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S.I.: PLURALISTIC PERSPECTIVES ON LOGIC

Logical pluralism without the normativity

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Abstract
Logical pluralism is the view that there is more than one logic. Logical normativism is the view that logic is normative. These positions have often been assumed to go hand-in-hand, but we show that one can be a logical pluralist without being a logical normativist. We begin by arguing directly against logical normativism. Then we reformulate one popular version of pluralism—due to Beall and Restall—to avoid a normativist commitment. We give three non-normativist pluralist views, the most promising of which depends not on logic's normativity but on epistemic goals.

Keywords Logical pluralism · Normativity of logic · Logical consequence · Inference and implication · Logic and reasoning · Telic logical pluralism

1 Introduction

Normativity and logical pluralism have interacted in two important ways. First, leading logical pluralists have built into their view the thesis that logic is normative,[1] and second, the normativity of logic has often been used in an argument against pluralism.[2] We are sympathetic to logical pluralism, but we don't think that logic is normative. In this paper we show how to reject the normativity of logic, and develop three versions of logical pluralism that do not require logic to be normative.[3]

[1] Beall and Restall (2006); Field (2009b).

[2] Priest (2006); Read (2006); Keefe (2014); Caret (2016); Kouri Kissel and Shapiro (2017); Steinberger (2017b).

[3] One of the authors has argued for a version of logical pluralism in (Russell 2008). We note here for clarity that that was a different kind of pluralism from the case-based pluralism which is the focus of the present paper. Still, we also hold that Russell's truth-bearer-relative pluralism is compatible with logic's not being normative.

Since universities and committees can sometimes use this information, the authors would like to note that §2 and §3 were written by Russell, and §4 and §5 by Blake-Turner.

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2 Normativity in logic

A broad spectrum of philosophers have thought that logic is normative, and there are two central things that they have meant by this. The first is that logic simply *is* a theory of what we ought to believe (perhaps given what we already believe) and how we ought to reason (where to reason is to change one's beliefs.)

> "In Logic we do not want to know how the understanding is and thinks, and how it has hitherto proceeded in thinking, but how it ought to proceed in thinking." (Kant 1800, 4)

> "...logic is a normative subject: it is supposed to provide an account of correct reasoning." (Priest 1979, 279)

If this is what a logic is supposed to do, then to the extent that a theory fails to include such normative claims it is not a logic, and to the extent that a theory does include such claims, it is normative. Hence any logic will be normative. On this first view, logic is normative in the same way that (normative) ethics is normative: just as a key characteristic of an ethical theory is that it tell us what we ought to do, so a key characteristic of a logic is that it tell us what we ought to believe.

A complication for this view is that formulations of modern logics—say first-order classical logic, or Strong Kleene logic—don't usually contain words like "ought", "believe" and "reason". Rather, they talk about sets of symbols, interpretations, models, and the relation of logical consequence. If a logic is supposed to tell us what we ought to believe and how we ought to reason, we will need to explain why the words "belief", "ought", and "reason" are absent from logical theories.

But there is a second, less explicit, way in which logic is thought to be normative. According to it, even if logic is not really about what we ought to believe and how we ought to reason, it has significant consequences for those things all on its own. If modus ponens is valid, for example, this might be thought to have the consequence that if you believe both P and $P \rightarrow Q$, then you *ought* to believe Q as well. Similarly, some logics tell us that any set that contains P and $\neg P$ is unsatisfiable, i.e. there is no way for them both to be true. From this, one might think, it follows that if one believes both P and $\neg P$, then one ought to give up one of these beliefs.[4]

> "Rules for asserting, thinking, judging, inferring, follow from the laws of truth. And thus one can very well speak of laws of thought too." (Frege 1918, pp. 289–290)[5]

[4] We use these two examples for the sake of simple illustrations, but we note that defenders of logic's normativity generally reject these simple accounts and instead endorse much more complicated accounts of the link between entailment and the norms of belief. For discussion see MacFarlane (2004), Field (2009a), Celani (2015), and Steinberger (2017a), but also our Sects. 3.2 and 3.3.

[5] Frege immediately goes on to consider and reject the idea that this means that logic is itself a theory of thought and reasoning (the first view considered above): "But there is an imminent danger here of mixing different things up. Perhaps the expression "law of thought" is interpreted by analogy with "law of nature" and the generalisation of thinking as a mental occurrence is meant by it. A law of thought in this sense would be a psychological law. And so one might come to believe that logic deals with the mental process of thinking and the psychological laws in accordance with which it takes place. This would be a misunderstanding of

> "...logical consequence is normative. In an important sense, *if* an argument is valid, *then* you somehow go wrong if you accept the premises but reject the conclusion." (Beall and Restall 2006, (our emphasis))

The principle sometimes called *Hume's Law* says that any set of premises which entails a normative conclusion must itself contain a normative premise. On the second view outlined above, claims about logical consequence have normative consequences all on their own, i.e. the arguments which take them as premises and have normative conclusions are not enthymemes. Hence it follows by Hume's Law that the claims about logical consequence are themselves normative. Since the job of a logic is to tell us when the relation of logical consequence holds between premise sets and conclusions, the job of a logic is again to tell us about something normative.

Call the first approach—the one on which the job of a logic is to give us an account of good reasoning—the Reasoning View. Call the second approach—the one on which the job of a logic is to tell us about logical consequence, and it turns out that any such theory will have normative consequences, and so itself be normative—the Consequences View. These two views are different but compatible. Assuming Hume's Law—as we will throughout this paper—the Reasoning View entails the Consequences View (assuming every theory has itself as a consequence) but one might accept the Consequences View without accepting the Reasoning View, provided one considered logical consequence and reasoning to be different—though related—subjects.

In this paper we will call both these views "logical normativism." We think both views are false.

3 Logic as a descriptive theory

In this section we are going to explain why we don't think that logic is normative. In doing so we aim to establish three theses:

1. Theories in logic are descriptive.
2. Theories in logic are not theories of how we ought to reason.
3. The normative consequences of logics stem not from a normativity inherent in the logics themselves, but from widely accepted background norms which, *together with* the descriptive facts provided by the logics, have consequences for what we ought to believe.

