# Granular Partitions and Vagueness

Thomas Bittner and Barry Smith
Department of Computer Science, Northwestern University,
bittner@cs.nwu.edu
Department of Philosophy and NCGIA, State University of New York at Buffalo,
phismith@buffalo.edu

Abstract — There are some who defend a view of vagueness according to which there are intrinsically vague objects or attributes in reality. Here, in contrast, we defend a view of vagueness as a semantic property of names and predicates. All entities are crisp, on this view, but there are, for each vague name, multiple portions of reality that are equally good candidates for being its referent, and, for each vague predicate, multiple classes of objects that are equally good candidates for being its extension. We provide a new formulation of these ideas in terms of a theory of *granular partitions*. We show that this theory provides a general framework within which we can understand the relation between vague terms and concepts on the one hand and correlated portions of reality on the other. We also sketch how it might be possible to formulate within this framework a theory of vagueness which dispenses with the notion of truth-value gaps and other artifacts of more familiar approaches.

Categories & Descriptors —  F.m, I.m
General Terms —  Theory, Human factors
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## 1. Introduction

Consider the proper name 'Mount Everest'. This refers to some mereological whole, a certain giant formation of rock. A mereological whole is the sum of its parts, and Mount Everest certainly contains its *summit* as part. But it is not so clear which parts along the foothills of Mount Everest belong to the mountain and which to its neighbors. Thus it is not clear which mereological sum of parts of reality actually constitutes Mount Everest. One option is to hold that there are multiple candidates, no one of which can claim exclusive rights to serve as the referent of this name. Each of these many candidates has the summit as part. They differ, however, regarding which parts are included among the foothills. Consider, analogously, the predicate 'is a bald male'. Bill Clinton certainly does not belong to the extension of this predicate, and Yul Brunner certainly does. But how about Bruce Willis? It would seem that there are some candidates for the extension of this predicate in which Bruce Willis is included, and certain others in which he is not.

Varzi [12] refers to the above as a *de dicto* view of vagueness. It treats vagueness not as a property of objects but as a semantic property of names and predicates. There are, for each vague name, multiple portions of reality that are equally good candidates for being its referent, and, for each vague predicate, multiple classes of objects that are equally good candidates for being its extension. There are some, for example Tye [11], who are happy to include in their ontology vague objects and regions and thus defend a *de re* view of vagueness. In a quantitative formalism this might result in what [4] calls fuzzy objects and regions. The important point is that on this *de re* view one needs to extend one's ontology in such a way as to include new, special sorts of regions and objects, in addition to the crisp objects and regions one has already recognized. This not only brings added ontological commitments but implies

also that one needs to investigate the question whether vague location (of vague objects in vague regions) is or is not the same relation as the more familiar, crisp location of old.

Given the *de dicto* point of view there is no need to extend our ontology in this way. One needs, rather, to reconceptualize the relationships between terms and concepts on the one hand, and crisp objects and locations out there in the world on the other. Such relationships are not one-one, but rather one-many, and we can think of their targets, tentatively, as multiple *products of demarcation*. Note that this reconceptualization is not intended as an account of what is involved cognitively when we use vague terms or predicates. Normal subjects in normal (which means: non-philosophical) contexts are not aware of the existence of such multiple targets. Rather, the simultaneous demarcation of a multiplicity of crisp referents or extensions takes place as it were behind the scenes. What we offer here is a proposal for dealing theoretically with the *ontology* of that particular type of relation which is involved between a cognitive subject and some correlated reality when vague terms or predicates are used. We are however able to throw some throw light on the subject's cognitive awareness when using such terms or predicates. This is because the very fact that many of the matters with which we deal are ones which fall beneath the threshold of concern of the cognitive subject is itself something which the approach here advanced is able to illuminate.
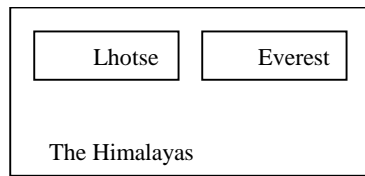


**Figure 1: Left: a partition, with cells *Everest, Lhotse* and *The Himalayas*. Right: A part of the Himalayas seen from space, with Mount Lhotse (left) and Mount Everest (right).**

The *de dicto* view of vagueness goes hand in hand with the doctrine of supervaluationism [5], [3], which is based on a redefinition of the notion of truth to accommodate the multiplicity of candidate precisifications associated with vague names or predicates. The basic idea is that, when determining the truth of an assertion containing a vague name or predicate, it is necessary to take into account all its candidate referents or extensions. In order to evaluate such an assertion semantically, we must effectively run through these candidates in succession and determine, for each particular choice, whether it makes the assertion true or false. An assertion such as 'Yul Brunner was bald' is *supertrue* because it is true for all such choices. An assertion such as 'Bill Clinton is bald' is *superfalse* because it is false for all such choices.

