

conflict. Supposing that the above conflict holds, 'P₁... and P_n and P_{n+1}' might nonetheless not conflict with 'not-C', since P_{n+1} is not one of the factors *because of which* 'not-C' fails to hold. That is, the 'because' would prevent the addition of irrelevant conjuncts. In this way the idea of the relevance of the premisses to the conclusion, as a condition for 'proper following from', would be part of the validity criterion.

*

In addition to validity, the Stoics assumed that arguments could have the properties of truth and falsehood. An argument is true (we would say 'sound') if, besides being valid, it has true premisses (D.L. VII.79, cf. S.E. M VII.418); an argument is false if it is either invalid or has a false premiss (D.L. VII.79). The predicates of truth and falsehood are here based on the concept of truth of assertibles, but are used in a derivative sense. The relevance of truth and falsehood of arguments is epistemic rather than logical: only a true argument guarantees the truth of the conclusion.

But the Stoics went further in assuming that arguments could also have modal properties: like assertibles, arguments can be possible, impossible, necessary and non-necessary (D.L. VII.79). The modal predicates, too, can only be used in a derivative sense here. The motivation behind such a classification is easy to make out; again, it is in the first place epistemic. The time dependency of assertibles affects the arguments. Since the concept of truth of arguments is based on that of truth of assertibles, and the latter can change their truth-value, so can arguments. For instance, the argument

If it is day, it is light.
Now it is day.
Therefore it is light.

will be true in the daytime but false at night.¹²¹ It seems that arguments with premisses that did (or could) change truth-value were called 'changing arguments' (μεταβαλλόμενα λόγια) (Epicr. Diss. 1.7.1; III.21.10). Chrysippus or some later Stoic wrote five books about changing arguments (D.L. VII.195-6). Now, if like the Stoics one is interested in knowledge gained by inference, one would focus on conclusions on which one can always rely, that is, on true arguments of which one can be sure that they are always true – or at least from the time onwards at which the argument was propounded. It is hence plausible to assume that the modalities

of arguments were introduced in order to have available a way of referring to arguments which do not change truth-value. For this purpose the Chrysippean modal accounts¹²² could have been used: for example, a necessary argument would be one that either cannot be false or can be false but is hindered by external circumstances from being false, and accordingly for the three remaining modalities.

6: Syllogistic

More important for logic proper are the divisions of valid arguments. These are based primarily on the form of the arguments. The most general distinction is that between syllogistic arguments (συλλογιστικοί λόγοι) or syllogisms (συλλογισμοί) and those called 'valid in the specific sense' (τροποῦτικοί εὐκρίδεις). The latter are concludent (ie they satisfy the general criterion of validity), but not syllogistically so (D.L. VII.78). Syllogisms are, first, the indemonstrable arguments, that is, those that are valid in virtue of having one of a limited number of basic forms, and secondly those that can be reduced to indemonstrable arguments by the use of certain rules called 'θεμματα'.¹²³

The indemonstrable syllogisms are called 'indemonstrable' (ἀντροδόμητος) because they are not in need of proof or demonstration (ἀτροδόμησις) (D.L. VII.79), since their validity is obvious in itself (S.E. M II.223). The talk of five indemonstrables alludes to classes of argument, each class characterized by a particular argument form in virtue of which the arguments of that class are understood to be valid. Chrysippus distinguished five such classes; later Stoics up to seven.

The Stoics defined the different kinds of indemonstrables by describing the form of an argument of that kind. The five Chrysippean types were described as follows.¹²⁴ A first indemonstrable is an argument that is composed of a conditional and its antecedent as premisses, having the consequent of the conditional as conclusion (S.E. M VII.224; D.L. VII.80). An example is

If it is day, it is light.
It is day.
Therefore it is light.

A second indemonstrable is an argument that is composed of a conditional and the contradictory of its consequent as premisses, having the

¹²¹ Two of several passages that take into account truth-value changes of arguments are S.E. M VII.418 and PH II.139. There the provisos 'given it is this night' and 'it being day' are added when a particular truth-value of an argument is stated.

¹²² For Chrysippus' modal accounts see above, p. 118.

¹²³ For a comprehensive discussion of Stoic syllogistic and its relation to modern logic see Bobzien 1996.

¹²⁴ For the terminology used cf. the section on non-simple assertibles.

contradictory of its antecedent as conclusion (S.E. *M* VIII.225; D.L. VII.80), e.g.

If it is day, it is light.

Not: it is day.

Therefore not: it is light.

A third indemonstrable is an argument that is composed of a negated conjunction and one of its conjuncts as premisses, having the contradictory of the other conjunct as conclusion (S.E. *M* VIII.226; D.L. VII.80), e.g.

Not: both Plato is dead and Plato is alive.

Plato is dead.

Therefore not: Plato is alive.

A fourth indemonstrable is an argument that is composed of a disjunctive assertible and one of its disjuncts as premisses, having the contradictory of the remaining disjunct as conclusion (D.L. VII.81), e.g.

Either it is day or it is night.

It is day.

Therefore not: it is night.

A fifth indemonstrable, finally, is an argument that is composed of a disjunctive assertible and the contradictory of one of its disjuncts as premisses, having the remaining disjunct as conclusion (D.L. VII.81), e.g.

Either it is day or it is night.

Not: it is day.

Therefore it is night.

Each of the five types of indemonstrables thus consists – in the simplest case – of a non-simple assertible as leading premiss and a simple assertible as co-assumption, having another simple assertible as conclusion.¹²⁵ The leading premisses use all and only the connectives that Chrysippus distinguished.

The descriptions of the indemonstrables encompass many more arguments than the examples suggest, and this for three reasons. First, in the case of the third, fourth and fifth indemonstrables the descriptions of the argument-form provide for 'commutativity' in the sense that each time it is left open which constituent assertible or contradictory of a constituent

assertible is taken as co-assumption. For instance, if we symbolize the constituent assertibles in a fourth indemonstrable as d_1, d_2 (disjuncts one and two), the two sub-types covered are of the following kind:

Either d_1 or d_2 , now d_1 ; therefore not d_2 .

Either d_1 or d_2 ; now d_2 ; therefore not d_1 .

Secondly, the descriptions are all given in terms of assertibles and their contradictories, *not* in terms of affirmative and negative assertibles. In all five cases, the first premiss can have any of the four combinations of affirmative and negative assertibles: for instance in the case of the first and second indemonstrable (if we symbolize affirmative assertibles by p, q , negative ones by not: p , not: q):¹²⁶

if p, q if not: p, q if p , not: q if not: p , not: q .

Thus, putting together these two points, we have four sub-types under the first and second description of indemonstrables and eight in the case of the third, fourth, and fifth, thirty-two subtypes in all.

The third reason for the multitude of kinds of indemonstrables is the fact that the descriptions, as formulated, permit the constituent assertibles of the leading premisses to be themselves non-simple. And indeed, we have an example in Sextus which is called a second indemonstrable and which is of the kind¹²⁷

If both p and q , r ; now not: r ; therefore not: <both p and $> q$.

In addition to describing the five types of indemonstrables at the meta-level, the Stoics employed a second way of determining their basic forms of arguments, namely by virtue of modes ($\tau\rho\acute{o}\tau\tau\omicron\upsilon$). A mode is defined – rather vaguely – as 'a sort of scheme of an argument' (D.L. VII.76; S.E. *M* VIII.227).¹²⁸ Diogenes Laertius adds the example

If the first, the second; now the first; therefore the second.

This is an example of the (or a) mode of the first indemonstrable. It differs from a first indemonstrable in that ordinal numbers have taken the place of

¹²⁵ Where not indicated otherwise, p, q, r , etc. symbolize affirmative and negative simple assertibles alike.

¹²⁷ S.E. *M* VIII.237; the text requires emendation: in the conclusion the first conjunct of the leading premiss has to be added, as is clear from 236.

¹²⁸ In later authors, $\tau\rho\acute{o}\tau\tau\omicron\upsilon$ and the Latin translation 'modus' are frequently used as synonyms either for 'indemonstrable' or for 'basic kind of indemonstrable' (e.g. Philo. *AP*, 244-9, 13, 33; 245-23, 26, 33; Cic. *Top*, 54-7; Martianus Capella IV 414-21), whereas the term 'indemonstrable' is not used at all in these texts. In contrast, what was called $\tau\rho\acute{o}\tau\tau\omicron\upsilon$ by the early Stoics is then called *forma* in Latin (e.g. Martianus Capella IV 420) and in Greek probably $\sigma\chi\eta\mu\alpha$ (Philo. *AP*, 246.10-12).