Theories in logic are descriptive theories. What they aim to describe are patterns of truth-preservation over sentences—usually by delivering a set of claims about which argument forms are valid and which are not.[6] For example, the following claims are included in the set delivered by many logical theories that we call classical:

Footnote 5 continued
the task of logic, for truth has not been given the place which is its due here." For Frege, laws of logic are laws of truth which have normative consequences for how we ought to reason.

[6] We think it is clear that logics deliver results about which arguments are *not* valid. It's even part of what it is for a logic to be a paraconsistent logic that it say that explosion ($\phi, \neg\phi \vDash \psi$) is not valid. Still, for some circumstances we might wish to distinguish a logic's positive validity claims from its negative ones, and for these purposes we suggest speaking of the positive and negative aspects of the logic.

$$\begin{array}{lcl} A, A \rightarrow B & \models & B \\ & \models & \neg(A \wedge \neg A) \\ & \not\models & A \wedge \neg A \\ A \rightarrow B, B & \not\models & A \end{array}$$

Call such sentences *E-sentences*—E for "entailment." Logical theories generate their E-sentences in a variety of ways—for example, they might take a model-theoretic or a proof-theoretic approach—and they often make use of various auxiliary assumptions, and posit the existence of additional objects and properties. The status of these—both the assumptions and the posits—will vary and different logicians may have different (or no) views as to their status. As an illustration, consider two assumptions made by the usual Tarski-derived model-theories for first-order classical logic: (i) that all domains of quantification are non-empty, and (ii) that all simple predicates have determinate extensions (the complements of their anti-extensions.) The former is presumably an idealisation, in the sense that it is an assumption which sacrifices some accuracy for the sake of simplicity. Few fans of classical logic[7] believe that the resulting E-sentence $\models \exists x(x = x)$ is a genuine logical truth or that there is a principled reason to ignore empty domains *other than* the way it provides us with a cleaner logic. It is an idealisation on a par with point particles or frictionless planes in physical theories.

On the other hand, the status of the second assumption—which says that all predicates are determinate—is much more controversial. Here one's opinion is likely to vary with one's philosophical views. If one believes that natural language predicates often have indeterminate extensions, then one might regard the assumption as another idealisation. But if one is an epistemicist about vagueness, one might believe that natural language predicates always have determinate extensions (even if we are sometimes ignorant of these) and hence that this part of the theory of classical logic is straightforwardly true.[8] Hence one can endorse a theory of classical logic while taking various stances about what it commits one to beyond the set of E-sentences that it generates.

A similar point can be made about the *objects* that are referred to in logical theories. The most famous theories for modal logics take models to contain a set of points—often referred to as "possible worlds"—at which the various sentences of the formal language are said to be true or false. This language provides a vivid image that humans frequently find helpful for thinking about modality.[9] Logicians, however, are often quick to reject the idea that the references to such sets of points in their theories commit them to the existence of anything exotic. (Not that mathematicans are in general leery of exotic objects.) But there are other objects posited by logical theories that one might take a more realist stance on. Strong Kleene logic is often presented via a theory on which sentences may have one of three possible truth-statuses: true, false and neither. Though it is possible to take an instrumental stance on these statuses, one

[7] Necessitists (as in Williamson (2013)) are one exception here.

[8] Williamson (1994).

[9] Of course, vivid images can also lead us astray.

might instead accept Strong Kleene logic precisely because one thinks that these are all and only the statuses for sentences of natural language.[10]

In sum, we can think of logical theories as having two parts. First, they possess a set of E-sentences. Second, they possess a superstructure. A logic's superstructure generates the logic's E-sentences. The superstructure will make assumptions that may be true or merely useful, and it will make reference to objects and properties (real or ideal) in order to generate the logic's E-sentences. Two different logical theories may generate the same set of E-sentences, either because they are trivial notational variants of each other, or because substantially different theories turn out to generate the same results, e.g. the model theory for S5 that makes use of a transitive, reflexive and symmetric accessibility relation, and the model theory for S5 that does without an accessibility relation entirely. In either case we will say that the two theories generate the same logic (and sometimes more carelessly that the two theories *are* the same logic).

There are many instances in which the question of whether or not an argument form is valid *according to one of these logical theories* is straightforward. For example, the model theories above determine (i) a set of models and (ii) what the truth-status of a sentence is in one of those models. Assume that an argument is valid according to model theory X iff there is no X-model in which every premise is true and the conclusion is not true. Then we are often able to demonstrate via model-theoretic proof or counterexample that an argument is or isn't valid according to X. For example, explosion is valid in classical logic, but not in the paraconsistent logic LP.

Still, classical logic is best thought of as an attempt to capture validity simpliciter, not merely validity-according-to classical logic. It is uncontroversial that explosion is valid according to classical logic, and not valid according to LP. Yet it remains controversial whether explosion is valid—that is, valid *tout court*. What would it take to settle the question? Two things seem clear: (i) if the superstructure for some theory of classical logic is accurate, then explosion is valid. And (ii) if the superstructure for some theory of LP is accurate then explosion is not valid. And more generally, if we could determine that the superstructure of a theory for a logic V was accurate, and explosion was among V's E-sentences, that would tell us whether explosion was valid tout court.[11] One reason to think that logic is descriptive then is that the sort of facts that will establish that a logical theory—such as LP or classical logic or Strong Kleene logic—is accurate are themselves descriptive facts.

Suppose, for example, that a superstructure for Strong Kleene logic is accurate. To get clearer on what that would mean, it will help to look at a particular theory of that logic.

1. List of symbols:
   (a) Sentence letters: P_1, P_2, P_3 ... etc.
   (b) Connectives: $\neg$, $\wedge$, $\rightarrow$, $\vee$

[10] More exotically, one could be a realist about the 'worlds' referred to in the logic for modal model theory, and an instrumentalist about the 'neither' truth-status, i.e. if one thought that Strong Kleene logic delivered the right set of E-sentences, even though in fact *true* and *false* are the only statuses available to sentences.

[11] As we will go on to explain, we mean *accurate* to be slightly stronger than just true. To say that the theory is accurate is to say that what it says is true and also that it is not missing out any important aspect of the world that would go into determining logical consequence.