The problems arise in regard to sentences which are *indeterminate*, in the sense that they come out true for some choices and false for others. The core of these problems is captured in the so-called Sorites paradox [6]. Consider Bill Clinton. He is certainly not bald, and losing one hair will not make him bald. This seems to hold quite generally: if Clinton is not bald and he loses one hair, then he is still not bald. Following this chain of reasoning if we start from a non-bald Clinton, then Clinton will still not be bald even if he has only 10 hairs left on his head. This is because, intuitively, losing *one* hair does not cause the transition to baldness. A similar chain of reasoning can be constructed in the case of Mount Everest. The summit is part of the mountain. If *x* is a part of a mountain, then every molecule that is connected to *x* is also part of the mountain. Following this chain of reasoning, we end up concluding that Berlin is part of Mount Everest. In this paper we will provide a framework for understanding how such chains of reasoning are broken in normal contexts of assertion.

We shall concentrate our attentions in what follows on *the case of singular reference*, i.e., reference via names and definite descriptions to concrete portions of reality such as mountains

and deserts. Thus we leave for another occasion the task of extending the account to the case of vague predication. We shall concentrate also on examples which are primarily spatial. It is, however, one advantage of the framework here defended that it can be generalized automatically beyond the spatial case.

## 2. Judgments, Supervaluation, and Context

The technique of supervaluation evolved as part of standard model-theoretic semantics. Thus it has been applied primarily to sentences of artificial languages conceived in context-free fashion. As the authors of [10] point out, however, the degree and type of vagueness by which the singular terms of natural language are affected varies in significant ways according to the contexts in which such terms are used. They therefore argue that, if the supervaluationistic method is to be extended to natural language, then it will be necessary to *contextualize* the theory by applying semantic evaluations not to *sentences* but to the *judgments* which such sentences express. It is, after all, through judgments – sentences as used assertively in specific contexts – that terms are projected onto reality by the subjects who make them.

This recognition of the context-dependence of vagueness has important consequences. For while it is easy to concoct examples of sentences neither supertrue nor superfalse when such sentences are treated out of context – much of the philosophical literature on vagueness is devoted to the discussion of examples of this sort – it is much less easy to find examples of such sentences when we confine ourselves to assertions which would naturally arise in the specific types of contexts which human beings inhabit. This is for reasons of pragmatics: such contexts have features which make it difficult, if not impossible, for judgments to occur within them which are marked by indeterminacy.

To get an idea of what we have in mind consider the sentence:

[A] This glass is empty,

and contrast the behavior of this sentence in two distinct contexts. In the first, $C_1$, it is used to express a judgment by a drunkard in a seedy bar just after taking the last sip of beer from his glass. In the second, $C_2$, it is used as the target of a negative judgment by a hygiene inspector inspecting the same glass just a few seconds later. We have here two distinct judgments, which we can abbreviate loosely as: $J_1 = (A, C_1)$ and $J_2 = (not-A, C_2)$. $J_1$ is supertrue, since the glass contains, on all precisifications, nothing left to drink. And $J_2$ is supertrue also: for the hygiene inspector sees all the bacteria inside the glass, and on no precisification consistent with what she sees would the sentence [A] be evaluated as true.

Judgments, to repeat, are always made in contexts. Hence to evaluate a judgment as to its truth (supertruth) or falsehood (superfalsehood) is to evaluate that judgment *in its context*. A judgment is supertrue if and only if it is true under all contextually appropriate ways of putting members of the pertinent 'many' into the extensions of the corresponding terms; and analogously for superfalsehood. Importantly, however, a sentence may be *unjudgeable* in a given context. It then does not even reach the point where it can serve as a proper object of semantic evaluation.

Can a sentence be judgeable in a context and yet still be indeterminate as to its truth-value? It is this question with which we shall deal in what follows. The notion of 'context' is of course itself notoriously problematic. The primary advantage of the framework here advanced is that it enables us to rephrase our question in a way which does not rely on the use of this problematic notion.

## 3. Granular Partitions

Consider the way in which every use of a referring term and every act of perception effects a partition of reality into a *foreground* domain, within which the object of reference is located,

and a *background* domain, which comprehends all the entities beyond. Our fundamental idea is that every use of language to make a judgment similarly brings about a certain context-dependent partition of reality.

As our attentions shift through time, such partitioning of reality is subject to what we might call *ontological regrouping*, as portions of reality that are in the background in one context are moved to the foreground in another. Sometimes our partitioning of reality is subject to *ontological zooming*, which occurs when we move, in relation to the same portion of reality, between partitions of different granularity. That is, we use a coarse-grained partition in one context and a fine-grained partition in another.

