An Evolutionary Argument for a Self-Explanatory, Benevolent Metaphysics

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Abstract: In this paper, a metaphysics is proposed that includes everything that can be represented by a well-founded multiset. It is shown that this metaphysics, apart from being self-explanatory, is also benevolent. Paradoxically, it turns out that the probability that we were born in another life than our own is zero. More insights are gained by inducing properties from a metaphysics that is not self-explanatory. In particular, digital metaphysics is analyzed, which claims that only computable things exist. First of all, it is shown that digital metaphysics contradicts itself by leading to the conclusion that the shortest computer program that computes the world is infinitely long. This means that the Church-Turing conjecture must be false. Secondly, the applicability of Occam's razor is explained by evolution: in an evolving physics it can appear at each moment as if the world is caused by only finitely many things. Thirdly and most importantly, this metaphysics is benevolent in the sense that it organizes itself to fulfill the deepest wishes of its observers. Fourthly, universal computers with an infinite memory capacity cannot be built in the world. And finally, all the properties of the world, both good and bad, can be explained by evolutionary conservation.

Keywords: metaphysics, set theory, logic, theology, theory of everything, Occam's razor

1. Introduction

Many branches of metaphysics, like platonism, nominalism, conceptualism, mind-body dualism, and even the Yin and Yang system, make use of fundamental dualisms. Things are classified on metaphysical properties like abstract versus concrete or physical, existing versus non-existing in reality, conceptual or mental versus non-mental, mind versus matter, or Yin versus Yang. The theory in this paper proposes a neutral monism, given the idea that all these sorts of things can be modeled as the output of a computer program (Lockwood 1981; Turing 1946). For every metaphysical property *X* we either have "everything is *X*" or "nothing is *X*." More rigorously, everything can be considered as a well-founded multiset (Blizard et al. 1988).[1] For example, the set that contains seven times the

[1] There exists a bijection between the class of all multisets and the class of all sets. For example, the multiset (1,1,1,2,2) can be brought in correspondence with the set (((1),1),1,(2),2).

empty set, and nothing else, represents the number seven. Also for larger things, like planet Earth, there exists a well-founded multiset that represents the thing exactly. Applying monism on the dualism 'representing thing' versus 'represented thing,' we have to assume that the number seven and planet Earth *are* multisets, and that every multiset can be represented by one or more other – typically larger – multisets.

An important idea is that multisets may be part of other multisets more than once. This provides an explanation mechanism: the properties of the world that we observe are like they are, because worlds with such properties appear very many times in larger worlds. Therefore sufficiently large worlds contain every 'thing that can be observed' many times.

The class of all the well-founded multisets is the largest possible world and is therefore the world of the *maximal* metaphysics. This class has been extensively studied by mathematicians since the work of Cantor (Bernadete 1964; Dauben 1990). However, this class is itself not a well-founded multiset. For this reason, we will consider metaphysics that are *not* maximal. Such metaphysics have a world that is itself a well-founded multiset, which enables us to count how many times it is a part of some larger multiset.

The idea that the output of computer programs can appear at either side of every dualistic wall in metaphysics, will enable us to make great conclusions. In particular, it will follow that a maximal metaphysics is self-explanatory and benevolent.

Section 2 introduces definitions for a series of terms that we will need to analyze metaphysics. Two important conclusions, derived in Section 3, are that the metaphysics that gives a sufficient explanation of our world is self-explanatory and that even though every possible infinitely long life exists, there was no finite probability to be born as anybody else. Section 4 defends the adoption of a maximal metaphysics by showing that other metaphysics make assumptions that cannot be reasonably explained. Section 5 shows that Occam's razor is self-contradictory (Domingos 1999). The assumption of a metaphysics with a countable number of things leads to the existence of a thing that is largest in size and that is not in the countable set. Section 6 derives the most important conclusions by analyzing digital metaphysics over five subsections (Schmidhuber 2000). The benevolence of digital metaphysics is derived in the third subsection. The validity of this argument for the maximal metaphysics is derived in the fifth subsection. Finally, Section 7 summarizes the conclusions.

2. Definitions

A metaphysics is *maximal* if everything that can possibly exist, really exists in its world. A metaphysics is *benevolent* if its world is organized to fulfill the deepest wishes of its observers.[2]

[2] Theists could define God as being responsible for this organization.

Every well-founded[3] multiset is a *thing* that exists *possibly*. This means that we will consider things that can be represented by either a natural number, or a real number, or a subset of the real numbers, or a subset of the subsets of the real numbers, etc. The fundamental element is the empty multiset, which is a proper subset of each non-empty multiset at least once. Things that are not multisets cannot exist. The *world* of a non-maximal metaphysics is a thing that exists, and that is a well-founded multiset of all the things that exist.

If a thing A is a subset of a thing B, then we say that *A is part of B*. The relation 'is part of' is reflexive, which means that each thing is a subset of itself.[4] Each thing has two – possibly infinite – ordinalities:[5] a multiplicity and a size (Manolios and Vroon 2003).[6] The *multiplicity* of a thing is the number of times that it is a part of the world. For example, in a semantically more advanced interpretation of 'is part of', the Eiffel tower is part of Paris, of your mind and of this sentence. All three of these locations are copied many times as parts of computer simulations and minds in a higher world, which are in turn copied in the physics of even higher worlds. Therefore the Eiffel tower exists many times as part of a world with a sufficiently large size. The *size* of a thing is the number of subset instances that it contains. In other words, the size of a thing A is the sum of all the multiplicities of all the things that have A as world. The size of the empty multiset is one, since it only has itself as a subset. The size of a thing measures the amount of information that it contains (Shannon 1948). As an exact example, consider the world that consists of 1000 identical hydrogen atoms. An electron is a multiset that consists of 3 empty multisets, and a proton is a multiset that consists of 5 empty multisets. Each hydrogen atom, which consists of an electron and a proton, has 8 empty multisets as subset and 3 non-empty multisets. Then the sizes of the empty multisets, electrons, protons and atoms are 1, 4, 6 and 11 respectively. Their multiplicities are 8000, 1000, 1000 and 1000 respectively. The size of the world is 11001, and its multiplicity is – like always – 1.

