Does everything resemble everything else to the same degree?

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Abstract

According to Satosi Watanabe's "theorem of the ugly duckling", the number of predicates satisfied by any two different particulars is a constant, which does not depend on the choice of the two particulars. If the number of predicates satisfied by two particulars is their number of properties in common, and the degree of resemblance between two particulars is a function of their number of properties in common, then it follows that the degree of resemblance between any two different particulars is also constant, which is absurd. Avoiding this absurd conclusion requires questioning assumptions about infinity in the proof or interpretation of the theorem, adopting a sparse conception of properties, or denying degree of resemblance is a function of number of properties in common. After arguing against both the first two options, this paper argues for a version of the third which analyses degree of resemblance as a function of properties in common, but weighted by their degree of naturalness or importance.

Introduction

"...the point of philosophy," according to Bertrand Russell, "is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it" (Russell, 1918, 514). The argument discussed in this paper, which proceeds from platitudinous or nearly provable premises about predicates and properties to the barely believable conclusion that any two different particulars resemble each other to the same degree, does not fall far short of Russell's goal. This paper assesses the metaphysical significance of the argument by considering which premise should be rejected, and whether any surrogate premise can capture its platitudinous aspects, without entailing a barely believable conclusion.

The first premise, which Satosi Watanabe (1969, 376) dubs "the theorem of the ugly duckling", is:

(1) The number of (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is a fixed (infinite) constant, which equals the number of (possible) predicates satisfied by only the first, as well as the number of (possible) predicates satisfied by only the second.

As Watanabe explains "The reader will soon understand the reason for referring to the story of Hans Christian Andersen, because this theorem, combined with the foregoing interpretation [or premises two and three below], would lead to the conclusion that an ugly duckling and a swan are just as similar to each other as are two (different) swans"

(Watanabe, 1969, 376).[1] The first section explains the rationale behind this premise.

The second premise follows from an abundant conception of properties, according to which a particular has a property if and only if it satisfies a (possible) predicate corresponding to that property, and so the number of properties is the number of (possible) predicates.[2] According to it:

(2) The number of (possible) predicates satisfied by two particulars is the number of properties they have in common, the number of (possible) predicates satisfied by only the first is the number of properties the first has not in common with the second, and the number of (possible) predicates satisfied by only the second is the number of properties the second has not in common with the first.

Snow, for example, has the property of being white, according to abundant conceptions of properties, if and only if snow satisfies the corresponding predicate 'is white'. Likewise, two peas in a pod have the properties of greenness, roundness and yuckiness in common, according to abundant conceptions of properties, if and only if they satisfy the corresponding predicates 'is green', 'is round', and 'is yucky'.

[1]See also Watanabe (1965) and Watanabe (1985, 82).

[2]The name "abundant" comes from Lewis (1983, 346); see also Lewis (1986b, 59). Some writers prefer "deflationary", following Hale, who writes "According to the *abundant* or, as I prefer to call it, *deflationary* conception of properties, every meaningful predicate stands for a property or relation, and it is sufficient for the *actual* existence of a property or relation that there *could* be a predicate with appropriate satisfaction conditions" (Hale, 2013, 132); see also Cook (2019) and Hale (2015). I prefer "abundant" not only because of the contrast with "sparse", but also because I do not wish to imply that, according to the abundant conception, properties are any less real or fundamental.

The second premise is plausible on the hypothesis that the meaning of a (possible) predicate is a property: the meaning of the predicate 'is white', for example, is simply the property of being white. So even though the second premise could also be motivated by assuming the doctrine of predicate nominalism, according to which a particular has a property in virtue of satisfying a corresponding (possible) predicate, or the doctrine of class nominalism, according to which an individual has a property in virtue of being a member of the class of (possible) individuals which have that property, it is also independently plausible: it requires for its motivation no stronger metaphysical doctrine than the thesis that properties are the meanings of (possible) predicates.[3]

Proponents of sparse conceptions of properties, according to which there is not a property corresponding to every (possible) predicate, will find this the obvious premise to reject.[4] But I will argue in the second section that many sparse conceptions of properties which are otherwise well motivated nevertheless fail to escape the barely believable conclusion that the degree of resemblance between two different particulars is a constant, independent of the choice of the two particulars. According to David Armstrong's influential conception, for example, there are instantiated conjunctive, but no negative, disjunctive or uninstantiated properties (Armstrong, 1978b). But later I will argue, following John Bacon (1986), that Armstrong's conception entails that every

[3]For predicate nominalism in this sense see especially Armstrong (1978a, 11-24). For class nominalism see, for example, Armstrong (1978a, 28-34), Lewis (1983) and Lewis (1986b, 50-69).

[4]See, for example, Niiniluoto (1987, 37), Armstrong (1989, 40), Rodriguez-Pereyra (2002, 66-7) and Ott (2016, 140).

particular has at most one property, and so the degree of resemblance between any two different particulars is *zero*, regardless of the choice of the two particulars.

The third premise is supported by the analysis of resemblance as having properties in common, which suggests that the more properties particulars have in common, the more they resemble each other, and the more properties particulars have not in common with each other, the less they resemble each other. According to it:

(3) The degree of resemblance between two particulars is a function of the number of properties they have in common, the number of properties the first has not in common with the second, and the number of properties the second has not in common with the first.

It's natural to suggest, for example, that two peas in a pod resemble each other to a high degree because they have many properties in common and few properties not in common. Likewise, it's natural to suggest that the degree of resemblance between a raven and a writing desk is low because a raven and a writing desk have few properties in common and many properties not in common.

For illustrative purposes, I will focus on the suggestion that the degree of resemblance between particulars is their proportion of properties in common or, in other words, their number of properties in common, divided by their number of properties in total (the sum of their number of properties in common and number of properties not in common). This is convenient because the degree of resemblance between particulars which have all of their properties in common is one, whereas the degree of resemblance between particulars which have none of their properties in common is zero. But nothing important depends on this choice of illustration.[5]

The first premise and the second in combination entail that the number of properties in common and the number of properties not in common between two different particulars is a constant, which in combination with the third premise entails that the degree of resemblance between two different particulars is a function of a constant, and so:

(4) The degree of resemblance between two particulars which do not satisfy all the same (possible) predicates, or which do not have all of their properties in common, or which differ from each other, is a fixed constant, which does not depend on the choice of the two particulars.

This conclusion is barely believable. According to it, a raven resembles a writing desk, for example, to the same degree as a raven resembles a magpie, and a cygnet resembles a duckling to the same degree as two different ducklings resemble each other.