Understanding how such regrouping and zooming work can help us, now, to understand how judging subjects deal, contextually, with vagueness. Consider again the judgments $J_1 = (A, C_1)$ and $J_2 = (not\text{-}A, C_2)$ referred to above. Corresponding to $J_1$ and $J_2$ are two partitions, $Pt_1$ and $Pt_2$, each projecting onto the same portion of realty – the glass in front of the drunkard. Both partitions contain cells labeled 'glass' and 'beer', similar to the cells in the partition in the left part of Figure 1. But $Pt_2$ has in addition cells labeled 'bacteria', 'mold', and 'chlorine'. This is what we mean by ontological regrouping. Parts or reality in the background in one partition are brought into the foreground in another. Moreover $Pt_1$ and $Pt_2$ do not differ only in their complement of cells; they differ also in the way in which the cells they share in common are projected onto reality. Here ontological zooming occurs. The cell labeled 'beer' in the drunkard's partition projects (tries to project) onto drinkable amounts of beer. The corresponding cell in the partition of the hygiene inspector projects even onto amounts of beer that are visible only under a microscope.

Ontological regrouping and zooming operate in such a way as to ensure that both judgments in their respective contexts are supertrue. In the case of the drunkard, the granularity of the selected partition *traces over* those tiny amounts of beer that could cause truth-value indeterminacy. In the case of the hygiene inspector ontological zooming ensures that these same tiny amounts of beer are recognized and not traced over. The third alternative – in which tiny amounts of beer are recognized under some but not other contextually appropriate ways of putting members of the pertinent 'many' into the extensions of the corresponding term – arises in neither case. For details see [10].

In some cases our granular partitions do not merely reflect objects and boundaries existing in the side of the reality towards which our cognitive acts are directed. Rather, they themselves impose fiat boundaries onto this reality, and they thereby carve out fiat objects. [2] Granular partitions are defined as systems of cells, which are to be conceived as projecting onto reality in something like the way in which a bank of flashlights projects onto reality when it carves out cones of light in the darkness. Consider, for example the simple partition of the Himalayas that is depicted in the left part of Figure 1 above. This partition contains cells labeled 'Everest' and 'Lhotse', together with one maximal cell labeled 'the Himalayas'. These cells project onto different parts of that portion of reality that is depicted in the right part of Figure 1. They carve *mountains* out of a certain formation of rock. They do not do this physically, but rather by establishing fiat boundaries in reality, represented by the black lines in the right part of the figure. (See [2], [9], [8].)

While fiat boundaries are in a way like the boundaries of the light-cone of a flashlight, there is one important difference, which turns on the fact that we cannot directly *see* fiat boundaries. The latter are, rather, analogous to the boundaries of a light-cone that is projected during daylight. Because we cannot see fiat boundaries, we have to use indirect means (for example maps and compasses and complex calculations) in order to discover where they lie. In some cases we may have good grounds to believe that we have crossed them. For example a sudden increase in slope may tell us that we have crossed the boundary of Mount Everest. In some cases fiat boundaries have become associated with suitable bona fide props or supports, for example with systems of pegs or fences in reality. Surveying is about establishing relations between fiat boundaries and real, physical landmarks of these sorts. [7], [1]

The problematic nature of the cases which concern us here, however, lies in the fact that the fiat boundaries do not exist singly, but rather only as parts of those entire systems of fiat boundaries which come to be projected onto reality as a reflection of the existence of our cognitive acts. Let us return to our partition of the Himalayas. There are, we can now say, multiple equally good ways of projecting the cell 'Mount Everest' onto the corresponding formation of rock. Each is slightly different as regards the location of the mountain boundary which is projected among the pertinent foothills. Each projection targets just one possible candidate precisification. Each has, in other words, an ontological correlate that is entirely crisp. The differences between these precisifications, however, and the very fact that there are such multiple targets, falls in this context beneath the threshold of the subject's concern.

Reflecting on such examples reveals ways in which partitions, by means of their cell structure and the way these cells project onto reality, can stand proxy for contexts in a theory of judgment designed to take account of the context-dependence of vagueness. The number and arrangement of cells within a partition and the ways in which these cells project onto reality serve as formally tractable surrogates for those features of contexts which are relevant to the understanding of vagueness as a semantic (*de dicto*) phenomenon.


## 4. A Theory of Granular Partitions: A Brief Outline

To produce an ontological theory of granular partitioning will be somewhat tricky. This is because the results of partitioning are granular in every case, and this means that they cannot be understood along any simple mereological lines. For if an object is included in the foreground domain of our partition, this does not at all imply that all the parts of this object are also included therein.