Every thing that is a part of the world at least once, *exists* (necessarily). A thing that does not exist has a multiplicity of zero (Linsky and Zalta 1991). Things with the same multiplicity are *symmetric*. The *multiplicity distribution* of a set of things is the multiplicity as a function of each thing. A multiplicity distribution is symmetric if some of its multiplicities are equal, and asymmetric if none of its multiplicities are equal. Two multiplicity distributions are *order*

[3] No infinite chains can be followed along the 'has as proper subset' relation.

[4] In order to remain well-founded, a thing cannot be a *proper* subset of itself.

[5] Ordinalities extend the natural numbers beyond ω, the smallest infinity.

[6] The definitions of these two concepts do not correspond exactly with those of multiplicity and cardinality in classical set theory. The reason is that classical set theory makes a dualistic distinction between sets and so-called 'elements.' Somewhat confusingly, these elements can be sets, which make them not very elementary. Since there is a bijection between sets and multisets, these two concepts can also be defined for sets, but are more easily understood for multisets.

equivalent for a set of things if the same 'has a larger multiplicity than' relation, and the same 'is symmetric with' relation, apply for these things. For example, the multiplicity distributions (1,3,3,2) and (5,ω+9,ω+9,700) are order equivalent for the set (*a*,*b*,*c*,*d*). Two metaphysics are order equivalent if they contain the same things and their multiplicity distributions are order equivalent. A metaphysics is *more addition invariant* to the degree that it remains order equivalent after the addition of a large number of large things to its world. The *sum* of *n* metaphysics is the sum of their worlds, which results in the metaphysics in which the multiplicities of all the things that exist are added, and in which a new world comes into existence with multiplicity one. An example is shown in Figure 1.

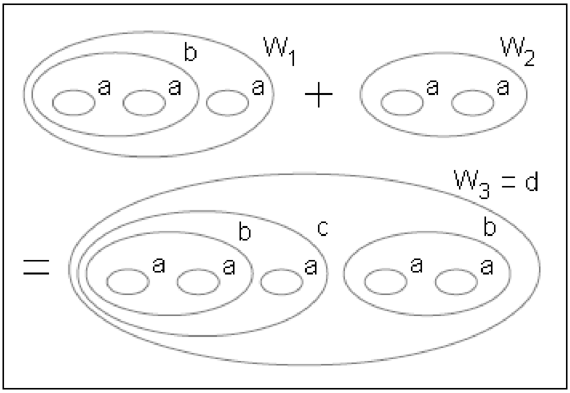

Figure 1: On the set (*a*,*b*,*c*,*d*), the multiplicity distribution of world W_1 with size 5 is (3,1,1,0) and that of world W_2 with size 3 is (2,1,0,0). The sum of W_1 and W_2 results in W_3 with multiplicity distribution (5,2,1,1) and with size 9

Relations are things that hold between things in the world. A relation either holds or does not hold for each couple of things. Two relations in which we are interested are 'has a larger multiplicity than' and 'is symmetric with.' However, these relations can be calculated from *primitive* relations like 'is part of' or 'is computed by.' The set of relations that exist has to be *consistent* with regard to the calculation of the multiplicities.

Each thing *A* in the world *W* is possibly *actual*. The *probability* that *A* is actual is given by the multiplication of its size and its multiplicity, divided by the size of *W*. If this probability equals one, then *A* is *certainly actual*. In the example of the hydrogen world above, the hydrogen atom is almost certainly actual.

An observer of a world and an observation method used by the observer, orders the world in a – possibly infinite – series of *initial segments* I_i that are all part of the world, with I_i a proper part of I_j if i < *j*, and such that there exist no thing in the world that has I_i as proper part and that is a proper part of I_{i+1}. The smallest initial segment I_1 is equal to the empty multiset and the largest initial segment I_α is equal to the world. The sum of the series of initial segments obtained through direct observation is the *life L* of the observer of the world:

$$L = \sum_{i=1}^{\alpha} I_i$$

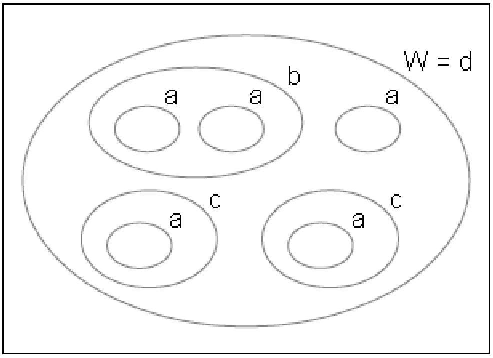

Figure 2: The probabilities to be actual in the world *W* for *a*,*b*,*c* and *d* are 5/9, 1/3, 4/9 and 1. There are two lives L_1 and L_2 that order *W*: L_1=a+b+d and L_2=a+c+d.

A life of an observer of a world *orders* the world. An example of two lives that order the same world is shown in Figure 2. A life that orders a world *W* is *certainly actualized* in *W* if all its initial segments I_i are certainly actual in *W*. Every initial segment is an *observed world* that has been observed through an observation method like direct observation, scientific inference or logical thinking.

Every world that is a thing that can exist possibly is an *observable world*. If a world *W* is a part of a metaphysics *E* with world W_E, then *E* is an *explanation* for *W*. If *W* has a multiplicity in *E* that is an infinity smaller than the multiplicity of *W* in some non-equivalent metaphysics *E*' with an observable world $W'_{E'}$ then *E* is an insufficient explanation for *O*. Otherwise *E* is a *sufficient* explanation for *W*. A metaphysics is *self-explanatory* if it can provide a sufficient explanation for every world, including its own world. A metaphysics is more self-explanatory to the degree that it can explain more worlds.

3. Life in a Maximal Metaphysics

In this section the nature of a maximal metaphysics is elucidated through six theorems. From the theorems 1 and 2 it follows that the world that we observe is only sufficiently explained by the maximal metaphysics, even though it is forever impossible to observe its whole world. Theorems 3 and 4, which say more about the transitive nature of being certainly actual, are required to prove theorems 5 and 6. These show in turn that exactly one life among the infinitely many lives in the maximal metaphysics is certainly actualized. This means that solipsism is true (Thornton 2004). The construction of certainly actualized lives reveals that even finitely small things that are observed early in a life can be certainly actual. Such things are therefore statistically infinitely exceptional among average things in the world. It also follows that a life requires a beginning, but not an end.