¹²⁵ The forms of the first and second indemonstrables correspond to the basic argument-forms later named *modus (genetivus) ponens* and *modus (callendiv) tollens* and those of the fourth and fifth to the basic argument-forms later called *modus ponendo tollens* and *modus tollendo ponens*.

the antecedent and consequent assertibles of the leading premiss, and the same ordinals are re-used where the antecedent and consequent assertibles recur in co-assumption and conclusion. It is always whole assertibles that correspond to the ordinals – as opposed to terms that correspond to the letters in Aristotelian logic. A mode is syllogistic when a corresponding argument with the same form is a syllogism. There can be little doubt that the modes played a prominent role in the Stoic theory of arguments. They feature in at least seven of Chrysippus' book titles (D.L. VII.193–5). But their exact status in Stoic logic is hard to make out. It seems that modes, and parts of modes, performed at least three distinct functions.

First, the modes functioned as forms in which the different indemonstrables – and other arguments – were propounded (S.E. M VIII.227; *PH* II.292). If for instance one wants to propound a first indemonstrable, the mode provides a syntactic standard form in which one has (ideally) to couch it. This is similar to the requirement of couching non-simple assertibles in a certain form, for example, of expressing a conjunction by using 'both . . . and . . .'¹²⁹ When employed in this way, the modes resemble argument-forms: the ordinal numbers do not stand in for particular assertibles; rather, their function is similar to that of schematic letters. So, any argument that is propounded in a particular syllogistic mode is a valid argument, but the mode itself is not an argument. The logical form presented by a syllogistic mode is the reason for the particular argument's formal validity. In this function the modes can be used to check the validity of arguments.

In the two other ways in which modes and ordinal numbers are employed the ordinals seem to stand in for assertibles and the modes are used as abbreviations of particular arguments rather than as argument forms. In the analysis of complex syllogisms, for purposes of simplicity and lucidity, ordinals may stand in for simple assertibles, in the sequence of their occurrence in the argument (S.E. M VIII.235–7). And in the so-called 'mode-arguments' (Μοδῶδῶτον) the constituent assertibles are given in full when first occurring, but are then replaced by ordinal numbers, as in

If it is day, it is light.

Now the first.

Therefore the second.

Here, too, the function is mainly one of abbreviation (D.L. VII.77). There are however a couple of pertinent questions on which the texts provide no

unambiguous information. First, it cannot be made out with certainty what kind of assertibles may correspond to the ordinals in the modes. On the one hand, in all our sources the ordinals correspond exclusively to simple affirmative assertibles. This holds even in those cases in which the illustrative arguments are indemonstrables with negative or non-simple assertibles as constituents in the leading premiss, such as

If both p and q , r ; but not r ; therefore not: both p and q .¹³⁰

On the other hand, two Chrysippean book-titles imply that one and the same argument can be classified in several modes (D.L. VII.194). Chrysippus may – but need not – have maintained that examples like the above could be classified not only in the mode

If both the first and the second, the third;

but not the third;

therefore not both the first and the second.

but also in the mode

If the first, the second; but not the first; therefore not the second.

A related point is the question of whether there was one (typical) mode for every basic type of indemonstrable, that is, one that fits all first indemonstrables, one that fits all second indemonstrables etc. Alternatively, there could have been several (typical) modes for each type of indemonstrable; that is, as many as there are subtypes. Again, the sources provide no answer, but if there were just one mode for each basic type of indemonstrable, this would raise a number of problems.¹³¹

*

For a full understanding of Stoic syllogistic it is essential to know what made Chrysippus choose the five types of indemonstrables; however, the sources do not permit a clear answer. All we are told expressly is that the indemonstrables were thought to be evident and hence not in need of demonstration, and that all other syllogisms could be reduced to them (D.L. VII.78; cf. above). And we can infer from the presentation of the

¹³⁰ Cf. S.E. M VIII.236, quoted below, p. 140; see also Mariannus Capella IV.420. I use p , q , r for affirmative simple assertibles in this case.

¹³¹ E.g. in the case of the third, fourth, and fifth indemonstrables, commutativity would not be catered for. For instance, how could

Either p , q ; therefore not: p ,

which is a fourth indemonstrable according to the general account, fit the mode
Either the first or the second; the first; therefore not the second?

¹²⁹ See above, p. 103.

types of indemonstrables that their evident validity is grounded on their form. No doubt the five types of indemonstrables are basic arguments and evident 'in some respects'. But so are other types of arguments.

In which respects then are all and only indemonstrables evident? We may approach this question in the first place negatively, by listing some ways of being basic and evident which Chrysippus cannot have had in mind. First we can see that all indemonstrables (and consequently all Stoic syllogisms) relate whole assertibles, and not terms as Aristotelian syllogisms do. These latter, which consist of three different categorical general statements,¹³² did not count as syllogisms, let alone as evident for the early Stoics. Second, it seems that Chrysippus was not entertaining the idea of minimizing connectives. Third, Chrysippus cannot have been concerned to minimize the number of types of indemonstrables: for, with the help of the first *thema*,¹³³ second indemonstrables can be reduced to first ones (and vice versa), and fifth to fourth ones (and vice versa), and this can hardly have escaped his attention. Fourth, Chrysippus seems not to have aimed at deducing the conclusions from premisses of the minimum possible strength. For any conclusion one can draw by means of a first or second indemonstrable (with a leading premiss 'If p , q ?), one could also draw from a corresponding third indemonstrable (with a leading premiss 'Not: both p and not- q ?). The extra requirement in the truth-criterion for the conditional – compared with the negated conjunction – that is, the element of conflict, seems completely irrelevant to the conclusions one can draw in Chrysippus' syllogistic.

For a conjecture as to what Chrysippus' positive criteria were it may help to consider the following points: in the indemonstrables – and consequently in all syllogisms – all and only the Chrysippean connectives ('and', 'if', 'or') and the negation ('not') are used to construct non-simple assertibles. Among these non-simple assertibles Chrysippus distinguished a particular class entitled 'mode-forming' or 'grounding assertibles' (τροπικὸν ἀξιόμας). These were apparently conditionals, disjunctions and negated conjunctions.¹³⁴ All the indemonstrables have as leading premiss such a 'mode-forming' assertible.

On the assumption that Chrysippus restricted the connectives to those mentioned above, the way the Stoics thought about the indemonstrables

¹³² A paradigm form is *modus Barbara*: A holds of every B; B holds of every C; therefore A holds of every C. ¹³³ See below, p. 138.

¹³⁴ Later logicians, in particular Peripatetics, would refer to such premisses as 'hypothetical propositions' (ὀρθολογικὰ προτάσεις). Accordingly, they often called standard Stoic arguments 'hypothetical', as opposed to the Aristotelian 'categorical' ones. This use of 'hypothetical' is not to be confused with Stoic 'hypothetical sayables' and 'hypothetical arguments', see above, p. 122 and Bobzien 1997a.

may have been like this. Of all non-simple assertibles, the mode-forming ones stand out in that they permit the construction of formally valid arguments. In the most basic cases they make it possible to infer, with a simple assertible as co-assumption, another simple assertible as conclusion. Thus one obtains exactly the five types of indemonstrables, with all the above-mentioned subtypes. Perhaps the deductive power of the indemonstrables was somehow thought to be grounded on the mode-forming assertibles.

But still, why single out the valid arguments composed of a mode-forming premiss and two simple assertibles? There are certainly other syllogisms that are fairly short and simple. What the indemonstrables seem to have in common (and not to share with others) is that no one could reasonably doubt their validity, simply because understanding the connectives that are used in their leading premisses implies knowing the validity of the corresponding forms of the indemonstrables. (Understanding 'Not: both p and q ' implies knowing that if one of them holds, the other does not; understanding 'If p , q ' implies knowing that (i) if p holds, so does q , and (ii) if q doesn't hold, neither does p ; and understanding 'Either p or q ' implies knowing that (i) if one of them holds, the other does not, and (ii) if one of them does not hold, the other does.)

This kind of criterion would for instance fail the following two candidates for indemonstrability, although they are simple and evident in some ways:

p, q , therefore p and q

would not rank as an indemonstrable since understanding p does not imply knowing that if q then ' p and q '. And

If p, q ; if q, r ; therefore if p, r

would not rank as an indemonstrable since understanding 'if p, q ' does not imply knowing that if ' q, r ', then ' p, r '.

*

The situation is slightly complicated by the fact that Chrysippus talked about a syllogism which he called 'fifth indemonstrable with several <disjuncts>' (ὁ πέμπτος δὲ πάλαιων ἀντιθέτικος) (S.E. PH 1.69). It is of the following kind (S.E. PH 1.69; cf. PH II.150; M VIII.434; Philp. AP. 246.3–4):

Either p or q or r

Now, neither p nor q

Therefore r .

