Is incommensurability vagueness?

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For: Ruth Chang (ed.) *Incommensurability, Comparability and Pracatical Reason*,
Harvard University Press, 1997.

This paper was written while I was a Visiting Fellow at the Australian National University in 1993. I owe a large debt to Adam Morton; his paper `Hypercomparatives' was the source of many of my arguments. I was also greatly helped by Linda Burns's book *Vagueness*, but unfortunately this paper is handicapped by having been written before Timothy Williamson's *Vagueness* was published. I have received very many valuable comments from the audiences to which I have presented the paper, and from many other people too. They include Luc Bovens, Ruth Chang, Dorothy Edgington, Sven Danielsson, James Griffin, Frank Jackson, Douglas MacLean, Philip Pettit, Joseph Raz, John Skorupski, Susan Wolf and Crispin Wright.

1 *Indeterminate comparatives*

Which is more impressive: Salisbury Cathedral or Stonehenge? I think there is no determinate answer to this question. The dyadic predicate `more impressive than' – the comparative of the monadic predicate `impressive' – seems to allow indeterminate cases. Many comparatives are like that. Amongst them are many evaluative comparatives, such as `lovelier than', `cleverer than', and the generic `better than'.

For many comparatives, the indeterminacy arises because the comparison involves several factors or dimensions, and it is indeterminate exactly how the factors weigh against each other. The impressiveness of a building depends on some combination of its size, its importance in the landscape, the technological achievement it represents, and more, and it is indeterminate how these factors weigh against each other. Many evaluative comparatives are indeterminate for this reason. They depend on a combination of values, and it is indeterminate how the values are to be weighed. The values are *incommensurable*, we say.

Not all indeterminate comparatives arise from incommensurable dimensions, however. `Redder than' can also be indeterminate, even though it presumably does not involve several factors. Compare some reddish-purple patch of colour with some reddish-orange patch, and ask which is redder. The answer may be indeterminate.

To make it clear: when I say a comparative `*F*er than' is indeterminate, I mean that for some pairs of things, of a sort to which the predicate `*F*er' can apply, there is no determinate answer to the question of which is *F*er than which. Neither is *F*er than the other, and nor are the two equally *F*. This paper is about the logical structure of indeterminate comparatives, and in particular whether they are vague. The analysis is intended to apply to all indeterminate comparatives, but my chief interest is in the structure of evaluative and moral comparatives. I am particularly interested in the structure of `better than'. I shall argue that the indeterminacy of betterness is commonly misrepresented; it does not take the form that is most commonly assumed.

2 *Standard configurations*

I am going to set up a standard framework to use in the investigation. Before doing so I need to make a few preliminary points about the structure of comparatives. First, two formal features of the structure:

> *Asymmetry of* `F*er than'*. For any *x* and *y*, if *x* is *F*er than *y*, then *y* is not *F*er than *x*.
> *Transitivity of* `F*er than'*. For any *x*, *y* and *z*, if *x* is *F*er than *y* and *y* is *F*er than *z*, then *x* is *F*er than *z*.

I think these are principles of logic. If `*F*er than' is indeterminate, that is no reason to doubt them. They are only conditional statements. Asymmetry requires that *if x* is *F*er than *y*, then *y* is not *F*er than *x*. With an indeterminate comparative, it may turn out for some *x*s and *y*s that *x* is not *F*er than *y* and nor is *y* *F*er than *x*; this is consistent with asymmetry. Transitivity requires that *if x* is *F*er than *y* and *y* is *F*er than *z*, then *x* is *F*er than *z*. If it turns out that neither *x* is *F*er than *y* nor *y* *F*er than *x*, then in this case transitivity is vacuously satisfied.

My next point is that no comparative I can think of is indeterminate between every pair of things. For some pairs of colour patches, it is indeterminate which is redder than which, but for other pairs, one of the pair is determinately redder than the other. A red patch is determinately redder than an orange patch, for instance. Salisbury Cathedral is determinately more impressive than Bath Abbey. For most comparatives `*F*er than', we can form whole *chains* of things, each of which is *F*er than the next in the chain. A well-chosen chain may run from things that are very

F to things that are not at all *F*. For instance, we could form a chain of colour patches, each redder than the next, starting from a pure red and running through orange to a yellow with no red in it at all. Churches could be arranged in a chain, each more impressive than the next. The chain might start with St Peter's, and end with some unimpressive chapel. Stonehenge would not be included in this chain.

Take some chain ordered by `*F*er than'. Then take something that is not included in the chain, and compare it with each of the things that are in the chain, to see which is *F*er. It may turn out that there is no determinate answer for any member of the chain. Compare Kantor's diagonal proof that the real numbers are uncountable with churches that make up a chain. Which is more impressive? You might conclude that proofs and churches in general are so different that they can never be determinately compared in terms of their impressiveness. Maybe so and maybe not. Things that are very different from the members of a chain are less likely to be comparable to these members than are similar things. But things that are not too different will normally be comparable to some extent. For instance, take a reddish purple patch and compare it with the chain of patches I described that runs from red to yellow. The purple patch will be determinately comparable with some members of the chain. Patches at the top of the chain are redder than the purple patch, and the purple patch is redder than patches at the bottom of the chain. But somewhere in the middle there may well be patches where the comparison is indeterminate. To take another example, I think St Peter's is more impressive than Stonehenge, and Stonehenge is more impressive than the gospel chapel in Stoke Pewsey, but for some churches in the chain the comparison with Stonehenge will be indeterminate. So as we move down a chain from top to bottom, comparing its members in *F*ness with some object outside the chain, we may start in a zone where the members of the chain are *F*er, then move into a zone where the comparison is indeterminate, and finally come to a zone where the other object is *F*er.