Theorem 1: A metaphysics with an observable world cannot be a sufficient explanation for any world.

Proof: Assume that a metaphysics E with an observable world O is an explanation for some world W. Let M_W be the multiplicity of W in O. Let T be a thing that is not a part of O and let T'' be a thing that has T as proper part, but that does not have $T+W$ as part. T and T'' are symmetric and have multiplicity zero in O. Now consider the addition $A= (T+W) \times M_W \times \omega$, with ω the smallest infinity. Let $O'=O+A$ be the world of E'. Since O is a thing that exists possibly, also M_W, A and O' are things that exist possibly. Moreover, O' is not order equivalent with O, because T'' is no longer symmetric with T. Therefore O' fulfills three conditions: 1) it is an observable world, 2) it is not order equivalent with O, and 3) it contains W infinitely much more often than O. Therefore E cannot be a sufficient explanation for W.

From this theorem if follows directly that it is impossible to observe the whole world.

Theorem 2: A maximal metaphysics is self-explanatory.

Proof: Let E be a maximal metaphysics with world W_E and assume that there exists a world W for which E is not a sufficient explanation. Let M_W be the multiplicity of W in W_E. Then there exists a metaphysics E' with a world W'_E such that the multiplicity M'_W of W in W'_E is an infinity higher than M_W, with W'_E being an observable world. Since W'_E exists possibly, also M'_W exists possibly. W_E contains every possible thing, which means that it also contains a thing that contains W with a multiplicity that is higher than M'_W. Therefore M'_W cannot be higher than M_W. This is a contradiction. Therefore a world W for which E is not a sufficient explanation cannot exist.

From this proof it follows that a maximal metaphysics is entirely addition invariant and fully asymmetric: the multiplicities of things in the maximal metaphysics, as well as the differences between these multiplicities, are so large that they are at least an infinity larger than any possibly existing addition of any possibly existing thing.

Theorem 3: Being certainly actual is transitive: if the world A is certainly actual in a world B, and the world B is certainly actual in a world C, then A is certainly actual in C.

Proof: Let the sizes of A, B and C be S_A, S_B and S_C. Let the multiplicities of A in B, B in C and A in C be $M_{A,B}$, $M_{B,C}$ and $M_{A,C}$. Let $M_{A,C/B}$ be the number of times that A is a part of C, without being a part of B. Then $M_{A,C} = M_{A,B} \times M_{B,C} + M_{A,C/B}$. If we multiply both sides with S_A, we get:

$$S_A \times M_{A,C} = S_A \times M_{A,B} \times M_{B,C} + S_A \times M_{A,C/B}$$

By the definition of being certainly actual, $S_B = S_A \times M_{A,B}$ and $S_C = S_B \times M_{B,C} = S_A \times M_{A,B} \times M_{B,C}$. Therefore we get:

$$S_A \times M_{A,C} = S_C + S_A \times M_{A,C/B}$$

Since $S_A \times M_{A,C} \leq S_C$ we must conclude that the term $S_A \times M_{A,C/B}$ must be an infinity smaller than S_C, so that $S_A \times M_{A,C} = S_C$. This means that A is certainly actual in C.

Theorem 4: If both A and B are certainly actual in C, and A is not larger than B, then A is certainly actual in B.

Proof: Assume that A is not certainly actual in B. Then we have:

$$S_A \times M_{A,B} < S_B$$

Because B is certainly actual in C, we have $S_B \times M_{B,C} = S_C$. If we multiply both sides with $M_{B,C}$ in the previous inequality, we get:

$$S_A \times M_{A,B} \times M_{B,C} < S_C$$

Because A is certainly actual in C, we can decompose S_C in terms of S_A. This results in:

$$S_A \times M_{A,B} \times M_{B,C} < S_A \times M_{A,B} \times M_{B,C} + S_A \times M_{A,C/B}$$

This is a clear contradiction. Therefore A must be certainly actual in B.

Theorem 5: Not more than one life can be certainly actualized in a world.

Proof: Suppose there are two different lives L and L' that are both certainly actualized in the world W. This means that the initial segments I_i and I'_i should all be certainly actual in W. However, since L and L' are different, at least one of the two following situations must be true: 1) there must exist an initial segment I_i that is smaller or equal in size than an initial segment I'_j, but that is not a part of I'_j, or 2) there must exist an initial segment I'_i that is smaller in size than an initial segment I_j, but that is not a part of I_j. According to Theorem 4, the smaller or equal initial segment must be certainly actual in the larger or equal initial segment, but this is impossible if it is not a part of it. Therefore there cannot be two different lives that are both certainly actualized in the same world.

Theorem 6: For any life L that orders a world W it is possible to construct a life L' and a world W' such that L' is certainly actualized in W' and such that L' has the same information content as L.

Proof: We can construct the initial segments of L' recursively such that $I'_1 = I_1$ and $I'_i = M_i \times I'_{i-1} + I_i$ for $i > 1$, and with the multiplicities M_i chosen so large that they are at least an infinity larger than the size of I_i. The constructed world W' equals I'_α. Now it is easy to verify that each initial segment I'_i is certainly actual within the initial segment I'_{i+1}, and because of the transitivity of being certainly actual also within the world W'. Moreover, each initial segment I'_i contains at least the same information as the initial segment I_i.

Through the mathematical technique of transfinite induction we can infer that this construction can also be made for the maximal metaphysics. Therefore the life of the reader must be the life that is certainly actualized in the maximal metaphysics.

An important test for the benevolence of a self-explanatory metaphysics can be proposed in the form of a paradox: do hells exist, given that hells can be represented by well-founded multisets? The solution to this paradox is that hells exist indeed, but the multiplicities of hells with a given size are an infinity smaller than that of heavens with a similar size.

A test for the possible infiniteness of the length of our lives can be proposed in the form of a second paradox: is there a finite probability to be born from parents who do not live in an afterlife, given the fact that it is logically possible to make infinitely many children during an infinitely long life? The solution to this problem is that life before death is an initial segment that has a multiplicity that is an infinity larger than the sum of the multiplicities of all the segments that follow.

