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If it's clear, then it's clear that it's clear, or is it? Higher-order vagueness and the S4 axiom*

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1. Introduction

In his pioneering essay on the treatment of vague expressions and the Sorites paradox in antiquity, Jonathan Barnes writes: 'If you don't know whether a_i is clearly F, then it is not clearly clearly F and hence not clearly F.'[1] This sentence contains two conditional claims about vague predicates. The present paper is about the second: if *a* is not clearly clearly F, then *a* is not clearly F. Or, contrapositively, if it is clear that *a* is F, then it is clear that it is clear that *a* is F. Formally, the principle at issue can be expressed thus:

(CC) $\quad CA \rightarrow CCA$.

In analogy with standard modal logic, this principle is sometimes called the S4 axiom. To indicate that the principle is about clarity rather than, say, knowledge, we call it the CC Principle. Instead of 'clear' and 'C', some use the expressions 'determinate' or 'definite' and the operators *Def*, DET, Δ, or similar. For the purpose of this paper, these will all be treated alike. (CC) does by no means find universal approval among vagueness theorists. Some take it for granted that it should be rejected. Thus Richard Heck writes: 'Surely, anyone who takes higher-order vagueness seriously is going to want to deny *Def*(P) $\rightarrow$ *Def*(*Def*(P))'.[2] In fact, it seems to be the prevalent view, shared by bivalence-preservers (like Williamson) and bivalence-discarders (like Dummett), that (CC) is to be rejected, since it is incompatible with higher-order vagueness.[3] Yet,

* It is my pleasure to dedicate this paper to Jonathan Barnes, from whom I have learned more than from anyone else, and from whose kindness and generosity I have benefited more than I can ever hope to return.

[1] Barnes (1982), p. 55, n. 78.

[2] Heck (1993), 203–4.

[3] For example, Dummett (1975), 311; Wright (1987) and (1992); Williamson (1994), pp. 159, 271–2 and (1999); Greenough (2003); cf. also Garrett (1991) 347.

in recent years, (CC) has found several supporters of rank among bivalence-discarders.[4] But it has not, so far, been shown how exactly the presence or absence of (CC) manifests itself within the various mainstream theories of vagueness.

The purpose of this paper is to challenge some widespread assumptions about (CC). *First*, we argue that, contrary to common opinion, higher-order vagueness and (CC) are perfectly compatible (Section 4). This is in response to claims like the one by Timothy Williamson that, if vagueness is defined with the help of a clarity operator that obeys (CC), higher-order vagueness disappears. *Second*, we argue that contrary to common opinion, (i) bivalence-preservers (such as epistemicists) can without contradiction condone (CC); and (ii) bivalence-discarders (such as open-texture theorists, supervaluationists) can without contradiction reject (CC) (Sections 5–7). To this end, we show how in the debate over (CC) two different notions of clarity are in play and what their respective functions are in accounts of higher-order vagueness (Section 3). *Third*, we rebut a number of arguments that have been produced by opponents of (CC)—in particular, by Williamson (Section 8). Since discussants of (CC) have employed rather heterogeneous nomenclatures, we introduce diagrams to facilitate comparisons between the various theories.

2. Higher-order borderline vagueness: some preliminary remarks

(i) This paper is concerned only with Sorites-vague predicates—that is, predicates that give rise to Sorites paradoxes. Every Sorites paradox runs on what we call its *dimension D*. Thus, for a paradox built on 'tall', the dimension *D* is height.[5] Multi-dimensional Sorites-vague predicates are considered only insofar as they give rise to one-dimensional Sorites paradoxes. The most basic case of (CC) is then

$$\mathrm{C}\mathrm{F}a \rightarrow \mathrm{CC}\mathrm{F}a,$$

with F for a simple, ordinary language, Sorites-vague predicate and *a* for a designator. An example would be 'if it is clear that Curly is bald, then it is clear that it is clear that Curly is bald.' Given the context sensitivity of Sorites-vague predicates, the semantic value of F*a* is always assumed to be relative to a context *C*.

(ii) We consider in the first instance those theories of higher-order vagueness that intend to capture higher-order borderline cases; that is, borderline borderline cases, borderline borderline borderline cases, and so on. This type of higher-order vagueness—which we also call higher-order *borderline* vagueness—is the one most frequently

[4] In addition to Barnes (1982), these include Field (2000) and Soames (2003). It is considered by Shapiro in his (2006), for example, ch. 5.6, and by Fine (1975). Fine does not accept (CC) outright, but he ponders the possibility that 'definitely *A*' is simply elliptical for '"*A*" is true'.

[5] For 'bald', typically the dimension is number of hairs, increasing by one; for 'red' it may be a series of color patches from red to orange, and so on.

discussed.[6] Here is an illustration. People may think that Curly is clearly bald, and that Moe is clearly borderline bald, but that with respect to baldness, a third man, Larry, falls somewhere between Curly and Moe; more precisely, somewhere on the border of clear baldness and borderline baldness. In that case, Larry is a borderline borderline case of bald. And we could imagine a fourth man, Shemp, who is a borderline borderline borderline case of bald.

Diagram 0

Clearly bald		**BL clearly bald / BL-bald**	**Clearly BL bald / not bald**	. . .
Curly	**Shemp**	**Larry**	**Moe**	
(Dimension *D*: number of cranial hairs, increasing from left to right).				

Note that Curly, Moe, Larry, and Shemp all belong to the same class of objects: men. More generally, if we take number of hairs as the dimension *D* of 'bald' that varies from person to person, and represent this dimension as a line, with increasing numbers of hairs from left to right, we can see that borderline cases of baldness of all orders can be mapped onto that one line. This is a typical case of higher-order *borderline* vagueness. An alternative way of expressing the idea of higher-order borderline vagueness is that, if it is borderline unclear whether F*a*, then *a* is a borderline borderline case of F, or second-order borderline F.[7]

(iii) It has become customary to define 'borderline case' with the help of a clarity (definiteness, determinateness) operator. Often, first a clarity operator is introduced, and then an account of 'borderline case' or 'borderline utterance' or similar[8] is given in terms of the *absence* of such clarity. Sometimes an unclarity operator is defined in terms of the respective clarity operator, for instance (with 'U' for 'it is unclear whether', 'it is indeterminate whether', and so on):

$$\mathrm{U}A \quad =_{\mathrm{df}} \quad \neg \mathrm{C}A \ \& \ \neg \mathrm{C}\neg A.$$

That is, 'it is unclear whether *A* iff it is neither clear that *A* nor clear that not *A*'. The most basic case is 'It is unclear whether F*a* iff it is neither clear that F*a* nor clear that not

[6] For example, in Sainsbury (1991), Wright (1992), Fara (2003), Shapiro (2005) and (2006) ch. 5, and Greenough (2005). We believe that the examples usually given, including our own, are best analysed not in terms of higher orders but as borderline nestings. However, in this paper we consider mostly how the mainstream theories of vagueness and *their* notions of higher-order vagueness relate to the S4 axiom, and for that reason retain the description in terms of higher-order vagueness.

[7] A quite different problem of higher-order vagueness, which we disregard in this paper, is the question of the vagueness of the predicate 'vague'. This is discussed, for example, in Sorensen (1985), Varzi (2003), and Hyde (2003).

[8] An utterance of F*a* is a borderline utterance of F*a* iff *a* is a borderline case of F. A sentence F*a* is a borderline sentence with regard to context *C* iff with regard to *C a* is a borderline case of F.

Fa.' The unclarity of Fa is then explicitly or implicitly equated with a being a borderline case of F. As the basic semantics of 'unclear whether' suggests, when it is unclear whether A, it is also unclear whether $\neg A$:

(U) $\quad$ $UA \leftrightarrow U\neg A$.[9]

Higher-order borderline cases can be described by saying that it is unclear whether it is unclear whether A, and so on, and formalized as UUA or $U^2A, U^3A, \ldots U^nA$.