The aim of this paper is to investigate what exactly goes on in the zone of indeterminacy. What is true of things in that zone, and what is not true? To focus the discussion, I shall concentrate on a particular type of example, which I shall call a `standard configuration'. Here is one of the type. Take a chain of colour patches ranging from red through orange to yellow, each patch redder than the next. In fact, let the patches form a continuum: a smooth band of colour graded from red at the top to yellow at the bottom. Compare small patches or points of colour from this band with a single reddish-purple patch and consider which is redder. At the top are points redder than the single patch and at the bottom points the single patch is redder than.

A *standard configuration* for a comparative `*F*er than' consists of a chain of things, fully ordered by their *F*ness and forming a continuum, and a fixed thing called the *standard* that is not itself in the chain. At the top of the chain are things *F*er than the standard, and at the bottom things the standard is *F*er than. I shall refer to the things that make up the chain as `points', whatever sort of thing they are.

Two matters of clarification. There may be a zone of indeterminacy between the members of the chain that are *F*er than the standard and those that the standard is *F*er than, and I am particularly interested in cases where there is, but the definition of a standard configuration does not require there to be one. The second point is that the continuum of points may not exist in fact; we may simply imagine it. For instance, actual churches do not form a continuum, but we can imagine a continuum of churches. Also, I have not defined what I mean by a `continuum'; I hope I may leave that to intuition.

The colour example is a standard configuration, and here is another. Suppose you have a choice between two careers, and your are wondering which is better. By this I mean: which would be the better one for you to take up. One offers travel and adventure, the other security and a regular income. Let us take the adventurous career as the standard, and form a continuous chain out of the other by varying the amount of income it offers. At the top of the chain are secure jobs with a very high income, and let us suppose these are better than the adventurous career. At the bottom are secure careers offering poor incomes, and let suppose the adventurous career is better than those. In the middle there may be a zone of indeterminacy.

A third example. A government is wondering whether to preserve a stretch of rainforest, or open it up for commercial exploitation. Take as the standard the policy of preserving the forest. We can make a chain out of the exploitation option by imagining a range of economic benefits that might arise. Suppose that, if the benefits are enormous, exploitation is better, and if the benefits are minute, preservation is better. Somewhere in between may be a zone of indeterminacy.

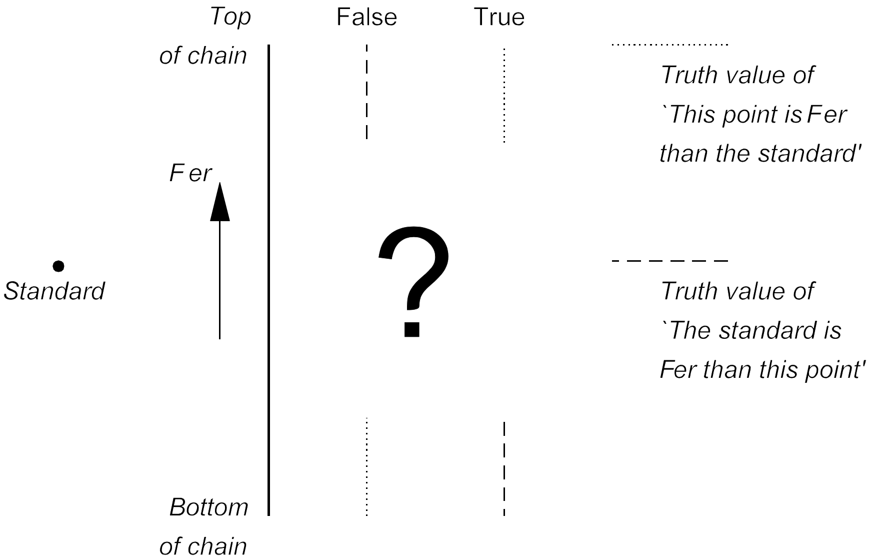

Figure 1. *A standard configuration*

A standard configuration is illustrated in Figure 1. Graphed against points in the chain, the diagram shows truth values for the statements `This point is *F*er than the standard' and `The standard is *F*er than this point'. At the top of the chain is a zone of points with the property that they are *F*er than the standard and the standard is not *F*er than them. At the bottom is a zone of points with the property that the standard is *F*er than them and they are not *F*er than the standard. That is a constant feature of any standard configuration. But what happens between the top and the bottom zones? There are a number of possibilities, which I shall describe in turn. In Sections 9 and 10, I shall discuss which of the possibilities are realized in actual English comparatives.

3 No indeterminacy

The first possibility is that there is no more than one point between the top zone and the bottom zone. It may be that, for every point in the chain, either it is Fer than the standard or the standard is Fer than it. Alternatively, there may be just one point for which this is not so. In both of these cases there is no indeterminacy. Let us call these type (a) determinacy and type (b) determinacy respectively. Figure 2 illustrates either.