Its form obviously differs from the form of the fifth indemonstrables as given above, which have two disjuncts only in their leading premiss. Some have thought, therefore, that this is a Stoic complex argument, to be analysed into two fifth indemonstrables. However, such a reduction does not work. Syllogisms of this form cannot be reduced in Chrysippus' system.¹³⁵ This might have been the reason why Chrysippus regarded such arguments as indemonstrables. However, as the name implies, he did not introduce them as 'sixth indemonstrables'; rather they are a special version of the fifth, that is, they *are* fifth indemonstrables.⁴¹

If we take this seriously, we have to revise our understanding of the fifth indemonstrable. In line with the account we should assume that the leading premiss in a fifth indemonstrable has two-or-more disjuncts, and that the 'basic idea' which one grasps when one understands the disjunctive connective is 'precisely one out of several' rather than 'precisely one out of two'. This understanding of the major premiss of the fifth indemonstrables has the consequence that one also has to modify one's understanding of the co-assumption: its description 'the contradictory of one of its disjuncts' becomes a special case of 'the contradictory of one-or-more of its disjuncts', the added possibility coming down to 'the conjunction of the negation of all but one of them'. There was a standard way of expressing such co-assumptions, namely by 'neither... nor...?' (οὔτε... οὔτε...;) (e.g. S.E. *PH* 1.69; cf. *PH* 11.150; *M* VIII.434; *Philp. AP*. 246.3-4).

If Chrysippus allowed non-simple conclusions in indemonstrables, we could have a further kind of 'syllogism with several disjuncts' in the case of the fourth indemonstrables - which, too, is irreducible in Chrysippus' syllogistic:

Either p or q or r ...; p ; therefore neither q nor r¹³⁶

There could also be third 'indemonstrables' with three or more conjuncts.¹³⁷ However these would be analysable into indemonstrables.

*

In Cicero and a number of later Latin authors we find a list of seven basic syllogisms which most probably is of Stoic origin (*Cic. Top.* 53-7; *Boeth. Cic. Top.* 355-8; *Martianus Capella* IV.414-21; *Cassiod. Inst.* II.3.13). In addition, we find mention of basic syllogisms other than Chrysippus' indemonstrables in Galen (*Gal. Inst. Log.* V.3-4; VI.7; XV.1-11; cf. XIV.4-8;

10-11) and in a *scholium* to Ammonius (*ΣΑμμων. AP*. XI.3-4; 13-36). They, too, may be of Stoic origin. Most of these texts adopt the Peripatetic terminology and refer to the basic syllogisms as hypothetical syllogisms. The presentation of the list of seven varies slightly from one source to another, but the first five types tend to correspond closely to the Chrysippean indemonstrables.¹³⁸

Difficulties arise with the sixth and seventh types of argument. Both have a negative conjunction with two conjuncts as leading premiss; they are of the kinds

Not: both p and q ; now p ; therefore not: q (6th)
 Not: both p and q ; now not: p ; therefore q (7th)

The problem is obvious: the sixth looks exactly like a Chrysippean third indemonstrable whereas the seventh, as it stands, is patently invalid. If one wants to make sense of them, perhaps the best guess is that the sixth and seventh basic syllogisms were those with pseudo-disjunctions as leading premiss.¹³⁹ For, with one exception, the additional basic hypothetical syllogisms in Galen and in the *scholium* are all of that kind, and several later sources suggest they are or should be formulated as (negated) conjunctions.¹⁴⁰

*

Not all Stoic syllogisms, or formally valid arguments, are indemonstrables. Non-indemonstrable syllogisms can be more complex than indemonstrables in that they have more than two premisses; but they can also have just two premisses. For example, in our sources we find Stoic non-indemonstrable syllogisms of the kinds:

If both p and q , r not r ; p ; therefore not: q (S.E. *M* VIII.234-5)
 If p , p ; if not: p , p ; either p or not: p ; therefore p (S.E. *M* VIII.281, 466)
 If p , if p , q ; p ; therefore q (S.E. *M* VIII.230-2).

The Stoics distinguished and discussed a number of special cases of syllogisms, both indemonstrable and non-indemonstrable. First, there is the class of indifferently concluding arguments (ἀδικοφόρος τετραπλοῦτες); as example we get

¹³⁵ Cf. *Lenidikonon* 1993a. ¹³⁹ For the Stoic pseudo-disjunction see above, p. 110.

¹⁴⁰ Cf. *Gal. Inst. Log.* V.1 (περὶ ἀπὸ τοῦ διελγυμένου) and XIV.6 and 11 for the sixth indemonstrable and *Ap. Dyc. Conj.* 230.18-19 and *Digesta Justiniani Augusti* 34.5.13. 5.6 for the seventh (cf. also *Philp. AP*. 246.5-6). Perhaps the leading premiss of the seventh was originally 'Not: both not: p and not: q and the second and third negation dropped out in the process of copying - as has been suggested by Becker 1957b, 47.

¹³⁶ For the Stoic method of reduction see below, pp. 137-48.

¹³⁷ Cf. *Gal. Inst. Log.* XV.9.

¹³⁸ Cf. *Cic. Top.* 54.

Either it is day or it is light.

Now it is day.

Therefore it is day. (Apul. *Int.* 201.4-7; Alex. *Top.* 10.10-12)

This argument is of the kind

Either p or q ; p ; therefore p .

The name of these arguments is presumably based on the fact that it is irrelevant for their validity what comes in as second disjunct. Often mentioned in tandem with the indifferently concluding arguments are the so-called 'duplicated arguments' (διεφορούμενοι λόγους) (Apul. *Int.* 201.4-7; Alex. *Top.* 10.7-10; *APr.* 18.17-18). It seems that their name rests on the fact that their leading premiss is a 'duplicated assertible'; that is, composed of the same simple assertible, used twice or several times (Cf. D.L. vii.68-9; S.E. *M VIII* 95, 108). The standard and only example is

If it is day, it is day.

Now it is day.

Therefore it is day.

It is of the kind

If p , p ; therefore p

and is a special case of the first indemonstrable. It is uncertain whether the use of the negation of the simple assertible was allowed, e.g. whether this argument was duplicated:

Either p or not- p ; p ; therefore p .

Such an example occurs in Alexander (*APr.* 19.3-10) but it is not called duplicated.¹⁴¹

A third type of syllogism were those with two mode-forming premisses (οἱ δὲ δὺο τροπικῶν), that is, arguments composed of two mode-forming assertibles as premisses and a simple assertible as conclusion: the examples we get are of the kind

If p , q ; if p , not- q ; therefore not- p .

A Stoic example is

¹⁴¹ The Aristotelian commentators characterized both the indifferently concluding arguments and the duplicated ones as those in which one premiss is identical with the conclusion, and usually argued that this fact is the reason why they were not syllogisms (Alex. *APr.* 18.12-18; Ammon. *APr.* 28.9-13; Philo. *APr.* 33.23-6). They seem to have been unaware of their special characteristics and as a consequence the two types seem to have occasionally been confounded (Zaist. *Top.* 294b23-9 Brindis).

If you know that you are dead, you are dead.

If you know that you are dead, not: you are dead.

Therefore not: you know that you are dead. (Orig. *Cels.* vii.15)

A related type of syllogism is that with three mode-forming premisses: the examples are all of the kind

If p , p ; if not- p , p ; either p or not- p ; therefore p (S.E. *M VIII*.281, 466),

that is, containing only one constituent assertible (and its negation), used several times. Generally, such syllogisms may have been of the kind

If p , q ; if r , q ; either p or r ; therefore q .

This is a simple constructive dilemma, which was used, for example, in paradoxes. The examples in Sextus would then be a special case of this kind. (A passage in Alexander (*APr.* 164.27-31) suggests that the Stoics distinguished further types of syllogisms.)

*

Arguments of all these kinds were syllogisms. And, since Diogenes reports that all syllogisms are either indemonstrable or can be reduced to indemonstrables (D.L. vii.78), this means that - if they were not indemonstrables themselves - these arguments, too, could be reduced to indemonstrables. The Stoic expression for reducing arguments was to analyse (ἀναλύειν) them into indemonstrables (D.L. vii.195; Gal. *PHP* II.3.188-90; S.E. *M VIII*.235; 237). What is the purpose of such a reduction or analysis (ἀνάλυσις)? It is a method of proving that certain arguments are syllogisms or formally valid, by showing how they stand in a certain relation to indemonstrables. This relation between the argument-to-be-analysed and the indemonstrables is basically either that the argument is a composite of several indemonstrables, or that it is a conversion of an indemonstrable, or that it is a mixture of both. The analysis or reduction was carried out by means of certain logical meta-rules which determined these relations. They were called *themata* (θέματα), sometimes translated as 'ground-rules'. They were argumental rules, i.e. rules that can only be applied to arguments. They reduce arguments to arguments, not, say, assertibles to assertibles.¹⁴² Our sources suggest that there were four of them (Alex. *APr.* 284.13-17; Gal. *PHP* II.3.188 De Lacy). We know further that the Stoics had some logical meta-rules, called 'theorems' (θεωρήματα), which were relevant for the analysis of arguments (D.L.