Diagram 1a represents the distribution on a dimension D in a context C of first-order borderline cases (**B**) with the respective—uninterpreted—clarity and unclarity operators indicated underneath.

Diagram 1a

Level 1	**¬B**	**B**	**¬B**	dimension $D \rightarrow$
	CFx	UFx	$C\neg Fx$	

On the left side, we have to imagine the a-s for which it is not the case that a is borderline F, and for which it holds that it is clear that a is F. In the middle, we have the a-s that are borderline F, and for which it holds that it is unclear whether a is F. On the right side, we have the a-s for which it is not the case that a is borderline F, and for which it holds that it is clear that a is not F. Imagine the number of hairs or grains, and so on, on the dimension D to increase or decrease respectively from the left to the right, with a borderline area in the middle.

Diagram 1b illustrates the distribution on a dimension D in a context C of second-order borderline cases ($\mathbf{B}^2$) with the respective combinations of—uninterpreted—clarity and unclarity operators indicated underneath:

Diagram 1b

Level 2	$\neg\mathbf{B}^2\wedge\neg\mathbf{B}$	$\mathbf{B}^2$	$\neg\mathbf{B}^2\wedge\mathbf{B}$	$\mathbf{B}^2$	$\neg\mathbf{B}^2\wedge\neg\mathbf{B}$	dimension $D \rightarrow$
	C^2Fx	U^2Fx	$CUFx$	U^2Fx	$C^2\neg Fx$	

Higher orders can be depicted in the same way, barring shortage of space.[10] The distribution of borderline cases and non-borderline cases on a dimension as represented

[9] $UA \leftrightarrow \neg C\neg A \wedge \neg C\neg\neg A$ $\quad$ definition of UA, substitution of $\neg A$ for A
$UA \leftrightarrow \neg C\neg A \wedge \neg CA$ $\quad$ double negation
(Substitution of logically equivalent formulae within the scope of C is taken to be permitted.)

[10] Diagram 1b shows that U^2Fa is equivalent to $\neg C^2Fa \wedge \neg CUFa \wedge \neg C^2\neg Fa$. Generally, U^nFa is equivalent to $\neg C^nFa \wedge \neg C^{n-(n-1)}U^{(n-1)}Fa \wedge \neg C^{n-(n-2)}U^{(n-2)}Fa \wedge \ldots \wedge \neg C^{n-(n-m)}U^{(n-m)}Fa \wedge \neg C^n\neg Fa$, with n+1 conjuncts and $m = (n-1)$.

in the diagrams is characteristic for higher-order borderline vagueness in its standard form, as it is most commonly understood.

In fact, the diagrams make visually explicit several assumptions that seem to hold of any Sorites-vague predicate F relative to a dimension *D*. First, two assumptions about penumbral connections: if an object *a* on *D* is *n* times clearly F, then all objects on *D* to the left of *a* are *n* times clearly F; and if an object *a* on *D* is *n* times clearly not F, then all objects on *D* to the right of *a* are *n* times clearly not F, with $n > 0$. These two assumptions can be expressed formally as:

(PC$_1$) $\quad C^{n}Fa_{m} \rightarrow C^{n}Fa_{m-1}$

(PC$_2$) $\quad C^{n}\neg Fa_{m} \rightarrow C^{n}\neg Fa_{m+1}$,

with the standard directionality from F to ¬F running from left to right. Second, there is the assumption ($C_{\exists}$) that there are clear cases of F and of ¬F. This reflects the basic intuition about simple Sorites-vague predicates that they have non-borderline cases. Third, there is the linearity assumption (LA) that at every level, every *a* falls into one of the categories listed for that level on *D*.[11] For level *n*, we can express this formally as

(LA) $\quad \forall x\,[C^{n}Fx \vee U^{n}Fx \vee CU^{n-1}Fx \vee C^{2}U^{n-2}Fx \vee C^{3}U^{n-3}Fx \vee \ldots \vee C^{n-1}UFx \vee C^{n}\neg Fx]$.

Finally, no assumption is made regarding the question of whether any of the middle sections at levels 2 and higher have instances.[12]

Higher-order borderline vagueness differs from what we call simple higher-order clarity. The latter has (HOC) instead of (LA):

(HOC) $\quad C^{m}\Phi a_{n} \rightarrow C^{m}\Phi a_{n-1}$,

(with $n \geq 1$, and increasing *n* in the direction towards the border between F and ¬F). Diagram 1c illustrates simple higher-order clarity relative to a Sorites dimension *D*.

[11] (LA) does not state that at every level every *a clearly* falls into one of the categories. Rather, for higher-order borderline vagueness, each higher level can be thought of as improving on the previous one. The lower levels may be considered as incomplete (inviting some precisification), or as provisional (inviting some revisionary specification). Either way, level *n* would be superseded and replaced by level *n*+1. Of course, given (T_c), the lower levels are all in *some* sense preserved in the higher ones. (LA) does, however, throw out theories which introduce more than two possible semantic values for (utterances of) sentences governed by the clarity operator (for example, true, false, indeterminate), rather than expressing lack of clarity, as clarity that is unclear. (Thus, for example, the option considered by Edgington (1993), 193–200, is precluded by (LA).)

[12] It has been suggested that this type of higher-order vagueness gives rise to the so-called 'higher-order vagueness paradoxes' if it is taken to be 'radical'—if it allows for an infinite number of higher orders (cf. Wright (1987) §5, Wright (1992), Fara (2003), Greenough (2005), and Shapiro (2006), ch. 5). Several philosophers (such as Heck (1993) and Edgington (1993)) have proposed ways of defusing these paradoxes. For a theory of radical higher-order borderline vagueness that avoids the paradoxes, see Bobzien (2010). In any event, no paradox ensues for theories that assume relatively small and finite numbers of higher orders.

Diagram 1c

Level 1			CFx	UFx	C¬Fx			dimension $D \rightarrow$
Level 2		C^{2}Fx		U^{2}Fx		C^2¬Fx		
Level 3	C^{3}Fx			U^{3}Fx			C^3¬Fx	
Level n C^{n}Fx				U^{n}Fx				C^n¬Fx

Theories of simple higher-order clarity cannot explain the phenomenon of higher-order borderline cases like Larry and Shemp in the example above. Nor do they allow for the possibility of clear borderline cases, or generally of clearly unclear cases. Theories of vagueness who cash out higher-order vagueness in terms of simple higher-order clarity (like Williamson's, for example) thus have an explanatory deficiency.[13]

(PC_1), (PC_2), ($C_\exists$), (LA), and (HOC) are not themselves part of the logical systems that we are considering—those which may or may not contain (CC). Rather, these assumptions allow us to check what consequences such a logical system would have, *if* we restrict our consideration to a set of objects on a dimension that form a Sorites series.

3. Self-revealing versus concealable clarity

With these preliminaries in place, we return to (CC). The dispute over (CC) concerns the logical relations that hold between the different orders of clarity. In terms of borderline cases, it can be rephrased thus: are we to allow for the possibility that something *a* is a second-order borderline case of F but not a first-order borderline case, but rather a first-order clear (definite, determinate) case of F? Epistemicists generally allow for this possibility, whereas bivalence-discarders often do not. Intuition has been invoked in defense of either view, and this fact can be used to direct us towards two distinct notions of clarity with which the discordant parties appear to operate: self-revealing and concealable clarity. We consider them in turn.

What we call self-revealing clarity is a clarity that cannot be concealed but always reveals itself. It shines through all higher-order levels—if you want.[14] If something is self-revealingly clear, it cannot be unclear whether it is clear. The fact that many people judge that, considered *in a single context*, paired sentences such as the following contradict each other, exemplifies how intuition supports this notion:

[13] See Williamson (1994) chs. 7–8, Williamson (1999), and the discussion of Williamson's view of higher-order vagueness in Sections 8.1 and 8.2 below.

[14] Alternatively, you could imagine that at any point on the dimension, higher-order unclarity casts, as it were, a shadow of unclarity down on all lower orders. This notion of self-revelation is similar to Williamson's notion of luminosity; cf., for example, Williamson (2000), ch. 4.