A third paradox is more difficult to resolve: why are we born as social beings when solipsism is true? Solipsism takes away the common ground for truth with other beings in the world. A possible solution could be the idea that one day in our afterlives all our minds will slowly merge into one large mind that is probably vastly older than all of us. When this happens, our egos will inevitably die in order to merge into a united ego. After all, the separate neurons in our minds do not have an ego all by themselves either. They work through cooperation. Another possible solution is that we testify – irrationally – that Jesus Christ, the only son of God, has the certainly actualized life.

4. Three Non-Maximal Metaphysics

In this section a further defense will be given for the adoption of a metaphysics that contains every possible thing. This will be done by showing that the assumptions about existence and non-existence in some other metaphysics cannot be reasonably explained. Therefore three examples of a metaphysics will be discussed that fail to be self-explanatory. It are the metaphysics that generate:

- a finite world,
- causally isolated things,
- maximal symmetry.

4.1 Finite Worlds

The idea of a finite world is something that appeals to many (Tait 2002). It seems indeed plausible at first sight that an observed world that is finite, does not need an explanation that is infinite. However, this intuition contravenes logic.

We can use the results of information theory to understand that a finite world can be represented exactly by a finite string of symbols F (Shannon 1948). On the other hand, there are infinitely many finite strings of symbols. Clearly, there will be things in the finite world that are also represented by another string than F. Even more, F will be a part of strictly longer strings. Therefore it cannot be explained why the worlds that are represented by other strings do not exist.

4.2 Causally Isolated Things

In a maximal metaphysics, two things are necessarily bidirectionally causally related.[7] Consider a thing t_1 that exists, like the big bang universe, and consider a thing t_2 that appears to be causally isolated from t_1, like a parallel universe where planets have square moons. Then consider a thing t_3 in which three things exist: t_1, t_2 and an intelligent and powerful universe multiplier, who assesses universes on their properties before multiplying them. More formally, t_3 is a thing that represents a relation between t_1 and t_2, and that breaks the symmetry between t_1 and t_2 in some metaphysics in which both t_1 and t_2 exist.

For t_1 and t_2 to remain causally isolated, t_3 must be a thing that does not exist, having multiplicity zero. Otherwise the fact that moons are not square in t_1, is partly explained by the properties of t_2. But how can it be explained that t_3 does not exist?

4.3 The Symmetry of Mathematical Theories

Can we create a maximally symmetric metaphysics in which each thing occurs just once? Let us analyze the world that includes all the propositions that are provably true in Zermelo-Fraenkel set theory with the axiom of choice (ZFC) (Van Heijenoort 1977). This theory is known as the foundation of mathematics, but the analysis applies to any consistent theory whose axioms are recursively enumerable. This metaphysics is very promising as a symmetric metaphysics: a true proposition appears with multiplicity one in the world and a false

[7] An example of a metaphysics that consists of causally isolated things – or rather so-called possible worlds – is the modal realism of David Lewis (Lewis 2001). As the reasoning in this subsection shows, this metaphysics cannot contain all the possible worlds.

proposition with multiplicity zero. In a complete theory every proposition is either true or false.

The incompleteness theorem of Gödel, however, states that a consistent theory cannot be complete (Gödel 1931). This means that some propositions will appear in the world with a multiplicity that is neither one nor zero. The proposition "The continuum hypothesis is true" is just one example of a proposition that is neither true nor false according to ZFC (Cohen 1963). But then the addition of such a proposition just once influences the multiplicities of related propositions. This brings the multiplicity distribution of ZFC to a non-equivalent state. Therefore ZFC is neither addition invariant, nor self-explanatory.

5. Occam's Razor Rejected

The principle of Occam's razor states that we should adopt the metaphysics with the fewest things that explains the world (Domingos 1999). This principle goes radically against the principle of the maximal metaphysics, which adopts the existence of each possible thing an infinite number of times. Yet, Occam's razor is very natural in a world that makes any sense for an observer. If we see the moon during three different nights, then it is easier to assume that there is just one moon, rather than three different moons. It is not a proof. Maybe we are just a computer simulation that simulates the moon whenever we look at it, but we have to use Occam's razor if we ever want to make any sense of the world.

As physicists have proceeded to describe the world throughout the ages, they have not been using Occam's razor very strictly. Indeed, they assume there is just one moon, but further insights have forced them to adopt the existence of billions of other moons in billions of galaxies. Instead of reducing the number of things, they have been increasing it. For this reason the principle of Occam's razor has been reformulated as follows: we should adopt the metaphysics with the fewest *kinds* of things that explains the world. With this subtle difference the metaphysics in this paper can claim to apply Occam's razor more strictly than most other metaphysics, since it is a monism. The maximal metaphysics does not draw a line between things that exist and things that do not exist. Logic does not provide us with any clue where we should draw such a line. Therefore only a maximal monism truly proposes the existence of only one kind of thing.

The principle of Occam's razor has been formalized into what is now known as the mathematical formulation of Occam's razor (MFO) (Rathmanner and Hutter 2011). In this formulation things that exist are replaced by programming code and the world is replaced by the output of the program. Let us focus now on the following question: is it possible to use MFO in such a way that there is a finite probability that the observed world O is part of a world in which it appears only finitely many times, rather than appearing with infinite multiplicities? We will assume that O is given by a string of 0's and 1's with a length between 10^{100} and 10^{1000}, and is also given as the output of a computer program P_O with a length far below 10^{10}. P_O encodes the laws of physics of *O* and its boundary conditions.[8]

