A NOTE ON LOGICAL PARADOXES AND ARISTOTELIAN SQUARE OF OPPOSITION

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ABSTRACT

According to Aristotle if a universal proposition (for example: “All men are white”) is true, its contrary proposition (“All men are not white”) must be false; and, according to Aristotle, if a universal proposition (for example: “All men are white”) is true, its contradictory proposition (“Not all men are white”) must be false.

I agree with what Aristotle wrote about universal propositions, but there are universal propositions which have no contrary proposition and have no contradictory proposition.

The proposition X “All the propositions that contradict this proposition are true” does not have the contrary proposition and does not have the contradictory proposition.

In fact: FEX “All the propositions that contradict this proposition are not true” has a different subject: the subject of the proposition X is constituted by all the propositions that contradict the proposition X; by contrast, the subject of the proposition FEX is constituted by all the propositions that contradict the proposition FEX.

And FOX “Not all the propositions that contradict this proposition are true” has a different subject: the subject of the proposition X is constituted by all the propositions that contradict the proposition X; by contrast, the subject of the proposition FOX is constituted by all the propositions that contradict the proposition FOX.

According to Aristotle, a singular proposition (in his example: "Socrates is white") which is true must have its negative proposition ("Socrates is not white") which is false.

I agree with Aristotle, but there are singular propositions which do not have the corresponding negative proposition.

The proposition (or, rather, the pseudo-proposition) L “This same statement is not true” does not have the negative proposition because the proposition FNL “This same statement is true” has a different subject from the subject of the proposition L: the subject of the proposition L

is the proposition L; by contrast, the subject of the proposition FNL is the proposition FNL. L and FNL cannot have the same subject.

By contrast, the proposition M “This mount is entirely in Swiss territory” and the proposition NM “This mount is not entirely in Swiss territory” can have the same subject (for example, the Mount Eiger): in case the subject of the proposition M and the subject of the proposition NM is the same, M and NM are opposite propositions, NM is the negative proposition of M.

By contrast, the proposition (or, rather, the pseudo-proposition) L "This same statement is not true" cannot have the corresponding negative proposition because FNL "This same statement is true" has a different subject: the subject of L is L ; by contrast, the subject of FNL is FNL.

Then the paper continues by analyzing some variants of the liar’s paradox: L1 “The statement L1 is not true”; the so-called liar cycle; and the so-called Yablo’s paradox.

1 - INTRODUCTION: ARISTOTELIAN SQUARE OF OPPOSITION AND INDUCTION

Aristotle states in his treatise "Peri Hermeneias" [chapter 7] that two universal propositions having the same subject are named "contrary" when they assert and deny the same predicate, for example: "All men are white" and "All men are not white".

In Aristotelian logic two contrary propositions cannot be both true.

What I’m going to write in this section is referred only to Hume’s problem, not to Goodman’s paradox, which is a more general problem that regards also induction but not only induction.

In order to be true a universal proposition must satisfy the necessary condition (not sufficient but necessary) that wants the contrary proposition to be false.

It is reasonable to prefer a (non-contradicted) universal proposition which satisfies this necessary condition, rather than a (non-contradicted) universal proposition which we do not know whether it satisfies this necessary condition.

Let’s compare two incompatible hypotheses A, which states that all emeralds are green, and B, which states that all emeralds are subdivided into green ones and blue ones (in other words, the hypothesis B affirms that there are not only green emeralds, but also blue emeralds).

The hypothesis B can be written in many different ways, not only in the way I'm going to write. Predicate "X" applies to an object if either the object is a green emerald and the set of emeralds contains a non-empty subset of blue emeralds, or the object is a blue emerald and the set of emeralds contains a non-empty subset of green emeralds.

Let's analyse the following four propositions:

A - “All emeralds are green"

B - “All emeralds are X"

EA -"All emeralds are not green"

EB - “All emeralds are not X".

The proposition EA is the only one that is falsified (if all emeralds are green the proposition EB is true and the proposition B is false).

The proposition A is not contradicted by the data in our possession and has the contrary proposition (EA), that we know is false; by contrast, the proposition B is not contradicted by the data in our possession, but has not the contrary proposition (EB) that is falsified.

We have a reason to prefer the proposition A rather than the proposition B, because the proposition A has the contrary proposition (EA) that we know is false.

Surely there are differences between Aristotelian logic and Frege-Russell logic, but in this case they are irrelevant because when we have to do with inductive generalizations of the form "All Fs are G" we know that the set of Fs (the set of emeralds in the example) is not empty; and so, we have that also using Frege-Russell logic a true universal proposition "All Fs are G" must have the contrary proposition "All Fs are not G" that is false.