- 'It is clear that Tallulah is tall'—'It is unclear whether it is clear that Tallulah is tall.'
- 'This is undoubtedly sterling silver'—'It is doubtful whether this is undoubtedly sterling silver.'
- 'It is somewhat indefinite whether this colour patch is definitely purple'—'This patch is definitely purple.'

And exploiting the fact that we are considering higher-order borderline vagueness, we can add cases such as:

- 'Baldwin is borderline borderline bald'—'Baldwin is not borderline bald but clearly bald.'
- 'Selma is definitely slim'—'Selma is a borderline borderline case of slim.'[15]

In terms of borderline cases, self-revealing clarity leads to the motto: what smacks of being borderline is borderline.[16] For, if something is in any way tinged by unclarity, it cannot be self-revealingly clear. The utility of this notion of self-revealing clarity lies in the fact that it enables us to separate the clear cases from any cases that may have any lack of clarity to them—even if it is just a hint of a trace of a smidgeon of unclarity. It is a notion of clarity for those who want to be able to express that some fact is surefire and utterly unquestionable.

By contrast, concealable clarity is a clarity that is not automatically detectable through higher-order levels of clarity or unclarity. In this case, something can be clear while it is unclear whether it is clear. Here, plausibility may be gained from intuitions about the semantics of 'unclear whether'. If it is unclear whether she is in the bedroom or the bathroom, then surely she *could be* in the bedroom. Now, if on a dimension D something a is on the border of clear cases and unclear cases of F (as, for example, Larry in Diagram 0), we may express this by saying that it is unclear whether it is clear that Fa or whether it is unclear whether Fa. Hence, analogously to the bedroom/bathroom example, if (i) it is unclear whether it is clear that Fa, or unclear whether Fa, then surely it *could be* the case (ii) that it is clear that Fa. But then (i) and (ii) are compatible; and

[15] *Objection*: In the examples we do not have true contradictions, but only some kind of pragmatic conflict, similar to the one we observe in Moore's paradox ('p. I don't believe that p.'). *Reply*: In Moore's paradox, the two sentences are in conflict only because they are (assumed to be) uttered by the same person. By contrast, 'it is clear that p' and 'it is not clear that it is clear that p', etc., neither contain an indexical (like 'I') nor do they have to be (assumed to be) uttered by the same person, or be in any way relative to a speaker's perception, view, or similar, for them to appear contradictory. (See also Section 8.1 below).

Since vague predicates are, as a rule, context sensitive, there may, of course, be no appearance of contradiction, if the contexts of the two assertions differ. Moreover, even in the same context, if we have what one may call negotiations regarding a temporary stipulation of the extension of a vague predicate, there need be no contradiction. For example, if in a conversational context (along the lines of the theories of conversation in Stalnaker (1979) and Lewis (1979)) the conversationists try to settle the relevant borderlines with regard to that context, tentative proposals may be made by different speakers (cf. Shapiro (2003) and (2006)). The appearance of contradiction is relative to the assumption that the utterance(s) are considered as assertions of how things are, regardless of whether the utterance(s) are made.

[16] This motto differs from Kit Fine's suggestion that anything that smacks of being borderline is treated as a *clear* borderline case; cf. Fine (1975), 297.

since (i) entails (iii), that it is not clear that it is clear that Fa, (ii) and (iii) are also compatible. Hence, with concealable clarity, (ii) and (iii) are not contradictories, as the notion of self-revealing clarity had suggested. This notion of concealable clarity is useful as follows: with it we can express for higher orders of vagueness that, if something is in the border area of a vague expression Φx (Fx, CFx, C^{2}Fx . . .), it may after all be Φ, even if this is not apparent.

4. Concealable and self-revealing clarity and modal logic

Before we consider how self-revealing and concealable clarity correlate with some of the more popular theories of vagueness, one prevalent prejudice against (CC) needs to be laid to rest. Many proponents of higher-order vagueness have remarked on the partial similarity between the logic of clarity (determinateness, definiteness) and modal logic, and in particular between necessity and clarity.[17] One theorem that is generally granted in modal logic,

(T) $\Box A \rightarrow A$

has an analogue in logics of clarity that is equally generally granted:

($\mathrm{T_c}$) $\mathrm{C}A \rightarrow A$;[18]

that is, like necessity, clarity is considered to be veridical. The converse,

$$A \rightarrow \mathrm{C}A$$

is assumed neither for concealable nor for self-revealing clarity. Similarly, most modal logics do not have a theorem

$$A \rightarrow \Box A.$$[19]

[17] For example, Fine (1975), Dummett (1975), Heck (1993), Williamson, (1994), (1999), McGee and McLaughlin (1995), Field (2000) and (2003), Shapiro (2006), and Wright (2010).

[18] Our observations should hold for all systems of higher-order borderline vagueness and of simple higher-order clarity. To provide a general idea of what an axiomatic system might look like, here is a set of rules and axioms which could be used to introduce a basic logic of borderline cases, or more generally, of the kind of clarity relevant to vagueness:

(PC)⋆	If A is a truth-functional tautology, then $\vdash A$
(MP)	If $\vdash A \rightarrow B$ and $\vdash A$, then $\vdash B$
($\wedge$I)	If $\vdash A$ and $\vdash B$, then $\vdash A \wedge B$
($\mathrm{T_c}$)	$\vdash \mathrm{C}A \rightarrow A$
(C$\wedge$)	$\vdash [\mathrm{C}A \wedge \mathrm{C}B] \rightarrow \mathrm{C}[A \wedge B]$

⋆ (PC) works for supervaluationists and epistemicists. For those who maintain that classical logic may not apply in the borderline zone, the rule can be replaced by one that makes truth-functional tautologies dependent on the clarity of their component formulae (cf. Bobzien (MS)).

[19] Most standard modal logics do have a meta-rule that if A is provable (from no undischarged premisses), then so is $\Box A$. The equivalent in a logic of borderline cases would be that if you can prove A, you can prove that A is not borderline, or, in short, that nothing provable is borderline. Here is not the place to discuss this point.

Where self-revealing and concealable clarity come apart is

(CC) $CA \rightarrow CCA$.

(CC) is granted only by proponents of self-revealing clarity. For systems with self-revealing clarity, we hence also have[20]

(C4) $CA \leftrightarrow CCA$,

which makes them resemble System S4.[21] This fact has led to unjustified vilification of self-revealing clarity, based on a misguided analogy. System S4 can be characterized along the following lines: 'In S4, the sentence $\Box\Box A$ is equivalent to $\Box A$. As a result, any string of boxes may be replaced by a single box, and the same goes for strings of diamonds. This amounts to the idea that iteration of the modal operators is superfluous. Saying that A is necessarily necessary is considered a uselessly long-winded way of saying that A is necessary.'[22] In the same vain, it has been suggested that self-revealing clarity amounts to a collapse of higher-order vagueness to first-order vagueness.[23] But the fact that

$$CA \leftrightarrow CCA$$

holds, does not have this effect at all.[24] In S4, not only is saying that A is necessarily necessary a long-winded way of saying that A is necessary, but also is saying that A is possibly possible just a long-winded way of saying that A is possible.[25] However, in the case of higher-order vagueness, the relevant two notions of clarity and unclarity (or non-borderlinehood and borderlinehood) are related in a different way. 'It is unclear whether A' is not synonymous to 'it is not (the case that it is) clear that not A'; that is, we do not have

$$UA \leftrightarrow \neg C\neg A.$$

Nor, by the by, is 'It is unclear whether A' synonymous with 'it is not (the case that it is) clear that A'; that is, we do not have

$$UA \leftrightarrow \neg CA$$

either. Rather, as we stated in Section 2, the relation between clarity and unclarity is

$$UA \leftrightarrow \neg CA \wedge \neg C\neg A.$$

20 We get $CCA \rightarrow CA$ by substituting CA for A in (T_c). From $CCA \rightarrow CA$ and (CC) we get (C4) by using the rule ($\wedge$I) and the definiens of $\leftrightarrow$, $[A\rightarrow B]\wedge[B\rightarrow A]$.