¹⁴² Cf. on this point Corcoran 1974b and Bobzien 1996.

vii.195; S.E. *M* vii.231; *PH* ii.3; cf. *Orig. Cels.* vii.15.166-7). Since the *themata* were regarded as sufficient for the analysis of all non-indemonstrable syllogisms (D.L. vii.78), the function of some of the theorems was presumably to facilitate or speed up the analysis.

It is important to see that Stoic analysis is strictly an upwards method (to the indemonstrables), rather than a downwards method (from the indemonstrables). Analysis always starts with a given non-indemonstrable argument, and with the question whether it can be analysed into indemonstrables by means of the *themata*. There are no signs that the Stoics ever tried to establish systematically (or otherwise) what kinds of formally valid non-indemonstrable arguments could be *deduced* or *derived* from their set of indemonstrables by means of the *themata*.

Related to this point is the fact that Stoic analysis was carried through with the arguments themselves, not with argument-forms or schemata, although, of course, the analysis depends precisely on the form of the arguments. This might strike one as odd, since it appears to imply that analysis had to be carried out again and again from scratch, each time the (formal) validity of a non-indemonstrable argument was in question. But this need not have been so: the Stoics seem to have introduced certain meta-rules, which would state that if an argument is of such and such a form, it is a syllogism or it can be analysed into indemonstrables in such and such a way (cf. S.E. *PH* ii.3 together with *Orig. Cels.* vii.15.166-7). Moreover, at least in complex cases, the modes were employed in order to facilitate the reduction; that is, ordinal numbers were used as abbreviations for constituent assertibles (S.E. *M* vii.234-6).¹⁴³ This abbreviation brings out the form of the argument and makes it easier to recognize which *themata* can be used.

*

How did Stoic analysis work in detail? How did the *themata* and theorems function, that is, how were they applied to arguments? Let us look first at the first *themata*. It ran:

When from two <assertibles> a third follows, then from either of them together with the contradictory of the conclusion the contradictory of the other follows. (*Apul. Int.* 209.10-14)

Here - as in the case of the last three indemonstrables - a formulation is chosen that leaves the order of the premisses undetermined. The rule may be presented formally as

$$\begin{array}{l} P_1, P_2 \vdash P_3 \\ \hline P_1, \text{ctrd } P_3 \vdash \text{ctrd } P_2 \end{array} \quad (T_1)$$

'ctrd' stands for 'contradictory'; ' \vdash ' for 'therefore'; $P_1, P_2 \dots$ mark places for assertibles that function as premisses. In an application of the rule the argument-to-be-analysed (or the original argument) would occupy the bottom line, the syllogism into which it is analysed the top line. For instance, if we have a non-indemonstrable argument of the kind

$p, \text{not-}q$; therefore not: if p, q

this can be reduced to a first indemonstrable of the kind

If p, q, p ; therefore q

by employing the first *themata* as follows: When from ' p ' and 'if p, q ' q follows (this being the indemonstrable), then from ' p ' and 'not: q ' 'not: if p, q ' follows (this being the non-indemonstrable argument). Or formalized:

$$\begin{array}{l} \text{If } p, q; p \vdash q \\ \hline p; \text{not-}q \vdash \text{not: if } p, q \end{array} \quad (T_1)$$

In all cases in which such a procedure leads to one of the five indemonstrables, the original argument is a syllogism.

By using the rule on all possible kinds of simple non-indemonstrable arguments, one obtains four new types of syllogisms, namely those of the kinds (with the indemonstrables into which the arguments are analysed in brackets)

$$\begin{array}{ll} p, \text{not-}q \vdash \text{not: if } p, q & \text{(first or second)} \\ p, q \vdash \text{not: either } p \text{ or } q & \text{(fourth)} \\ \text{not } p, \text{not } q \vdash \text{not: either } p \text{ or } q & \text{(fifth)} \\ p, q \vdash \text{both } p \text{ and } q & \text{(third)} \end{array}$$

These arguments may be called 'simple non-indemonstrable syllogisms'. In fact, no such arguments are handed down. As will be seen, the first *themata* can be used in one and the same reduction in combination with one or more of the other rules of analysis; it can also be employed several times in the same reduction.¹⁴⁴

*

¹⁴³ On this point see above, p. 130. Cf. also Frede 1974a, 136-44.

¹⁴⁴ For the analysis of some arguments with more than two premisses a more general version of the first *themata* is required; a passage in Galen (*Inst. Log.* vi.5) suggests that there was such a rule. It could have run: 'When from two or more assertibles something follows, then from all but one of them together with the contradictory of the conclusion, the contradictory of the remaining one follows.'

It will be helpful to consider the meta-rule which was known as a 'dialectical theorem' (S.E. *M* VIII.231) before the discussion of the remaining three *themata*. This theorem presumably did the same work as the second, third, and fourth *themata* together.¹⁴⁵ Sextus preserves the rule, which ran simply:

When we have (the) premisses which deduce some conclusion, we potentially have that conclusion too in those premisses, even if it is not expressly stated. (S.E. *M* VIII.231)

As it stands, this theorem does not fully determine a method of analysis. It is only a general presentation of a principle. But the Sextus passage illustrates how the analysis works, by applying it to two arguments (S.E. *M* VIII.230-8). In the second example the analysis is carried out first with the mode of the argument, then by employing the argument itself. Let us look at the former, which begins by presenting the mode of the argument-to-be analysed:

For this type of argument is composed of a second and a third indemonstrable, as one can learn from its analysis, which will become clearer if we use the mode for our exposition, which runs as follows.

If the first and the second, the third.

But not the third.

Moreover, the first.

Therefore not: the second.

For since we have a conditional with the conjunction of the first and the second as antecedent and with the third as consequent, and we also have the contradictory of the consequent, 'Not: the third' we will also deduce the contradictory of the antecedent, 'Therefore not: the first and the second', by a second indemonstrable. But in fact, this very proposition is contained potentially in the argument, since we have the premisses from which it can be deduced, although in the presentation of the argument it is omitted. By putting it together with the remaining premiss, the first, we will have deduced the conclusion 'Therefore not: the second' by a third indemonstrable. Hence there are two indemonstrables, one of this kind

If the first and the second, the third.

But not: the third.

¹⁴⁵ This can be inferred from the facts that it allows reduction of the same arguments as the so-called 'synthetic theorem' which was used by the Peripatetics (cf. Mignucci 1993, 218-21), and which in turn did the work of the second to fourth *themata* (Alex. *APr.* 284.10-17). We are told that Antipater facilitated Stoic analysis (Gal. *PHP* II.3.190). Perhaps it was he who introduced the synthetic or the dialectical theorem.

Therefore not: the first and the second.

which is a second indemonstrable; the other, which is a third indemonstrable, runs like this

Not: the first and the second.

But the first.

Therefore not: the second.

Such is the analysis in the case of the mode, and there is an analogous analysis in the case of the argument (S.E. *M* VIII.235-7).

The general procedure of reduction by means of the dialectical theorem then is as follows: take any two of the premisses of the argument-to-be analysed and try to deduce a conclusion from them, by forming with them an indemonstrable. Then take that 'potential' conclusion and look whether by adding any of the premisses, you can deduce another conclusion, again by forming an indemonstrable. (The old premisses are still in the game and can be taken again, if required, as is plain from Sextus' first example, S.E. *M* VIII.232-3.) Proceed in this manner until all premisses have been used at least once and the last assertible deduced is the original conclusion. In that case you have shown that the original argument is a syllogism.