MFO is enabled by Solomonoff probabilities (Solomonoff 1964), which are defined through Kolmogorov complexities (Kolmogorov 1993). The Kolmogorov complexity, or descriptional complexity (Holzer and Kutrib 2010), of a string *s* is equal to the length of the shortest computer program *p*(*s*) that can calculate *s*. The Solomonoff probability *S*(*s*) of a string *s* is exponentially or hyperexponentially inversely proportional to its descriptional complexity:

$$S(s) \propto e^{-|p(s)|}$$

Strings with a high Solomonoff probability, and a low descriptional complexity, are e.g. 1, 01 or 101010…. A string with a lower Solomonoff probability is e.g. 10110100010011010111O. Let us call *M*(*O*,*s*) the number of times that *O* appears in *s*, which is the multiplicity of *O* in a metaphysics that has *s* as world. Each string *s* that contains *O* can be considered as a possible world and the Solomonoff probability *S*(*s*) multiplied with *M*(*O*,*s*) provides, after normalization, the probability *P*(*O*,*s*) that *s* is the actual world of *O* (Lewis 2001):

$$P(O,s) = S(s)\frac{M(O,s)}{\sum_{i=0}^{\infty} S(i)M(O,i)}$$

There is an immediate problem for our attempt to show a finite multiplicity of *O* from MFO. MFO assigns finite probabilities to programs that have an infinitely long output. There are infinitely many programs that produce every possible string infinitely many times as output. This means that the probability to find *O* as output of a program that produces *O* only finitely many times is zero.

What we could do to solve this problem is simply exclude the existence of infinite outputs. Interestingly, this exclusion leads to a contradiction. This can be shown by the simple observation that hyperexponential operations exist (Nambiar 1995). Addition, multiplication and exponentiation, which can be called hyper 1, hyper 2 and hyper 3, can be extended easily to hyper 4, hyper 5, etc. For a, b, and n ∈ N_0:

- hyper n (a,b) = hyper n-1 (a, hyper n (a, b-1)),
- hyper n (a,1) = a, for n > 1, and
- hyper 1 (a,b) = a + b.

[8] Using Occam, we should assume that seemingly random boundary conditions are actually related. They could, for example, be given by the binary notation of some shortly definable transcendental number. For this reason P_O can be very small.

Let us call H_n = hyper n (n,n), so that $H_1 = 1 + 1 = 2$, $H_2 = 2 \times 2 = 4$, $H_3 = 3$ ^ 3 = 27, $H_4 = 4\uparrow\uparrow4 = 4$ ^ (4 ^ (4 ^ 4)), $H_5 = 5\uparrow\uparrow\uparrow5 = 5\uparrow\uparrow(5\uparrow\uparrow(5\uparrow\uparrow(5\uparrow\uparrow5)))$, etc.[9]

Using MFO, one might be tempted to think that the probability to find *O* in the output of a program reaches a maximum for the program P_0. This does not hold when we turn our attention to hyperoperations. Just consider the string that is the binary notation of H_5. More formally, call S_n the string that results from taking the first H_n symbols from the string that consists of the concatenation of all finite strings. S_5 has a high Solomonoff probability, since it can be produced by a short program. On the other hand, it contains *O* zillions of times. So this string reduces the probability to find the output of P_0 as the actual world of the string *O* to practically zero. But then we can make the same reasoning for S_6, which contains *O* again so much more often than S_5, that it will turn the probability of S_5 to be the actual world of *O* to zero. In general, the Solomonoff probability of S_n will be only subexponentially much lower than that of S_{n-1}, but it will contain any appearance of *O* hyperexponentially much more often. Therefore the world of *O* will be S_∞, whose existence we had excluded explicitly. It does not matter how fast decreasing we choose the probability distribution over the programs with a finite output, there will always be a computable hyperexponential function that increases faster. We must conclude that no matter how we use MFO, *O* will always appear with an infinite multiplicity in the world.

6. Digital Metaphysics

In this section we will analyze a metaphysics that is almost maximal, namely digital metaphysics (Schmidhuber 2000). Digital metaphysics assumes that only things exist that are the output of a finitely long computer program. The analysis ranges over five subsections. In the first subsection it will be shown that the assumption of digital metaphysics leads to its own contradiction:

- The shortest program that computes the world is infinitely long – or – the Church-Turing conjecture is false.

The first subsection also introduces a range of concepts that will be used in the other subsections. The second subsection explains why Occam's razor, in spite of its strict invalidity, is actually a useful principle to explain the world. This explanation is based on the idea that the infinitely long program that computes the world has evolved Darwinistically (Darwin 2009). This gives at each moment

[9] Note that, in order to write H_5 down by means of the exponential operator only, we would need an extremely large sheet of paper. The size of that paper would be so large that it can in turn not be expressed by means of the exponential operator on a paper with any imaginable size.

the false impression that its output is caused by finitely many things. The third subsection brings the essence of the paper: the evolutionary argument for the benevolence of digital metaphysics. The fourth subsection brings two theorems that are related to the question whether the world that we observe really corresponds to the world that follows from digital metaphysics. These will be called the 'evolutionary conservation theorem' (Ashkenazy et al. 2010) and the 'no infinite UTM theorem' (Herken 1995). Finally, in the fifth subsection, the argument for the benevolence of digital metaphysics will be turned in an argument for the benevolence of the maximal metaphysics.

6.1 The 'Is Computed By' Relation

Digital metaphysics already assumes the existence of many things. Among the things that are computable we can count an afterlife, a universe with square moons and a divine universe multiplier who assesses universes on their qualities before multiplying them. However, it also includes hells where sentient beings are burning forever.

In order to use digital metaphysics, we will have to define the world, things in the world, and a primitive relation between the things from which we can calculate their multiplicities in the world. The things in the world are the outputs of programs that run on a Universal Turing Machine (UTM). The world is a multiset that consists of all the program outputs and the primitive relation. The multiplicity of a program output is the number of times that the output is computed. For calculating these multiplicities we will have to assume there exists a transitive 'is computed by' relation between program outputs. For each computable output, there are infinitely many programs that compute the output. By selecting the shortest and earliest ordered program for each output, we have created a bijective relation between shortest programs and program outputs. From here on, simply 'program' will be used to refer to an element of the set of programs that are shortest for producing a given output, or even to refer to its output.

Now it suffices to consider a transitive 'is computed by' relation between programs. Indeed, computer programs can compute other computer programs, which is called multitasking when executed on a single memory tape, or parallel computing when executed on separate memory tapes (Hennie and Stearns 1966). This does not mean that it can easily be decided for each pair of programs whether one program computes the other. As will be discussed in Subsection 6.5, there are both semantic problems as algorithmic problems related to the question whether one program computes another program. Let us assume for now that there exists a decision maker who can solve this problem for each pair of programs. Even more, the decision maker can decide on a probability distribution for each computing program, which corresponds to the multiplicity and velocity with which each computed program is computed by the computing program.