21 We say 'resemble', since nothing has been said so far about whether a logic of clarity must include counterparts for the axiom (K) and rule (RN); or for the rule that if A is derivable from premises, all of which begin with '$\Box$', then $\Box A$ is derivable from those very premises. The equivalent for the latter in a logic of clarity would be what Crispin Wright christened 'rule DEF' (Wright (1992), 131), and the validity of which is questioned by Edgington (1993) and Heck (1993).

22 Quoted from Garson (2009), *Stanford Encyclopedia of Philosophy*, 'Modal logic', Section 2.

23 For example, Williamson (1994), p. 160.

24 The only author we are aware of who has commented on this fact is Hartry Field, in Field (2000), §V.

25 $\Diamond A =_{df} \neg\Box\neg A$

$\Diamond\Diamond A \leftrightarrow \neg\Box\neg\Diamond A \leftrightarrow \neg\Box\neg\neg\Box\neg A \leftrightarrow \neg\Box\Box\neg A \leftrightarrow \neg\Box\neg A \leftrightarrow \Diamond A$:

This difference to the necessity/possibility relation is marked by the fact that we typically say 'clear *that* A' but 'unclear *whether* A'. The logic that governs unclarity is thus analogous to the logic of contingency,[26] not of possibility. Acknowledging this, we can introduce an operator parallel to the non-contingency operator. We simply define 'it is clear whether A' or $\mathrm{C_w}A$ by $\neg \mathrm{U}A$. Using the definition of $\mathrm{U}A$, it follows that

$$(\mathrm{C}/\mathrm{C_w}) \qquad \mathrm{C}A \to \mathrm{C_w}A.$$ [27]

We have seen above that it is $\mathrm{UF}a$—and not $\mathrm{CF}a$—that is used directly in the account of 'borderline case': a is a borderline case of F iff $\mathrm{UF}a$. Now it becomes apparent that with a notion of self-revealing clarity, and (CC) granted, saying that A is clearly clear is just a uselessly long-winded way of saying that A is clear.[28] Yet, saying that it is unclear whether it is unclear whether A is *not* a uselessly long-winded way of saying that it is unclear whether A. For—at least with the theorems introduced so far—it is not the case that

$$\mathrm{UU}A \leftrightarrow \mathrm{U}A.$$

It may hold that

$$(\mathrm{UU/U}) \qquad \mathrm{UU}A \to \mathrm{U}A$$

But, or so most vagueness theorists assume, it certainly does not hold that

$$\mathrm{U}A \to \mathrm{UU}A.$$

For it seems possible for there to be a and F such that a is a clear borderline case of F, or, in other words, such that it is possible that it is both unclear whether Fa and clear that it is unclear whether Fa; that is

$$\mathrm{UF}a \wedge \mathrm{CUF}a.$$ [29]

By equivalence transformation we obtain

$$\neg\, [\mathrm{UF}a \to \neg \mathrm{CUF}a]$$

[26] Cf., for example, Kuhn (1995), and Montgomery and Routley (1966).

[27]

$\mathrm{U}A \leftrightarrow \neg \mathrm{C}A \wedge \neg \mathrm{C}\neg A$	definition $\mathrm{U}A$
$\mathrm{U}A \to \neg \mathrm{C}A$	by $[A \leftrightarrow B \wedge C) \to [A \to B)$
$\mathrm{C}A \to \neg \mathrm{U}A$	by contraposition, double negation
$\mathrm{C}A \to \mathrm{C_w}A$	by definition of $\mathrm{C_w}A$.

[28] This seems intuitively plausible. When someone says 'it is clearly clear that Simone is small', it is hard to detect in it any substantial addition to 'it is clear that Simone is small'. But intuitions vary, and those at work here are obviously those that support self-revealing clarity.

[29] When this paper was originally conceived (in 2006), it was almost universally assumed by vagueness theorists that if there is higher-order borderline vagueness, then there are clear borderline cases. This assumption has been challenged in Bobzien (2010). Those who doubt the very existence of true higher-order vagueness (for example, Sainsbury (1991), Wright (1992), (2003 implied) and (2010), Fara (2003), and Shapiro (2006)—to which, in response, Bobzien (2009)—Smith (2009), and Raffman (2010)) are invited to consider the argument in the main text *ad hominem*.

from which, by (C/C_w), it follows that

$$\neg\,[UFa \rightarrow \neg C_w UFa]$$

which, by the definition of C_w is equivalent to

$$\neg\,[UFa \rightarrow UUFa].$$

We can infer that

$$UA \rightarrow UUA$$

does not hold. Hence (CC) does not make higher-order vagueness disappear.[30] Diagram 1b in Section 2 illustrates that this is the result we would expect for Sorites series on *D*. At level 2, there are sections for clearly clear cases, and for two kinds of borderline cases: U^2 cases and CU cases. They are distributed on the dimension *D* as follows:

$$C^2Fx \qquad U^2Fx \qquad CUFx \qquad U^2Fx \qquad C^2\neg Fx.$$

At level 3, we have nine sections with four different cases: C^3, U^3, CU^2 and C^2U, distributed on D as follows:

$$C^3Fx \quad U^3Fx \quad CU^2Fx \quad U^3Fx \quad C^2UFx \quad U^3Fx \quad CU^2Fx \quad U^3Fx \quad C^3\neg Fx.$$

The center sections on *D*, $CUFx$ from level 2 and C^2UFx from level 3, would be equivalent. This is, in fact, just an instance of

$$CA \leftrightarrow CCA$$

but neither U^3Fx nor CU^2Fx is equivalent with $CUFx$, nor are they equivalent to each other. And although U^3Fx entails U^2Fx, the converse does not hold. At higher levels we have increasingly more different kinds of borderline cases, with an increase of one kind per level. It follows that it is simply wrong to assume that just because a logic of self-revealing clarity adopts the characteristic axiom of S4, higher-order borderline vagueness disappears, or that all statements of higher-order unclarity collapse into those of first-order unclarity. Rather, at each level we obtain just as many different types of higher-order borderline cases as we expect.[31] Thus the structural identity of

[30] For a more complex argument that shows that even if there are no clear borderline cases, (CC) does not make higher-order vagueness disappear, see Bobzien (2010).

[31] Åkerman and Greenough (2010a), p. 287, n. 37, observe that if we use the logic of S4 (KT4), 'then there is only a finite number of modalities (in fact at most fourteen distinct modalities . . .). Consequently, there cannot be borderline cases to borderline cases ad infinitum.' However, this seems to be just another instance of the mistaken analogy of borderlinehood with possibility rather than with contingency. We saw in Section 2 that the modal operator for borderlinehood (the unclarity operator U) is defined as $\neg CA \wedge \neg C\neg A$. Iterations of the operator U (together with its combination with the clarity operator C) thus do not collapse into the fourteen distinct basic modalities of the standard modal system S4.

(CC) with the S4 axiom alone provides no reason for dismissing a logic of higher-order borderline vagueness that includes (CC).[32]

5. Comparison between concealable and self-revealing clarity

There is thus no difference between concealable and self-revealing clarity with regard to the possible number of types of higher-order borderline cases per order. The difference between logics of vagueness with and without (CC) is more subtle. As mentioned above, it concerns the relation between different orders. We have just seen that both with and without (CC) in place, the extensions (or quasi-extensions)[33] of the borderline cases, borderline borderline cases, and so on, do not necessarily coincide. For example, something can be a borderline case without being a borderline borderline case. But with (CC), the extensions (or quasi-extensions) of clear cases and clearly clear cases, and so forth, do coincide, even though the boundaries between the clear cases and the unclear cases need not be sharp. By contrast, without (CC), the extensions (or quasi-extensions) of clear cases and clearly clear cases need not coincide. We can show the impact of (CC) on higher-order borderlinehood by considering Shemp in Diagram 0. There, Shemp was borderline borderline borderline bald. Without (CC), for all we know, Shemp could be neither first-order nor second-order borderline bald, but instead both clearly and clearly clearly bald, despite the fact that he is third-order borderline bald. By contrast, if (CC) is taken to hold, Shemp would be a borderline case of bald, though only borderline borderline so; and he would be a borderline borderline case, though only borderline so. It is the coextensionality of clear and clearly clear cases by which (CC) endears itself to some supervaluationists and open-texture theorists, but proves unpopular with epistemicists—as will be seen presently.