In other words:

Let's consider a universal proposition K "All Fs are G" and its contrary proposition EK "All Fs are not G". There are only three possible cases: 1) K is true and EK is false; 2) K is false and EK is true; 3) K and EK are both false.

Thus, let’s consider the propositions A ‘All emeralds are green”, EA “All emeralds are not green”, B “All emeralds are X”, EB “All emeralds are not X”.

In the case of the propositions B and EB there are three possible cases:

1) B is true and EB is false; 2) B is false and EB is true; 3) B and EB are both false.

In one case out of three B is true (in case 1: B is true and EB is false).

By contrast, let’s consider the case of the propositions A and EA:

1) A is true and EA is false; 2) A is false and EA is true; 3) A and EA are both false.

We know that the case 2 (the case “A is false and EA is true”) must be eliminated:

since we know that EA is false, there are only two possible cases: in one case out of two the proposition A is true.

So, we have to choose between a proposition (proposition A) that is true in a case out of two, and a proposition (proposition B) that is true in a case out of three: we have a reason to prefer a proposition that has one chance out of two over a proposition that has one chance out of three, there is an asymmetry between the propositions A and B, and we can use the aforementioned asymmetry to justify our preference for A over B.

2 - ON SOME LOGICAL PARADOXES AND ARISTOTELIAN SQUARE OF OPPOSITION

As we have seen in the previous section, having to choose between two incompatible and non-contradicted universal propositions, it is rational to prefer the proposition that not only is not contradicted but also has the contrary proposition that is contradicted.

We can check whether, in some way, this method can be adapted to singular propositions.

According to Aristotle, a true singular proposition must have its negative proposition that is false.

We can try to adapt the criterion reported at the beginning of this section to singular propositions: a singular proposition is true if, in addition to not being contradicted, it has its negative proposition which is contradicted.

For example: "Socrates is dead" is a true proposition because, in addition to not being contradicted, it has its negative proposition "Socrates is not dead" which is contradicted.

Symmetrically: a singular proposition is false if, in addition to being contradicted, it has its negative proposition which is not contradicted.

About universal propositions: a universal proposition is true if, in addition to not being contradicted, it has the contradictory proposition which is contradicted.

Symmetrically: a universal proposition is false if, in addition to being contradicted, it has the contradictory proposition which is not contradicted.

The aforementioned criterions (the criterion of truthfulness and the criterion of falsehood) can be useful for the treatment of some logical paradoxes; for example, in the Russell's paradox (or Russell's antinomy), let's call R the set of non-self-membered sets. Let's consider the two following sentences:

B - "R is a member of itself".

EB - "R is not a member of itself".

Since we are not able to tell which of the two sentences is contradicted and which one is not contradicted, both the sentences B and EB don't satisfy the aforementioned criterion of truthfulness and the aforementioned criterion of falsehood.

Let's return briefly to universal propositions: according to Aristotle if a universal proposition (for example: "All men are white")is true, its contrary proposition ("All men are not white") must be false; and, according to Aristotle, if a universal proposition (for example: "All men are white") is true, its contradictory proposition ("Not all men are white") must be false.

I agree with what Aristotle wrote about universal propositions, but there are universal propositions which have no contrary proposition and have no contradictory proposition.

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Now let's go back to the singular propositions: according to Aristotle, a singular proposition (in his example: "Socrates is white") which is true must have its negative proposition ("Socrates is not white") which is false.

I agree with Aristotle, but there are singular propositions which do not have the corresponding negative proposition.

The proposition (or, rather, the pseudo-proposition) L "This same statement is not true" does not have the negative proposition because the proposition FNL "This same statement is true" has a different subject from the subject of the proposition L: the subject of the proposition L is the proposition L; by contrast, the subject of the proposition FNL is the proposition FNL. L and FNL cannot have the same subject.

By contrast, the proposition M "This mount is entirely in Swiss territory" and the proposition NM "This mount is not entirely in Swiss territory" can have the same subject (for example, the Mount Eiger): in case the subject of the proposition M and the subject of the proposition NM is the same, M and NM are opposite propositions, NM is the negative proposition of M.

By contrast, the proposition (or, rather, the pseudo-proposition) L "This same statement is not true" cannot have the corresponding negative proposition because FNL "This same statement is true" has a different subject: the subject of L is L ; by contrast, the subject of FNL is FNL.