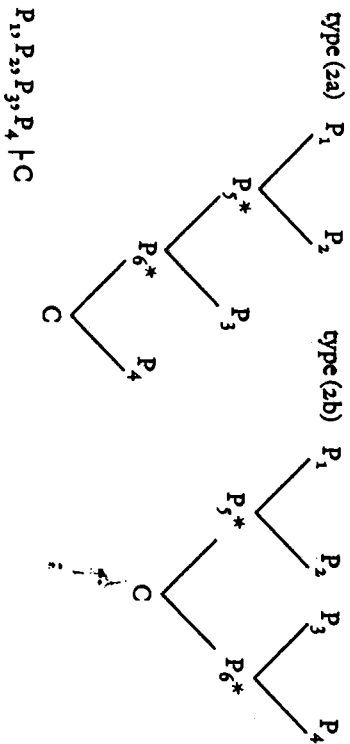
Thus, the dialectical theorem turns out to be a rule for chain-arguments by which a complex non-indemonstrable is split up into two component arguments. The theorem should suffice to analyse all composite arguments, i.e. all arguments with any of the following as underlying or 'hidden' structures. (A triangle gives the form of a simple two-premiss argument with the letter at the bottom giving the place of the conclusion. $P_1 \dots P_n$ give the places of the premisses, C that of the conclusion of the argument-to-be-analysed; P_{n^*} that of a premiss that is a 'potential conclusion' and hence does not show in the argument-to-be-analysed. The type of argument-to-be-analysed has been added underneath each time.)



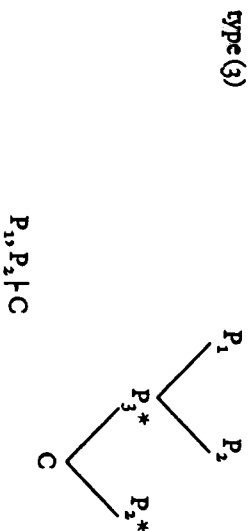
$P_1, P_2, P_4 \vdash C$

The argument in the above quotation for instance, is of this type.

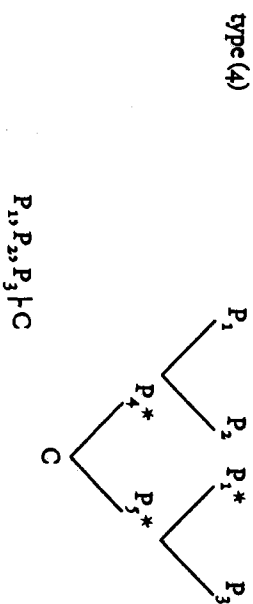
type (2) (four premiss arguments)



One can analyse all the expansions of these types which one gains by adding further two-premiss syllogisms which have one of the explicit premisses as conclusions. These conclusions would thereby become 'potential', i.e. would no longer appear in the argument to be analysed, and would accordingly get an '*'. As is clear from Sextus' first example of analysis (S.E. *M* VIII.232-3), the dialectical theorem also covers inferences in which the same premiss is implicitly used more than once, but occurs only once in the original argument. The most basic type of these is:



Sextus' first example, which is of the kind 'If p , if q , q ; $p \vdash q$ ', is of this type. A more complex case is



Again, all expansions and variations of these types, and moreover all their combinations with type (1) can be analysed by repeated use of the theorem. If one takes together the first *thema* and the dialectical theorem, with their help at least all Stoic syllogisms of which we know can be analysed into Stoic indemonstrables.

*

Next I consider the second, third, and fourth Stoic *thematia*. Formulations of the third *thema* have survived in two sources (Simp. *Cael.* 237.2-4; Alex. *APr.* 278.12-14). The second and fourth are not handed down. However, a tentative reconstruction of them and of the general method of analysis by means of the *thematia* is possible, since there are a number of requirements which these three *thematia* have to satisfy; they are:

- the second, third, and fourth *thematia* together should cover the same ground as the dialectical theorem¹⁴⁶
- the *thematia* have to be applicable, in the sense that by using them one can actually find out whether an argument is a syllogism
- they have to be simple enough to be formulated in ordinary Greek
- the second *thema*, possibly in tandem with the first, is said to reduce among others the indifferently concluding arguments and the arguments with two mode-premisses¹⁴⁷

- the third and fourth *thematia* should show some similarity or should be used together in some analyses (Gal. *PHP* II.3.188 De Lacy).

The following is a reconstruction which satisfies these requirements reasonably well.¹⁴⁸

*

The two formulations of the third *thema* that have survived in Alexander and in Simplicius present in fact two different versions of it. Alexander has

When from two <assertibles> a third follows, and two external assumptions syllogize one of the two, then the same <i.e. third> one follows from the remaining one and the external ones that syllogize the other. (Alex. *APr.* 278.12-14)

¹⁴⁶ See previous note.

¹⁴⁷ Gal. *PHP* II.3.188 De Lacy; Alex. *APr.* 164.27-31. For these kinds of arguments see above, pp. 135-7.

¹⁴⁸ For details of this reconstruction of Stoic analysis see Bobzien 1996; for alternative reconstructions cf. Frede 1974a, 174-96 and Ierodiakonou 1990, 60-75.

And Simplicius reads

When from two <assertibles> a third follows, and from the one that follows <i.e. the third> together with another, external assumption, another follows, then this other follows from the first two and the externally co-assumed one. (Simp. *Cacl.* 237.2-4)

Both formulations reveal that, like the dialectical theorem, the third *thema* is a kind of chain-argument rule which allows one to break up a complex argument into two component arguments. But the two versions of the *thema* differ essentially, in that in Alexander the assumptions or premisses that are taken 'from outside' (ἐξώθεν) deduce one of the premisses of an argument that deduces the conclusion of the argument-to-be-analysed; whereas in Simplicius the external premiss comes in, together with the conclusion of another inference, in order to deduce the conclusion of the argument-to-be-analysed. Formally this difference between Alexander and Simplicius can be made clear as follows: ($P_1, P_2 \dots P_3 \dots$ give the places for non-external premisses, $E_1, E_2 \dots$ for external premisses, C for the conclusion of the argument-to-be-analysed).

Simplicius' version:

$$\frac{P_1, P_2 \vdash P_3 \quad P_3, E \vdash C}{P_1, P_2, E \vdash C}$$

Alexander's version:

$$\frac{P_1, P_2 \vdash C \quad E_1, E_2 \vdash P_1}{P_2, E_1, E_2 \vdash C}$$

By comparing these versions, one can see that they allow us to reduce exactly the same arguments, and that they differ only with respect to the premisses that count as 'external'.¹⁴⁹ I assume that Chrysippus' version of the third *thema* was closer to Simplicius' version.¹⁵⁰

For the analysis of arguments with more than three premisses one needs an expanded version of the third *thema*, in which one of the component arguments has more than two premisses. One obtains such an expansion if one modifies Simplicius' version in such a way that the second component argument can have more than one 'external premiss'.¹⁵¹

¹⁴⁹ For example, one can get from Simplicius' to Alexander's version by substituting E_1 for P_1, E_2 for P_2, P_1 for P_2 and P_2 for E_1 .

¹⁵⁰ For a detailed discussion of this point see Bobzien 1996, 145-51.

¹⁵¹ Chrysippus' book title 'On the <question of> which <premisses> syllogize something together with another <assumption> or with other <assumptions>' (D.L. VII.194) – which is part of a group of titles on arguments and their analysis – might refer to the third *thema*. It has 'with another <assumption>' or with other <assumptions>' instead of Simplicius' 'with another <assumption>', that is, it would refer to a plural of external premisses.

Perhaps Simplicius mentioned only one external premiss because the example he uses has only one. The expanded version of the third *thema* then runs:

When from two assertibles a third follows, and from the third and one or more external assertibles another follows, then this other follows from the first two and those external(s).

Or formalized:

$$(T_3) \quad \frac{P_1, P_2 \vdash P_3 \quad P_3, E_1 \dots E_n \vdash C}{P_1, P_2, E_1 \dots E_n \vdash C}$$

There are two types of composite arguments the reduction of which is not covered by the third *thema*, namely first those in which there are no 'external' premisses, but instead one of the premisses used in the first component argument is used again in the second component argument; and secondly those in which both a premiss of the first component argument and one or more external premisses are used in the second component argument. One may conjecture that the remaining two *themata* covered these two cases. They hence could have run:

the second *thema*:

When from two assertibles a third follows, and from the third and one (or both) of the two another follows, then this other follows from the first two.

formalized:

$$(T_2) \quad \frac{P_1, P_2 \vdash P_3 \quad P_1, (P_2), P_3 \vdash C}{P_1, P_2 \vdash C}$$

the fourth *thema*:

When from two assertibles a third follows, and from the third and one (or both) of the two and one (or more) external assertible(s) another follows, then this other follows from the first two and the external(s).

formalized:

$$(T_4) \quad \frac{P_1, P_2 \vdash P_3 \quad P_3, P_1, (P_2), E_1 \dots E_n \vdash C}{P_1, P_2, E_1 \dots E_n \vdash C}$$

Each of the second to fourth *themata* thus has a typical kind of argument to which it applies; but they can also be used in combination or more than once in one reduction. Going back to the types of arguments distinguished when discussing the dialectical theorem one can see that arguments of type (1) take the third *thema* once; those of types (2a) and (2b)

take it twice. More complex ones – without implicitly multiplied premisses – take it more often. Arguments of type (3) take the second *thema* once; those of type (4) take the fourth and third each once. More complex arguments may take combinations of the second, third, and fourth *thema*. Occasionally the first *thema* is needed in addition. Taken together the second, third, and fourth *thema* cover precisely the range of the dialectical theorem.