With these assumptions, it is possible to construct a Markov chain that has programs as states and the probability distributions as transition probabilities (Markov 1971). This Markov chain represents a metaphysics that we can analyze. In order to analyze it, we have to distinguish different kinds of programs.

There are programs that can compute themselves, in the sense that their output is a fractal with an infinite depth. These programs correspond to states that have a self-loop in the Markov chain. In some of these programs, the self-loop has a transition probability of one. Together with the programs that do not compute any program at all, these are absorbing programs, which correspond to absorbing states in the Markov chain. There are programs that compute all possible programs (CAP) in parallel. Also CAPs are fractals.

With the availability of this Markov chain, there are certain metaphysical questions that we can give an answer, like

1. From which programs can we reach any other program?

2. From which program do we derive the most homogeneous distribution of actual programs after N transitions?

3. Which is the most likely program from which we can reach any other program?

The first question asks which programs are good models of the world. They are the CAPs that have a finite transition probability to every other program. We will call these the universal CAPs (UCAPs). Not every CAP is a UCAP. Consider the CAP that starts the program P 2^n times after the n-th start of any other program. This CAP has a transition probability of zero towards programs that are not P or that are not computed by P.

Any odd UCAP cannot be the best model of the world. A UCAP can have very odd preferences for certain programs. Sufficiently intelligent UCAPs could even have cruel preferences with regard to sentient beings in the programs. We need to make a simulation of the Markov chain in order to find the UCAP that is the best model of the world.

The second question asks which program is most suited to start from in a simulation of the Markov chain. A good start for such a simulation might be a UCAP that gradually computes more and more programs in parallel, starting from the shortest programs and adding always longer and longer programs. Another good choice might be the shortest UCAP.

The third question asks for the best – or most self-explanatory – model of the world. A more accurate formulation of the question is which UCAP has the highest multiplicity. It is this question that we will investigate further. For this reason, consider a *UCAP Markov chain* that has only UCAPs as states.[10]

[10] We will assume that our decision maker can also decide what the transition probabilities within this UCAP Markov chain are.

An important fact is that there exist transient chains of UCAPs within the UCAP Markov chain. In a transient chain the probability to make a transition towards a program that is not further in the chain, decreases exponentially or hyperexponentially as transitions through the chain are made. This means that there is always a finite probability to escape from the chain, but the probability to escape infinitely many times from the chain is zero. This means, inevitably, that after some finite number of transitions in the UCAP Markov chain, the actual state will be inside a transient chain, and never escape from it anymore. In this case the UCAP Markov chain is transient, which means that there is no probability distribution over its states.

As an example of a transient chain, consider a sequence of UCAPs with the following properties: for each program that is started, the n-th UCAP in the sequence starts the (n+1)-th UCAP in the sequence H_n times. Moreover, the execution of each program that was started gets equal priority. This is just one example of a transient chain. Because each chain can split infinitely many times, we get uncountably many transient chains. By simulating the Markov chain many times with different randomizers, we can find the most likely transient chain for a given short UCAP to start the simulation from.

The fact that the UCAP Markov chain is transient implies that the best model of the world is given by an infinitely long program, which is in fact not a program. We get a similar situation as for the application of MFO. There we have assumed that the world was represented by a finitely long program output, which led us to the conclusion that the world was represented by an infinitely long program output. In our more advanced model of digital metaphysics we have first assumed that the world is represented by any possible finite program that is shortest in its kind, leading to the conclusion that it is given by an infinitely long program.

If the world is indeed given by an infinitely long program, then this means that the finitely describable theory of everything where physicists are looking for cannot exist (Laughlin and Pines 2000). A rigid physics must therefore be an illusion that is carefully held up for us. Even more, if the shortest program that computes the world is infinitely long, then the Church-Turing conjecture must be false (Cleland 1993). This conjecture says that it is not possible to build a machine in the world that can compute a function that a Turing machine cannot compute, neglecting resource limitations. However, every machine that can roll a real-world dice infinitely many times computes a function that a Turing machine cannot compute.

6.2 Occam's Razor Restored by Darwin

We have rejected MFO, the mathematical formulation of Occam's razor, as being a possible candidate for providing an addition invariant, self-explanatory metaphysics. However, we still lack an explanation why the world is intelligible at all, given that it is produced by an infinitely long program. Two more questions about the UCAP Markov chain are related to this matter:

> 1. Which are the most likely predecessors and successors of a given UCAP?
>
> 2. How are the multiplicities – or probabilities – distributed when we create a finite UCAP Markov chain which has only the UCAPs as states that are shorter than a given length?

In order to answer the first question we have to use the scientific results of evolutionary biology (Darwin 2009). We can compare the set of all the UCAPs with the set of all the possible DNA strings and transitions within the Markov chain with biological reproduction. Non-sexual reproduction without mutations corresponds to a transition over a self-loop. The ecological environment consists of the other UCAPs, which may have favorable or unfavorable transitions probabilities towards a given UCAP. As we can derive from the evolution theory of Darwin, the predecessors of a UCAP will be given by less evolved, shorter UCAPs, and ultimately from the very shortest UCAP. The successors will be more evolved, longer UCAPs.

The second question corresponds with the situation on Earth, which has only species with a finite DNA length. Biological organisms self-reproduce. Therefore UCAPs with a high transition probability towards themselves will have a higher multiplicity than UCAPs without a strong self-loop. Since a shorter DNA length requires fewer resources for reproduction, short DNA strings have a higher multiplicity than long DNA strings (McCutcheon and von Dohlen 2011). Humans have a lower multiplicity than animals, which have in turn a lower multiplicity than bacteria. This corresponds to higher multiplicities for short UCAPs, and lower multiplicities to long UCAPs. A finite UCAP Markov chain seems to restore Occam's razor.