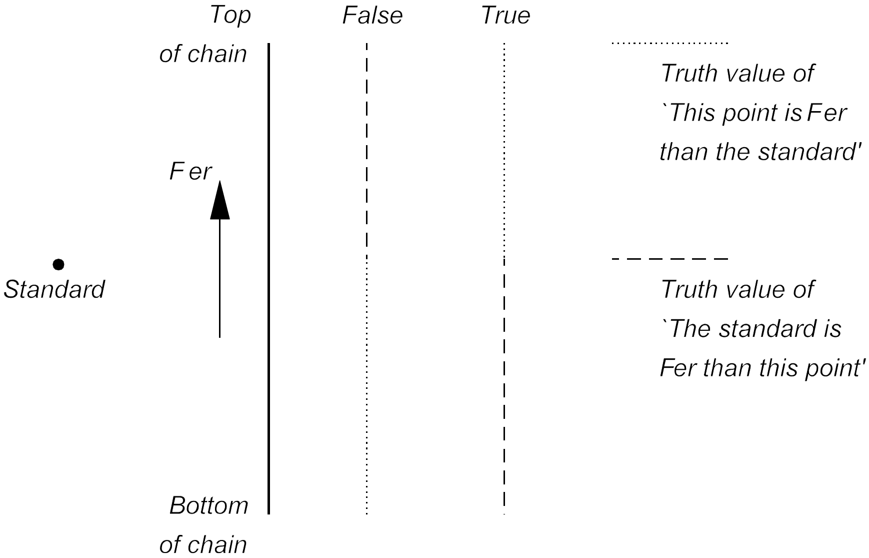

Figure 2. *No indeterminacy*

In type (b) determinacy, I assume the unique point must be equally as F as the standard. That is why I say these cases have no indeterminacy: for every point, either it is Fer than the standard, or the standard is Fer than it, or it is equally as F as the standard. What justifies this assumption? Not much, I am sorry to say, but here is my reason for adopting it.

Take a standard configuration, and suppose one point in the chain is equally as F as the standard. Since higher points are Fer than this point, and this point is equally as F as the standard, it follows that higher points are Fer than the standard. This is an application of the principle:

> *Extended transitivity of* `F*er than'*. For any x, y and z, if x is Fer than y and y is equally as F as z, or if x is equally as F as y and y is Fer than z, then x is Fer than z.

This is a small extension of the transitivity principle, and I take it too to be a principle of logic. The same extended transitivity implies that if one point in a chain is equally as F as the standard, then the standard is Fer than all lower points.

So if one point is equally as F as the standard, then for every other point in the chain, either it is Fer than the standard or the standard is Fer than it. I am inclined to believe the converse. That is: if, for every point in a continuous chain bar one, either it is Fer than the standard or the standard is Fer than it, but for one point this is not so, then I am inclined to believe this one point must be equally as F as the standard. In fact, I am inclined to believe that `equally as $F$ as' is not an independent notion at all, but can be defined in terms of $F$er than. I would say that `x is equally as F as y' means that x is not Fer than y, and y is not Fer than x, and anything that is Fer than x is also Fer than y, and y is Fer than anything x is Fer than. This is the basis of my assumption that, in type (b) determinacy, the unique point is equally as good as the standard. I have no argument for it, but I can think of no counterexample.

Also, I can think of no examples of type (a) determinacy. I suspect it does not exist. This must depend on precisely what is meant by saying that the chain of points in a standard configuration is *continuous*. It has no gaps in it, but in what sense, precisely? Since I have not given a precise definition, I cannot be sure whether or not type (a) determinacy exists. It does not matter in this paper.

4 Hard indeterminacy

The next possibility is that there is more than one point in the chain such that it is not *F*er than the standard and the standard is not *F*er than it. All the points with this property form a central zone.

None of them can be equally as *F* as the standard. If one was, I showed in Section 3 that all points above it would be *F*er than the standard and the standard would be *F*er than all points below it. Then only this one point would have the property that it is not *F*er than the standard and the standard is not *F*er than it. But our assumption is that there is more than one point with this property.

The whole central zone, then, constitutes a zone of indeterminacy. If there is more than one point in the chain such that it is not *F*er than the standard, and the standard is not *F*er than it, I shall say that `*F*er than' has *hard indeterminacy*.

Hard indeterminacy is not vagueness. It is definite that points within the zone of indeterminacy are not *F*er than the standard and the standard is not *F*er than them. However, it is natural to think there is often something vague about an indeterminate comparative. Take `redder than', for instance. `Redder than the standard' is a monadic predicate, and it is natural to think it may be vague, like the predicate `red'. Start from the top of the chain in the standard configuration I described, which runs from red to yellow. Points at the top of the chain are red, and it seems implausible that as we move down the chain we encounter a sharp boundary that divides these points that are red from those that are not red. Similarly, points at the top are redder than the standard (which is reddish-purple and not in the chain itself), and one might find it implausible that, as we move down the chain, we encounter a sharp boundary that divides these points that are redder than this standard from points that are not redder than it. Instead, it seems there may be a zone of vagueness between the points that are redder than the standard and those that are not. For many comparatives `*F*er than', it seems plausible there is an zone of vagueness between points that are *F*er than the standard and points that are not. There seems also to be an area of vagueness at the bottom of the zone of indeterminacy, between points the standard is not *F*er than, and those it is.

Is vagueness like this compatible with hard indeterminacy? At first it seems it should be. Hard indeterminacy simply says there is a zone where points are not *F*er than the standard and the standard is not *F*er than them. The vagueness I have just described is around the borders of this zone. The suggestion is that the borders are vague rather than sharp. The zone of indeterminacy is bordered by zones of vagueness – at first, there seems nothing wrong with that. However, oddly enough, it turns out that this cannot be so. The boundaries of the zone of indeterminacy must be sharp rather than vague, for the following reason.

Take any point somewhere around the top boundary of the zone of indeterminacy. Clearly, it is false that the standard is *F*er than this point, since this is false for all points in the zone of indeterminacy and above. If it is also false that this point is *F*er than the standard, then the point is squarely within the zone of indeterminacy. If, on the other hand, it is true that this point is *F*er than the standard, then it is squarely within the top zone. So if there is really a zone of vagueness, for points in this zone it must be neither true nor false that they are *F*er than the standard. But now we can apply something I call the *collapsing principle*.

> *The collapsing principle, special version*. For any *x* and *y*, if it is false that *y* is *F*er than *x* and not false that *x* is *F*er than *y*, then it is true that *x* is *F*er than *y*.

This principle is crucial to the argument of this paper. I shall defend it in the next section, and here I shall simply apply it. I have just said that, for a point in the zone of vagueness, if there is such a zone, it is false that the standard is *F*er than it, but not false that it is *F*er than the standard. Then according to the collapsing principle, it is true that it is *F*er than the standard. This implies it is not in a zone of vagueness after all. So there is no such zone.