(c) Parentheses: (,)

2. Formation Rules:
   (a) All sentence letters are wffs.
   (b) if ϕ and ψ are wffs then $\ulcorner\neg\phi\urcorner$, $\ulcorner(\phi \wedge \psi)\urcorner$, $\ulcorner(\phi \rightarrow \psi)\urcorner$ and $\ulcorner(\phi \vee \psi)\urcorner$ are wffs.
3. An interpretation I is an assignment of exactly one of the truth-statuses *true*, *false* and *neither* to every sentence letter in the language.
4. A valuation is an extension of that interpretation which obeys the following rules:
   (a) for all sentences letters ϕ, $V_I(\phi) = I(\phi)$
   (b) $V_I(\neg\phi) = \begin{cases} true & \text{if } V_I(\phi) = false \\ false & \text{if } V_I(\phi) = true \\ neither & \text{otherwise} \end{cases}$
   (c) $V_I(\phi \wedge \psi) = \begin{cases} true & \text{if } V_I(\phi) \text{ and } V_I(\psi) = true \\ false & \text{if } V_I(\phi) \text{ or } V_I(\psi) = false \\ neither & \text{otherwise} \end{cases}$
   (d) $V_I(\phi \rightarrow \psi) = \begin{cases} true & \text{if } V_I(\phi) = false \text{ or } V_I(\psi) = true \\ false & \text{if } V_I(\phi) = true \text{ and } V_I(\psi) = false \\ neither & \text{otherwise} \end{cases}$
   (e) $V_I(\phi \vee \psi) = \begin{cases} true & \text{if } V_I(\phi) \text{ or } V_I(\psi) = true \\ false & \text{if } V_I(\phi) \text{ and } V_I(\phi) = false \\ neither & \text{otherwise} \end{cases}$
5. A sentence ϕ is a logical consequence of a set of sentences Γ just in case for all interpretations I such that $V_I(\gamma)$ = true for all $\gamma \in \Gamma$, $V_I(\phi)$ = true as well.

This superstructure is one way to generate the set of E-sentences for *Strong Kleene logic*. Now suppose that this superstructure is accurate. By this we mean, roughly, that it is true, and that it doesn't leave anything important out. Part of what is required is that the assumptions the theory makes—such as that atomic sentences have exactly one of three truth-statuses—are true of natural languages and that any objects and properties posited by the theory—such as sets and truth-statuses—exist and behave as the theory says they do. And part of what is required is that the theory is not leaving anything else true out which would change the set of E-sentences.

The non-logical expressions specified in clause (1a) (here P_1, P_2 …etc.) are uninterpreted sentences. Of course, natural language expressions are interpreted. We usually say that a sentence in the formal language (such as $\neg P_1 \rightarrow P_2$) merely represents the logical form of some natural language sentences (say, *if Alice isn't tall then Alice is strong* and *if Bob isn't clever then Bob is good* etc.) Hence accuracy for clause (1) does not require that natural language sentences be uninterpreted, but only that there *are* (interpreted) natural language sentences which have the logical forms specified.[12]

[12] It is perhaps even easier to see what this requires if the formal language has a more complex syntax: for example, if it has relational predicates and individual constants, then accuracy requires that the interpreted language have relational predicates and individual constants.

Most natural languages contain a lot more expressions. For example, there might be names, predicates and functors. To the extent that clause (1) fails to posit these, we can regard it as inaccurate due to being incomplete—the theory is merely partial.

Clause (1b) lists logical expressions whose truth-conditions are given later. For these clauses to be accurate requires both that the natural language have connectives which behave syntactically as these connectives do, and also that the truth-conditions of complex sentences containing the connectives are determined from the truth-conditions of the simple sentential parts as the theory says they are. If there is no material conditional in the natural language, for example, then clause (4d) is inaccurate.

Clause (2) specifies the syntax of the language and, for example, claims that whenever something is a sentence, so too is the result of writing a negation in front of that sentence. For a very strict interpretation of accuracy, we might require that the natural language form negations in exactly the same way (e.g. by putting the word for "¬" *at the beginning* of the sentence to be negated) but we might also think that such strict accuracy is not really crucial, perhaps because theories which are inaccurate in this regard—but otherwise accurate—are mere notational variants of accurate theories.

Clause (3) is important. It will be accurate if *true*, *false*, and *neither* are all and only the possible truth-statuses for the atomic sentences of the language. And indeed, thinking that this is the case—for reasons related to vagueness, reference failure and even future-contingents—has been one of the main motivations for adopting a strong Kleene logic.

Clause (4) we have discussed already, but it is worth emphasising that its content is a locus of controversy. If one thinks that a natural language conditional composed of an antecedent and a consequent that both possess the *neither* truth-status possesses the truth-status *true*—as Łukasiewicz did—then one thinks that clause 4d) is false, and hence that the logic is inaccurate. If one thinks that natural language contains another conditional—in addition to the one with the truth-conditions given in (4d)—with different truth-conditions which affect the validity of arguments which contain it—then one thinks that clause (4) is partial and hence inaccurate. And if one thinks that natural languages contain additional logical constants like quantifiers, modal operators, and indexicals, then one will also regard clause (4) quite generally as inaccurate due to partiality.

We might decide to count inaccuracy due to being partial as a less serious fault than inaccuracy due to falsity (much as we regard lack of completeness in a proof system as a less serious fault than lack of soundness.) The sentential version of the right logic is prima facie less problematic than the first-order version of the wrong one—at least its positive E-sentences are true, even if it misses some out.

Clause (5) gives the definition of logical consequence for the theory, and it allows us to extract a set of E-sentences from the rest of the superstructure. One might expect there to be less controversy here, at least when it comes to model-theoretic approaches to consequence. Other kinds of superstructure (e.g. those cashed out in proof theory) might provide different ways of identifying the extension of $\models$, but these need not be regarded as in conflict with (5) but rather as fixing the extension of $\models$ via a different definition.

Summarizing: the key things we would have to determine in order to determine the accuracy of (1)–(5) are (i) whether (1) and (2) accurately reflect the structure

of the language, (ii) whether (3) accurately reflects the possible interpretations for the simplest, non-logical, parts of the language, and (iii) whether the truth-statuses of complex sentences are indeed functions of the truth-statuses of their parts as expressed in clause (4).