Through Nelson Goodman's later work, especially *Seven Strictures on Similarity*, the argument has become extremely familiar. As Goodman writes "...any two things

[5]For analyses of degree of dissimilarity as a function of number of properties in common and not in common see, for example, Niiniluoto (1987, 22-35), Oliver (1996, 52), Rodriguez-Pereyra (2002, 65-9), Paseau (2012, 365), Blumson (2014b, 179-93), Paseau (2015, 110), Blumson (2018), and Blumson (2019b). Yi (2018) criticises some of these analyses. For scepticism of whether resemblance is measurable by numerical degree at all, see Lewis (1973, 50), Williamson (1988, 457-60), Blumson (2019a) and Paseau (2020); for defence see, for example, Bigelow (1976, 1977), Tversky (1977), Suppes et al. (1989, 159-225), Weisberg (2012), Kroedel and Huber (2013, 459-462) and Enflo (2020). Section (3) is also a partial defence of this presupposition. Morreau (2010) argues against the cogency of overall comparative similarity on different grounds.

have exactly as many properties in common as any other two. If there are just three things in the universe, then any two of them belong together in exactly two classes and have exactly two properties in common... Where the number of things in the universe is n, each two things have in common exactly 2^{n-2} properties out of the total $2^n - 1$ properties; each thing has 2^{n-2} properties that the other does not, and there are $2^{n-2} - 1$ properties that neither has" (Goodman, 1972, 443-4; see also Quinton, 1957, 36). But despite its familiarity, the argument is worth revisiting for four reasons.

First, this version of the argument assumes the controversial doctrine of class nominalism, according to which there is a property corresponding to every set of (possible) particulars, and so if n is the number of (possible) particulars, the number of properties is 2^n. While this is a much quicker route to the absurd conclusion, the assumption of class nominalism is itself in need of justification. As Goodman himself admits "...as a [predicate] nominalist, I take all talk of properties [and classes] as slang for a more careful formulation in terms of predicates" (Goodman, 1972, 443). The presentation below assumes neither class nor predicate nominalism (although it does assume the existence of classes and properties), and so is a more careful formulation in terms of predicates of the kind Goodman alludes to. As a result, it reveals more than Goodman's presentation about the underlying sources of the problem.

Second, discussions of the problem almost invariably focus on the case in which the number of (possible) predicates or properties is infinite. David Lewis, for example, writes "Because properties are so abundant, they are undiscriminating. Any two things share infinitely many properties, and fail to share infinitely many others. That is so whether the two things are perfect duplicates or utterly dissimilar. Thus properties do nothing to capture facts of resemblance" (1983, 346). In a similar vein, David Armstrong writes "We should perhaps take quick and unfavourable notice of the view sometimes encountered that degrees of resemblance are quite arbitrary because with respect to any two things at all we can find an indefinite number of resemblances and an indefinite number of differences and that, as a result, no two things are more, or less, alike than any other two" (1989, 40).[6]

Since the number of (possible) predicates is infinite, the focus on the infinite case is unsurprising. But the focus on the infinite case also makes it hard to avoid the impression that infinity is the source of the problem, which in turn suggests that clear thinking about infinity may be the route to a solution. In section (1), I will explain how the rationale for the first premise depends on two assumptions about infinity. But since the argument goes through in the finite case without either assumption, I will also argue that rejecting either of these assumptions is not the right approach to solving the problem. In section (3), I will note that some problems with infinity affecting the analysis of similarity are paradoxes of measure, familiar from philosophy of probability.

Third, despite the absurdity of the conclusion, the argument has been extremely influential. In a discussion of concepts in cognitive science, Peter Gardenfors (2000, 111), for example, writes:

> The problem is pressing because given only a moderate generosity, any two objects can be shown to share an infinite number of properties. ... If it

[6]For similar passages see Guigon (2014, 390), Cowling (2017, 4) and the references in footnote 7.

> is the number of shared properties that determines the similarity of objects, then any two objects will be arbitrarily similar. ... Restricting the problem to natural properties ... does not help – there are still arbitrarily many. ... I know of no theory of properties that furnishes a satisfactory solution to this problem. Consequently, I see no way of defining similarity as the number of shared properties.

It is clear from this passage that Gardenfors would respond to the argument by outright rejecting the third premise. But he also rejects the second, arguing that "From the point of view of cognitive science the abundant properties are totally worthless" (Gardenfors, 2000, 67). We may hope for a more conservative solution, which at least retains some plausible surrogate for the abandoned premises.

Other writers come close to simply accepting the absurd conclusion, by taking the argument to throw doubt on whether overall similarity makes sense at all. In a discussion of biodiversity, James Maclaurin and Kim Sterelny (2008, 14), for example, come close to accepting the conclusion when they write emphatically that:

> ... the project of building a classification based on overall similarity is hopeless. If any characteristic at all counts in determining similarity relations among (say) a horse fly, a fruit fly, and a bee, then they are all equally similar and equally unlike one another. For every individual has, and lacks, an infinity of characteristics. ... Overall similarity is not a well defined concept, as Nelson Goodman vigorously remarked in *Seven Strictures on Similarity.*

An argument purported to have such radical and far-reaching consequences deserves to be considered extremely carefully.[7]

1 The First Premise

A predicate is a sentence with a name removed. The predicate 'is white', for example, is the sentence 'snow is white' with the name 'snow' removed. A (named) particular satisfies a predicate if and only if replacing the gap in the predicate by a name of the particular results in a true sentence. Snow satisfies 'is white', for example, because the sentence 'snow is white' is true. According to abundant conceptions of properties, there is a property corresponding to every (possible) predicate: corresponding to the predicate 'is white', for example, is the property of being white, and corresponding to 'is red' is the property of being red.

There are some properties that do not correspond to any actual predicate. As David Armstrong writes "It is clearly possible, and we believe it to be the case, that particulars have certain properties and relations which never fall under human notice" (Armstrong, 1978a, 21). Nevertheless, if these properties were to fall under human notice, we could introduce predicates to talk about them, so corresponding to every property is a possible

[7]The argument is also raised outside metaphysics by, amongst others, Towster (1975), Watanabe (1985), Oddie (1986, 164-5), Niiniluoto (1987, 35-7), Medin et al. (1993, 255), Mundy (1995, 35-6), Feldman (1997, 150), Byrne (2003, 641), Priest (2008, 97), Sterrett (2009, 803), Decock and Douven (2011, 68), Isaac (2013, 685), Blumson (2014b, 182-96), Ott (2016, 140-1) and Harnad (2017, 36-7). Most of these authors stress the infinite case, and many draw radical conclusions.

predicate. Because of their great number, possible predicates are a complicating factor at some points in the argument, but a simplifying factor at others. When possible, I may omit to mention them.