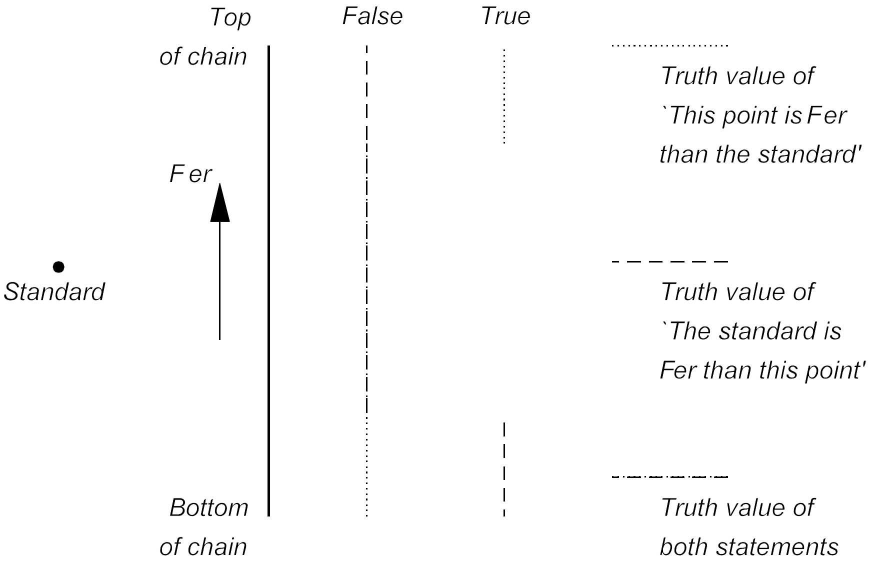

Figure 3. *Hard indeterminacy*

Figure 3 shows hard indeterminacy. As we move down the chain from the top, starting with points that are *F*er than the standard, we suddenly come to ones that are not *F*er than the standard. Later, having found only points that the standard is not *F*er than, we suddenly encounter ones that the standard is *F*er than.

A comparative that has hard indeterminacy is a strict partial ordering without vagueness.

5 The collapsing principle

The proof in the previous section depended crucially on the collapsing principle. Now I need to justify it. My only real argument is this. If it is false that *y* is *F*er than *x*, and not false that *x* is *F*er than *y*, then *x* has a clear advantage over *y* in respect of its *F*hess. So it must be *F*er than *y*. It only takes the slightest asymmetry to make it the case that one thing is *F*er than another. One object is heavier than another if the scales tip ever so slightly towards it. Here there is a clear asymmetry between *x* and *y* in respect of their *F*hess. That is enough to determine that *x* is *F*er than *y*.

I find this obvious, and here is a thought-experiment to reinforce its obviousness. Suppose you had to award a prize to either *x* or *y* for its *F*hess. Suburbs in Canberra are named after great Australians, and each new suburb has to go to the greatest Australian who does not yet have a suburb. Suppose there are two candidates for the next suburb, and you have to decide between them. Suppose that, on investigating their cases, you conclude it is false that Wye is a greater Australian than Exe, but that it is not false that Exe is a greater Australian than Wye. This is not at all like the case where you conclude that Wye and Exe are equally great Australians, because then it is not clear who should get the suburb; you should probably toss a coin. Nor is it like the case where you conclude that neither Wye nor Exe is greater than the other and they are not equally great either. This is a case squarely in the zone of indeterminacy. In this case, it is once again not clear who should get the suburb, just because neither candidate is better than the other. Perhaps you should toss a coin in this case too, or perhaps some other procedure would be right. But when it is false that Wye is greater than Exe, but not false that Exe is greater than Wye, you need not hesitate. It would be quite wrong to give the suburb to Wye. Since the prize was for being the greater Australian, it could not be so obvious who should win unless that person was the greater Australian.

When it is false that *y* is *F*er than *x* but not false that *x* is *F*er than *y*, then if you had to award a prize for *F*ness, it is plain you should give the prize to *x*. But it would not be plain unless *x* was *F*er than *y*. Therefore *x* is *F*er than *y*. This must be so whether actually you have to give a prize or not, since whether or not you have to give a prize cannot affect whether or not *x* is *F*er than *y*.

6 Soft indeterminacy

What other options are there for the structure of a comparative `*F*er than'? No other option can allow the existence of points in the chain such that they are not *F*er than the standard and the standard is not *F*er than them (that is: such that it is false that they are *F*er than the standard and false that the standard is *F*er than them). That would be hard indeterminacy. But it might be possible for there to be points such that it is neither true nor false that they are *F*er than the standard, and neither true nor false that the standard is *F*er than them. There could be a zone of indeterminacy composed of points like this. I call this case *soft indeterminacy*.

None of the points in the zone of indeterminacy can be equally as *F* as the standard, since if a point was equally as *F* as the standard it would be false that it is was better than the standard, whereas it is not false. Therefore, the zone of indeterminacy must have more than one point in it. If there were only one point, I argued in Section 3 that it would have to be equally as *F* as the standard. But I have just said this is impossible.

Unlike hard indeterminacy, which has no vagueness, soft indeterminacy is entirely vagueness. The zone of indeterminacy is also a zone of vagueness. For any point in the zone, it is vague whether or not it is *F*er than the standard, and vague whether or not the standard is *F*er than it.