*

How were the *thematata* applied? Before I describe the general method of analysis, here are a few examples. First, take again the second example from the Sextus passage (S.E. M VIII.230–8, used there to illustrate the dialectical theorem). The argument-to-be-analysed is of the kind

If both p and q , r ; not r ; $p \vdash$ not q .

It has three premisses and takes the third *thema* once. By simply 'inserting' this argument into the *thema* we obtain:

When from two assertibles

i.e. If both p and q ; not r :

a third follows

i.e. not: both p and q (by a second indemonstrable)

and from the third and an external one

i.e. p

another follows

i.e. not: q , (by a third indemonstrable)

then this other

i.e. not: q

also follows from the two assertibles and the external one.

Or, using the formalized version of the *thema*:

If both p and q , r ; not r ; both p and q ; not both p and q ; $p \vdash$ not q

(T₃)

If both p and q , r ; not r ; $p \vdash$ not q

Examples of the use of the second *thema* we obtain from some of the special types of non-indemonstrable arguments. Indifferently concluding arguments like

Either it is day or it is light.

Now it is day.

Therefore it is day.

use the second *thema* once and reduce to one fourth and one fifth indemonstrable

Either p or q ; $p \vdash$ not q ; Either p or q ; not $q \vdash p$

(T₂)

Either p or q ; $p \vdash p$

Syllogisms with two mode-premisses like those of the kind

If p , q ; if p , not q ; therefore not p

take the first *thema* twice, the second once and reduce to two first indemonstrables. The analysis works again step by step from the bottom line

(a) to the top line (d):

(d) p , if p , not $q \vdash$ not q

(T₁)

(c) If p , q ; $p \vdash q$; $p, q \vdash$ not: if p , not q

(T₂)

(b) If p , q ; $p \vdash$ not: if p , not q

(T₁)

(a) If p , q ; if p , not $q \vdash$ not p

In general then, the method of analysis into indemonstrables by means of the *thematata* appears to have worked as follows. In a very first step, you check whether the argument-to-be-analysed, or original argument, happens to be an indemonstrable. If so, it is valid. If not, the next thing to do is to try to pick from the set of premisses of the argument-to-be-analysed two from which a conclusion can be deduced by forming an indemonstrable with them.

If the original argument is a syllogism, this conclusion, together with the remaining premiss(es) (if there are any), and/or one or both of the premisses that have been used already, deduces the original conclusion – either by forming an indemonstrable or by forming an argument that by use of the four *thematata* can be analysed into one or more indemonstrables. Hence you see whether one of the remaining premisses plus this conclusion yields the premisses to another indemonstrable (in which case you apply the third *thema*); if there are no remaining premisses, or none of them works, you look whether one of the premisses already used in the first step is such a premiss (in which case you apply the second or the fourth *thema*).

If the second component argument thus formed is an indemonstrable too, and all premisses have been used at least once and the last conclusion

is the original conclusion, the analysis is finished, the original argument a syllogism. If not, the same procedure is repeated with the argument which is not an indemonstrable (i.e. the second component argument, which has the original conclusion as conclusion); and so forth until the premisses of the second component argument imply the original conclusion by forming an indemonstrable with it.

If at any point in the analysis no indemonstrable can be formed, the first *thema* might help: namely if the negation of the conclusion would produce a premiss you need, i.e. a premiss which together with one of the available premisses makes up a pair of premisses for an indemonstrable. If at any step the application of none of the *themata* leads to two premisses that can be used in an indemonstrable, the argument is not a syllogism.

This method of reduction is practicable and easy. All one has to know is the *themata* and the five types of indemonstrables, plus those four types of simple arguments which can be reduced to indemonstrables. The number of steps one has to go through is finite; they are not very many, even in complex cases. The method appears to be effective, the system decidable.

*

Stoic syllogistic is a system consisting of five basic types of syllogisms and four argumental rules by which all other syllogisms can be reduced to those of the basic types. In Sextus Empiricus (*PH* II.156f, 157f, 194) we find Stoic claims that can be understood as the assertion of some kind of completeness in their logical system. We learn that the valid non-indemonstrable arguments have the proof of their validity from the indemonstrables (194), that the indemonstrables are demonstrative of the validity of the other valid arguments (156) and that those other arguments can be reduced (*ἀναφέρεσθαι*) to the indemonstrables (157).

The implication that the proof of the validity of the non-indemonstrables is given by reduction is confirmed by Diogenes (D.L. VII.78) who reports that all syllogisms are either indemonstrables or can be reduced to indemonstrables by means of the *themata*. We may then assume that the claim of 'completeness' in Sextus is that (at least) all non-indemonstrable syllogisms can be reduced to indemonstrables by the *themata* (or by related theorems). One could take this as the - trivial - claim that the *themata* (or theorems) lay down or determine whether an argument is a syllogism. But this is unlikely. Rather, we should assume that the Stoics had - independently of the *themata* - some pretechnical notion of syllogismhood, and that the indemonstrables plus *themata* were devised in order to 'capture' this notion, perhaps also to make it more pellucid and precise.

This is a plausible assumption. It leaves us with the following problem: how can we find the independent Stoic criteria for syllogismhood? that is, how can we decide which peculiarities of the Stoic system preceded their choice of logical rules and which peculiarities are simply a result of their introducing these rules? The paucity of evidence does not allow us to fully answer this question. *A fortiori*, we cannot decide whether the Stoics achieved completeness, i.e. were successful in devising their rules in such a way that they adequately covered their pretechnical notion of syllogismhood; and consequently, whether they were successful in demonstrating the completeness of their syllogistic.

Still, it is possible to determine a number of features of the Stoic system that are relevant to its completeness, and thus to narrow down considerably the number of possible interpretations of what completeness they wanted. It is safe to assume that the Stoic system shared the following condition of validity with modern semantic interpretations of formal logic: it is necessary for the validity of an argument that it is not the case that its premisses are true and its conclusion is false. Accordingly, it is a necessary condition for formal validity (syllogismhood) that no syllogism or argument of a valid form has true premisses and a false conclusion. To this we can add a number of necessary conditions for Stoic syllogismhood which are not requirements for formal validity in the modern sense, and which show that the class of Stoic syllogisms can at most be a proper subclass of valid arguments in the modern sense.

First, there is a formal condition which restricts the class of syllogisms not by denying validity to certain arguments, but by denying the status of argumenthood to certain compounds of assertibles. Stoic syllogistic is interested in formally valid *arguments*, not in *propositions* or *sentences* that are logically true. And their concept of argument is narrower than that of modern logic: an argument must have a minimum of two premisses and a conclusion. That is, Stoic syllogistic considers (tests etc.) only arguments of the form

$$\Delta \vdash A$$

in which Δ is a set of premisses with at least two (distinct) elements. Stoic syllogistic does not deal with arguments of the forms

$$\vdash A \quad A \vdash B \quad \text{or} \quad \Delta \vdash.$$

A consequence of this is that there is no one-to-one correspondence between valid arguments and logically true conditionals. Such a correspondence exists only between a proper subclass of the latter - those

which have the form 'If both A and B and . . . , then C' - and valid arguments.¹⁵²

Second, there is the restriction of validity through the requirement of non-redundancy of the premisses:¹⁵³ an argument is invalid according to redundancy if it has one or more premisses that are added to it from outside and superfluously (S.E. *M* II.431). For cases of non-indemonstrable arguments one may interpret the clause 'from outside and superfluously' as meaning that there is no deduction in which this premiss, together with the others of the argument, deduces the conclusion.¹⁵⁴ The requirement of non-redundancy leads to the exclusion, for instance, of the following kinds of arguments from being syllogisms:

p, q ; therefore p
 If $p, q; p; \neg$; therefore q

although they are valid in all standard propositional calculi.

A third restriction known to us - independently of the *themata* - concerns the wholly hypothetical 'syllogisms'. (Their prototype is 'If p, q ; if q, \neg ; therefore if p, \neg ') There are some hints that the Stoics considered such arguments as valid but not as syllogisms.¹⁵⁵ We do not know whether this restriction was part of the Stoic pretechnical notion of a syllogism, or whether these arguments were excluded because they were not analysable in the system.