A predicate *entails* another predicate if and only if necessarily any particular which satisfies the former also satisfies the latter. The predicate 'is white' entails the predicate 'is coloured', for example, because it's necessary that any particular which satisfies 'is white' also satisfies 'is coloured'. Entailment between predicates is a reflexive and transitive relation: since necessarily any particular which satisfies 'is white' satisfies 'is white', for example, 'is white' entails 'is white'. And since 'is scarlet' entails 'is red' and 'is red' entails 'is coloured', 'is scarlet' entails 'is coloured'.

It's also convenient to stipulate that the relation of entailment between predicates is antisymmetric. In other words, if a predicate entails a second predicate and the second predicate entails the first, then they are the same predicate. Since 'is white' entails 'is not unwhite' and 'is not unwhite' entails 'is white', for example, 'is white' and 'is not unwhite' are the same predicate. This stipulation is convenient because it ensures that there is only one (possible) predicate corresponding to each property, so if there is a (possible) predicate corresponding to every property, then there is exactly one (possible) predicate corresponding to every property (Armstrong, 1978a, 7).

Two clarifications. First, for many purposes it's desirable to distinguish between predicates which entail each other, and between their corresponding properties (see, for example, Lewis (1986b, 55-59)). Although 'is triangular' and 'is trilateral', for example, entail each other, it's often desirable to distinguish between them, as well as to distinguish between the corresponding properties of being triangular and being trilateral. But it's not desirable to draw this distinction for the purpose of analysing resemblance. We should not count being triangular and being trilateral, for example, as distinct similarities between two shapes, since that would be to count one similarity twice over.

Second, consider the purported predicate 'is not satisfied by itself'. If an abundant conception of properties is correct, a predicate satisfies 'is not satisfied by itself' if and only if it satisfies the corresponding property of not being satisfied by itself or, in other words, if and only if it does not satisfy itself. So 'is not satisfied by itself' satisfies itself if and only if it does not satisfy itself, which is a contradiction. This is a serious problem for abundant conceptions of properties (Field, 2004), but not the problem under discussion in this paper. To avoid it, I consider only predicates applying to individual particulars, and not to other predicates, properties, or sets.

A relation which is reflexive, antisymmetric and transitive is called a partial ordering relation, and a partially ordered set is an ordered pair $\langle A, \leq \rangle$ of a set A and a partial ordering relation $\leq$ between elements of A (Gratzer, 2011, 1). Since entailment between predicates is a reflexive, antisymmetric and transitive relation, it follows that entailment between predicates is a partial ordering relation, and predicates are a partially ordered set under the relation of entailment. (As usual I will say that $a < b$ if and only if $a \leq b$ but it is not the case that $b \leq a$, and $a = b$ if and only if $a \leq b$ and $b \leq a$.)

The conjunction of two elements in a partially ordered set can be defined in terms of its ordering relation as follows: an element $a \in A$ is the conjunction $b \wedge c$ of two elements $b, c \in A$ of a partially ordered set $\langle A, \leq \rangle$ if and only if (i) $a \leq b$ and $a \leq c$ and (ii) for all $d \in A$ if $d \leq b$ and $d \leq c$, then $d \leq a$. The predicate 'is red and square' is the conjunction of 'is red' and 'is square', for example, because 'is red and square' entails 'is red' and entails 'is square', and because every predicate which entails 'is red' and entails 'is square' entails 'is red and square'.

Likewise, an element $a \in A$ is the disjunction $b \vee c$ of two elements $b, c \in A$ of a partially ordered set $\langle A, \leq \rangle$ if and only if (i) $b \leq a$ and $c \leq a$ and (ii) for all $d \in A$ if $b \leq d$ and $c \leq d$, then $a \leq d$. The predicate 'is Czechoslovakian' is the disjunction of 'is Czech' and 'is Slovakian', for example, because 'is Czech' and 'is Slovakian' entail 'is Czechoslovakian', and every predicate which is entailed by 'is Czech' and entailed by 'is Slovakian' is also entailed by 'is Czechoslovakian'.

Notice that neither definition requires 'or' or 'and' to appear in a predicate for it to be a conjunction or disjunction. This is important, since it means that whether a predicate is a conjunction or disjunction of possible predicates in this sense is not contingent on which language we speak. So if, as we consider in the next section, we deny that there are properties corresponding to disjunctive or conjunctive predicates in this sense, which properties we are denying the existence of likewise is not contingent on language.[8]

A *lattice* is a partially ordered set $\langle A, \leq \rangle$ in which every pair of elements $a, b \in A$ has a conjunction $a \wedge b$ and a disjunction $a \vee b$. Since every pair of (possible) predicates has a conjunction which may be formed with 'and' and a disjunction which may be formed with 'or', the partial ordering of (possible) predicates under the relation of entailment is a lattice. In order to complete the rational for the first premise, it remains to be shown

[8]For a related point see, for example, Oddie (2005, 151-2).

that it is a boolean lattice, which is both complete and completely distributive, as the rest of this section explains.

An element $a \in A$ is the maximum $\top$ of a partially ordered set $\langle A, \leq \rangle$ if and only if $b \leq a$ for all $b \in A$ (Gratzer, 2011, 5). The predicate 'exists' or 'is white or not white', for example, is the maximum element of the set of predicates under the relation of entailment, because necessarily, if a particular satisfies any predicate, then it satisfies 'exists' or 'is white or not white'. Likewise, an element $a \in A$ is the minimum $\perp$ of a poset $\langle A, \leq \rangle$ if and only if $a \leq b$ for all $b \in A$ (Gratzer, 2011, 5). The predicate 'does not exist' or 'is white and not white', for example, is the minimum element of the set of predicates under entailment, because 'does not exist' or 'is white and not white' entail every predicate.

An element $a \in A$ is the negation $\neg b$ of an element $b \in A$ if and only if $a \vee b = \top$ and $a \wedge b = \perp$ (Gratzer, 2011, 97). And A lattice $\langle A, \leq \rangle$ is *complemented* if and only if it has a minimum $\perp$, a maximum $\top$ and every $a \in A$ has a negation $\neg a$ (Gratzer, 2011, 98). Notice that this definition of negation does not require 'not' to appear in a predicate in order for it to be a negation. In fact, in a complemented lattice, every element is a negation, since every element is the negation of its negation. Just as 'is foreign', for example, is the negation of 'is local', 'is local' is the negation of 'is foreign'. So if, as we consider in the next section, we deny that there are "negative properties", we must be especially careful to be clear what we mean by this.[9]

A lattice $\langle A, \leq \rangle$ is *distributive* if and only if for all elements $a, b, c \in A$, $a \wedge (b \vee c) =$

[9]For a related point see, for example, Oddie (2005, 151-2).