The possible structures of a softly indeterminate comparative are severely limited. Once again, it is chiefly the collapsing principle that limits them. One limit is that, for any point, it is neither true nor false that it is *F*er than the standard if and only if it is neither true nor false that the standard is *F*er than it. So the entire zone of indeterminacy necessarily has vagueness of both sorts; the comparison is vague taken either way. Here is the proof. First, suppose some point is *F*er than the standard and it is neither true nor false that the standard is *F*er than it. Since this point is *F*er than the standard, the asymmetry of `*F*er than' implies the standard is not *F*er than it. This contradicts that it is neither true nor false that the standard is *F*er than it. Second, suppose a point is not *F*er than the standard and it is neither true nor false that the standard is *F*er than it. From this supposition, the special version of the collapsing principle, stated in Section 4, implies that the standard is *F*er than the point. That contradicts that it is neither true nor false that the standard is *F*er than it. The rest of the proof is a matter of ringing the changes.

Another limit on the structure of softly indeterminate comparatives is imposed by a more general version of the collapsing principle. My statement of this version needs a preface. I intend it to be neutral between competing theories of vagueness. It uses the expression `more true than', but this is not to be read as implying the existence of degrees of truth as they are usually understood. Its meaning is wider. For instance, if P is true and Q is false, then I would say P is more true than Q. If P is neither true nor false, and Q is false, again I would say P is more true than Q. In this case, P is certainly less false than Q, and I intend `more true' to include `less false'. If P is definitely true and Q is not definitely true, I would again say that P is more true than Q. In general, I will say P is more true than Q whenever P in any way rates more highly than Q in respect of its truth. Now, the principle is:

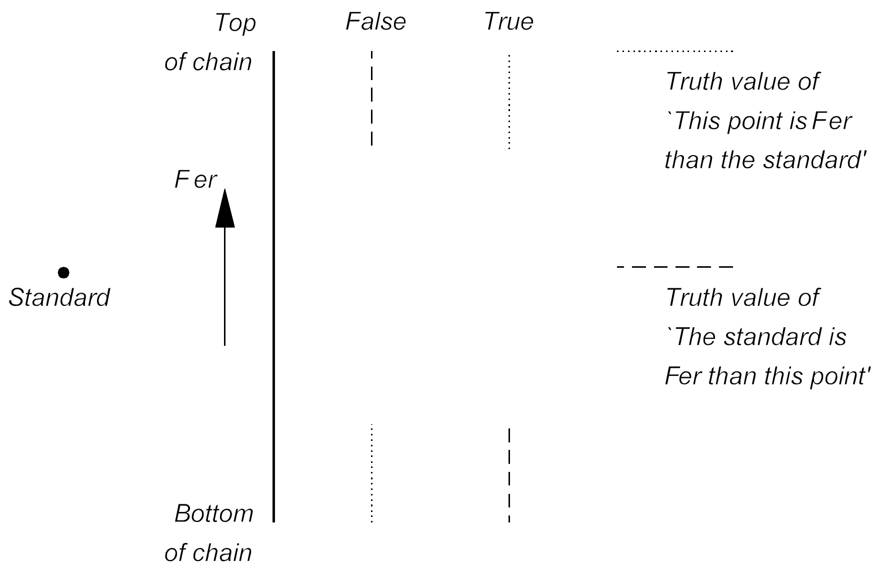

Figure 4. *Soft indeterminacy type (a)*

> *The collapsing principle, general version*. For any x and y, if it is more true that x is Fer than y than that y is Fer than x, then x is Fer than y.[1]

The argument for it is the same as the one I gave in Section 5 for the special version. Remember this principle is specifically about reciprocal comparative statements. I do not endorse the general claim that if any statement P is more true than another statement Q then P is true, nor the different specific claim that if P is more true than its negation $\neg P$ then P is true. If someone is more dead than alive, I do not believe she is necessarily dead.

The general version of the collapsing principle implies that for no point in the zone of indeterminacy can it be more true that the point is *F*er than the standard than that the standard is *F*er than it. Nor can it be less true. This leaves open several possibilities, corresponding to different theories of vagueness. One theory is that when statements are vague they have no truth

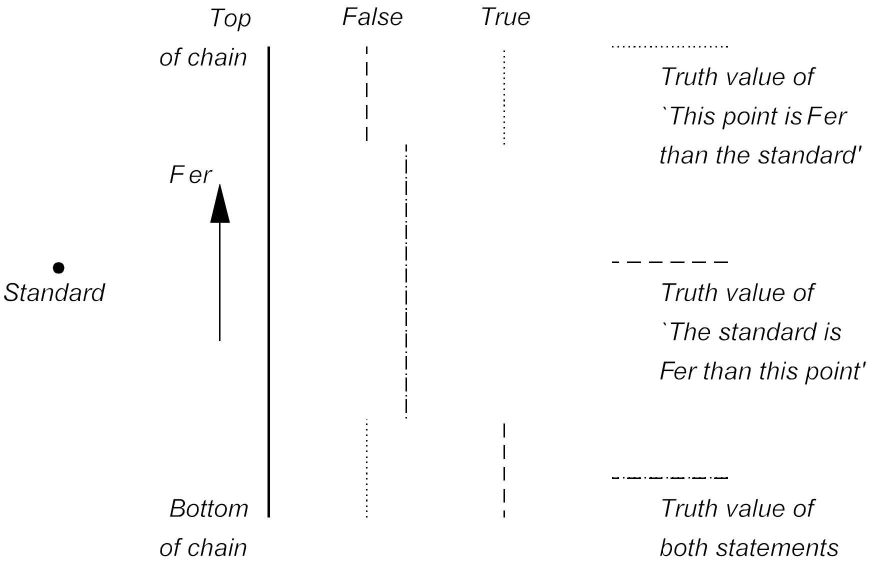

Figure 5. *Soft indeterminacy type (b)*

value. Figure 4 illustrates how soft indeterminacy will appear from this point of view; I have called it type (a) soft indeterminacy. Evidently, it is consistent with the collapsing principle. According to other theories, vague statements may have some other truth value besides true or false. It would be consistent with the collapsing principle for `This point is *F*er than the standard' and `The standard is *F*er than this point' to have the same truth value throughout the zone of indeterminate. It might be `indefinite', say, or some particular degree of truth. Figure 5 illustrates this possibility. I have called it type (b) soft indeterminacy.