Our own suspicion is that the Strong Kleene superstructure detailed above is likely to fall short of accuracy. An accurate superstructure would need to replace clauses (1)–(4) with more accurate ones. What's important for our purposes is that deciding between each of these clauses and more accurate ones is not a normative matter, but a descriptive one. The accuracy of clauses (1) and (2) is a matter of what sorts of expressions languages contain. Similarly, settling whether (3) and (4) are accurate requires determining whether the truth statuses of *true*, *false* and *neither* are all and only the statuses a sentence may have, as well as detailing how the statuses of complex expressions are determined by the statuses of their parts. These too are descriptive matters.

If the superstructure of Strong Kleene logic is accurate, then the set of E-sentences it gives us is the right one.[13] Assuming this holds for the perfectly accurate superstructure, and not just for Strong Kleene logic (and it does), determining the correct set of E-sentences can be done by determining a number of descriptive claims—the ones which make up an accurate superstructure for the correct logic.[14] And hence—since it follows from descriptive claims alone—the correct set of E-sentences is itself descriptive.

[13] Do we not also require that clause (5) is correct? The definition of entailment in clause (5) is quite standard, but there are a few alternatives that might lead to different results. For example, some logics define entailment in terms of degrees of truth. If one of those logics turned out to be correct, clause (5) would probably need to be revised, and the extension of the entailment relation might then be different. So one might think that clauses 1–4 were not sufficient to determine the entailment relation after all.
We don't think that. We think this would be a case in which 'entailment' means something slightly different. Assuming that 'entailment' expresses the relation defined in clause (5), a correct superstructure does determine the right logic. We might be wrong about that meaning-assumption, but even if we are, given the correct definition of entailment, (i) the accurate superstructure will determine the correct logic relative to that, and (ii) which superstructure is accurate will be a descriptive matter. That's because the sensible alternatives to clause 5 simply bring in other descriptive features—such as degrees of truth—and because logical theories are designed to determine an entailment relation. Hence whatever entailment is (assuming it is the kind of relation logicians study) an accurate superstructure will determine the correct entailment relation.

[14] Here our argument rests on the assumption that an accurate superstructure exists (even though it is likely that we have not discovered it yet.) We suppose there are two ways this assumption might fail. One, there might be some aspect of the way that the syntax and semantics of a language determine a relation of logical consequence that is somehow ineffable—perhaps it uses concepts that humans can never acquire—and so we simply cannot formulate an accurate theory of it. In this case the descriptive facts still determine the correct logic but we can't write them all down in our logical theory. And two, perhaps the relation of logical consequence is sui generis or brute; it is not determined by the syntax and semantics of the language but rather blooms randomly on arguments, with no explanation, so that the best a logic can hope to do is give the correct set of E-sentences, not explain them in terms of an accurate superstructure. For now, we are content to add the negation of this view as a premise in our arguments.

3.1 Logic isn't about reasoning

It is fairly clear, once we examine a particular logic like Strong Kleene, that it is not explicitly a theory about how we ought to reason, but rather one about truth-preservation over truth-bearers and sets of truth-bearers—it is a theory of entailment. But those who support the Consequences View of the normativity of logic have a fallback position: they can claim that E-sentences have consequences for how we ought to reason. For example, they might make the very simple claim that if modus ponens is true, then if someone believes P and they believe $P \rightarrow Q$, then they ought to believe Q as well.

Harman (1986) argued against this view and although the defenders of the consequence view have gone on to develop more sophisticated versions which avoid Harman's counterexamples, we think that the original problems remain of interest. One of the problems Harman points out is that that if you believe P and you believe that $P \rightarrow Q$, and Q itself turns out to be inconsistent, then it is false that you ought to believe Q.[15]

Similarly, if you believe every sentence in the book you have just finished writing, but also, quite reasonably, believe that you are not infallible and will have made at least one mistake in the book—hence that one of the sentences in your book is false—then you have inconsistent beliefs. But it is far from clear that you are being irrational, reasoning badly, or doing something that you ought not to. You have no evidence that would support giving up any of the beliefs in your book, but the only other option would be to give up the belief that at least one of those sentences is false—and you shouldn't give that up, because that would be reckless hubris.

Later defenders of the normativity of logic have developed more sophisticated accounts of the connection between logic and reasoning that avoid the letter of these particular objections[16] but we think Harman's examples are enough to make two things plausible. First, logics don't really have the right kind of structure to be theories of reasoning. For example, most logics are *monotonic* in the sense that their consequence relation satisfies Thinning: if $\Gamma \vDash \phi$, then $\Gamma, \psi \vDash \phi$ too. If the entailment relation were a guide to what you ought to believe, that would seem to suggest that learning new things can only add to the things you believe—it won't ever give you a reason to give up a belief. But genuine reasoning should tell you when you ought to abandon beliefs as well as when you ought to add new ones. The entailment relation isn't set up to give this kind of information: it tells you what follows logically from a set of premises, but in case of conflict it does not even try to tell you which of the conflicting beliefs you should drop. And second, this isn't really surprising, once we have distinguished between the study of entailment and the study of reasoning: modern logic focuses on the former, not on the latter; it is neither a problem nor a surprise that it doesn't give us a good theory of the latter.

[15] For simplicity here we ignore complications related to dialetheism.

[16] See especially MacFarlane (2004); Field (2009a) and criticisms in Celani (2013, 2015).