[32] *Objection*: a logical system of the kind suggested that contains (CC) is inconsistent:

(1)	Assume	UUFa	assumption.
(2)	Hence	UFa	(1), by (UU/U), assuming this holds.
(3)	Hence	CUFa	since surely anything that can be inferred must be clear in the sense defined.
(4)	Hence	C_wUFa	(3), (C/C_w).
(5)	Hence	$\neg$ UUFa	(4), definition of C_w.

But (5) contradicts (1). Hence the system is inconsistent. *Reply*: step (3) is problematic. It assumes that the clarity of inference is identical with or entails the clarity of non-borderline utterances. Or, expressed differently, it assumes that the clarity operator C is governed by a rule that allows one to infer CA from A in natural deduction modal logics. However, we have no reason to believe that this is the case.

[33] A quasi-extension is for a theory of predicates without sharp boundaries what an extension is for a theory of predicates with sharp boundaries; cf. for example, Soames (2003).

6. Bivalence-discarders and concealable clarity

Among bivalence-discarders, those who prefer systems with self-revealing clarity either hold that

(C4) $\quad CA \leftrightarrow CCA$

is compatible with higher-order vagueness, or deny the existence of higher-order borderline cases, in which case (C4) is vacuously true.[34] For instance, some supervaluationists assume that all borderline utterances (or borderline sentences) are neither true nor false, and consider the possibility that 'clearly *A*' is merely elliptical for '"*A*" is true'.[35] On this assumption, (CC) becomes immediately convincing. For, 'it is clear that it is clear that *A*' translates into '"'*A*' is true" is true'; and by (CC), in those cases in which '*A*' is true, '"*A*" is true' is true too. And this is immensely plausible. Proposing quite a different theory of higher-order vagueness, based on his analysis of vague predicates as partially defined, Scott Soames has also reached the conclusion that (CC) holds.[36]

However, it is by no means necessary for bivalence-discarders to accept (CC). In particular, those who wish to allow for an epistemic element in their theory may find a notion of concealable clarity useful for expressing this. To give an example, they may define first-order borderline cases as those where it is unclear whether F*a* in the sense that one can't tell whether F*a*,[37] and hold that among those cases there are some $a_m - a_n$ which make F*x* true, some $a_p - a_q$ which leave F*x* indeterminate,[38] and some $a_s - a_t$ which make F*x* false, as depicted in Diagram 2a. (The curly brackets '{' indicate the absence of a sharp border.)

Diagram 2a

Level 1	¬B	{		B		{	¬B	
	CF*x*	{		UF*x*		{	C¬F*x*	
F*x*:		True	{	Indeterminate	{	False		dimension *D* →

Moving up one level, such epistemically minded bivalence-discarders may define second-order borderline cases as those where it is unclear whether it is unclear

[34] For example, Field (2000), Soames (2003), Shapiro (2005), Shapiro (2006) ch. 5, and considered by Fine in his (1975), §5.

[35] For example, Fine (1975), 296, with 'definitely' instead of 'clearly'. Fine points out two problems with this 'ellipsis' view.

[36] Soames (2003); see especially Section 4.2.

[37] Here we leave aside the question of who exactly it is who cannot tell. In Section 8 we show that the interpretation of UF*x* as inability to tell whether F*x* does not commit one to the KK Principle.

[38] We here use 'indeterminate' as a substitute for whatever alternative semantic status a bivalence-discarder may wish to assign to these cases.

whether Fa in the sense suggested above. Thus they may hold that among the second-order borderline cases there are (or may be in any case) some $a_m - a_n$ which make CFx true, some $a_p - a_q$ which leave CFx indeterminate, and some $a_s - a_t$ which make CFx false. This is illustrated in Diagram 2b, which corresponds to the left half of the previous diagram. (The grey areas indicate concealed clarity, and the ellipses indicate the missing right half of the diagram.)

Diagram 2b

Level 2	$\neg\mathbf{B}\wedge\neg\mathbf{B}^2$	{	$\mathbf{B}^2$	{	$\mathbf{B}\wedge\neg\mathbf{B}^2$	. . .
	C^2Fx	{	U^2Fx	{	CUFx	. . .
CFx:		True }	Indeterminate {	False		dimension D →

Higher orders work in the same way. In this manner, the epistemically inclined non-epistemicists could use concealable clarity in order to express that for every vague predicate of the form Φx (including Fx, CFx, C^2Fx, . . . C^nFx) there is a margin such that Φx but not clearly Φx.[39] For those margins they would permit facts of the kind that color patch number n is clearly blue, but it is unclear whether it is clearly blue.

7. Bivalence-preservers and self-revealing clarity

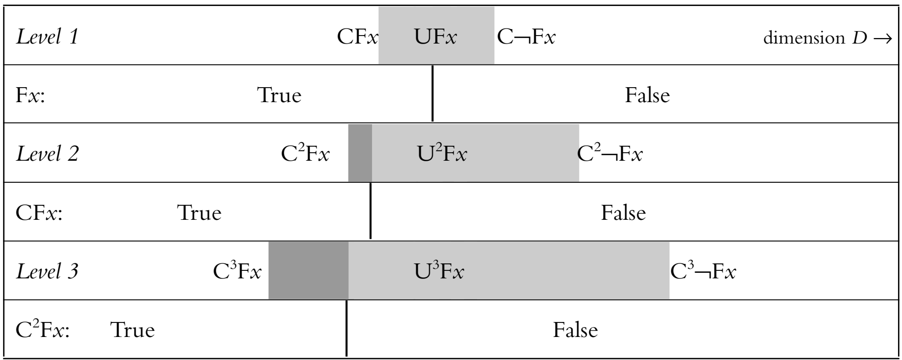
Epistemicists—the most notorious of bivalence-preservers—tend to maintain that the only workable notion of clarity is concealable clarity. For example, Timothy Williamson suggests 'knowably A' as an interpretation of 'it is clear that A' or 'definitely A'.[40] (CC) then becomes 'if it can be known that A, then it can be known that it can be known that A', and is consequently rejected.

With regard to a dimension D, at the first level, the epistemicist assumption is that all first-order unclarity utterances are *de facto* either true or false; we just cannot know which one. That is, there are on D some $a_m - a_n$ which make Fx true and some $a_p - a_q$ which make Fx false, but we cannot know that they do so. At the second level, there are assumed to be on D some $a_m - a_n$ which make CFx true and some $a_p - a_q$ which make CFx false, but of which, again, we cannot know that they do so. For these a it is unclear whether they are clearly F. Higher orders work accordingly. (At each level, it is possible for there to be cases which make C^nFx true but C^{n+1}Fx false.) Thus we have a theory with concealed clarity, as illustrated in Diagram 3.

[39] For example, modeled on Williamson (1994), pp. 271–2.

[40] Williamson (1994), pp. 164 and 195. We noted in Section 2 that Williamson's logic of clarity is not a logic of higher-order borderline vagueness, but of simple higher-order clarity, and that it precludes clear borderline cases (cf. 1c). With classical logic and standard modal logic, it seems impossible to provide a satisfactory model for higher-order borderline vagueness that permits clear borderline cases.