$(a \wedge b) \vee (a \wedge c)$ and $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ (Gratzer, 2011, 14-5). The lattice of predicates is plausibly distributive – 'is married and a man or a woman', for example, is equivalent to 'is a husband or a wife' and 'is red and round or square' is equivalent to 'is red or round and red or square'. Since it's relatively uncontroversial in this context that the predicates ordered under the relation of entailment is a distributive lattice, I won't stress its role, except when it comes to its generalisation to the infinite case below.

A lattice is *boolean* if and only if it is distributive and complemented, and it has a minimum and maximum element (Gratzer, 2011, 15). So the lattice of predicates under the relation of entailment is boolean, because predicates under the relation of entailment are distributive and complemented (since every element has a negation), and it has a minimum and maximum element ('does not exist' or 'is white and not white' and 'exists' or 'is white or not white'). In order to establish the first premise, we need to show that the lattice of predicates under entailment is not only boolean, but also *atomic*.

An element $a \in A$ of a lattice $\langle A, \leq \rangle$ is an *atom* if and only if $\perp < a$ and for all $b \in A$ if $b < a$ then $b = \perp$ (Gratzer, 2011, 101). In other words, an element is an atom if and only if no element is smaller, except the minimum. In a lattice of predicates an atom is a predicate entailed only by itself or inconsistent predicates – in the lattice of predicates which apply to a die in virtue of the number it lands, for example, there are six atoms: 'lands one', 'lands two', 'lands three', 'lands four', 'lands five' and 'lands six'.[10] A lattice is atomic if and only if every element except $\perp$ is greater than or equal

[10] Note that an "atom" in this context is not a syntactically simple predicate: 'lands', for example, is not an atom, because it is strictly entailed by 'lands one', whereas 'lands on an even number and lands

to an atom (Davey and Priestley, 2002, 113).

To generalise to the infinite case, the definitions of conjunction and disjunction need to be generalised as follows. An element $a \in A$ is the disjunction $\bigvee B$ of the elements in a subset $B \subseteq A$ if and only if (i) for all $b \in B$, $b \leq a$ and (ii) for all $c \in A$ if $b \leq c$ for all $b \in B$, then $a \leq c$ (Gratzer, 2011, 5). The predicate 'is red', for example, is the disjunction of 'is scarlet', 'is crimson', 'is maroon', ... and so on, because 'is scarlet', 'is crimson', 'is maroon', ... and so on all entail 'is red', and every predicate which entails 'is scarlet', 'is crimson', 'is maroon', ... and so on also entails 'is red'. Likewise, an element $a \in A$ is the conjunction $\bigwedge B$ of the elements in a subset $B \subseteq A$ if and only if (i) for all $b \in B$, $a \leq b$ and (ii) for all $c \in A$ if $c \leq b$ for all $b \in B$, then $c \leq a$ (Gratzer, 2011, 5).

Notice that neither of these definitions requires a predicate to be infinitely long for it to be the conjunction or disjunction of an infinite set of predicates – 'is prime', for example, is the disjunction of the infinitely many predicates 'is two', 'is three', 'is five', 'is seven', 'is eleven' ... and so on, but 'is prime' is nevertheless only two words long.[11] Nor does one have to be able to enumerate an infinite list of predicates to be able to understand their infinite disjunction – I am perfectly capable of understanding 'is red' and 'is prime', for example, even though I'm unable to enumerate all the shades of red or all the prime numbers.

If $\langle A, \leq \rangle$ is a finite lattice, then every element $a \in A$ must be greater than or equal

on a prime number' is an atom, because it is strictly entailed by 'does not land' but not by any other predicate. Atoms are akin to Sider's "profiles" (1993, 50); see also Dorr and Hawthorne (2013, 23).

[11] In contrast to predicates in the infinitary languages discussed by Cook (2019).

to some atom $b \in A$. Moreover, every element $a \in A$ in a finite boolean lattice $\langle A, \leq \rangle$ is equivalent to the disjunction $\bigvee \{b \in B \mid b \leq a\}$ of a subset of the atoms $B \subseteq A$ and for every subset $C \subseteq B$ of the atoms $B \subseteq A$ in the lattice, $C = \{b \in B \mid b \leq \bigvee C\}$ (Davey and Priestley, 2002, 114-5). In other words, there is a one to one correspondence between the elements of a finite boolean lattice and the subsets of the atoms in the lattice which the elements are disjunctions of. But in the infinite case, we need to further assumptions to obtain this result.

The first further assumption needed to generalise to the infinite case is that the lattice of predicates is complete. A lattice $\langle A, \leq \rangle$ is *complete* if and only if every subset of elements $B \subseteq A$ has a conjunction $\bigwedge B$ and a disjunction $\bigvee B$ (Gratzer, 2011, 50). Every finite lattice is complete, since the disjunction or conjunction of its elements can be formed by successively disjoining or conjoining each pair of its elements. So if there were only a finite number of predicates, then the partial ordering of predicates under the relation of entailment would be a complete lattice. So the assumption of completeness is not controversial in the finite case, which strongly suggests that completeness is not the source of the underlying problem.

But what about the infinite case? There *is* an infinite number of predicates. The predicates 'is one year old', 'is two years old', 'is three years old', ... and so on, for example, are countably infinite. Moreover, although every pair of predicates has a disjunction and conjunction, it's controversial whether every infinite subset of predicates does, since it's controversial whether the disjunction or conjunction of even a countably infinite set of predicates can be formed by joining every predicate in the set with the words ‘or’ or ‘and’, or whether only finitely long strings of words can form grammatical predicates of natural language.[12]

However, recall that according to abundant conceptions it is really *possible* predicates that correspond to properties, and so what matters for the purposes of the argument is whether every subset of predicates has a *possible* predicate as its conjunction or disjunction. So even if not all infinite disjunctions and conjunctions of arbitrary sets of predicates exist in natural languages, it suffices to show that infinite disjunctions and conjunctions of arbitrary sets of predicates are possible, which is a much lower bar. Even if the set of actual predicates under the relation of entailment does not form a complete lattice, it’s still plausible that the set of possible predicates does.

One argument which would suffice to show this proceeds from the possibility of enunciating an infinitely long predicate by completing a supertask.[13] It would be possible, for example, to enunciate an infinitely long predicate such as ‘is one year old or is three years old or is five years old ...’ and so on by enunciating the first disjunct in half a minute, the second disjunct in a quarter of a minute, the third disjunct in an eighth of a minute, ... and so on, until the whole disjunction is enunciated within a minute. The same goes for any arbitrary infinite disjunction or conjunction, and so it would follow that the lattice of possible predicates under the relation of entailment is complete.

[12]For discussion of whether there are infinite sentences in natural language see, for example, Langendoen and Postal (1986), Collins (2010) and Blumson (2014a).

[13]See especially Cook (2019, 2572-81). For linguistic supertasks see also Blumson (2015, 129-30), and for supertasks in general see Benacerraf (1962) and Thomson (1954).