7 Incomparable truth values

Both types (a) and (b) of soft indeterminacy fail to capture one natural intuition. For any point in the zone of indeterminacy, it is neither true nor false that it is *F*er than the standard. But intuition suggests that in some sense or other the statement `This point is *F*er than the standard' is more true for points near the top of the zone than for points near the bottom. For points near the top, indeed, it seems it must be almost true they are *F*er than the standard, and for points near the bottom it seems this must be almost false.

This intuition clearly implies that statements can be true to a greater or lesser degree. So in order to discuss it, I shall have to give up my attempt to be neutral amongst competing theories of vagueness. For this discussion, I shall take it for granted that there are degrees of truth. Anyway, the study of comparatives suggests there are. Suppose x is redder than y. This implies that y has redness to a greater degree than y has it, so there are degrees of redness.[2] But it seems also implies that x is red more than y is red, which in turn surely implies that `$x$ is red' is truer than `y is red', so there are degrees of truth. I shall say more about this in Section 8. Other sorts of comparisons perhaps point more clearly to degrees of truth, because they more clearly compare statements. `It's raining more than it's snowing' seems clearly to imply that `It's raining' is truer than `It's snowing.

One intuition, then, is that as we move from point to point up through the zone of indeterminacy, `This point is *F*er than the standard' becomes progressively more true and `The standard is *F*er than this point' progressively less true. At first, this intuition seems to be inconsistent with the collapsing principle. It seems to imply that, as we move up through the zone, we must eventually encounter points for which `This point is *F*er than the standard' is more true than `The standard is *F*er than this point'. But then the collapsing principle will kick in and say these points must actually be determinately *F*er than the standard: they are not in the zone of indeterminacy after all.

However, a point made by Adam Morton can be used to show this need not happen.[3] It may be that the truth values of `This point is *F*er than the standard' and `The standard is *F*er than this point' are incomparable throughout the zone of indeterminacy, but nevertheless one of these statements may increase in truth value as we move up through the zone and the other decrease. The reason this seems impossible at first is that we traditionally think of degrees of truth as numbers between zero and one. At least, we assume they are linearly ordered. If degrees are linearly ordered, every degree is comparable with every other. But degrees are not linearly ordered, and there really are incomparable degrees.

The evidence that there are incomparable degrees is this. I have already assumed that, if *x* is redder than *y*, `*x* is red' is truer than `*y* is red'. We know already that in many cases it is indeterminate which of *x* and *y* is redder. In those cases it must be indeterminate which of `*x* is red' and `*y* is red' is truer. So, because comparatives can be indeterminate, degrees of truth can be incomparable. Given that, there is no reason why it should not be indeterminate which of `This point is *F*er than the standard' and `The standard is *F*er than this point' is truer. This gives us a type (c) of soft indeterminacy. I have illustrated it as well as I can in Figure 6, but the illustration is inadequate because I can find no way of showing the truth values of the two statements varying

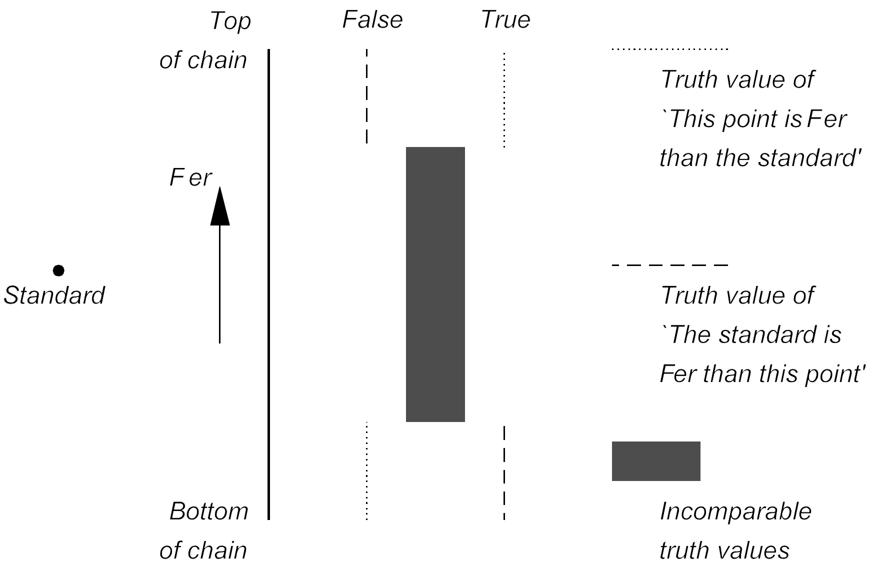

Figure 6. *Soft indeterminacy type (c)*

through the zone of indeterminacy.

Type (c) indeterminacy raises a more complex question. If it is indeterminate which of two statements – specifically, `This point is *Fer* than the standard' and `The standard is *F*er than this point' – is truer, just what sort of indeterminacy is this? This is a question about the structure of the comparative `truer than': does it have hard or soft indeterminacy in this application? I suggest it has soft indeterminacy, on grounds of uniformity. `*x* if *F*er than *y*' implies `*x* is *F*' is truer than `*y* is *F*'. So if `*F*er than' is softly indeterminate, `truer than' must be softly indeterminate between statements of the form `*x* is *F*'. My suggestion is that it is also softly indeterminate between statements of the form `*x* is *F*er than *y*' (such as, `This point is *F*er than the standard') or equivalently between statements of the form ``*x* is *F*' is truer than `*y* is *F*''. I also suggest it is softly indeterminate between statements at the next level of complexity: statements of the form \`\`\`*x* is *F*' is truer than \`*y* is *F*'' is truer than \`*y* is *F*' is truer than \`*x* is *F*''. In fact, I suggest we find only soft indeterminacy as we iterate to infinity.[4] My reason is that I would expect \`truer than' to have a uniform structure wherever it is used.