The theory of granular partitions has two parts: (A) a theory of the relations between cells and the partitions in which they are housed, and (B) a theory of the relations between cells and objects in reality. (For formal details see [2].)

Theory (A) studies the properties granular partitions have in virtue of the relations between and the operations performed upon the cells from out of which they are built. All such partitions involve cells arranged together in some grid-like structure. This structure is intrinsic to the partition itself; that is to say, it is what it is independently of the objects onto which it might be projected. As we shall see this part of the theory applies equally well to crisp as to vague partitions.

The cells in a partition may be arranged in a simple side-by-side fashion, for example in our partition of *the Beatles* into *John*, *Paul*, *George* and *Ringo*. Cells may also be nested one inside another in the way in which, for example, the species *crow* is nested inside the species *bird*, which is nested in turn inside the genus *vertebrate* in standard biological taxonomies. The possibility of this nesting is one mark of granular partitions as here understood which distinguishes them from partitions in the more familiar mathematical sense (partitions generated by equivalence relations).

We define the cell structure, A, of a partition, Pt, as a system of cells, $z_0, z_1, \ldots,$ . We write $Z(z, A)$ as an abbreviation for 'z is a cell in the cell-structure A'. We write $z_1 \subseteq_A z_2$ to designate this relationship between two cells $z_1$ and $z_2$ belonging to the cell structure A when the first is a subcell of the second. ($z_1 \subset_A z_2$ then abbreviates: $z_1$ is a proper subcell of $z_2$.) In the remainder of this paper we omit subscripts wherever no ambiguity will result.

We now impose four axioms (or 'master conditions') on all partitions, as follows:

MA1: The subcell relation $\subseteq$ is refle*x*ive, transitive, and antisymmetric.

MA2: The cell structure of a partition is always such that chains of nested cells are of finite length.

MA3: If two cells overlap, then one is a subcell of the other.

MA4: Each partition contains a unique maximal cell.

These conditions together ensure that each partition can be represented as a tree (a directed graph with a root and no cycles), in which each node corresponds to a cell of the partition with which we begin.

The second component (Theory B) of the theory of granular partitions arises in reflection of the fact that partitions are more than just systems of cells. They are constructed in such a way as to project upon reality. Intuitively, this projection corresponds to the way proper names project onto or refer to the objects they denote and to the way our acts of perception are related to their objects. (Projection is close to what philosophers call 'intentionality'.) When projection is successful, then we shall say that the object targeted by the pertinent cell is *located in* that cell. We then write 'P(z, o)' as an abbreviation for: cell z is projected onto object o, and 'L(o, z)' as an abbreviation for: object o is located in cell z. Intuitively, being located in a cell is like being illuminated by a spotlight.

That location is not simply the converse of projection follows from the fact that a cell may project without there being anything onto which it is projected (as a spotlight can cast its beam without striking any object). Because location is what results when projection succeeds, location presupposes projection. An object is never located in a cell in a partition unless as a result of the fact that this cell has been projected upon that object. This is the first of our master conditions for theory (B):

MB1 $L(o, z) \rightarrow P(z, o)$.

Partitions are cognitive artifacts. Objects can come to be located in their cells only if we have constructed cells of the appropriate sort and have targeted them in the right direction. We then say that the partition in question is *transparent to* the corresponding portion of reality. We can formulate this condition of transparency as follows:

MB2 $P(z, o) \rightarrow L(o, z)$.

In what follows we shall assume conditions MB1 and MB2 as master conditions governing all partitions. Thus, for the restricted purposes of this paper, MB1 and MB2 collapse to: $L(o, z) \leftrightarrow P(z, o)$. MB2 serves to guarantee that objects are actually located at the cells that project onto them. In a more general theory of granular partitions, MB2 will be weakened to allow for misprojection, for example where an object is wrongly named or wrongly classified.

In order to ensure that projection and location satisfy the intuitions underlying our spotlight analogy, we demand further that projection and location be functional relations, i.e., that every cell projects onto just one object and every object is located in just one cell:

MB3 $P(z, o_1)$ and $P(z, o_2) \rightarrow o_1 = o_2$
MB4 $L(o, z_1)$ and $L(o, z_2) \rightarrow z_1 = z_2$

For partitions satisfying MB3, each cell is projected onto one single object: there is no overcrowding. For partitions satisfying MB4 objects are in every case located at single cells: thus there is no redundancy (of the sort which would be involved where a single partition would contain distinct cells, for example labeled 'Mount Everest' and 'Chomlungma', both projecting onto the same formation of rock). Notice also that 'object' here is used in a very wide sense, to include also scattered mereological sums. Thus a partition of the animal kingdom might involve a cell labeled *cat*, which projects onto that single object which is the mereological sum of all live cats.