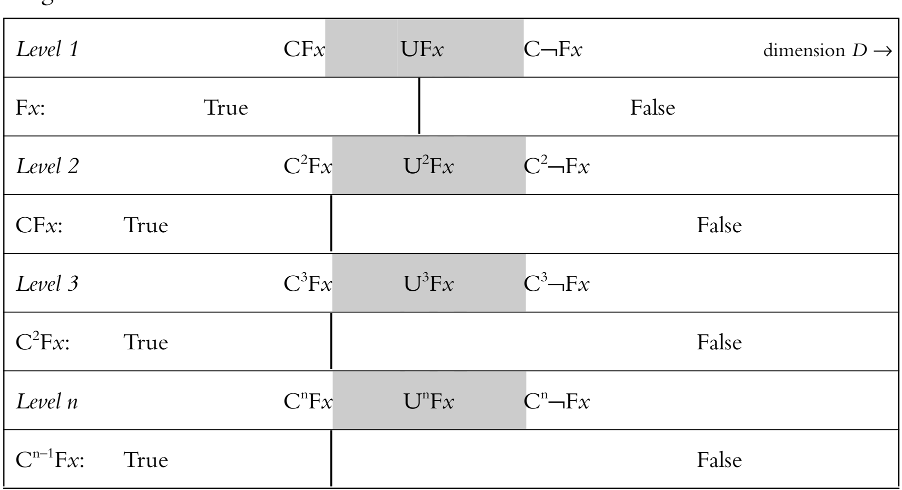
Diagram 3



(The vertical lines '|' indicate the sharp boundaries between the true and the false cases, the grey areas indicate the sections on *D* where the truth-value is unknowable, and the darker grey areas indicate the sections on *D* where clarity is concealed.)

Still, bivalence-preservers, including epistemicists, *need not* postulate concealable clarity to keep their theory consistent for higher orders of unclarity. The following scenario shows how this works. We retain the assumptions (i) that classical logic and semantics hold for vague predicates; (ii) that there are sharp boundaries between the clear and the unclear cases at every order of clarity; and (iii) that the sharp boundaries are unknowable. With regard to any dimension *D*, we keep the requirements for simple higher-order clarity, as introduced in Section 2 (and accepted, for example, by Williamson). Now we simply add (CC), thereby trading concealable clarity for self-revealing clarity. The result is perfectly consistent. The only change of note is that instead of having the higher orders of clarity—possibly—staggered with regard to the lower ones, now the sharp boundaries between the clear and unclear cases of all orders of clarity coincide, as illustrated in Diagram 4.

Diagram 4



In addition to the feature of Williamson's theory that there are no clear unclear cases, it now also holds that if it is clear that Φa, then it is also clearly clear that Φa. One may be tempted to object that as a result we can tell, or know, where the last clear case is (or where the first unclear case is); but this is not so. As long as we assume that higher-order vagueness is radical, it is still not possible for us to know or tell where the boundary lies between the clear and the unclear cases. We cannot tell or know where the border between the C^2Fx and the U^2Fx cases is, since it holds of all U^2Fx cases that it is unclear whether they are U^2Fx cases or C^2Fx cases. And we cannot tell or know where the border between the C^3Fx and the U^3Fx cases is, since it holds of all U^3Fx cases that it is unclear whether they are U^3Fx cases or C^3Fx cases. And so it goes on. Hence we cannot tell or know where the border is between the clear and the unclear cases. Thus we have constructed a bivalence-preserving theory of higher-order vagueness which contains (CC) and can be interpreted as epistemicist. (Elsewhere we call this kind of higher-order vagueness COLUMNAR.)

If we suspend judgment about the question whether, at any level, the boundaries are sharp, we obtain an agnostic position like the one developed by us in detail elsewhere.[41] Either way (agnostic or epistemicist), the theory has an advantage over Williamson's own in that it can be interpreted as a theory of *higher-order borderline vagueness* (see Section 2), with the proviso that there are no instances of clear borderline cases at any level. In its agnostic version, it has the attractive advantage over bivalence-discarding theories that it preserves radical higher-order vagueness but escapes the various paradoxes of higher-order vagueness.[42]

In its epistemic version, this theory is open to any bivalence-preserver who is happy to give up at least one of Williamson's assumptions that force him to reject (CC), as there are:

(i) The C-operator is governed by rule (RN).
(ii) The C-operator is governed by axiom (K).
(iii) Clarity is cashed in directly in terms of a notion of knowability that by definition includes a margin for error, such that for any Sorites series it holds that $CFa_n \rightarrow Fa_{n+1}$.
(iv) If $CFa_n \rightarrow Fa_{n+1}$, then also $C^mFa_n \rightarrow C^{m-1}Fa_{n+1}$, for all m>1.

Since each one of (i) to (iv) can be questioned, there may well be epistemicist takers for the option here presented.

8. Discussion of some arguments against (CC)

We conclude the paper by considering the main arguments that have been marshaled against (CC).

[41] In Bobzien (2010).

[42] Cf. Bobzien (2010), Section 11. Elsewhere we provide an explanation of the common assumption that there are clear borderline cases.

8.1. (CC), (KK), and propositional attitudes

Assume that the KK Principle—that if I know something, then I know that I know it[43]—concerns propositional attitudes and that it is false. One standard objection to (CC) is that when it is cashed in in epistemic terms, the difficulties encountered in (KK) reoccur for (CC). For example, if C*p* is interpreted as 'it is known (by some person *s*) that *p*'[44] or 'a person *s* can tell that *p*', we appear to deal with propositional attitudes. CC*p* then becomes 'it is known (by *s*) that it is known (by *s*) that *p*' or 'a person can tell that they/a person can tell that *p*', respectively. For any philosopher who rejects (KK) and accepts an interpretation of C*p* like the ones given, (CC) would appear to be unacceptable too.

However, first of all it is not at all obvious that in the context of vagueness, clarity must be defined in epistemic terms. And even if it *is* thus defined (as we believe it should be), it does not follow that we are dealing with propositional attitudes. For example, CF*x* could be defined roughly along the lines of '*x* is such that all relevantly competent and informed speakers of English, if asked, could tell whether F*x*'.[45] CF*x* would then not express a propositional attitude, but rather a property of items *a* that involves (but cannot be reduced to) certain human dispositions to react to certain things. In that respect, CF*x* would be on a par with expressions such as 'amusing', 'amazing', 'understandable', 'comprehensible', 'instructive', 'funny', 'gloomy', 'readable', '(il)legible' (said of a story, for example), '(un)illuminating', '(un)intelligible', 'boring', 'tedious', 'uninspired' (said of a performance, for example). Just as 'I find it amusing that *p*' and 'he finds it boring that *p*' express propositional attitudes, but 'it is amusing that *p*' and 'it is boring that *p*' often do not,[46] so 'I cannot tell whether *p*' and 'I can tell that *p*' may express propositional attitudes, but 'all relevantly competent and informed speakers of English, if asked, could tell whether *p*' need not. Hence, whenever C*A* and U*A* are defined in epistemic terms, but do not express propositional attitudes, the standard objections against (KK) fail when transferred to (CC).

8.2. Williamson's argument from intuition

In the Appendix of his book *Vagueness*, Timothy Williamson bases his *rejection* of (CC) on intuition:[47] He argues as follows: 'Intuitively, any formula [any propositional

[43] This is Williamson's version of the principle (Williamson (1994), p. 223. For a more formal version see, for example, Greenough (2003), 275.

[44] So Greenough (2003), for example, 259, 275–8.

[45] For example, Shapiro takes this line in his (2003) and (2006), but gives up compositionality for C (definiteness, in his terminology). In Bobzien (2009) we show that compositionality can be preserved for this general type of clarity.

[46] In the simplest case, such expressions can be analyzed along the lines of standard secondary quality predicates; for example, 'is boring' as 'apt to cause, in a normal way, boredom in normal subjects under normal conditions'; more subtle alternatives that involve normative or relativist elements are also available.