In order to obtain a proposition that has a meaning opposite to the meaning of the proposition L we have to resort to a stratagem, a different way of writing L "This same statement is not true" (which is devoid of the corresponding negative sentence):

L1 "The statement L1 is not true"

EL1 "The statement L1 is true".

L1 is logically equivalent to L in the sense that L and L1 have the same paradoxical truth values: each is true when it is not true; L is true when L is not true, and L1 is true when L1 is not true: in this sense L and L1 are logically equivalent.; and L1 has the negative proposition, which is EL1.

Let us consider the following propositions:

L1 "The statement L1 is not true"

L2 "The statement L1 is not true"

L3 "The statement L1 is not true".

L1 has truth values different from L2 and L3: L2 is true if L1 is not true; L3 is true if L1 is not true; by contrast, L1 has paradoxical truth values.

Why does L1, although written as L2 and L3, have truth values different from L1 and L3?

Aristotle in chapter 8 of "Peri Hermeneias" gives the example of something that has the form of a proposition but is actually a set of two sentences.

In my opinion, L1 has truth values different from L2 and L3 because L1 has only the form of a proposition, but is actually a set of sentences..

Thanks to the acronym L1 outside the quotation marks, the additional information that L1 refers to itself, and therefore is logically equivalent to L, is smuggled into the proposition L1. For this reason the truth values of L1 differ from the truth values of L2 and of L3.

What makes the difference between L1 and L2, L3 in truth values is not what is explicitly stated by the statement (which is identical in L1, L2, L3) but what is implicitly stated.

L2 and L3 only state that L1 is not true; L1 also claims that L1 refers to itself and therefore claims that L1 is logically equivalent to a sentence, L "This same statement is not true", which has not the negative proposition, which is devoid of the corresponding negative proposition.

In my opinion, the proposition (or rather, the pseudo-proposition) L "This same statement is not true" is actually a set of sentences: there is an implicit sentence which warns that L "This same statement is not true" has only the form of a proposition because L is devoid of the corresponding negative proposition (because "This same statement is true" is not its negative proposition).

In the pseudo-proposition L1 "The statement L1 is not true" the supplementary information is not entirely implicit in L1 and it is necessary to import, smuggle into L1 the piece of information (the acronym L1 preceding the quotation marks) which warns that L1 is logically equivalent to a proposition (better: a pseudo-proposition: L “This same statement is not true”) which is devoid of the corresponding negative proposition.

In the sentence L "This same statement is not true", unlike in the sentence L1, the supplementary information is implicit in the statement itself: the supplementary information warns us that L does not have the negative proposition.

In the so-called liar cycle we have:

P "The statement S is true"

S “The statement P is not true”.

P and S are camouflages of L "This same statement is not true".

If we consider P without knowing what S states, we have that P is true in case S is true.

If we consider S without knowing what P states, we have that S is true in case P is not true.

But when we learn that P and S refer to each other, the truth values change and become paradoxical: P is true if P is not true; S is true if S is not true.

What has changed? Within each of the two sentences P and S a supplemental piece of information has been imported, smuggled.

We have:

P "The statement S is true"

S “The statement P is not true”.

Once we surreptitiously introduce into P the supplementary information that S refers to P, and once we surreptitiously introduce into S the supplementary information that P refers to S, we obtain that P is equivalent to asserting "The affirmation S is true and claims that this affirmation is not true", which is equivalent to asserting "This same statement is not true".

And S is equivalent to asserting "The affirmation P claims that this affirmation is true, but this affirmation claims that the affirmation P is not true (and therefore this same affirmation is not true)", which again is equivalent to asserting "This same statement is not true" (which, as we have seen, is a supposed proposition which is devoid of its corresponding negative proposition).

S and P have the same paradoxical truth values as L "This same statement is not true":

L is true when L is false; P is true when P is false; S is true when S is false.

About the so-called Yablo's paradox:
"All the following sentences are untrue" means " All the sentences following this same sentence are untrue": it is a universal proposition (rather, a pseudo-proposition) that does not have the contrary proposition, it is a proposition which is devoid of the contrary proposition.
Y "All the sentences following this same sentence are untrue".
FEY "All the sentences following this same sentence are true".
FEY is not the contrary proposition of Y because Y and FEY have different subjects: the sentences following Y are not the same subject as the sentences following FEY.

A specification:
G "This same sentence is written in English".
The sentence G is actually a set of sentences (because it is implicit the affirmation that G lacks the corresponding negative proposition, given that FEG "This same sentence is not written in English" has a different subject), but it is true that every word written in G (in the set of sentences G) is written in English.

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