3.2 Normative force without normative logic

But if logic isn't normative, what are we to make of the widespread intuition that E-sentences do have normative consequences for belief and reasoning? We think we can account for such normative consequences of logic without supposing that logic is itself normative. Descriptive theories—such as theories in physics and logic—have normative consequences when taken alongside other theories which *are* normative. In particular, if there is a widely accepted background norm according to which one ought to believe only *true* things, then descriptive theories—which purport to tell us truths—will, in conjunction with this background norm, entail normative consequences. If this is right, then we would expect purely descriptive theories like those in physics to also have normative consequences in this way. And they do. For example, a famous equation in classical mechanics tells us that $F = ma$ and this has the consequence that if you believe that $F = 24\text{N}$ then you should not believe that $ma \neq 24\text{N}$ too. This normative consequence does not come from physics alone, but rather from physics plus the normative premise that one ought only to believe true things. The same goes for the entailment claims of logic. Perhaps this has been harder to see in the case of logic because of the more abstract nature of the things being described (e.g. truth and sentence-schemata, as opposed to physical objects and their properties—some have even claimed that logic has no subject matter) but there is no need to impute normativity to logic itself, in order to explain its importance to what we ought to think. Descriptive theories frequently have important consequences for what we ought to think.

3.3 Proof, truth, and bridges

Above we argued that neither the Reasoning, nor the Consequences Views of the normativity of logic are correct. In this section we consider three possible responses to our arguments.

The first is that while we don't normally employ normative terms in defining model-theoretic consequence, it might be easier to think of the *proof* rules for a logic as having normative force.[17] This is perhaps because they come with at least an implicit "if you have (premises) then you *may* infer (conclusion)." (We write 'may' rather than 'ought to' because you have not violated a proof rule if you do not infer its conclusion from its premises).

We agree that proof rules might naturally be thought of, or explained to someone, using such normative expressions, but we also think that is another example of the derivative normative force that we discussed in Sect. 3.2. If one may infer P given that one has $P \wedge Q$, then that is not because the logic itself is normative, but because there is a background norm in place that says that one may infer the consequences of true things. Teachers also introduce rules for solving equations in algebra, or physical mechanics, by saying things like 'if you have a fraction over n on one side, then you may/should/need to multiply both sides by n.' But neither algebra nor mechanics are normative. Rather, we speak this way because of a background normative assumption,

[17] We would like to thank Reviewer 1 for this suggestion.

perhaps that one may/should/needs to infer the algebraic or mechanical consequences of truths.

The second objection we will consider is that it is not because logic is about how we ought to reason that it is normative, but rather because the correct definition of logic employs some *other* normative expression.[18] The most plausible candidate here is perhaps *truth*, given the ubiquity of defining consequence in terms of truth across models (indeed, that has been our own approach) and the fact that truth seems like something which might plausibly be taken to be normative.

However, "truth is normative" is true on some construals, and not on others, and we hold that it is not true on the construals on which its truth would affect our thesis. For example, someone might want to say that truth is normative because truth is a goal, e.g. something that we aim at or should aim at. Many people aim to have true beliefs and we tend to think that (under some precisification) they are right to. So in one sense, truth is itself a norm. We could even express this with the sentence "Truth is normative."

But then, many people aim to have lots of money—they might even be right to do this, given that money translates into power, and power can be used to do good in the world. So there is a sense in which we might think that having lots of money is normative. But it isn't normative in the sense of the normative/descriptive distinction in play in this paper: to say that someone has lots of money is not yet to say anything beyond the descriptive; it can be determined by purely descriptive facts (like facts about their bank accounts and pockets) and it is not yet to say anything about what we are permitted or ought to do, or to evaluate anything as good or bad. In the sense which is in play in the present debate, having lots of money is descriptive and not normative.[19]

But a second reason one might think that truth is normative is that it has consequences concerning *correctness*. For example, if *snow is white* is true, then to utter *snow is white* is to utter something correct, while to reject that sentence is to make a certain kind of mistake. So the argument would run: claims about correctness are normative claims, claims about truth have claims about correctness as consequences and so (invoking Hume's law), claims about truth are themselves normative.

This is indeed the sense of normative under dispute in this paper, but this time we deny that the argument is sound, just as we denied that the isomorphic argument was sound when it was used to support the Consequences View earlier, and for the isomorphic reason: the argument is an enthymeme and we are able to draw the normative

[18] We would like to thank Reviewer 2 for this suggestion.

[19] It might be objected that truth and having lots of money are not analogous in the sense that truth is arguably a constitutive goal of belief, whereas having lots of money is not a constitutive goal of working. We'd like to remain neutral here on these issues about constitutivism, but we note it doesn't follow from aiming at Y's being constitutive of X that Y is normative. For instance, it might be constitutive of being an aspiring circus performer that one aim at juggling, but this doesn't make juggling normative in the relevant sense—it's just a goal like making lots of money. Of course, if one already has reason to X, then one might thereby have derivative reason to aim at Y in such a case. But this is just the kind of derivative normative force that we already discussed in Sect. 3.2. We thank Reviewer 2 for pressing us to think more carefully about this issue.

conclusion 'To utter *snow is white* is correct' only because of a suppressed normative premise, namely: uttering true things is correct.[20,21]

The third objection that we will consider here is intended to be a weakening of the Consequences View so as to escape our argument: the idea is that rather than having normative consequences all on its own, logic constrains how we ought to reason in conjunction with sophisticated bridge principles. It seems wrong to say—so this response runs—that the validity of modus ponens is completely irrelevant to what we ought to believe and how we ought to reason, and we can take the best contemporary defences of the normativity of logic, e.g. MacFarlane (2004) and Field (2009a), to be proposing bridge principles between logic and the normative. These principles seek to articulate the complicated constraints that logic puts on correct reasoning, and they take into account Harman's objections to simpler articulations of the connection between logic and reasoning.

Our response to this objection is that it depends on how you think of the bridge principles: either they are entailed by true claims about logical consequence alone, or they are not. If they are, then by Hume's Law the original entailment claims must be normative—and hence subject to our arguments at the beginning of Sect. 3, that the true entailment claims follow from descriptive facts alone, and hence *can't* be normative. On the other hand, if the bridge principles do not follow from the entailment claims alone, then the view collapses into a version of what we called "normative force without normative logic" in Sect. 3.2. That is, logic is just as normative as as arithmetic and physics: in conjunction with normative principles, it has normative consequences. We agree that, if modus ponens is true, then this constrains how you ought to think and reason. Just as we think that, if ZFC or Special Relativity is true, this constrains how you ought to think and reason. It makes as much sense to develop special bridge principles for contemporary mathematics and physics as it does to develop special bridge principles for logic.