Type (c) soft indeterminacy satisfies our intuitions to some extent, but not completely. One intuition is that, as we move up through the zone of indeterminacy, it becomes progressively more true that the points we encounter are *F*er than the standard, and progressively less true that the standard is *F*er than them. Type (c) indeterminacy reconciles this idea with the collapsing principle. But another intuition is that, as we get near the top of the zone, it becomes nearly true that the points we encounter are *F*er than the standard, and nearly false that the standard is *F*er than them. I think this really is inconsistent with the collapsing principle. If one statement is nearly true and another nearly false, it is surely undeniable that the first is truer than the second. Other degrees of truth might be incomparable, but the degrees \`nearly true' and \`nearly false' are plainly comparable. So the intuition suggests that, for points near the top of the zone, it is truer that they are *F*er than the standard than that the standard is *F*er than them. This drags in the collapsing principle in the usual way, and leads to a contradiction. So, even in type (c) indeterminacy, at the boundary of the zone of indeterminacy, \`This point is *F*er than the standard' changes its truth value sharply. At the top boundary its value drops from true to some degree that is not nearly true.

8 Supervaluation

The most widely held theory of vagueness is supervaluation theory.[5] If we take this theory on board, what can we add to the analysis?

When a term is vague, there are various ways it could acceptably be made sharp. Suppose \`*F*er than' is vague – softly indeterminate that is – and in particular suppose it is vague whether *x* is *F*er than *y*. Then if \`*F*er than' was sharpened in one direction, *x* would be *F*er than *y*, and if it was sharpened in another direction, *y* would be *F*er than *x*. I shall confine the word \`sharpening' to sharpenings that are *complete*: they are not themselves vague, that is to say. According to supervaluation theory:

> *Supervaluation*. A statement containing a vague term is true if and only if it is true under all acceptable sharpenings of the term.

The sharpenings of a vague comparative \`*F*er than' will be comparatives. Since they are not themselves vague, they must either be fully determinate, or have hard indeterminacy. Which will it be? I cannot rule out either possibility on logical grounds. A vague comparative may have as its sharpenings both determinate comparatives and comparatives with hard indeterminacy. Alternatively, a vague comparative may have only determinate comparatives as its sharpenings. However, I can show that no vague comparative can have only comparatives with hard indeterminacy as its sharpenings; at least some of its sharpenings must be determinate.

Here is the demonstration. Suppose every sharpening of *F*er has hard indeterminacy. Then each sharpening has two \`switch points' as I shall call them. The lower switch point is where \`The standard is *F*er than this point' switches from true to false as we move up the chain; the upper switch point is where \`This point is *F*er than the standard' switches from false to true. For every sharpening there is a gap between these switch points. Now think about the top end of the zone of indeterminacy. Every sharpening must have its upper switch point within the zone. So every sharpening must have its lower switch point a definite distance below the top of the zone. Consequently, there will be some points within the zone that are above the lower switch point of every sharpening. (I shall ignore the mathematical possibility that there might be a sequence of sharpenings such that the distance between their switch points tends to zero; this is uninteresting.) For these points, every sharpening makes it false that the standard is *F*er than them. That is, every sharpening makes it true that the standard is not *F*er than them. Consequently, it is definitely true that the standard is *F*er than them. But in the zone of indeterminacy this is not so. So there is a contradiction.

Why does it matter what sort of indeterminacy the sharpenings have? If all the sharpenings of a softly indeterminate comparative were determinate, we could draw a striking conclusion. In every sharpening of `*F*er than', it would be true of every point that either it is *F*er than the standard, or the standard is *F*er than it, or that it and the standard are equally *F*. Since this would be true of every point in every sharpening, according to supervaluation theory it would simply be true of every point. Even when `*F*er than' is softly indeterminate, it would nevertheless be the case for every point that either it is *F*er than the standard, or the standard is *F*er than it, or it and the standard are equally *F*. This may seem puzzling, since for any point in the zone of indeterminacy, we know already it is not true that the point is *F*er than the standard, and it is not true that the standard is *F*er than the point, and it is not true that the point and the standard are equally *F*. Since none of these three things is true, how can their disjunction be true? The answer is: in the same way as the law of the excluded middle is true according to supervaluation theory. If F is vague, then for some x it may not be true that x is F nor true that x is not F. Nevertheless, according to supervaluation theory, the disjunction is true: x is either F or not F, since this disjunction is true in every sharpening.[6]

So, if every sharpening of a softly indeterminate comparative were determinate, the contrast between hard determinacy and soft determinacy would be particularly stark. In hard indeterminacy, no point in the zone of indeterminacy is *F*er than the standard, and nor is the standard *F*er than it, and nor are it and the standard equally *F*. In soft indeterminacy, for every point, either it is *F*er than the standard, or the standard is *F*er than it, or it and the standard are equally *F*.

Now, what version of soft indeterminacy does supervaluation theory support? Many supervaluationists refuse to recognize degrees of truth,[7] and that suggests they would adopt type (a) as their version. But I propose a small addition to the theory that I think should be readily accepted. I propose:

> *Greatervaluation*. One statement P, containing a vague term, is truer than another Q if P is true in every sharpening that makes Q true, and also true in some sharpening that does not make Q true.

When people dislike the idea of degrees of truth, I think that is generally because they assume degrees must be linearly ordered. Greatervaluation does not imply that.

I only offer greatervaluation as a sufficient condition for P to be truer than Q. It is not a necessary condition, as this example shows. Compare a reddish-orange patch with a pure orange patch. The former is redder than the latter. Consequently, it is more true that the former is red than that the latter is red. However, there is no sharpening of the language in which it is true that the former is red.