But although the argument from the possibility of supertasks would suffice to show that the lattice of possible predicates under the relation of entailment is complete, it is not necessary. As I noticed above, it is not necessary for a predicate to be infinitely long for it to be a disjunction or conjunction of infinitely many predicates – the disjunction of 'is one year old', 'is three years old', 'is five years old', ... and so on, can be expressed simply by 'is an odd number of years old', for example. Of course, not every infinite set of predicates has an actual finitely-long predicate as its conjunction or disjunction. But there is no reason why not every infinite set of predicates should have a *possible* finitely-long predicate as their conjunction or disjunction.

The second further assumption needed to generalise to the infinite case is that the lattice of possible predicates under the relation of entailment is *completely distributive*, which is a generalisation of the distributivity condition stated above to infinite disjunctions and conjunctions (Davey and Priestley, 2002, 239-40). Since 'is red', for example, is the disjunction of 'is scarlet', 'is crimson', 'is mauve', ... and so on, complete distributivity implies, for example, that 'is square and red' is equivalent to 'is square and scarlet or square and crimson or square and mauve ...' and so on. Like the condition of distributivity above, complete distributivity of the lattice of possible predicates ordered under entailment is relatively uncontroversial in this context.

It can be proven that all and only complete and completely distributive lattices are complete and atomic, and so just as in the finite case, there is a one to one correspondence between the elements of a finite boolean lattice and the subsets of the atoms in the lattice which the elements are disjunctions of (Davey and Priestley, 2002, 240-1). And this is true of the lattice of predicates under the relation of entailment, even if there are infinitely many predicates, so long as it remains complete and completely distributive. It is also this correspondence between predicates and subsets of the set of atoms which underlies the appeal of the doctrine of class nominalism, and its identification of properties with subsets of the set of individuals.[14]

Let n be the number of atoms in a complete and completely distributive boolean lattice $\langle A, \leq \rangle$. Then since there is a one to one correspondence between elements in A and subsets of atoms in $B \subseteq A$, the number of elements in A is the number of subsets $|\mathcal{P}(B)| = 2^n$ of the atoms. The number of predicates which apply to a die in virtue of the number it lands, for example, is 2^6, since there is a predicate which applies for each combination of the six atoms 'lands one', 'lands two', 'lands three', 'lands four', 'lands five' and 'lands six'. More importantly, if n is the number of atoms in the complete boolean lattice of possible predicates, then the total number of possible predicates is 2^n.

It follows that the number of predicates a particular satisfies is 2^{n-1}, since each particular must satisfy exactly one atomic predicate, but may satisfy the disjunction of that atomic predicate with any combination of the remaining $n-1$ atomic predicates. The number of predicates satisfied by two particulars which do not satisfy all the same predicates is 2^{n-2}, since each two particulars satisfy in common the disjunctions of exactly two atomic predicates with any combination of the remaining $n-2$. And the number of predicates which are satisfied by only the first of two particulars which do not satisfy all the same predicates is also 2^{n-2}, since the first particular satisfies not in

[14]Bjerring and Schwarz (2017, 26) make a similar point about analysing propositions as sets of worlds.

common with the second the disjunctions of the atomic predicate it satisfies with any combination of the remaining $n-1$ atomic predicates, except for the atomic predicate satisfied by the second particular (Watanabe, 1969, 377).

So, as the first premise of the argument states, the number of (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is a fixed constant, which equals the number of (possible) predicates satisfied by only the first and which equals the number of possible predicates satisfied by only the second (Watanabe, 1969, 376-377). So if there is a property corresponding to every (possible) predicate, and degree of resemblance is number of properties in common divided by number of properties in total, then it follows that the degree of resemblance between two particulars is $\frac{2^{n-2}}{2^{n-2}+2^{n-2}+2^{n-2}}$, which is $\frac{1}{3}$ if n is finite and undefined otherwise. This is something so paradoxical that no one will believe it.

2 The Second Premise

Sparse conceptions of properties deny that there is a property corresponding to every (possible) predicate, and so deny that the number of properties is the number of (possible) predicates. So it's natural for a proponent of the sparse conception to resist the conclusion of the argument by denying its second premise. As Gonzalez Rodriguez-Pereyra (2002, 66-7), for example, writes:

> ... what Watanabe proved is not a problem ... for it is essential to his proof that the properties in question (or "predicates" to use his terminology)

> are the members of the smallest complete Boolean lattice of a given set of properties ... Thus if the properties of *being red* and *being square* are among the given sparse properties, their Boolean lattice will contain properties like *being red and square*, *being red or not being square*, *being neither red nor square*, etc. In general the lattice will contain negative, disjunctive, and conjunctive properties. But these are not sparse or natural ...

Nevertheless, I will argue in this section that many conceptions of sparse properties cannot escape the absurd conclusion.

Two clarifications. First, sparse conceptions of properties typically maintain that whether a property corresponds to a (possible) predicate is an *a posteriori* question. The existence of the property of being white, for example, cannot be deduced *a priori* from the existence of the corresponding predicate 'is white' (Armstrong, 1978b, 7-9). But even if whether a property corresponds to a (possible) predicate is an *a posteriori* question, it does not follow that there is nothing to be said in answer to that question in logical terms, including in terms of negation, disjunction and conjunction.

Consider an analogy with probabilities. Although whether a proposition has a certain probability is plausibly an *a posteriori* question, there is much which can be said about the relationship between probabilities in logical terms – for example, that if two propositions are inconsistent, then the sum of their probabilities is the probability of their disjunction. In this section of the paper, I want to focus particularly on sparse conceptions which articulate the relationships between properties in broadly logical terms. Many sparse conceptions of this kind, I will argue, do not escape the absurd conclusion.

Second, some sparse conceptions of properties maintain that whether there is a property corresponding to a (possible) predicate is not only *a posteriori*, but revealed by fundamental physics. As David Lewis, for example, writes "Physics has its short list of 'fundamental physical properties': the charges and masses of particles, also their so-called 'spins' and 'colours' and 'flavours', ... an inventory of the *sparse* properties of this-worldly things" (Lewis, 1986b, 60). But fundamental physical properties are not the respects in which ordinary macroscopic objects typically resemble each other, so this conception of sparse properties is poorly suited to feature in the analysis of resemblance (Schaffer, 2004, 94).

A natural way for proponents of sparse conceptions of properties to address this concern is to postulate the existence, in addition to the fundamental physical properties, of structural or complex properties, which depend on or derive from the fundamental physical properties. Then although ordinary macroscopic objects do not typically resemble each other in respect of fundamental physical properties, they may resemble each other in respect of the structural or complex properties which derive from the fundamental physical properties.