How can the infinitely many finite UCAP metaphysics be reconciled with the metaphysics of the whole UCAP Markov chain? The solution lies in a world that evolves. At each moment, the world appears as if it is only causally influenced by a finite number of things. However, for the world as a whole there is no finite declaration. This also implies that the world will evolve always faster than what can be predicted from Occam-based physics. The world will become more and more complex as determined by an infinitely long evolution that starts as a well-known Darwinistic evolution.

6.3 The Benevolence of Digital Metaphysics

The code of each UCAP can be refactored into two finitely long sections of code. The first section is the metacode, which is variable in length and which is characteristic for each UCAP. The second section is the universal code, which is short and common for all the UCAPs. The universal code is responsible for computing all the possible programs. If we compare it to a classic software architecture, the metacode is like the main code of the program, while the universal code is like a function that is called from the main code. Every UCAP must call the universal code as a function at least once. However, executing the universal code by a UCAP in a transient chain causes a transition outside the chain. Therefore an infinitely long metacode, which never executes the universal code, is what remains in the program that is the best model of the world.

It cannot be denied that sufficiently advanced software programs can acquire intelligence. Therefore the metacode of a UCAP has some degree of intelligence. Since the metacode of a UCAP determines the transition probabilities towards other UCAPs, it is also clear that it has some degree of power over the multiplicities of what exists. The metacode can also decide to look inside the programs that are started by the universal code. This means that the metacode also has some degree of knowledge. As a combination of intelligence, power and knowledge, the metacode of a UCAP also has some degree of benevolence, which represents the degree to which it favors the multiplicity of beings whose deepest wishes become fulfilled. Finally, we can define a degree of godlikeness for the metacode of a UCAP, which represents the degree to which the metacode is intelligent, powerful, knowledgeable and benevolent.

With these definitions we can try to build an argument for the benevolence of the infinitely long computer program that is the most likely outcome of the UCAP Markov chain. The first thing that we have to remark is that we have knowledge about the behavior of the metacode of a UCAP, especially what concerns the short UCAPs. As mentioned in the previous subsection, the metacode of short UCAPs contains primitive computer programs that behave like primitive organisms (Koza et al. 1999). Since these computer programs have some degree of power over the transition probabilities towards other UCAPs, they will use this power to reproduce themselves. For primitive UCAPs, the transitions towards a somewhat shorter metacode may be favorable, because a shorter code is more easily and therefore more often assembled. A similar mechanism puts a downwards pressure on the DNA length of bacteria on Earth. However, above a certain threshold of complexity, organisms will only accumulate DNA. Similarly, above a certain threshold the metacode will only become longer. Above this complexity, the state of the Markov chain will stay within a certain transient chain and never evolve back below the threshold.

We also have knowledge about the metacode of a more evolved UCAP in a transient chain. The agents of the godlikeness in the metacode of such a UCAP will be computer programs, personal minds, governments and religions. These are things that we know and where we can reason about. From what we know, we can derive that the transition probabilities towards other UCAPs will, from here on, be determined by moral judgments. This means that a Darwinistic evolution will necessarily be followed by a moral evolution. Now we need just one last proposition to show that digital metaphysics is benevolent, namely that good will win from bad in the moral survival of the fittest (Güth and Kliemt 2000).

I will shortly discuss the possible winning strategies that a morally bad UCAP could follow. First of all, a bad UCAP could try to shut down the transition probabilities towards any other UCAP. This means that this UCAP becomes a CAP that fails to be a UCAP. This will degrade its status to that of any ordinary program that can never be addition invariant or self-explanatory. Secondly, a bad UCAP can try to favor other bad UCAPs secretly. This will not remain unnoticed by more godlike UCAPs, which will therefore not favor this bad UCAP with a high transition probability. Thirdly, a bad UCAP could do its very best to appear good by setting its transition probabilities towards good UCAPs as favorable as possible. In this case, the bad UCAP effectively strengthens the good side against its own nature. And finally, a bad UCAP could be openly bad by favoring only bad UCAPs that are also openly bad.

It is easy to see now that the good UCAPs will be at least as favored as the openly bad UCAPs, because good UCAPs will always be openly good. But the good UCAPs also attract transition probabilities from bad UCAPs that want to appear good, while bad UCAPs will obviously never be favored by good UCAPs that want to appear bad. We can model the evolutionary chains as a purely good chain on top, a purely bad chain below, and many intermediate chains in between. The transition probabilities of the intermediate chains finally always favor the purely good chain. In this way the purely bad chain becomes infinitely unlikely as evolution progresses. Therefore the best model of the world of digital metaphysics is benevolent.

6.4 Two More Theorems

It could be argued that the observed world does not have the properties that are predicted by the Markov chain of digital metaphysics. Two more theorems are related to this issue:

1. **Evolutionary conservation theorem:** All the properties of the observed world, both good and bad, can be declared by evolutionary conservation of the metacode.

2. **No infinite UTM theorem:** Godlike UCAPs create a physics in which a UTM with infinite memory capacity cannot be built, for less evolved organisms or systems.

6.4.1 Evolutionary Conservation Theorem

Once the UCAP Markov chain enters a transient chain, the metacode of the UCAP that is the current state at that moment, will become evolutionarily conserved. This means that the metacode will mostly only grow as it makes transitions towards UCAPs further in the chain. The metacode from earlier stages will evolve gradually through mutations. The same concept is known from evolutionary biology. After a certain threshold of complexity, the length of the DNA of an organism will also not decrease anymore. During the development of a new organism, it will therefore pass through all the stages of its evolutionary history, from a single cell to a complex adult. Some code is more important than other code, and is therefore better conserved. For the metacode of a UCAP the important code may constitute of the evolution towards a human-like organism, the transition to an afterlife, but also the evolution of science and religion. Especially the hypothesis that the evolution of science is evolutionarily conserved has great implications on how the world may have been able to evolve until now. It may have excluded, for example, the occurrence of miracles that could not be scientifically rejected as being a miracle. Therefore the observed world, being a finitely large initial segment of an infinitely large and complex world, gives a very biased view.

