation of the negation of A: either both are true or neither is." (pg. 157/389) However, this quotation does not establish the point. Mathematicians often use language such as "Our two numbers n and m are thus seen to be identical". Personally, I am hesitant to say that Frege changed his mind on this point in his very next paper, the third paper in the series.

[32]Personally, I am willing to accept this in the case of abstract objects. The reader is welcome to take their own position on the matter.

separate essay, so here I will merely state the interpretations without argument.

Regarding truth and the hierarchy of senses, Frege often says that any thought that p and the thought expressed by 'the thought that p is true' express the same thought (e.g. 1892a, pg. 34/164). Frege also says that the *indirect* sense which refers to the thought that p in a belief ascription is the sense expressed by the expression 'the thought that p' (1892a, pg. 37/166). These two positions can be made compatible by seeing any thought that p as decomposable into its indirect sense and the sense expressed by the truth predicate.[33] This implies that the truth predicate refers to a function from thoughts to their truth-values.

Finally, Frege states in his (1918-19a) that the utterances 'Today it is raining' and 'Yesterday it was raining', where those utterances are made in the same place and are separated by an appropriate amount of time, in fact express the same thought. Frege explicitly points to multiple decompositions as a way to make this view tenable.[34]

In future work I will argue extensively for these interpretations, and further argue that these positions in fact lead to tenable contemporary positions on the content of indexicals and the hierarchy of senses.

These various remarks on multiple decompositions suggest that thoughts are grasped in particular *ways* corresponding to particular decompositions of those thoughts into parts. And note that some decompositions of a thought will make various inferences from that thought salient while masking others. For example, if I grasp the thought that John loves Mary as consisting of the object-sense expressed by 'John' composed with the predicative sense expressed by 'loves Mary', this will immediately make salient a logical inference to the thought that *someone* loves Mary, while if I grasp this very same thought as consisting of the object-sense expressed by 'Mary' composed with the predicative sense expressed by 'John loves', this will instead immediately make salient a logical inference to the thought that John loves *someone*, while masking the inference to the thought that someone loves Mary.

Does the possibility of grasping thoughts in different ways undermine part of the motivation for the sense/reference distinction? I think not. Grasping a thought does not require having a *representation* of that thought. This would lead to a regress. If one *does* explicitly represent a thought, in a case of belief ascription for example, one's higher-order sense will *itself* be grasped in a particular way that thereby fixes the particular analysis one is representing that thought as allowing.

The point is the following: *In the context of making an inference, we must grasp the thoughts involved in particular ways.* Each way of grasping a thought makes various possible inferences salient while masking others. Hence Dummett was right to link multiple decompositions to possible inferences, but still wrong to treat analysis as a separate enterprise revealing the true structure of the

---

[33]As I stated in the introduction, it is controversial whether Frege thought of the truth predicate as expressing a sense. I believe he was at least *committed* to this, given his more fundamental commitments.

[34]Both claims are made in (1918-19a, pg. 64/358).

thought. Much like the real line, the thought is structured in such a way that it can be decomposed into parts in many distinct ways, each of which reveals a different aspect of its underlying structure.

5   Conclusion

I have extensively discussed Frege's positions that thoughts can be decomposed into parts in multiple ways and that thoughts can be seen as built out of parts. While most authors consider these two positions to be in serious conflict with each other and have proposed revisions of Frege's views, I have argued that Frege can and did consistently hold both positions and hence revision is unnecessary.

My primary argument for this point relies on textual evidence that Frege thought that wholes can be decomposed into parts in multiple ways, and (more importantly) a thought can be *built up out of* parts in multiple ways. Furthermore, Frege was quite explicit that the notion of a thought being built up out of parts is metaphorical; thoughts are timeless, abstract entities and have no origin. Thus the puzzle dissolves.

I have also argued that Geach's function/argument analysis of the structure of thoughts is not only a view Frege was committed to, it is also a perfectly acceptable position on the structure of thought. Frege's commitment follows from his treatment of the hierarchy of senses. For example, in the sentence 'John believes that Mary is happy', 'is happy' is an incomplete expression and thereby denotes a function. In particular, it denotes a function from the sense of 'Mary' to the thought expressed by 'Mary is happy'.

I concluded with reflections on how far multiple decompositions extend. I believe that multiple decompositions can help us make sense of Frege's positions on truth, indexicals, and the hierarchy of senses. Not only do they lead to correct interpretations of Frege (or, at least, of Frege's commitments); they are also defensible contemporary positions. I will argue for this latter claim in future work. Given the wide interest in such topics in the contemporary literature on Fregean propositions, I believe Frege's account of multiple decompositions deserves broad contemporary discussion.

References

[1] Bell, David (1996), "The Formation of Concepts and the Structure of Thoughts", *Philosophy and Phenomenological Research*, 56(3): 583-96.

[2] Bermúdez, José (2001), "Frege on Thoughts and their Structure", *Logical Analysis and History of Philosophy*, 4: 87-105.

[3] Currie, Gregory (1985), "The Analysis of Thoughts", *Australasian Journal of Philosophy*, 63(3): 283-98.

[4] Dummett, Michael (1981a), *Frege: Philosophy of Language, Second Edition*, Cambridge, MA: Harvard University Press.