[47] Williamson (1994), pp. 271–2. Williamson suggests (in his (1994), p. 274) that he has presented '*the* logic of clarity' (italics ours). We do not believe that there is precisely one correct logic of clarity, even in the restricted sense in which the notion of clarity is introduced to illuminate what borderline cases

formula with or without clarity operator in the logic of clarity that Williamson is presenting] permits a margin in which it is true but not clearly true, unless it takes up all or no conceptual space.'[48] This recourse to intuition is unconvincing. Let us admit for the sake of argument that any first-order vague expression (say, F*x*) permits a margin in which it is true but not clearly true; that is, that there are *a* such that F*a* is true but not clearly true. This may have some plausibility. Still, it is not at all intuitively true that a second-order vague expression (say, CF*x*) permits a margin in which it is true but not clearly true; that is, that there are *a* such that CF*a* is true but not clearly true. This becomes obvious when we reformulate this case on the object-level: it is not at all intuitively true that any second-order vague expression 'it is clear that F*x*' permits a margin for which it holds that 'it is clear that F*x* but it is not clear that it is clear that F*x*'. In fact, here many people experience a strong intuition to the contrary.[49] Williamson's presumed 'intuition' will be shared only by those who, like himself, already think of clarity in terms of concealable clarity. But whether this is how we *should* think of clarity in the context of vagueness is exactly the point under dispute. Intuition does not settle the dispute. There are some philosophical frameworks in which a notion of concealable clarity is suitable, and others in which a notion of self-revealing clarity is suitable. Accordingly, when confronted with the question whether (CC) holds, we can summon intuitions either way.

In his 'On the structure of higher-order vagueness',[50] Williamson uses (a model that corresponds to) standard modal logic to analyze the notion of definiteness relevant to vagueness, claims to provide a system that clarifies structural issues regarding this notion 'without addressing deep questions about the nature of vagueness' (128),[51] and suggests that his theory covers supervaluationist theories as well as epistemicist theories. He writes that 'for the supervaluationist, definiteness is truth under all . . . admissible sharpenings' and 'for the epistemicist, definiteness is truth under all sharp interpretations of the language indiscriminable from the right one' (128). About the S4 axiom he says that 'to deny 4 is to deny that accessibility is transitive; intuitively, there is a close connection between the non-transitivity of indiscriminability and higher-order vagueness' (134) and that 'the addition of 4 alone to KTB . . . gives S5' and then suggests—correctly, in our view—that S5 is not compatible with second-order vagueness (134).[52] This notwithstanding, Williamson's arguments are, again, not compelling. First, the 'close connection between the non-transitivity of indiscriminability and higher-order vagueness' that he mentions exists only if the

are. Rather, as the present paper illustrates, depending on to what features of borderline cases one wishes to draw attention, different logical systems can be used.

[48] Williamson (1994), p. 272.

[49] Cf. above, Section 3.

[50] This is Williamson (1999).

[51] Here we ignore the fact that the question of whether classical logic is the appropriate logic for vagueness is itself a 'deep question about vagueness', and that Williamson simply assumes that it is.

[52] So also Shapiro, who in his (2005) and (2006) bites the bullet and denies the existence of true higher-order vagueness, if at the cost of giving up compositionality. For details, see Bobzien (2009).

indiscriminability is based on his own margin-for-error principle.[53] However, non-epistemicists need not accept this principle (see Section 8.3 below). But then, contrary to what Williamson suggests, his argument is not compelling for supervaluationists. Second, the fact that 'the addition of 4 alone to KTB . . . gives S5' is irrelevant. For, Williamson himself suggests, four pages later, that axiom B should be abandoned (138);[54] and axioms K and T together with the S4 axiom do not give S5.[55]

8.3. Williamson's argument from his margin-for-error principle

In both his (1992) and his (1994), Williamson argues that (CC) does not hold for vague sentences, since it is incompatible with his margin-for-error principle.[56] Here we present a condensed form of his proof. Since Williamson cashes out clarity in terms of knowledge, he uses K rather than C as operator. K denotes 'it is known that'.[57] His analogue to the CC Principle—the KK Principle—can then be expressed as:

(KK) $\mathrm{KF}a_{\mathrm{n}} \rightarrow \mathrm{KKF}a_{\mathrm{n}}$,

with F for a first-order vague predicate and a_{n} for a member of a Sorites series. His margin-for-error principle, insofar as it is relevant to the Sorites paradox, can be formalized as

(ME) $\mathrm{KF}a_{\mathrm{n}} \rightarrow \mathrm{F}a_{\mathrm{n+1}}$,[58]

where a_{n} and $a_{\mathrm{n-1}}$ mark adjacent objects of a Sorites series. Roughly, the margin-for-error principle is justified by the fact that it precludes that F*a* could be false in circumstances very similar to those in which it is known that F*a*. Williamson then 'substitutes' KF*x* for F*x* in (ME):

($\mathrm{ME_K}$) $\mathrm{KKF}a_{\mathrm{n}} \rightarrow \mathrm{KF}a_{\mathrm{n-1}}$

[53] See Williamson (1999), 137, his reference to his (1994), where he argues that the acceptance of the margin-for-error principle leads to the rejection of KK. For details, see Section 8.3 below.

[54] In the same vein, in his (1994), p. 272, Williamson states that axiom B 'of KTB is unobvious', and suggests a model with KT instead of KTB.

[55] The only other support that Williamson provides for his rejection of (CC) is in footnote 5 of his (1999), where he states that 'Schema 4 [(CC)] plays a critical role in the supposed paradox of higher-order vagueness of Wright (1987) and (1992); it is likely to be rejected in any plausible account of higher-order vagueness.' However, Edgington (1993) and Heck (1993) have plausibly argued that the relevant critical role is played by Wright's 'DEF' rule rather than by (CC). Moreover, it is possible to both keep (CC) and avoid Wright's paradox, as is shown in Bobzien (2010), Section 11. It is also worth noting that none of the authors whom Williamson mentions in the footnote actually provides an *argument* against (CC).

[56] Cf. Williamson (1992), Sections 1, 2, and 5, and Williamson (1994), ch. 8. We say *his* margin-for-error principle, since a margin-for-error constraint can be introduced into a theory of vagueness by different principles; see, for example, Glanzberg (2003), Section 4.3, and Bobzien (2010), Section 4.

[57] In Williamson (1992); in Williamson (1994) 'I know that' is sometimes used instead.

[58] Williamson uses n–1 instead of n+1, but the difference is insignificant. What matters is that the *a* in the consequent of (ME) is closer to the borderline zone than the *a* in the antecedent, as is clear from his assumptions (S1) and (S2); see below.

and forms its contraposition

$(ME_{Kcontra})\quad \neg KFa_{n-1} \rightarrow \neg KKFa_n$.

Next, he introduces two suppositions: (S1) 'KFa_n is true', and (S2) 'KFa_{n-1} is false'. Applying *modus ponens* to (S2) and ($ME_{Kcontra}$), he derives $\neg KKFa_n$. He concludes that there is a possible situation in which we have both KFa_n (by (S1)) and $\neg KKFa_n$ (by derivation); and that hence, given his margin-for-error principle, (KK) does not hold in all cases.