4 The current state of pluralism and normativity

Despite the preceding, logical normativism is a popular view, especially in debates about logical pluralism. For instance, Colin Caret has recently argued that "the pluralist cannot divorce the normativity of logic from her view" (Caret 2016, p. 7). We deny this. In particular, we argue both that pluralism and normativism can come apart, and that they should. There are two reasons for the second claim.

First, given our arguments in Sect. 3, no one needs to embrace the thesis that logic is normative. If we're correct, excising normativism from logical pluralism is the only

[20] Dag Westerståhl contacted us by email and reminded us that there is a similar debate in the philosophy of language, concerning whether meaning is normative. e.g. Kripke (1982); Boghossian (1989); Fodor (1987); Glüer and Wikforss (2018).

[21] One might argue that if "Uttering true things is correct" is true, then that's good reason to think that truth is normative. But the truth of the claim doesn't establish truth's normativity in the sense that's at issue in this paper, namely of having consequences for correct behavior all on its own. Rather "Uttering true things is correct" is a normative claim in the same way that "Giving to the poor is correct" is a normative claim: both behaviors are worth aiming at; they make for good norms. We thank Reviewer 2 for inviting us to clarify this point.

way to save the latter view from a commitment that is superfluous at best, and false at worst. Second, logic's normative status has been at the center of the most influential criticism of logical pluralism: the normativity objection.[22] Roughly, the challenge is for the pluralist to explain how all of her endorsed logics can successfully impose normative constraints on deliberators. If the pluralist rejects the normative status of logic, she needn't be fazed by this difficult problem.[23]

Pluralists would therefore be in a bind if they could not divorce normativism from their view. But we argue that they can do so. In Sect. 5, we'll illustrate this by setting out three non-normativist versions of the most popular form of logical pluralism, due to Beall and Restall. Before doing so, we need to get clear on their original view.

Like all pluralists, Beall and Restall hold that there is more than one, distinct relation of logical consequence. They claim that these relations arise due to different specifications of the core notion of logical consequence, which is captured by the GENERALIZED TARSKI THESIS:[24]

> (GTT) An argument is valid$_x$ iff in every case$_x$ in which the premises are true, so is the conclusion.

Since there are different acceptable ways of sharpening the notion of a case in (GTT) and since these result in different logics, pluralism ensues.

Importantly, not just any kind of case can be plugged into (GTT) to yield a genuine consequence relation. The pluralist needs to preclude notions of case that yield disunified or degenerate consequence relations. For instance, for some language $\mathcal{L}$ let a *Trivcase* be a model with the stipulation that, in each *Trivcase*, $\bot$ is true, where $\bot$ entails every sentence of $\mathcal{L}$—for example, a contradiction in classical logic. Call the resulting notion of consequence T, for trivial: any sentence is a consequence$_T$ of any set of premises whatsoever.[25] The pluralist must impose constraints on the types of cases that can be admitted in order to preclude (GTT)'s generating unpalatable consequence relations such as T.

According to Beall and Restall, there are three constraints on admissible cases.[26] A type of case is admissible only if it yields a consequence relation that is *necessary*, *formal*, and *normative*. The latter condition embodies Beall and Restall's commitment to logical normativism; the former two conditions require a little more explanation. Let Γ and ϕ be the premises and conclusion respectively of a valid argument. Then a consequence relation is necessary just in case it's necessary that: if every member of Γ is true, then ϕ is true.[27] A consequence relation is formal just in case it holds between

[22] Priest (2006); Read (2006); Keefe (2014); Caret (2016); Kouri Kissel and Shapiro (2017); Steinberger (2017b).

[23] This is in fact a more complicated issue than we can address here. See Russell (2017) for discussion.

[24] Beall and Restall (2006, p. 29).

[25] *Proof.* Let Γ and ϕ be an arbitrary set of sentences and an arbitrary sentence of $\mathcal{L}$, respectively. Let t be an arbitrary *Trivcase* such that each $\gamma \in \Gamma$ is true in t. By assumption, $\bot$ is true in t and $\bot \vDash_T \psi$ for any sentence ψ. In particular, $\bot \vDash_T \phi$. So ϕ is true in t.

[26] Beall and Restall (2006, pp. 14–23).

[27] As one of us has argued, it's not in fact clear that consequence relations are necessary Russell (2012). Here, we follow Beall and Restall and grant the necessity constraint.

Γ and ϕ in virtue of their form, not their content.[28] Beall and Restall argue that all three constraints are met by at least the following specifications of case: Tarskian first-order models, situations, and worlds in Kripke models. Thus they endorse at least the following consequence relations: classical, relevant, and intuitionistic.[29]

It's important to note that *Trivcase*—the notion of case that yields the trivial consequence relation—meets both the necessity and the formality constraints; the normativity constraint is what rules it out. The thought is that one doesn't make a *mistake* when one reasons in ways that violate consequence$_T$, for example by failing to draw a conclusion that one should have drawn by the lights of the trivial consequence relation. Hence consequence$_T$ fails to meet the normativity constraint and *Trivcases* are inadmissible.

This explains why Caret insists that the logical pluralist must embrace logical normativism. Without doing so, he argues, the pluralist will be unable to rule out unpalatable relations of logical consequence, like the trivial one. Normativism is thus part of what "provides the pluralist with much-needed criteria of admissibility".[30] Despite this, we'll now argue that the logical pluralist can in fact embrace non-normativism. She can either dismiss the need for another criterion of admissibility in addition to necessity and formality, or replace the normativity constraint with a non-normativist criterion.

5 Non-normativist pluralism

To illustrate non-normativist pluralism, we're going to set out three versions of Beall and Restall's view. Each drops the normativity constraint while keeping the necessity and formality requirements. *Flat-footed logical pluralism* leaves matters there. *Pragmatic logical pluralism* adds a pragmatic constraint on admissible cases. *Telic logical pluralism* adds an epistemic constraint instead. Let's consider each in turn.