[5] Dummett, Michael (1981b), *The Interpretation of Frege's Philosophy*, Cambridge, MA: Harvard University Press.

[6] Frege, Gottlob (1879), *Begriffsschrift*, translated by T. Bynum as "Conceptual Notation", in: T. Bynum (ed.), *Conceptual Notation and Related Articles*, Oxford: Oxford University Press, 1972, pp. 101-203.

[7] Frege, Gottlob (1881), "Boole's Logical Calculus and the Concept-Script", translated by P. Long and R. White, in: Frege (1979), pp. 9-46.

[8] Frege, Gottlob (1884), *Die Grundlagen der Arithmetik*, translated by J. L. Austin as *The Foundations of Arithmetic*, Northwestern: Basil Blackwell, 1980.

[9] Frege, Gottlob (1891), "Function and Concept", translated by P. Geach, in: Frege (1984), pp. 137-56.

[10] Frege, Gottlob (1892a), "On Sense and Reference", translated by M. Black as "On Sense and Meaning", in: Frege (1984), pp. 157-77.

[11] Frege, Gottlob (1892b), "On Concept and Object (Preliminary Draft)", translated by P. Long and R. White, in: Frege (1979), pp. 87-117.

[12] Frege, Gottlob (1892c), "On Concept and Object", translated by P. Geach, in: Frege (1984), pp. 182-94.

[13] Frege, Gottlob (1893), *Grundgesetze der Arithmetik: Band I*, translated by P. Ebert and M. Rossberg as *Basic Laws of Arithmetic: Volume I*, Oxford: Oxford University Press, 2013.

[14] Frege, Gottlob (1895), "A Critical Elucidation of Some Points in E. Schröder, *Vorlesungen über die Algebra der Logik*", translated by P. Geach, in: Frege (1984), pp. 210-28.

[15] Frege, Gottlob (1897), "Logic", translated by P. Long and R. White, in: Frege (1979), pp. 126-51.

[16] Frege, Gottlob (1903), *Grundgesetze der Arithmetik: Band II*, translated by P. Ebert and M. Rossberg as *Basic Laws of Arithmetic: Volume II*, Oxford: Oxford University Press, 2013.

[17] Frege, Gottlob (1906a), "On Schoenflies: *Die Logischen Paradoxien der Mengenlehre*", translated by P. Long and R. White, in: Frege (1979), pp. 176-83.

[18] Frege, Gottlob (1906b), "Introduction to Logic", translated by P. Long and R. White, in: Frege (1979), pp. 185-96.

[19] Frege, Gottlob (1906c), "A Brief Survey of My Logical Doctrines", translated by P. Long and R. White, in: Frege (1979), pp. 197-202.

[20] Frege, Gottlob (1914), "Logic in Mathematics", translated by P. Long and R. White, in: Frege (1979), pp. 203-50.

[21] Frege, Gottlob (1918-19a), "Logical Investigations: Part I: Thoughts", translated by P. Geach and R. H. Stoothoff, in: Frege (1984), pp. 351-72.

[22] Frege, Gottlob (1918-19b), "Logical Investigations: Part II: Negation', translated by P. Geach and R. H. Stoothoff, in: Frege (1984), pp. 373-89.

[23] Frege, Gottlob (1919), "Notes for Ludwig Darmstaedter", translated by P. Long and R. White, in: Frege (1979), pp. 253-57.

[24] Frege, Gottlob (1923-26), "Logical Investigations: Part III: Compound Thoughts", translated by P. Geach and R. H. Stoothoff, in: Frege (1984), pp. 390-406.

[25] Frege, Gottlob (1979), *Posthumous Writings*, H. Hermes, F. Kambartel, and F. Kaulbach (eds.), Chicago, IL: The University of Chicago Press.

[26] Frege, Gottlob (1980), *Philosophical and Mathematical Correspondence*, B. McGuinness (ed.), translated by H. Kaal, Chicago, IL: The University of Chicago Press.

[27] Frege, Gottlob (1984), *Collected Papers on Mathematics, Logic, and Philosophy*, B. McGuinness (ed.), Oxford: Basil Blackwell.

[28] Garavaso, Pieranna (1991), "Frege and the Analysis of Thoughts", *History and Philosophy of Logic*, 12: 195-210.

[29] Geach, Peter (1975), "Names and Identity", in: S. Guttenplan (ed.), *Mind and Language*, Oxford: Clarendon Press, 1975, pp. 139-58.

[30] Heck, Richard and Robert May (2011), "The Composition of Thoughts", *Nous*, 45(1): 126-66.

[31] Kemmerling, Andreas (2011), "Thoughts without Parts: Frege's Doctrine", *Grazer Philosophische Studien*, 82: 165-88.

[32] Künne, Wolfgang (2007), "A Dilemma in Frege's Philosophy of Thought and Language", *Rivista di Estetica*, 34: 95–120.

[33] Levine, James (2002), "Analysis and Decomposition in Frege and Russell", *The Philosophical Quarterly*, 52: 195-216.

[34] Penco, Carlo (2003), "Frege: Two Theses, Two Senses", *History and Philosophy of Logic*, 24: 87-109.

[35] Textor, Mark (2009), "A Repair of Frege's Theory of Thoughts", *Synthese*, 167: 105-23.