At first blush, this argument may seem plausible for Sorites-vague predicates. However, neither epistemicists nor bivalence discarders are bound by it. *Epistemicists* can question both (ME) and the step from (ME) to (ME_K). The purpose of (ME) was to introduce a margin for error for knowledgability by precluding that F*a* could be false in circumstances very similar to those in which it is known that F*a*. But the truth of (ME) requires more than the existence of a margin for error for knowledgability. It additionally requires that the size of the margin for error that comes with the knowledge of Fa_n always equals or exceeds the distance on dimension *D* between two adjacent a_n and a_{n+1} of a Sorites series. Yet for all we know, this second requirement may be false in any number of cases.[59] There are obvious examples—such as cases where the number of objects in the Sorites series or in the borderline zone is very small ('a couple of us stayed for dinner' stated by one of a group of five afternoon tea visitors). But even in common-or-garden Sorites arguments there seem to be no compelling reasons why this second requirement should always be satisfied.[60] Second, the step from (ME) to

[59] Thus, for Williamson's theory to succeed in solving the Sorites paradox for some F, it requires the additional constraint on simple higher-order clarity that the number of orders of clarity is smaller or equal to the number of clear cases of F on *D*. Since standard Sorites paradoxes have a finite number of members of the Sorites series, the numbers of orders will be finite (and hence cannot converge towards some point on *D*). As Williamson himself describes the non-clear (non-definite) cases of the various orders as borderline cases (for example, Williamson (1999), 132), this leads to the awkward result that for 'bald', for example, there would be an *n*th order borderline case of being bald who has zero hairs (with *n* finite). When Williamson in his *Theoria* interview says that 'epistemicism can easily handle higher-order vagueness' (Williamson and Chen (2011), 9), this may be true. However, his simple and elegant theory of higher-order vagueness has no explanatory power when it comes to borderline cases of vague predicates as they are commonly understood.

[60] Could we not restrict our attention to Sorites series in which the difference between adjacent items is (much) smaller than the margin for error? We could. But since Williamson's solution to the Sorites is tied to the size of the margin for error, this would have the unhappy consequence that his solution only works for a fraction of Sorites paradoxes. And if we would not want this fraction to be determined arbitrarily, the answer to the question of how small '(much) smaller' would have to be is: so small that the distance on *D* between adjacent a_n and a_{n+1} of the Sorites series is not larger than the size of the margin for error that comes with the knowledge of Fa_n—which is just the point that we made in the main text.

In Williamson's theory, (ME) is needed to explain why people—mistakenly—believe that the inductive premise of the Sorites is true. In simplified terms, they overlook the difference between the true (ME) and the false inductive premise. However, contextualist theories, which have become the predominant theories of vagueness, offer a sheaf of alternative explanations for why people mistakenly accept the inductive premise, so that Williamson's (ME) is not required. (In addition to the groundbreaking work by Diana Raffman (1994), see Soames (1999), Fara (2000), Shapiro (2003), (2005), and (2006), and the more recent clarifications and defenses in Åkerman and Greenough (2010a), Åkerman and Greenough (2010b), and Åkerman (forthcoming)).

(ME_K) is not a *logical* step. It may seem like some kind of logical substitution, but it is not.[61] Rather, the step relies on a philosophical assumption which Williamson never makes explicit. This is the assumption that with the addition of the two K-operators in (ME_K) the margin for error never *decreases* to a size less than the distance between a_n and a_{n+1} on *D*. This assumption, too, can be challenged. It follows that even epistemicists are not forced to accept Williamson's conclusion that (CC) does not hold for vague sentences. This result is in line with our argument in Section 7 above.

Next, we show that *bivalence-discarders* have no reason to grant that Williamson's margin-for-error principle (ME) is relevant to vagueness in the way he suggests. Take any bivalence-discarding theory that accepts the following assumptions (A1) and (A2):

(A1) If in a context *C a* is borderline F, then with regard to *C* (an utterance of) the sentence F*a* either has a semantic value other than Truth or Falsehood, or has no semantic value at all.

(A2) For any Sorites series of F, there is always at least one a_n in the borderline zone of F.

In most theories of vagueness, including Williamson's own, (A2) is trivially true. (A1) is accepted by most bivalence discarders, including supervaluationists and open-texture theorists. Given (A1), F*x* has a more-than-two-valued semantics. Consequently, the formalization of the margin-for-error principle needs modification, if Williamson's argument is to fly *at all*. The least intrusive and most plausible change may be to

(ME)′ $\quad CFa_n \rightarrow -[Fa_{n-1}$ is false$]$ (with '–' for exclusion negation),[62]

meaning roughly 'if (in *C*) it is clear that Fa_n, then it is not the case that (in *C* the utterance of) the sentence Fa_{n-1} is false. In fact, (ME)′ follows from the conjunction of (A1) and (A2). Thus there is *de facto* no need for invoking any idea of a knowledge margin for error for the acceptance of (ME)′. The step analogous to Williamson's step from (ME) to (ME_K) would then be from (ME)′ to

(ME_K)′ $\quad CCFa_n \rightarrow -[CFa_{n-1}$ is false$]$.

Just like the step form (ME) to (ME_K), so the step from (ME)′ to (ME_K)′ is not a case of substitution and does not hold for logical reasons. Moreover, leaving margin-for-error considerations aside,[63] philosophically there seem to be no overriding considerations

[61] Cf. the following examples: $[KFa \rightarrow Fb] \rightarrow [KKFa \rightarrow KFb]$; [[Clearly *p*] → *q*] → [[Clearly clearly *p*] → [clearly *q*]]; [[Clearly 2+2=4] → [This is red]] → [[Clearly clearly 2+2=4] → [Clearly, this is red]]; [[I know that 2+2=4] → [4+4=8]] → [[I know that I know that 2+2=4] → [I know that 4+4=8]]. Of course, Williamson does not claim that the step is a logical step.

[62] The choice of exclusion negation ('–S is true if and only if S is not true') is motivated by the goal to cover as many theories as possible. It allows for theories in which, if in *C* a_{n-1} is borderline-F, then an utterance of the sentence Fa_{n-1} in *C* is the kind of utterance of which a truth-value cannot be meaningfully predicated. (Cf., for example, Glanzberg (2003), Bobzien (2010), Sections 5 and 9, Rayo (2010), pp. 42–3, and Iacona (2010) for consideration of this option.)

[63] Their dispensability was noted above.

that prevent $(\mathrm{ME_K})$' from being false. The truth-value of $(\mathrm{ME_K})$' depends simply on which model of clarity (self-revealing or concealable) and which type of higher-order vagueness (borderline or simple clarity) one chooses. With higher-order borderline vagueness, $(\mathrm{ME_K})$' could turn out false (see above Section 2, Diagram 1b, for an illustration). The same would hold for weak, simple higher-order clarity: it is possible that the last $\mathrm{C^nF}x$ case borders the first $\mathrm{C^{n-2}F}x$ case (with $n \geq 3$).[64] Thus bivalence-discarders need not accept $(\mathrm{ME_K})$'. Hence, *even* with the relevant modifications added, bivalence-discarders are not bound by Williamson's argument. Therefore, bivalence discarders too are not forced to accept Williamson's conclusion that (CC) does not hold for vague sentences.[65]

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[64] See Section 2 and Diagram 1c. To preclude this possibility, we would need something along the lines of *strong* simple higher-order clarity (for example, Dummett (1975), p. 311), which differs from *weak* simple higher-order clarity by the added constraint that for any $n \geq 1$, $\exists x[\mathrm{C}^n\Phi x \wedge \neg\mathrm{C}^{n+1}\Phi x]$. Rules $(\mathrm{PC_1})$ and $(\mathrm{PC_2})$ would guarantee that the cases singled out by this constraint are in the right place on D to make $(\mathrm{ME_K})$' true.

[65] A different argument against (CC) on the basis of an epistemic interpretation of 'It is clear that' comes from Greenough ((2003), 274–8). He uses the phrase 'It is known that p (to a speaker s)' to characterize borderline cases, and consequently considers (KK) relevant. He suggests that if (KK) fails for all orders n, then there must be radical higher-order vagueness (276). He reconfigures Crispin Wright's so-called paradox of higher–order vagueness in epistemic terms, and subsequently develops a complex quattro-lemma from which he concludes that (KK) fails for all orders n (276–8). His argumentation is not persuasive, for two reasons. First, in his proposal, 'borderline case' is relativized to a speaker, which is not how we usually understand the term. Second, his adaptation of Wright's argument against (KK) works only because it lumps together two distinct parameters: it does not distinguish between very similar objects a_n, a_m and very similar contexts, C_1, $C2$, but only between very similar 'normal cases of judgment conditions for the speaker s' α, β (260; 276–7). Greenough's argument deserves detailed discussion, but this goes beyond the scope of this paper.

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