We will assume that partitions are *complete* in the sense that every cell projects onto at least one object, i.e., that there are no empty cells (no cells projecting outwards into the void):

MB5 $Z(z, A) \rightarrow \exists o: L(o, z)$

Consequently, projection is a total function.

Location, however, is typically a partial function. This is because human beings are not omnipotent in their partitioning power. In the context of this paper we will assume that the constraints MB1–5 are always satisfied, i.e., projection and location are always functional, and there are no empty cells.

Each partition has a certain *domain*, which we can define as that portion of reality upon which its maximal cell is projected. By functionality of projection and location, there can be only one such object. That every partition has a non-empty domain follows from MB5.

We now can define a granular partition as a triple Pt = (A, P, L) where (i) A is a system of cells such that MA1–4 hold and (ii) P and L are projection and location relations which satisfy MB1–5. Partitions can reflect the basic part-whole structure of their domains in virtue of the fact that the cells in a partition can themselves stand in the relation of part to whole. This means that, given the master conditions expressed within the framework of theory (A) above, partitions have at least the potential to reflect the mereological structure of the domain onto which they are projected; and in felicitous cases this potential is realized. (For details see, again, [2].)

## 5. Vague Granular Partitions

The framework presented above can now be used to yield a formal account of granularity. Since it is the granularity of our partitions which allows questionable parts to be traced over in our cognitive directedness to objects, and since it is this tracing over of questionable parts which allows reference to be vague, this formal account of granularity will help in turn in formulating a theory of vagueness.

If projection is vague then, to pursue our earlier spotlight analogy, it is not only the case that the fiat boundaries carved out by projections are invisible; it is also as if every spotlight sends out *multiple*, slightly distinct, cones of light. Thus it is as if there are many cone-shaped portions of reality carved out by a single (vaguely projecting) spotlight. There are many alternative ways in which fiat boundaries for Mount Everest might be carved out among its foothills. Each of these boundaries must be such that it encloses the summit. There is then no fact of the matter that specifies where *the* boundary of Mount Everest lies. (And this is not merely an epistemological problem. Even an omniscient being would not know where this boundary lies, because *there is no such boundary*.)

Supervaluation theory in its standard form provides an instrument for the semantic evaluation of sentences involving vague terms and predicates. What we offer here is a modification of this theory designed to take account of the different ways in which our terms and concepts project – vaguely or crisply – onto corresponding portions of reality in different sorts of contexts. We proceed by extending the theory of granular partitions in order to take account of vague partitions in a way that is modeled on the contextualized supervaluationist understanding of vagueness described in [10]. In the crisp case, each partition is characterized by a *single* projection relation and a *single* location relation. In order to accommodate the supervaluation idea, we give up the constraint that each partition is associated with a single projection/location relation. Theory (A) is unaffected by this change, but we will need to provide modified axioms for theory (B) in such a way that crispness is included as just one special case.

A *vague* granular partition $Pt^v = (A, P^v, L^v)$ is a triple such that A is a system of cells for which MA1–4 hold and $P^v$ and $L^v$ are *classes* of projection and location relations satisfying the conditions set forth below.

Consider a vague partition $Pt_V = (A, P^v, L^v)$ of the Himalayas, with a cell structure A as shown, again, in the left part of Figure 1. In contrast to a single crisp projection of the sort indicated in the right part of this figure, vague partitions have a multiplicity of candidate projections for their cells, indicated by boundary regions which can be imagined as cloudy ovoids around the two mountains in the right of the figure. The boundaries of the actual candidates onto which the cells 'Lhotse' and 'Everest' are projected under the various $P_i$ in $P^v$ are included somewhere within the corresponding cloud of regions.

The projection and location relations in these classes form pairs $(P_i, L_j)$, which are such that each $P_i$ has a corresponding unique $L_j$ and vice versa, satisfying the following conditions (where the notation '$\exists$!' abbreviates: 'there exists one and only one i'):

$\text{MB1}^V \qquad \forall j, \exists ! i: L_j(o, z) \rightarrow P_i(z, o)$

$\text{MB2}^V \qquad \forall i: P_i(z, o) \rightarrow \exists ! j: L_j(o, z)$

In the context of this paper $\text{MB1}^V$ and $\text{MB2}^V$ can be simplified as: $\forall i \exists ! j: P_i(z,o) \leftrightarrow L_j(o,z)$.