In addition to these three requirements the Stoics apparently maintained that an argument cannot have two identical premisses. That is, compounds of assertibles of the form

$\Delta, \Delta, A \vdash B$

were, it seems, considered as a non-standard way of putting the argument $\Delta, A \vdash B$,

that is, as an argument in which the same premiss is stated twice rather than in which two premisses of the same form and content are stated.¹⁵⁶

¹⁵² For instance, although any conditional of the form 'If A, A ' is true for the Stoics, there is no valid argument of the form ' $A, A \vdash A$ ', since no compound of assertibles of that form is an argument.

¹⁵³ For redundancy see above, p. 125.

¹⁵⁴ So the premiss 'Either p or not: p ' is not redundant in the argument 'If p, p ; if not: p, p ; either p or not: p ; therefore p ' since there is a deduction of the conclusion in which it is used. Namely when one considers the argument as a special case of the simple constructive dilemma.

¹⁵⁵ Cf. for instance Frede 1977b, n. 5 (B) and (C); see also Alex. *APr.* 265.2-8-31. For the status of wholly hypothetical 'syllogisms' in Stoic logic see above, p. 133 and below, p. 156.

¹⁵⁶ At least this is implied by a passage in Alexander (Alex. *APr.* 18.5-7), which is most certainly Stoic, and which maintains that an 'argument' of the form ' $A, \text{not not: } A \vdash B$ ' is in actual fact of the form ' $A \vdash B$ ', and hence no argument at all.

Hence there can be no 'structural rules' which allow us to indiscriminately eliminate or introduce doublets of premisses - as there often are in rule-deductive systems.

We are now in a position to examine whether the Stoic system of syllogisms, as containing indemonstrables and *themata*, captures the - at least partly - pretechnical notion of syllogismhood as determined by the three requirements stated. And we can see that their system does not permit reduction of any of the arguments that are precluded by them from being syllogisms. First, no one- or zero-premiss arguments are reducible, since every indemonstrable has two premisses, and every *thema* can be applied only to arguments with two or more premisses. Secondly, redundant arguments cannot be reduced: the indemonstrables have no 'redundant' premisses, and the *themata* require that all premisses of the argument-to-be-analysed are components of the indemonstrables into which it is analysed - either as premiss or as negation of a conclusion. Thirdly, no wholly hypothetical 'syllogisms' are indemonstrables, nor can they be reduced to indemonstrables; for the last three *themata* require that one splits off one two-premiss argument each time they are used, and this two-premiss argument must contain at least one simple proposition,¹⁵⁷ because it must be either an indemonstrable itself or reducible into one by the first *thema*. And any reduction to an indemonstrable by means of a single application of the first *thema* also requires that the argument-to-be-analysed contains at least one simple proposition. So far then Stoic syllogistic coincides with what might have been their pretechnical concept of syllogismhood.¹⁵⁸

7: Arguments valid in the specific sense

Finally, the second group of valid arguments distinguished by the Stoics, the arguments called 'valid in the specific sense' or 'specifically valid' (D.L. VII.78). The surviving information on these arguments is sparse and many details are under dispute. At least two subclasses were distinguished. One were the so-called 'subsyllogistic arguments' (*Υποσυσλογιστικοί Λόγοι*),

¹⁵⁷ Or a substitution instance of a simple proposition.

¹⁵⁸ Can we state positively what the claim of completeness could have been? Maximally, the claim could have been that the class of arguments that either are indemonstrables themselves or can be analysed into indemonstrables by means of the *themata* contains precisely all arguments of the form ' $\Delta \vdash A$ ', with $\Delta = \{P_1, \dots, P_n\}$ and $n \geq 1$, which (i) because of their form can never have true premisses and a false conclusion, (ii) contain - as relevant to their form - only the Stoic logical constants 'not . . .', 'either . . . or . . .', 'if . . . then . . .', 'both . . . and . . .', (iii) contain no premiss doubts and no redundant premisses, and (iv) are not wholly hypothetical, and (v) are, or are composed of, nothing but self-evidently valid arguments. Perhaps a proof of this kind of completeness is possible.

another the arguments named 'unmethodically concluding' (ἀμεθοδῶς πρὸς τοὺς). There might have been a third group; there might have been more. How was the validity of the specifically valid arguments explained or justified? At D.L. VII.79 we read that all (valid) arguments were constructed by means of the indemonstrable syllogisms. If we take this at face value, the validity of the specifically valid arguments might have been grounded on or justified by the validity of syllogisms. This justification one would expect to vary from subclass to subclass.

*

Only two short passages explicitly talk about subsyllogistic arguments (Alex. *APr.* 84.12-14; Gal. *Inst. Log.* XIX.6), and a further passage briefly discusses them without naming them (Alex. *APr.* 373.28-35). From these texts it emerges that a subsyllogistic argument differs from a corresponding syllogism only in that one (or more) of its component assertibles, although being equivalent to that in the syllogism, diverges from it in its linguistic form. Examples are of the types

' p ' follows from ' q '; but p ; therefore q . (Alex. *APr.* 373.31-5)

instead of a first indemonstrable and

It is false that 'both p and q '; but p ; therefore not: q . (D.L. VII.78)

instead of a third indemonstrable. We may assume that the reason why subsyllogistic arguments were not syllogisms was that they did not share their canonical form. This distinction displays an awareness of the difference between object- and meta-language: a conditional is indeed not the same as a statement that one assertible follows from another. The validity of a subsyllogistic argument might have been established by constructing a corresponding syllogism and pointing out the equivalence to it.¹⁵⁹

The unmethodically concluding arguments are slightly better attested.¹⁶⁰ Stoic examples are:

Dio says that it is day.

But Dio speaks truly.

Therefore it is day.

¹⁵⁹ Training in recognizing which kinds of assertibles were equivalent to which seems to have been part of some Hellenistic logic courses (cf. Gal. *Inst. Log.* XVII.5). The manuscript text of the first Galen passage mentioned (Gal. *Inst. Log.* XIX.6) is corrupt. It can be read as implying that the Stoics distinguished a further class of specifically valid arguments which were linguistic mutations of syllogisms. Cf. Barnes 1993d, 38-43, 52.

¹⁶⁰ Gal. *Inst. Log.* XI.6; Alex. *APr.* 21-2; 68-9; 345-6; *Top.* 14-15; [Ammon.] *APr.* 70.11-15; Philo. *APr.* 35-6.

and

It is day.

But you say that it is day.

Therefore you speak truly.¹⁶¹

These arguments, as they stand, are not Stoic syllogisms. They are neither indemonstrables nor can they be reduced to them. For they contain no non-simple assertible as component. What was the reason for their validity? Perhaps they were dubbed 'unmethodically concluding' because there is no formal method of showing their validity; but even then their validity must have been justified somehow – and if we take the remark at D.L. VII.79 seriously, these justifications should have involved some suitably related syllogisms.

We have no direct evidence for a way of detecting 'corresponding syllogisms', as in the case of the subsyllogistics. One foolproof method is of course to add as leading premiss a conditional formed by the conjunction of the premisses as antecedent and the conclusion as consequent (and to conjoin the former premisses). For instance, add:

(p and $q \rightarrow r$) If you say that it is day and you speak truly, then it is day.

to the conjunction of the premisses and the conclusion

(p and q) You say that it is day and you speak truly.

(r) Therefore it is day.

This operation makes any argument into a syllogism, namely into a Chrysippean first indemonstrable. But, obviously, this cannot be the method devised to justify the validity of the unmethodicals. For it would equally work for invalid arguments.

Still, this might be a step in the right direction. First, Chrysippus used just such a first indemonstrable (with 'it is light' instead of 'it is day') in the discussion of the Liar,¹⁶² as a parallel argument to

If you say that you are lying and you speak truly, you are lying.

But you say that you are lying and you speak truly.

Therefore you are lying.¹⁶³

¹⁶¹ Alex. *APr.* 345.24-30; 22.17-19. The first example occurs with 'you' instead of 'Dio' in Gal. *Inst. Log.* XVII.2.

¹⁶² For the Stoic discussion of this paradox see below, p. 165.

¹⁶³ Cic. *Acad.* II.96. Equally, the second example is parallel to the second argument of the paradox of the Liar, cf. [Acro] *Scholae veterum in Horatii Epist.* II.1.45 (= *EDS* 1215): *alio me mentiri et mentior verum quia dico, see also Placidus Liber Glossarum* 95.14 (= *EDS* 1217). The liar might thus have provided the context in which these examples arose. Cf. below, p. 166.