But the existence and nature of sparse structural or complex properties is extremely controversial (see, for example, Lewis (1986a)). The least controversial case is the example of conjunctive properties. But they are the source of the last and most serious problem I discuss below. I take it that if even the least controversial example of structural or complex properties is problematic, then it's likely that the more controversial examples of structural or complex properties will be even more problematic. In general, although I won't show that there is no sparse conception of properties which avoids the problems I raise, I will consider enough to show that those problems are no accident.[15]

I will begin with various toy examples which, while philosophically motivated, do not correspond to positions in the literature. I will then turn to the sparse conception of properties favoured by David Armstrong, which is one of the most influential positions in the literature. In another context, it may seem as if the problems with Armstrong's conception are an accident due to careless formulation (this seems to be John Bacon's (1986) view, for example). But in the context of these toy examples, it is apparent that the problems with Armstrong's conception are deep and robust.

Some conceptions of sparse properties cannot escape the absurd conclusion that the degree of resemblance between any two different particulars is a fixed constant because they are not sparse enough. According to the *principle of instantiation*, for example, there is a property corresponding to a (possible) predicate only if some particular satisfies that (possible) predicate (Armstrong, 1978a, 113). So according to the principle of instantiation there is no property corresponding to the predicate 'is faster than the speed of light', for example, because nothing is faster than the speed of light. This suggests a sparse conception of properties according to which there is a property corresponding to a (possible) predicate if and only if some particular satisfies that predicate.

[15]Gardenfors (2000, 59-100) and Oddie (2005, 152-158), for example, present a sparse conception of properties which is like Armstrong's in accepting that all conjunctions of sparse properties are properties, but differs from Armstrong's in only denying that some negative and disjunctive properties are. But Blumson (2019b) proves this conception does not overcome the problems with Armstrong's.

But just as every (possible) predicate corresponds to a disjunction of the n atoms, every unsatisfied (possible) predicate corresponds to a disjunction of the r unsatisfied atoms. So if there is a property corresponding to a (possible) predicate if and only if some particular satisfies that predicate, then the number of properties is not 2^n, the number of (possible) predicates, but $2^n - 2^r$, the number of (possible) predicates minus the number of unsatisfied (possible) predicates.

Nevertheless, the number of properties a particular satisfies is still 2^{n-1}, since none of the (possible) predicates it satisfies are unsatisfied, the number of properties two different particulars have in common still 2^{n-2}, since none of the (possible) predicates satisfied by two particulars are unsatisfied, and the number of properties instantiated by only the first still 2^{n-2}, since none of the (possible) predicates satisfied by two particulars are unsatisfied. So if the degree of resemblance between two particulars is their proportion of properties in common, the degree of resemblance between two different particulars is still $\dfrac{2^{n-2}}{2^{n-2}+2^{n-2}+2^{n-2}}$ or $\dfrac{1}{3}$ if defined.

The conception of sparse properties which incorporates only the principle of instantiation cannot escape the absurd conclusion roughly because it is not sparse enough – there are still sufficiently many properties to ensure that any two distinct individuals resemble each other to the same degree. Other conceptions of sparse properties cannot escape the absurd conclusion that the degree of resemblance between any two different particulars is a fixed constant roughly because they are *too* sparse – according to them there are insufficiently many properties to ensure that any two distinct individuals resemble each other to a different degree.

Consider, for example, a sparse conception of properties according to which there is a property corresponding to every atomic (possible) predicate – in other words, those predicates which are not disjunctions of any others. Then the number of properties is n, the number of atomic predicates. Since the atomic (possible) predicates are all inconsistent with each other, no particular satisfies more than one. And since the disjunction of the atomic (possible) predicates is tautologous, every particular satisfies at least one. So the number of atomic (possible) predicates a particular satisfies is one, and the number of atomic (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is zero. So if properties correspond to atomic (possible) predicates and degree of resemblance is proportion of properties in common, then the degree of resemblance between any two different particulars is zero.

So whereas conceptions according to which the sparse properties correspond to the instantiated (possible) predicates are not sparse enough to escape the conclusion that the degree of resemblance between two different particulars is constant, the conception according to which the sparse properties correspond to the atomic (possible) predicates is too sparse to escape the conclusion that the degree of resemblance between two different particulars is constant.[16] This suggests pursuing a conception of properties of intermediate sparseness.

[16] Although no conceptions in the literature correspond exactly to this conception, some are just as unremittingly sparse. According to Rodriguez-Pereyra (2002, 48-52), for example, the sparse properties are "lowest determinate properties", and there are no negative, conjunctive or disjunctive sparse properties. I intend to consider Rodriguez-Pereyra's theory in more detail elsewhere.

I now turn to consider the intermediately sparse conception of properties favoured by David Armstrong, which is not merely a toy example, but one of the most influential in the literature. I will begin with a simplified version of Armstrong's conception, which ignores the principle of instantiation. I will then discuss Armstrong's actual conception, which includes the principle of instantiation. I will argue that neither conception succeeds in escaping the absurd conclusion that the degree of resemblance between two different particulars is a fixed constant, which does not depend on the choice of the two particulars.

If we ignore the principle of instantiation, then Armstrong favours a conception of properties which meets the following three conditions: (negation) if there is a property corresponding to a (possible) predicate a, then there is no property corresponding to its negation $\neg a$, (disjunction) if there is a property corresponding to two (possible) predicates a and b and $a \neq b$, then there is no property corresponding to their disjunction $a \vee b$, and (conjunction) if there is a property corresponding to two (possible) predicates a and b, then there is a property corresponding to their conjunction $a \wedge b$ (Armstrong, 1978b, 20-30).

This conception of properties is too sparse, because it follows that a property corresponds to only one possible predicate. For suppose that there is a property corresponding to a and a property corresponding to b. Then according to (conjunction) there is a property corresponding to $a \wedge b$. But since a is equivalent to $a \vee (a \wedge b)$ and b is equivalent to $b \vee (a \wedge b)$, and because we have stipulated (as Armstrong (1978a, 7) agrees) that equivalent predicates are the same, there is a property corresponding to $a \vee (a \wedge b)$ and a property corresponding to $b \vee (a \wedge b)$. Then according to (disjunction) $a = a \wedge b$ and $b = a \wedge b$ (otherwise $a \vee (a \wedge b)$ and $b \vee (a \wedge b)$ would correspond to disjunctive properties), so $a = b$. So if there is a property corresponding to a and a property corresponding to b, then according to this conception, $a = b$ (Bacon, 1986, 49).

Because Armstrong accepts the principle of instantiation, he does not accept (conjunction) in full generality, and so it does not follow from Armstrong's conception that there is a property corresponding to only one (possible) predicate. Instead, Armstrong endorses: (instantiated conjunction) if there is a property corresponding to two (possible) predicates a and b and some particular satisfies both a and b, then there is a property corresponding to their conjunction $a \wedge b$ (Armstrong, 1978b, 30).