The flat-footed pluralist adopts only the necessity and formality constraints on admissible cases, and embraces the ensuing plethora of consequence relations that (GTT) generates. Such a pluralist admits that even *Trivcases* generate a genuine relation of logical consequence, since such cases meet the constraints. She concedes that we never deploy the trivial consequence relation, but insists that just because we don't put a consequence relation to use doesn't undermine its claim to being genuine. After all, she urges, logical consequence relations just *are* those relations that (GTT) yields

[28] For present purposes, we can leave this gloss of the formality constraint intuitive. For more, see Etchemendy (1999), MacFarlane (2009).

[29] There are many different relevant logics. Here we mean the logic that Beall and Restall obtain by substituting situations for cases in (GTT), Beall and Restall (2006).

[30] Caret (2016, p. 7).

when constrained by necessity and formality.[31] The flat-footed pluralist thus endorses Carnap's slogan that "*In logic, there are no morals*".[32]

Pragmatic pluralism goes further than flat-footed pluralism in adopting an additional pragmatic constraint on admissible cases. Roughly, as well as necessity and formality, a case must deliver a relation of logical consequence that is *useful*.[33] There are many questions that need answering here, not least of which is: what does useful amount to? But, even leaving this notion undeveloped, the pragmatic pluralist can presumably rule out *Trivcases*, since the trivial consequence relation is not useful in any obvious sense.

The normativist pluralist might object at this point: isn't the notion of *usefulness* normative, and so isn't pragmatic pluralism committed to the thesis that logic is normative? We agree that usefulness is a normative notion, but in a very different sense from that claimed for logic by logical normativists. As explained in Sect. 2, normativists hold that logic is about reasoning, or that logical consequence has normative upshot all by itself. But this is a far cry from being useful in a pragmatic sense. To see that usefulness and logic's normativity in the normativist's sense are different, notice that they can come apart. Suppose, for instance, that the logic of paradox is the one true logic (Priest 2006). Since modus ponens is not LP-valid, logic can make no normative claims on reasoners who fail to draw the conclusions it recommends. It might nonetheless be useful to draw such conclusions. Thus there can be pragmatic demands where there are no normative demands in the stricter sense that normativists have in mind.

We sketch the foregoing pluralisms to show that a non-normativist pluralist has a variety of options. But these views have serious limitations. Flat-footed pluralism embraces a panoply of strange and unpalatable consequence relations. Pragmatic pluralism avoids that, but at the cost of tying the metaphysics of logic to what is useful for humans (or perhaps reasoners), a result that many will find unattractive. A more promising variety of non-normativist pluralism avoids these problems. Like pragmatic pluralism, telic logical pluralism rules out unwanted cases by replacing Beall and Restall's normativity constraint. But rather than replacing it with a parochial connection to human interests, telic pluralism replaces it with an epistemic constraint: logical consequence relations are best suited to meeting epistemic goals. The epistemic constraint rules out *Trivcases* since there are no epistemic goals that the trivial consequence relation is most conducive to achieving.

What exactly are epistemic goals? They're the *raisons d'être* of epistemic activity. They are what belief formation, reasoning, giving testimony, etc., *aim at*. The easiest way to flesh this out is by ostension. Here are three candidate epistemic goals: truth, relevance, and demonstrability. Now consider the practice of giving testimony. The

[31] Perhaps the flat-footed pluralist could impose other constraints on genuine consequence relations, constraints that nonetheless fall short of those adopted by pragmatic and telic pluralists. For instance, she might insist on an *actualist* constraint, such that admissible notions of case must count the actual world as a case. How much this would constrain consequence relations depends on what the correct metaphysics is. If there are no actual true contradictions, for example, then paraconsistent consequence relations will be ruled out. For discussion see Beall and Restall (2006, pp. 82–83).

[32] Carnap (1937, p. 52, original emphasis).

[33] There's a distinctly Carnapian flavor about this, too. But Carnap stopped short of letting pragmatic considerations constrain what counts as a logic in the first place.

success criteria for testifying depend on which epistemic goals the testimony aims at. An example will help make this clear:

> *Ice Cream*. Ali and Beata are arguing about whether to order chocolate or vanilla ice cream. Three people overhear their conversation and offer testimony. Tariq testifies truly that *either Goldbach's conjecture is true or Goldbach's conjecture is false*. Raúl testifies truly that *the diner's chocolate ice cream is better than its vanilla*. And Delia testifies truly that *the diner's vanilla ice cream is cheaper than the chocolate*, and she produces a price list to demonstrate her claim.

Whose testimony is best depends on the epistemic goal or goals.[34] Suppose the goal is truth. Then, since it's stipulated that they are all true, the testimonies are all equally good. Suppose the goal is relevance. Then Tariq's truthful, irrelevant testimony clearly falls down. Raúl's testimony does well in achieving this goal, as does Delia's (though hers may turn out to be irrelevant if money is no issue). For obvious reasons, Delia's testimony does best with respect to the goal of demonstrability.[35] Moreover, we can ask whose testimony is better with respect to a combination of epistemic goals. Take the combination, C, of all three goals: *truth*, *relevance*, and *demonstrability*. Delia's claim best meets C. But suppose money is no object for Ali and Beata, making Delia's testimony completely irrelevant. Then, assuming that relevance and demonstrability are equally weighted, there is a plurality of best testimonies with respect to C: Raúl's and Delia's are both best.

The epistemic constraint imposed on admissible cases by the telic logical pluralist precludes notions of case that yield consequence relations that fail to best meet epistemic goals. One might want to strengthen the epistemic constraint, and require that consequence relations be best suited to meeting not just *any* epistemic goals, but something like the core epistemic goals of argumentation. This would capture the widespread intuition that the primary purpose of logic is bound up with reasoning or argumentation.[36] Perhaps this is what motivates logical normativists who hold the Reasoning View, on which the task of logic is to give an account of reasoning. If so, the intuition that logic and reasoning are connected can be preserved without resorting to normativism. Here, however, we leave the epistemic constraint unstrengthened.

Exactly which consequence relations the telic pluralist endorses will depend on what the epistemic goals turn out to be. We'll consider three plausible candidates and the logics they give rise to.