So after all, there is some indication that the Stoics adduced syllogisms that correspond to unmethodicals.¹⁶⁴ Still, the Stoics must have ensured somehow that invalid arguments could not be justified as valid by forming a parallel syllogism with such a conditional added as leading premiss. To achieve this they might simply have stipulated that the added conditional must be true. For the truth of a conditional formed by the conjunction of the premisses of an argument as antecedent and the conclusion as consequent is nothing but the Stoic general criterion for the validity of an argument.¹⁶⁵

But we are still left with the question: what was the ground for the validity of the unmethodicals? For, although the conditional, if added, makes the unmethodical into a formally valid argument, it does not provide a reason or explanation for its validity. The reason for the validity – and for the truth of the added conditional – should rather be the truth of one or more ‘universal’ assumptions on which the argument is based, and which have not been made explicit in the argument. And indeed Galen reports that the Stoic Posidonius called at least some of the unmethodicals ‘concludent on the basis of the power of an axiom’ (*Gal. Inst. Log.* xviii. 8). Moreover, both Alexander (e.g. *Alex. APr.* 344–5) and Galen (*Inst. Log.* xvii. 1–4) state that the arguments the Stoics call unmethodicals depend on some universal statement or principle. Now it is likely that they took over the idea of an implicit universal assumption from the Stoics together with the category of unmethodicals. For the Stoics, universal propositions are standardly formulated as conditionals.¹⁶⁶ In our example a plausible candidate for such a universal would be:

(ii) If someone says something and that thing obtains, that one speaks true.

However, note that if one actually added such a Stoic universal to an unmethodical, one would not get a formally valid argument or syllogism; and that there is no reason to think that the Stoics wanted it to be added.

An argument would then be unmethodically concluding if the following requirements are fulfilled:¹⁶⁷ a corresponding syllogism can be constructed by adding a conditional formed with the conjunction of the premisses as antecedent, the conclusion as consequent. This conditional must be true, and it is true, since the unmethodical argument is valid. However, it does not provide any reason for the argument’s validity.

¹⁶⁴ *Plat. Barnes* 1990a, 81.

¹⁶⁵ See above, p. 123.

¹⁶⁶ See above, p. 113.

¹⁶⁷ For different views see *Frede* 1974, 121–3; *Barnes* 1990a, 78–81.

Rather, the ground for its validity is the truth of some universal assumption which is not made explicit in the argument.

*

There are a number of arguments which were regarded as valid by some Stoics, some of which might have been counted as specifically valid arguments.

There are first Antipater’s single-premiss arguments (*μονοσημιατοι*).

The orthodox Stoic view was that arguments must have at least two premisses. However, Antipater admitted single-premiss arguments, and he presumably regarded at least some of them as valid. If we trust Apuleius, Antipater adduced arguments like

You see.

Therefore you are alive. (*Apul. Mt.* 200.15–18)

Another example is ‘You are breathing. Therefore you are alive’ (*Alex. Top.* 8.19). What reasons he had for admitting such arguments, we are not told. But it is unlikely that Antipater proposed that these arguments were syllogisms (as *Alex. Top.* 8.16–17 has it). For they are certainly not valid in virtue of their form. Thus Antipater might have thought of them as unmethodically concluding, perhaps with a non-explicit assumption of the kind ‘if someone is breathing, that one is alive.’¹⁶⁸

Secondly, there are the arguments with an indefinite leading premiss and a definite (or middle) co-assumption, which were mentioned earlier in the context of non-simple assertibles.¹⁶⁹ Chrysippus’ work ‘Of arguments constructed from an indefinite and a definite <premiss>’ (*D.L.* vii. 198) may have dealt with such arguments. A typical example is

If someone walks that one moves.

This man walks.

Therefore this man moves. 170

Despite the similarity, this is not a straightforward first indemonstrable, as would be

If Plato walks, Plato moves.

Plato walks.

Therefore Plato moves.

¹⁶⁸ Cf. *Alex. Top.* 8.20–22 where ‘someone’ who is breathing is alive (ὁ διασπναιὸν ζῷ) and ‘Everyone who is breathing is alive’ (πᾶσι ὁ διασπναιὸν ζῷ) are given as alternative reasons for the conclusivity of the single-premiss argument: the first is a later Stoic non-standard formulation of universals, the second is Peripatetic.

¹⁶⁹ Above, p. 114.

¹⁷⁰ *Aug. Dialect.* iii. 84–6 Pinborg; cf. *Cic. Acad.* 11–15.

Preoccupied with linguistic form as the Stoics were, they must have noticed this. So if they did not simply smuggle such arguments into the class of syllogisms, how did they justify their validity? Presumably by referring to the truth-conditions of the leading premiss. Since its truth implies the truth of all subordinated assertibles, one can always derive the particular conditional one needs ('If this one walks, this one moves') and thus form the needed syllogism, in this case a first indemonstrable. This relation between the indefinite conditional and the corresponding definite and middle ones might have been counted as an implicit assumption by which validity was justified (but which if added would not make the argument formally valid). For similar reasons one may conjecture that Stoic arguments of the kind

If someone ϕ 's, ψ ; this one ϕ 's, therefore ψ ¹⁷¹

were regarded as unmethodically concluding.

Although it is unlikely that the early Stoics discussed Aristotelian logic, later Stoics were confronted with Peripatetic forms of arguments, in particular with categorical 'syllogisms' and wholly hypothetical 'syllogisms'.¹⁷² We know that some 'moderns' (*νεώτεροι*) – who may well have been Stoics – claimed that the unmethodically concluding arguments resembled categorical 'syllogisms' (Alex. *APr.* 345-13-17).

For the Stoics, following their policy concerning the formulation of universal statements, an argument in *modus Barbara*

'A holds of every B; B holds of every C; therefore A holds of every C' becomes:

If something is A, that thing is B.

If something is B, that thing is C.

Therefore, if something is A, that thing is C.

This is still not a Stoic syllogism. So, if anything, categorical 'syllogisms' could only have had the status of specifically valid arguments.

There is no direct evidence that the Stoics discussed wholly hypothetical 'syllogisms', i.e. arguments of the type

If ψ , then q ; if q , then r ; therefore if ψ , then r .

There are two such examples that use typically Stoic constituent sentences.¹⁷³ All one can say is that wholly hypothetical 'syllogisms' should

¹⁷¹ Cf. S.E. *PH* II.141; *M* VIII.313.

¹⁷² For these kinds of argument see above, p. 150. Here I put 'syllogism' in quotes to indicate that although the Peripatetics considered them syllogisms, presumably the Stoics did not.

¹⁷³ Zambon. *APr.* II.1-3; Alex. *APr.* 374-23-35. Ironically, the latter is employed to discredit this type of argument.

not have counted as syllogisms, since they cannot be analysed into indemonstrables by the *themata*. One would expect them to have been mentioned, had they been regarded as syllogisms. But if the Stoics reformulated *modus Barbara* in the way suggested, and considered the resulting arguments valid, they must also have considered wholly hypothetical 'syllogisms' as valid.

8. Paradoxes by Mario Miguélez

In the Stoic classification of arguments sophisms, the ancient counterparts of modern paradoxes, are put under the head of false arguments. A false argument is an argument which either has something false in its premisses or is formally incorrect (D.L. VII.79). What makes a false argument a sophism is that its conclusion is evidently false and it is not clear on what the falsity of the conclusion depends (Gal. *Pec. Dig.* V.72-3). As Sextus explains, in a sophism we are solicited to approve a clearly false conclusion by having endorsed premisses which look plausible and seem to yield the unacceptable conclusion (S.E. *PH* II.229).

To understand the meaning of this characterization it must be remembered that sophisms are supposed to be part of a (real or fictitious) discussion. One is asked to accept some propositions from which an overtly false conclusion is derived, and in this way the answerer is left in the embarrassing position of admitting a completely unacceptable statement. The many situations described by Plato in the *Euthydemus* caricature this sort of context.

This account explains the role that, according to the Stoics, dialectic should play regarding sophisms. It should not only distinguish sophistical from good arguments but also be able to solve them by showing what is wrong with them in such a way that any embarrassment is dispelled (S.E. *PH* II.229, 232). Their being classified among false arguments offers an indication of what one must look for in solving them: either the conclusion does not follow from the premisses or at least one of the premisses is false.

The Stoic characterization of paradoxes looks traditional and reminds us of the Aristotelian definition of contentious deductions at the beginning of the *Topics* (100b23-6). This impression is reinforced by Galen when he points out that sophisms resemble true arguments and stresses that a trained dialectician, being acquainted with good arguments, can easily detect and solve bad ones (Gal. *Pec. Dig.* V.73). Quintilian echoes this way of thinking when he reports that a training in solving paradoxes is part of the formation of the Stoic wise man because he cannot be mistaken even in trifles (Quint. *Inst.* I.10.5), and so does Seneca when he dismisses