We also demand that all $P_i$ and $L_j$ are functional, by analogy with their counterparts in the original theory:

$\text{MB3}^V \qquad P_i(z, o_1) \text{ and } P_i(z, o_2) \rightarrow o_1 = o_2$

$\text{MB4}^V \qquad L_j(o, z_1) \text{ and } L_j(o, z_2) \rightarrow z_1 = z_2$

We also demand that cells project onto some object (are non-empty) under every projection:

$\text{MB5}^V \qquad Z(z, A) \rightarrow \forall j \, \exists o: L_j(o, z)$

We call all partitions $Pt_i = (A, P_i, L_j)$ with pairs $(P_i, L_j)$ satisfying $\text{MB1}^V - 5^V$ *crispings* of the vague partition $Pt^V$. From $\text{MB5}^V$ it follows that the domain of each crisping is non-empty, i.e., $\forall i, \exists o: o = D(Pt_i)$, and we define the domain of a vague partition as the mereological sum of the domains of all constituent crispings.

Consider a partition with one or more cells labeled with vague proper names. Intuitively, each pair of projection and location relations $(P_i, L_j)$ then recognizes exactly one precisified referent for each such cell. The precise candidates carved out by each $(P_i, L_j)$ are all slightly different. But each is perfectly crisp, and thus it has all of the properties of crisp partitions discussed in the previous sections. This means that even under conditions of vagueness the principal properties of partitions are preserved. Vagueness *de dicto* is captured at the partition level via *multiple ways* of projecting crisply.


## 6 Judgments

A judgment is a pair $J = (S, Pt)$ where S is a sentence and Pt is a granular partition standing proxy for the context in which the judgment is made. It will take us too far afield to provide a partition-theoretic account of truth for judgments here. It will be sufficient for illustrative purposes to provide examples of truth conditions for sentences of the form 'a is part of b'.

Given a judgment $J = (S = $ 'a is part of b', Pt$)$, the relationship between S and Pt is provided by a *labeling function*, which assigns the names of the objects referred to in S to cells of $Pt = (A, P, L)$. We say that $\lambda$ is a *labeling* relating the partition Pt to the sentence S if and only if the following holds: (1) $\lambda$ maps the sentence S as a whole onto the root cell of the partition Pt; (2) $\lambda$ maps proper names appearing in S to cells in A in such a way that each cell gets uniquely labeled and each name has a unique corresponding cell, (3) the co-domain of $\lambda$ exhausts the cell-structure of Pt.

Condition (1) ensures that the judgment as a whole has a well-defined scope, namely the domain of Pt. Consider the judgment $J_1 = (S_1, Pt) = ($'Mount Everest is part of the Himalayas', Pt$)$, where Pt is the partition shown in the left part of Figure 1. The sentence $S_1$ as a whole is mapped by $\lambda$ onto the root cell of this partition. Condition (2) ensures, in conjunction with the assumption that there are no empty cells (MB5), that each cell is uniquely labeled by a name contained in S. The limitation of partition cells to the names actually occurring in the corresponding judgment corresponds to our discussion of ontological regrouping above. The judgment $J_1$ brings into the foreground Mount Everest, the Himalayas, and the part-of relation which holds between them and it forces everything else, including Mount Lhotse, into the background of our attentions. Condition (3) ensures that the corresponding partition contains the cells 'Everest' and 'The Himalayas' but not a cell labeled 'Lhotse'. In this sense the labeling function always maps onto partitions that are *minimal* with respect to the sentence used in making the corresponding judgment.

We now say that a judgment of the form 'a is part of b' is *true in the context represented by* Pt if and only if (i) Pt represents a partition of reality in such a way that MA1–4 and MB1–5 hold; (ii) there is a labeling function λ satisfying the conditions specified above, and (iii) the cell labeled 'a' is a subcell of the cell labeled 'b' in the partition Pt.

We can now define the notions of supertruth, superfalsehood, and indeterminacy for judgments, $J = (S, Pt^V)$ with respect to vague partitions $Pt^V = (A, P^V, L^V)$. We assume that the cell structure A satisfies MA1–4 and that all of its crisp $Pt_k = (A, P_i, L_j)$ are such that $MB1^V$–$5^V$ hold. A judgment J is then *supertrue* with respect to a vague partition $Pt^V$ if and only if it is true with respect to all of the crisp partitions $Pt_k = (A, P_i, L_j)$. A judgment J is *superfalse* with respect to $Pt^V$ if and only if it is true with respect to *none* of the crisp partitions $Pt_k = (A, P_i, L_j)$. It is *indeterminate* otherwise.

We should like to be able to prove that the indeterminate case cannot occur in naturally occurring contexts. To provide a sketch of such an argument we shall show how a proper understanding of the context-dependent projection of fiat boundaries rules out any truth-value indeterminacy for judgments of the form $J^V$ = ('a is part of b', $Pt^V$), where b is the vague proper name 'Mount Everest'.


## 7. Unity and Vagueness

When recognizing wholes as sums of parts, we draw upon unity conditions that specify what sums of parts we are concerned with. In the case of Mount Everest, the pertinent unity condition might be formulated, in first approximation, along the following lines:

U1    (1) The summit is part of Mount Everest. (2) If x is a part of Mount Everest and y is connected to x then y is a part of Mount Everest.