But Armstrong's conception is still too sparse, since it follows that each particular instantiates only one property. For suppose that there is a property corresponding to a and a property corresponding to b, and that some particular satisfies both a and b. Then according to (instantiated conjunction) there is a property corresponding to $a \wedge b$. But since $a = a \vee (a \wedge b)$ and $b = b \vee (a \wedge b)$, there is a property corresponding to $a \vee (a \wedge b)$ and a property corresponding to $b \vee (a \wedge b)$. Then according to (disjunction) $a = a \wedge b$ and $b = a \wedge b$, so $a = b$. So if there is a property corresponding to a and a property corresponding to b and some particular satisfies both a and b, then, according to Armstrong's conception, $a = b$ (Bacon, 1986, 49).

So Armstrong's conception of properties has similar consequences to the conception according to which the properties correspond to the atomic (possible) predicates. Supposing that that there are no bare particulars, or in other words that at least one of the (possible) predicates satisfied by a particular corresponds to a property, it follows from Armstrong's conception that exactly one of the (possible) predicates satisfied by each particular corresponds to a property, or that the number of properties a particular instantiates is one. The number of properties instantiated by two different particulars is zero, since the one property each satisfies must be different if they are different. So if their degree of resemblance is their proportion of properties in common, then their degree of resemblance is zero.

3 The Third Premise

A natural way for proponents of abundant conceptions of properties to avoid the conclusion of the argument is to revise the premise that the degree of resemblance between two different particulars is a function of their number of common and uncommon properties. Different properties, according to this revision, have different weights in determining degrees of resemblance: the degree of resemblance between two different particulars is a function of the weights of the properties they have in common, the weights of the properties the first has not in common with the second, and the weights of the properties the second has not in common with the first.

Revising the premise that the degree of resemblance between two different particulars is a function of their number of common and uncommon properties to the thesis that the degree of resemblance between two different particulars is a weighted function of their common and uncommon properties must be at least as good a way to avoid the barely believable conclusion as adopting a given sparse conception of properties, because if the weights of the properties are given by the function $w : A \to \{0, 1\}$ which takes each abundant property in A to one if it corresponds to a property according to the sparse conception but to zero if it does not, then the revision to the premise can give the same number (if defined) as the degree of resemblance between two particulars as the given sparse conception of properties did.

Note that proponents of abundant conceptions of properties disagree over the question of whether the weights should be interpreted as subjective degrees of importance (as Nelson Goodman (1972), for example, argues) or objective degrees of naturalness (as David Lewis (1983), for example, argues). Nevertheless, I will argue in this section that even without resolving this question, there is more to be said in logical terms about the nature of weights than there is to be said in logical terms about which predicates correspond to properties according to an appropriate sparse conception.

Reconsider the analogy with probabilities. Even though there is disagreement about whether the probability of a proposition should be interpreted in terms of subjective credences or objective chances, there is much to be said about the relationship between probability in logical terms – for example, that if two propositions are inconsistent, then the sum of their probabilities is the probability of their disjunction – which is independent of this issue. In this section of the paper, I want to focus particularly on what can be said about the nature of weights in logical terms.

The analogy with probabilities suggests that the weights should be given by a function $w : A \to \mathbb{R}$ from the set of predicates A to the real numbers such that for all $a \in A$, $w(a) = 1 - p(a)$, where $p : A \to \mathbb{R}$ is a function from the set of predicates A to the real numbers which meets the following three conditions: (non-negativity) for all $a \in A$, $0 \leq p(a)$, (normalisation) $p(\top) = 1$ and (finite additivity) for all $a, b \in A$ such that $a \wedge b = \bot$, $p(a \vee b) = p(a) + p(b)$. The weight of a property, according to this idea, is the opposite of its peculiarity, where the minimum degree of peculiarity is zero, predicates which apply to everything have the maximum degree of peculiarity, and the peculiarity of a disjunction is the sum of the peculiarity of its disjuncts, when they are inconsistent.

This characterisation of the weighting function captures the desired asymmetry between conjunctive and disjunctive properties, whereas sparse conceptions which maintained that properties exist corresponding to conjunctive but not to disjunctive predicates were unable to. This is because for any pair of predicates $a, b \in A$ such that $a \leq b$ or a entails b, it follows that $p(a) \leq p(b)$. And since for all $a, b \in A$, $a \wedge b \leq a \leq a \vee b$ it follows that for all $a, b \in A$, $p(a \wedge b) \leq p(a) \leq p(a \vee b)$ or, in other words, that the weight of a conjunction is greater than or equal to the weight of the conjuncts, whereas the weight of a disjunction is less than or equal to the weight of the disjuncts.

But this characterization of the weighting function also has three counterintuitive consequences. First, if $p(a) = 0$ and $p(b) = 0$ it follows from finite additivity that $p(a \vee b) = 0$. But if 'is red' and 'is green', for example, are not at all peculiar or in other words perfectly natural or important, it shouldn't follow that their disjunction 'is red or green' is not at all peculiar or perfectly natural or important, since there is a wider diversity between the things which are red or green than between the things which are red or than between the things which are green.

Second, if $p(a) = p(b)$, $p(c) = p(d)$, $a \wedge c = \bot$, and $b \wedge d = \bot$, it follows from finite additivity that $p(a \vee c) = p(b \vee d)$. But if 'is red' and 'is yellow' are peculiar or natural to the same degree, and 'is orange' and 'is purple' are peculiar or natural to the same degree, it shouldn't follow that 'is red or orange' is peculiar or natural to the same degree as 'is yellow or purple'. Rather, 'is red or orange' should have a higher weight in determining degree of resemblance and a lower degree of peculiarity than 'is yellow or purple', since red and orange particulars are similar with respect to colour whereas yellow and purple particulars are not.

Third, suppose finite additivity is strengthed to (ultra-additivity), according to which for every subset, finite or infinite, $B \subseteq A$ such that $a \wedge b = \perp$ for all $a, b \in B$, $p(\bigvee B) = \sum_{b \in B} p(b)$.[17] Then if there is an infinite subset $B \subseteq A$ such that $a \wedge b = \perp$ for all $a, b \in B$ and every predicate $b \in B$ is peculiar to the same non-negative degree, the sum $p(\bigvee B) = \sum_{b \in B} p(b)$ must be zero or infinite. But if $p(\bigvee B) = \sum_{b \in B} p(b)$ is infinite, this contradicts the fact that for all $a \in A$, $p(a) \leq 1$. So if there is an infinite subset $B \subseteq A$ such that $a \wedge b = \perp$ for all $a, b \in B$ and every predicate $b \in B$ is peculiar to the same non-negative degree, then the sum $p(\bigvee B) = \sum_{b \in B} p(b)$ is zero.