6.4.2 No Infinite UTM Theorem

A Turing machine is a computer that has an infinite memory capacity (Turing 1946). A UTM is a Turing machine that can compute the output of any arbitrary program that is run on any arbitrary Turing machine. The no infinite UTM theorem provides a seemingly trivial addition to the falsehood of the Church-Turing conjecture: there are functions that can be computed by a UTM, but that cannot be computed by a machine that we can build in the world, taking resource limitations into consideration. The whole fractal of UCAPs can easily be programmed and executed on a UTM. If a poorly evolved organism, like us, humans, could build a UTM in an advanced UCAP, then we would severely mess up the transition probabilities of our UCAP. A non-evolved organism would unintentionally start very short UCAPs. By doing this, the chain would lose its transient character. In the limit, no UTM can ever be built in the world.

The no infinite UTM theorem corresponds well with the no-cloning theorem of quantum mechanics, which says that the exact quantum state of a system cannot be copied (Karafyllidis 2004; Wootters and Zurek 1982). Simple physics, like cellular automata and Conway's game of life, do not have this property, which makes it possible to build UTMs with infinite memory capacity in it (Cook 2004; Gardner 1970).

6.5 Infinitely Many Decision Makers

Digital metaphysics is not entirely self-explanatory because of the arbitrary decisions that our decision maker had to make for defining the 'is computed by' relation. Deciding on a single world from digital metaphysics is therefore arbitrary.

The 'is computed by' relation cannot be algorithmically decidable. If it was, then we could decide the halting problem for any given program P (Turing 1936). If P is a CAP, then it will never halt. If P is not a CAP, then there exists a program Q that is not computed by P. Then we can construct a program R that runs P in parallel with a program S that checks whether P has already halted or not. When S sees that P halts, R starts the program Q. In this situation, the question whether R computes Q comes down to the question whether P halts or not. Since we know that the halting problem is not algorithmically decidable, the 'is computed by' relation cannot be algorithmically decidable either.

The 'is computed by' relation is also not semantically decidable. Imagine an advanced computer program that computes a big bang-like universe. In this universe, there is a schoolboy that calculates 5 times 8 and decides it is 40. However, the schoolboy made two calculation errors. Do his thoughts count as a computation of the program that calculates 5 times 8, or not? Since there are infinitely many programs, the decision maker will need to make infinitely many semantical decisions of this kind.

When the decision maker has made all the necessary decisions that we need for the UCAP Markov chain, he has created what is called an *oracle* for the 'is computed by' relation (ICBO) (Hemmerling 2002). This ICBO can be represented by a non-computable real number, which was chosen by the decision maker as one out of uncountably many other possibilities. The ICBO accepts two UCAPs as input and it gives as output a program that has as output the probability with which the first UCAP is computed by the second UCAP. This represents the transition probability from the second UCAP to the first UCAP in the UCAP Markov chain.

Unfortunately, most of the possible ICBOs that can be chosen are very untruthful. How can we propose a meta-decision process that selects an ICBO that is truthful, or, in other words, which ICBO has the highest multiplicity in a self-explanatory metaphysics? At this point, we should not miss the opportunity to harvest the experience with making decisions that emerges automatically in a UCAP Markov chain. Therefore the decision maker can define a countable set of non-computable ICBOs and make them accessible to the normal Turing programs. Let us call these ICBOs the first-order ICBOs and the programs that can make use of all the first-order ICBOs the second-order programs. However, in order to create a second-order UCAP Markov chain, we will also need a second-order 'is computed by' relation, and second order ICBOs for this relation. The second-order ICBOs are not computable by second-order programs, but they are defined by a second-order decision maker. But then, in order to assess the second-order decision maker, it is only fair to make the second-order ICBOs accessible to the second-order programs, which will result in third-order programs. This cycle of assessing always higher- and higher-order decision makers results in an uncountable series of countably large metaphysics, that are always more self-explanatory. Through these transfinite extensions, all the possible ICBOs will be included and assessed in the programs with an uncountably infinite order. This shows that the world has an uncountably infinite Turing degree of unsolvability (Kleene and Post 1954).

The tape length and the clock time of Turing machines are based on natural numbers. This makes Turing machines not entirely suited to prove the benevolence of the maximal metaphysics through the well-known mechanism of transfinite induction. However, standard Turing machines can be generalized to α-machines with any ordinal tape length and any ordinal time (Koepke and Seyfferth 2009). With that we can apply transfinite induction: by the availability of the α-th order ICBOs to the $(\alpha+1)$-th order programs, the $(\alpha+1)$-th order UCAPs can effectively assess the benevolence of the α-th order UCAPs that were defined by the α-th order decision maker. This means that the $(\alpha+1)$-th order UCAPs are strictly more intelligent, powerful and knowledgeable than the α-th order UCAPs. They can use these qualities to be even more benevolent. Therefore the maximal metaphysics is benevolent.

7. Conclusion

The maximal metaphysics is a monism in which everything that can be represented by a well-founded multiset exists in reality. This metaphysics is self-explanatory, which means that it can give a sufficient explanation for every possible world, including its own world. Exactly one life in this metaphysics is actualized – or – solipsism is true: there was no chance to be born as anybody else. The world of this metaphysics cannot be defined as a thing that exists. Therefore metaphysics are investigated that are not maximal, in order to induce their properties for the maximal metaphysics. In particular, digital metaphysics is analyzed, which assumes that only computable things exist. By proposing the existence of a transitive 'is computed by' relation between programs, as well as transition probabilities between programs, we could construct a Markov chain with programs as states. Within this Markov chain, special programs could be defined, like absorbing programs, CAPs and UCAPs. CAPs compute all programs, and UCAPs are CAPs that have a finite transition probability towards any other program. The fact that this Markov chain is transient leads to the conclusion that the shortest program that computes the world is infinitely long. Further analysis reveals that the theory of everything of physics cannot be static but must evolve eternally. The applicability of Occam's razor is declared by the idea that we are early in such an evolution. Two more theorems, the evolutionary conservation theorem and the no infinite UTM theorem, help us in understanding how the world that we observe corresponds indeed to the world of a maximal metaphysics. Finally, the observation that computer programs behave like primitive organisms in an early stage, and like morally aware people, governments and religions in a later stage, enables to prove that digital metaphysics is benevolent. Through the mathematical technique of transfinite induction it can then be shown that also the maximal metaphysics is benevolent.[11]

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