We can assume for present purposes that clause (1) is unproblematic. Not so for clause (2). For this clause makes the unity condition incapable of determining which outlying portions of reality are parts of Mount Everest, and it is because of this that paradoxes of the Sorites type can arise. U1 has the structure of an inductive definition. It specifies a start condition and a condition on how to add parts to Mount Everest, but it does not specify where to *stop* adding parts. This means that if we take (1) and (2) in U1 as true premises, then you can infer that portions of reality are parts of Mount Everest that clearly are not.

However we cannot simply dismiss U1, for clause (2) captures the *continuous structure* of the formation of rock to which the concept *mountain* applies, that is, it captures the fact that we can form chains of connected parts $a_1, a_2, a_3, \dots$, or in other words that mountains are never scattered wholes. But what determines the outer limits of such chains of connected parts? Where does the mountain stop? As will by now be clear, there is no generally applicable and context-independent stop condition that can be inferred from a general concept such as *mountain*.

Consider now the relationship between the unity condition U1 and a judgment of the form $J^V$ = ('a is part of Everest', $Pt^V$). The two are related in the following sense: U1 governs the way in which $Pt^V$ projects onto reality in the sense that the cell 'Everest' must project onto a topologically connected whole which contains the summit. On the other hand judgment $J_V$ in its context $Pt^V$ places limits on the range of admissible precisifications in the way in which it projects boundaries onto reality. These limits are of such a sort that they serve to break the unlimited chains of connected parts and thus remove the associated Sorites problem. However, these limits are subject to vagueness themselves, and it is this which threatens the possibility of truth-value indeterminacy. Our task will be to show how this possibility is prevented from becoming actual by the sorts of partitions actually used in natural contexts, and thus to show that even judgments in which vague terms are used have determinate truth-values.

To this end, we need to discuss the range of relevant kinds of contexts. Two cases in particular are of importance, distinguished by the *kinds of boundaries* that can provide stop conditions for the unity condition U1 introduced above:

I:    Contexts in which our use of the corresponding term brings a single crisp
boundary into existence.
II:   Contexts in which our use of the corresponding term brings a vague boundary
(i.e., a multiplicity of crisp boundary candidates) into existence.

**I: The single (crisp) boundary case** Contexts of the first type are illustrated by those cases where we ourselves have the *authority* (the partitioning power) to bring a precise boundary into existence. For example suppose that you have been delegated by some competent government agency to establish the boundaries of Mount Everest for purposes of regulating the activities of climbers. Your partition – we can imagine that it is set forth in some document *D* – would then come very close to being fully crisp, i.e. only one single projection relation would be involved, and the boundary of Mount Everest would then in relevant contexts coincide with the boundary imposed by you. This has the consequence that, in the given contexts, the incomplete unity condition that comes with the underlying general concept is completed contextually, as follows:

U2      (1) The summit is part of Mount Everest. (2′) x is part of Mount Everest if and
only if: (i) there is some y which is part of Mount Everest and x is connected to
y and (ii) x is part of the projection of the cell 'Everest' in the partition
determined by the document *D*.

U2 has the advantage of blocking the unlimited transitivity of our original condition U1. Moreover U2 still enforces the continuity of parts of the mountain in the spirit of U1.

**II: The multiple (vague) boundary case** Contexts where we ourselves have the authority and the need to bring a precise boundary into existence are very rare. On the other hand, however, there is in most contexts no need for the high degree of precision which such contexts represent. In most contexts, that is to say, the created boundary is *just precise enough*, it is precise only to the degree to which it matters where it lies. In most cases, therefore, it will manifest a certain degree of vagueness, and the actual degree of vagueness (or the degree of precision) will depend on the context. Where vagueness is involved indeterminate cases threaten to arise. To this end we must show, following [10], that in naturally occurring contexts where boundaries are *just precise enough*, sentences which would have indeterminate truth-values are unjudgeable.

In instructing your staff to set up the tables in your restaurant each evening you establish where the line between smoking and non-smoking zones is to be drawn by using a sentence like:

[B]  The boundary of the smoking zone goes here,

while pointing with your finger in such a way to bisect the restaurant floor. You thereby also indicate on which tables the ashtrays are to be placed. You specify vaguely where the boundary lies. This means that with your vague gesture you bring a whole multitude of equally good boundary-candidates into existence.

The question then arises whether a judgment of the form J = ('This table is part of the smoking zone', $Pt^V$) can be such as to have an indeterminate truth-value. Our concept of a smoking zone is, after all, one of a non-scattered whole with boundaries which are often not precisely defined by sharp lines, fences, or walls. Inspection reveals however that this apparent vagueness of the boundary-specification does not affect the determinacy of the judgments restaurant staff or customers might actually make. Whether an ashtray is or is not placed on a table is, after all, a completely determinate matter.