However, it seems as if there are infinite sets of predicates which all have the same non-negative degree of peculiarity. It seems, for example, that there is an infinite number of determinate shades of colour, which all have equal weight in determining degree of resemblance. Their equal degree of peculiarity cannot be positive, since then the degree of peculiarity of their disjunction 'is coloured' would be an infinite number greater than one, contradicting the fact that all degrees of peculiarity are real numbers less than one.

[17]The name "ultra-additivity" comes from Skyrms (1983, 227).

So their equal degree of peculiarity must be zero, and the sum of their zero degrees of peculiarity or the degree of peculiarity of their disjunction 'is coloured' must be zero too. But since there is a great deal of heterogeneity amongst the things that are coloured, the degree of peculiarity of 'is coloured' should be greater than zero.

This third problem is the least pressing, since it arises equally for the case of probability and measurement in general, where the denial of ultra-additivity is the modern solution to Zeno's paradox of measure (Skyrms, 1983, 235). Moreover, it is just one of many other counterintuitive results concerning infinity and connected with the axiom of choice – such as, for example, the Vitali and Banch-Tarski paradoxes – which afflict probability and measurement in general (Skyrms, 1983, 242-5). But since the first two problems arise even in the finite case, infinity is not the whole source of the problem, and a problem arises even if ultra-additivity is denied. Moreover, the first two problems do not have any probabilistic analogue.

In order to escape these problems with this approach, one not unnatural proposal is to weaken finite additivity to require that the peculiarity of the disjunction of inconsistent predicates is not strictly equal to but merely greater than or equal to the sum of the disjunctions or, in other words, to: (finite superadditivity) for all $a, b \in A$ such that $a \wedge b = \perp$, $p(a \vee b) \geq p(a) + p(b)$ (Wang and Klir, 2009, 67). The peculiarity of a disjunction such as 'is red or orange', for example, is greater than or equal to the sum of the peculiarities of the disjuncts 'is red' and 'is orange'.

This characterisation still captures the desired asymmetry between conjunctive and disjunctive properties. For for all predicates $a, b \in A$ such that $a \leq b$, $b = a \vee (b \wedge \neg a)$, and so it follows from finite superadditivity that $p(b) \geq p(a) + p(b \wedge \neg a)$ and so $p(b) \geq p(a)$. So it still follows that for all $a, b \in A$, $p(a \wedge b) \leq p(a) \leq p(a \vee b)$ or, in other words, that the weight of a conjunction is greater than or equal to the weight of the conjuncts, whereas the weight of a disjunction is less than or equal to the weight of the disjuncts.

But the three counterintuitive consequence don't follow. First, even if $p(a) = 0$ and $p(b) = 0$, it doesn't follow that $p(a \vee b) = 0$, but only that $p(a \vee b) \geq 0$. If 'is red' and 'is green', for example, are perfectly natural or not at all peculiar, 'is red or green' may still be less than perfectly natural or somewhat peculiar. Second, even if $p(a) = p(b)$, $p(c) = p(d)$, $a \wedge c = \perp$ and $b \wedge d = \perp$, it doesn't follow that $p(a \vee c) = p(b \vee d)$, but only that $p(a \vee c) \geq p(a) + p(c) = p(b) + p(d) \leq p(b \vee d)$. So even if 'is red' and 'is yellow' are peculiar or natural to the same degree, and 'is orange' and 'is purple' are peculiar or natural to the same degree, it doesn't follow that 'is red or orange' is peculiar or natural to the same degree as 'is yellow or purple'.

Third, suppose finite superadditivity is strengthened to (ultra-superadditivity), according to which for every subset, finite or infinite, $B \subseteq A$ such that $a \wedge b = \perp$ for all $a, b \in B$, $p(\bigvee B) \geq \sum_{b \in B} p(b)$. Then even if there is an infinite subset $B \subseteq A$ such that $a \wedge b = \perp$ for all $a, b \in B$ and every predicate $b \in B$ is natural to the same non-negative degree, the sum $\sum_{b \in B} p(b)$ must still be zero or infinite. Since $\sum_{b \in B} p(b)$ cannot be infinite, it must be zero. However, it only follows from this and ultra-superadditivity that $p(\bigvee B) \geq 0$, which is unexceptionable.

Supposing, for example, that there is an infinite number of determinate shades of colour, which all have equal non-negative weight in determining degree of resemblance.

Their equal degree of peculiarity still cannot be positive, since then the degree of peculiarity of their disjunction 'is coloured' or the sum of their equal degrees of peculiarity would be an infinite number greater than one. So their equal degree of peculiarity must be zero. But it follows from this and ultra-superadditivity only that the degree of peculiarity of their disjunction 'is coloured' is greater than or equal to zero, which is unexceptionable.

Some problems with infinity remain. Consider, for example, a two kilogram weight, which is more similar in respect of mass to a three kilogram weight than it is to a ten kilogram weight. If this is to be so, then some property which the two kilogram weight has in common with the three kilogram weight but not in common with the ten kilogram weight must have a positive degree of naturalness or importance. For the sake of illustration, say it is the property of weighing between one and four kilograms. It follows that the infinitely many properties which entail this property, such as weighing between one and r kilograms for any r between three and four, will have at least as high a degree of naturalness or importance.

Since all these properties are in common between the two and three kilogram weights, the degrees of importance and naturalness of their properties in common will sum to an infinite number, and their weighted proportion of properties will be undefined. One may try to avoid this problem by assigning positive weight only to properties which are entailed by a finite number of other properties (in other words only to properties corresponding to predicates which are finite disjunctions of atoms). But this is unacceptably *ad hoc* – after all being between one and four kilograms in weight is intuitively a property which makes for resemblance. Moreover, it is difficult if not impossible to assign the degrees in such a way that things closer together in respect of weight resemble each other more.[18]

Nevertheless, analysing degree of resemblance as a function of properties in common and not in common, weighted by degrees of naturalness or importance, represents a significant improvement over analysing degree of resemblance simply as a function of number of properties in common and not in common. Most importantly, it provides a precise way to accommodate the intuition that conjunctive properties make for resemblance more than disjunctive properties do. While difficulties remain in the infinite case, the same is true for rival theories of degree of dissimilarity, such as those which attempt to treat it in analogy with spatial distance (Blumson, 2019a). The finite case is hard enough on its own.[19]

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[18]This problem, suggested by anonymous referee, generalises objections to Rodriguez-Pereyra (2002) given by Yi (2018). Blumson (2018, 34-6) also raises similar problems of infinity for the analysis of degree of similarity as proportion of properties in common, some of which generalise to analysis of degree of similarity as weighted proportion of properties in common. I intend to take this up again elsewhere.

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