When reasoning deductively, one aims at preserving truth. But one might also aim at something more specific, such as drawing true conclusions that are relevant, or demonstrable. This line of thought yields the following three candidates for the

[34] For ease of exposition we talk about epistemic goals as if they're the kind of thing an agent gets to pick for herself. But telic logical pluralism can accommodate a constitutivist account of epistemic activity (Field 2000). On such a view, epistemic activities are partly constituted by the goals they aim at. This is compatible with telic pluralism as long as: either (i) multiple goals can constitute whatever epistemic activity a deliberator engages in when she deploys logic; or (ii) multiple logics are best suited to meeting the constitutive goal or goals.

[35] Assume for the sake of the example that Tariq's claim cannot be proved constructively.

[36] Priest (2006, p. 105).

epistemic goals in agents' sights when they deploy logic: truth-preservation, relevant truth-preservation, and demonstrable truth-preservation. Which logics are best suited to meeting these goals? A natural answer is classical logic, relevant logic, and intuitionistic logic, respectively. The classical consequences of Γ are those sentences that are true (in all classical cases) if each $\gamma \in \Gamma$ is true; the relevant consequences of Γ are those sentences that are true if each $\gamma \in \Gamma$ is true, and that are relevant to the members of Γ; and the intuitionistic consequences of Γ are those sentences that are true if each $\gamma \in \Gamma$ is true, and that can be constructively demonstrated. So, assuming those goals and that those are the logics best suited to meeting them, telic logical pluralism endorses the same logics as Beall and Restall's pluralism.

Our argument doesn't depend on the epistemic goals turning out to be all or any of truth-preservation, relevant truth-preservation, or demonstrable truth-preservation, however. Nor do we need it to be the case that classical, relevant, and intuitionistic logic are in fact best suited to meeting those goals. Our aim is not to defend this specific version of telic pluralism, but rather to set out the broad commitments of the view.[37] Most importantly, although these commitments do not include logical normativism, telic pluralism has the resources to rule out unpalatable consequence relations—like the trivial one—without invoking pragmatism. The result is a view that captures the attractive features of Beall and Restall's pluralism, without requiring a commitment to logic's normative status.

We'll end by dealing with an objection. The objection is that the notions of an epistemic goal, or of being best suited to meeting one, are normative. So telic pluralism fails to excise normativism from pluralism after all. Our reply is similar to the pragmatic pluralist's reply to the parallel objection to that view. It is to admit that telic logical pluralism uses normativity to constrain the admissible instances of (GTT), so normativity plays some role in the view. But this is very different from the role assigned to normativity by normativist pluralist accounts. Whereas those hold that the truths of logic themselves have direct normative upshot, the telic pluralist shifts the normative burden from logic to epistemology: logic per se is normatively inert. Thus logic is on a normative par with physics, or sociology. Its truths may give rise to normative claims, but only when coupled with basic tenets of epistemic normativity, such as that one ought to believe only true things.

Hence the distinction between normativist pluralism and non-normativist pluralism is no mere verbal difference. It is a dispute about the very foundations of normativity. Here's an example to bring this out. Suppose that the telic pluralist endorses at least classical and intuitionistic logic, as Beall and Restall do. Suppose A is deliberating about whether to draw the conclusion P from her known premise $\neg\neg P$. Suppose she fails to believe that P even though she knows $\neg\neg P$. The normativist might think that classical logic generates normative pressure to believe P; she ought to believe

[37] This invites a monist line of response to the effect that there is some one privileged epistemic goal that only one logic is best suited to meet. We are content to leave this possibility open; we take it to be a strength of the telic approach that it allows for a monist version of the view. Rather than arguing that telic pluralism is *the* correct account of logic, we've shown that it's a promising way to endorse non-normativist logical pluralism. We thank Reviewer 1 for pressing us to clarify this point.

the classical conclusion of a known premise.[38] Perhaps this pressure is defeated by other considerations, such as the intuitionistic invalidity of the argument. Or perhaps it isn't, but the mistake A makes is excusable on some other grounds. Either way, there is normative pressure to believe P simply in virtue of classical logic. According to telic logical pluralism, however, whether there is normative pressure for A's believing P depends not just on the correctness of any logic, but on the epistemic goal at issue. If the goal is truth-preservation simpliciter, then A makes a mistake in going against the recommendation of classical logic, which is best suited to meeting that goal. But if the goal is demonstrable truth-preservation, she makes no mistake at all, not even one that is excusable. There's no normative pressure whatsoever for her to believe P.

One might worry that this fails to dispel the suspicion of a merely verbal dispute.[39] In particular, the normativist might insist that logic is normative because consequence is built into the notion of an epistemic goal: talk of "truth-preservation" smacks of logic. But the telic logical pluralist denies that epistemic goals incorporate the notion of logical consequence. Epistemic goals are the *teloi* of epistemic activity. They are what our epistemic practises of reasoning, giving testimony, and so on, aim at: truth, relevance, demonstrability, and so on. Outside the context of a particular kind of epistemic activity—deductive reasoning—preservation of any particular epistemic quality has nothing much to do with anything. Part of logic's value is that it helps an epistemic agent in her pursuit of her epistemic goals, but it is logic that is subordinate to the goals and not vice versa. Deploying a logic helps an agent achieve her goals when she reasons deductively, but that does not mean that logical consequence is built into the concept of an epistemic goal.

Thus telic pluralism and Beall and Restall's pluralism are importantly distinct, even if they ultimately endorse the same logics. Beall and Restall endorse logical normativism, whereas the telic pluralist avoids that commitment.

6 Conclusion

Logical normativists hold that logic is normative; they might think its job is to provide an account of reasoning, or that logical consequences have normative upshot all on their own, or both. We deny normativism. Logic is a descriptive enterprise, the task of which is to delineate entailment relations. Moreover, it has no normative consequences unless combined with other normative claims, such as that one ought only to believe true things. In that respect, its normative status is no different from physics. We also deny that logical pluralists must be logical normativists. We illustrated this by developing three non-normativist versions of Beall and Restall's pluralism. These versions reject the normative constraint on admissible instances of (GTT). There's more work to be done on non-normativist versions of logical pluralism. But here we have cleared the way by vindicating the prospect of logical pluralism without the normativity.

[38] If normativists object to this particular example, similar ones can be given to accommodate their preferred formulation. See also n. 4.

[39] We thank Reviewer 2 for pressing us to clarify this point.

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