## 8. Degrees of Vagueness and Crispness

In our discussion of unity conditions we have seen that the *appropriate degree* of vagueness or crispness is critical for avoiding truth-value indeterminacy. In this section we discuss a range of examples which further strengthen this point.

Imagine two neighboring countries, one with the death penalty and the other without. Even if the border between the two countries is fiat in nature (no wall, no fence), still, if you murder somebody on one side of the border you will be liable to die, and if you commit your crime on the other side of the border you will be liable to go to jail. Here it does not seem that indeterminacy can arise. This will hold even if you commit the crime while your body spans the border of the two countries (a one-dimensional fiat spatial entity whose location can nowadays be determined with considerable accuracy). This is because, since this is the sort of case where your exact location relative to the boundary *matters to the proceedings of the courts*, these courts will themselves have developed mechanisms to remove indeterminacy by fiat from its judgments, in light of the fact that the same person cannot both be hanged, and not hanged, for the same crime.

Imagine that you are wandering across the desert somewhere in the borderlands between Libya and Egypt pointing towards a grain of sand on the ground, and that you pronounce the sentence:

[C] This grain of sand belongs to Egypt.

No corresponding judgment will have been made, according to the view we are here defending. This is the case not because the specification of the boundary between Libya and Egypt is vague. Rather, it is because speaker and audience would not take the given sentence seriously as expressing a judgment.

If, on the other hand, the need to determine the ownership of every grain of sand were to arise (for example because sand has become more valuable than gold), then means would be devised to determine the truth-value of corresponding judgments in such a way that we could at least in principle determine unequivocally, for each given grain of sand, whether it belongs to Libya or to Egypt. For so long as this is not the case, however, there is no way to determine the truth-value of a judgment like [C]. Consequently, too, any attempt to make a judgment of this kind must fail on pragmatic grounds.

Imagine that you are with a party of climbers somewhere in the foothills of Mountain Everest and that one of your number, pointing to some imaginary line on the ground, uses the sentence:

[D] This is the boundary of Mount Everest

in order to make a judgment. We argue that in the given context (a context in which it is obvious to all parties that there is no law or treaty which establishes where, in or around its foothills, the boundary of the mountain lies) someone using [D] would not succeed in making a judgment. Rather, he would be seen as making some sort of joke. This is because a judgment $J = (D, Pt)$ of this form would invoke a *crisp* partition $Pt = (A, P, L)$, and it is pragmatically impossible to invoke crisp partitions in contexts where both speaker and audience know that vague partitions are the best that can be achieved. Corresponding attempts to make judgments will not be taken seriously.

It is, though, possible to conceive of contexts in which it is necessary to refer to the boundary of Mount Everest no matter how vague it might be. Suppose you make a judgment of the form:

[E] We will cross the boundary of Mount Everest within the next hour.

The admissible candidate boundaries for Mount Everest are hereby delimited as falling within a certain range, projected out onto the path ahead and determined as a function of travel time (all under the assumption that the judgment in question is true).

In this case you do not care where precisely the border is crossed because you are aware that you yourself are in a sense creating this border. The judgment concerns the *approximate* location of the boundary: that it is such that it can be crossed within the next hour. It is then easy to see how it might be either supertrue or superfalse. It is supertrue if, after a few minutes, you embark on a steep rise, which continues uninterrupted until you reach the summit. It is superfalse if you discover (or could discover), two hours after making your judgment, that you were over-optimistic: a new, wide valley suddenly appears between you

and the mountain. The crucial question is: under what conditions might the given judgment be indeterminate in truth-value? Bear in mind that there is here no crisply pre-established boundary; it is you the judger who determines – roughly – where the boundary lies. Can you determine that the boundary will be located in such a way as to dissect the family of admissible precisifications associated with the judgment you express by [E] into two disjoint sub-families? We think not. There is here only *just enough precision*. The necessary degree of precision to give rise to indeterminacy is again not available.

## 9. Conclusions

In this paper we proposed an application of the theory of granular partitions to the phenomenon of vagueness seen as a semantic property of names and predicates. We argued that it is insufficient to consider vague names and predicates as these occur in sentences considered in abstraction. Rather, it is necessary to consider vague names and predicates in judgments as these occur in natural contexts. This move then helps to resolve some of the problems in the semantic treatment of vagueness. We have sketched an argument to the effect that judgments made in natural contexts, even judgments involving vague terms, are not marked by truth-value indeterminacy. The argument should be conceived as a challenge to the reader to provide counterexamples to this claim.

## Acknowledgements

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