Now, take a pair of points, one higher in the zone of indeterminacy than the other. Any sharpening that makes the lower point *F*er than the standard will also make the higher point *F*er than the standard, but there will be some sharpenings that make the higher point *F*er than the standard and the lower point not *F*er than the standard. So according to greatervaluation, it is more true that the higher point is *F*er than the standard than that the lower point is. Similarly, it is more true that the standard is *F*er than the lower point than that it is *F*er than the higher point. As we move up through the zone, then, it becomes progressively more true that the points we encounter are *F*er than the standard, and progressively less true than the standard is *F*er than them. This implies type (c) indeterminacy, so greatervaluation supports this type.

9 Ordinary comparatives are softly indeterminate

I have now laid out the possible structures a comparative might have. What structures do we find amongst our ordinary comparatives? I particularly want to ask what is the structure of `better than'.

Without doubt, there are comparatives that have no indeterminacy. `Heavier than' is one.

Are there comparatives with hard indeterminacy? A comparative with hard indeterminacy is a strict partial ordering without vagueness. There are plenty of such orderings around; the question is whether any of them are the comparatives of monadic predicates.

It may be possible to construe some of them artificially as comparatives, by deriving monadic predicates from them. For instance, let us say that one object is mucheavier than another if and only if it is more than one kilo heavier than the other. Then the relation `mucheavier than' is a strict partial ordering without vagueness. Let us say it is the comparative of a monadic predicate `mucheavy'. Then `mucheavy' is a predicate whose comparative has hard indeterminacy. However, I am not convinced that the predicate `mucheavy' really exists. The only property of objects that is involved in the relation `mucheavier than' is their heaviness. The objects have no separate property that a predicate `mucheavy' could refer to, and that suggests there is no such predicate. I do not think `mucheavier than' is the comparative of any predicate; instead it is a fragment of the genuine comparative `heavier than', the comparative of `heavy'.

Or take another example of a strict partial ordering without vagueness. Let us say that one option is `Pareto-better' than another if and only if it is better for someone and not worse for anyone. Pareto-betterness defined this way is a strict partial ordering, and for the sake of argument let us assume it is not vague. Could we define a monadic predicate `Pareto-good' to have `Pareto-better than' as its comparative? Once again, I am doubtful. The only property of options that seems to be in play is their goodness. Pareto-betterness is best understood as a partial, sufficient criterion for `better than', the comparative of goodness, rather than as itself the complete comparative of anything. There are people who think that Pareto-betterness is actually a *complete* criterion for betterness, so that one option is better than another if and only if it is better for someone and not worse for anyone. These people do not try to construct an artificial predicate out of the relation `Pareto-better than'. Instead they think that `better than', the comparative of the natural predicate `good', happens to be extensionally equivalent to `Pareto-better than'. If they were right, `better than' would have hard indeterminacy, but I shall say in Section 10 why I think are wrong.

These examples suggest to me that constructing artificial comparatives with hard indeterminacy may not be as easy as it seems. But I do not insist they cannot be constructed. Perhaps they can. I am interested in our ordinary, natural comparatives. Do any of them have hard indeterminacy?

I find it implausible that any do. My reasons are these. First, the difficulty of finding artificial comparatives with hard indeterminacy suggests to me there is something fishy about hard indeterminacy in general. A comparative with hard indeterminacy is incomplete, and that suggests it may be a fragment of a complete comparative rather than a comparative in its own right. Second and more important, I showed in Section 4 that if a comparative has hard indeterminacy it cannot be vague. I find it implausible that indeterminate comparatives are not vague at all.

If a comparative has hard indeterminacy, then as we move down from the top of a chain in a standard configuration, there comes a point where `This point is *F*er than the standard' suddenly changes from true to false. So the monadic predicate `*F*er than the standard' is sharp and not vague. I find this implausible for indeterminate comparatives. Why? I do not think it is as obvious as the vagueness of many predicates. Take `red', for instance. If we move down a continuous chain of colour-patches from pure red to yellow, it seems obvious there is no point in the chain where the patches suddenly cease to be red. Therefore, `red' is vague. But a predicate of the form `*F*er than the standard' cannot be quite so obviously vague, because it will not be vague everywhere. Taker `redder than the standard', for instance. Take the chain of colours from red to yellow, and for a moment let the standard be a patch that exactly matches some point in the chain. Then higher points are redder than the standard and lower points are not redder than the standard, and the transition is sharp. Since there is a sharp transition for certain standards, it is not perfectly obvious that there is no sharp transition for other standards.

Notice first about this example that it has no indeterminacy. The point in the chain that exactly matches the standard is equally as red as the standard. Higher points are redder than the standard, and the standard is redder than lower points. So this is not an example of hard indeterminacy. Moreover, I am sure there is no sharp transition when the standard is, say, reddish purple, and does not match a point in the chain. The reason is that, if there is a sharp transition, I cannot tell where it is. I know no way of detecting where it is, and I know of no one who can detect where it is. If there is a point of transition, it is undetectable, and I do not believe redness can have an undetectable boundary of this sort. The same goes for every other natural indeterminate comparative I can think of, including evaluative ones.

I think ordinary indeterminate comparatives are softly indeterminate. Admittedly, even soft indeterminacy has a sharp boundary of a sort. There is a sharp boundary between points where `This point is *F*er than the standard' is true and points where it is not true. I find it hard to believe in a sharp transition of this sort. It is the collapsing principle that demands it, and this implication of the principle is paradoxical. The principle implies there are sharp boundaries where there seem to be none. I am recommending soft rather than hard indeterminacy as the lesser of two evils. It certainly is the lesser evil. In soft indeterminacy the sharp transition is between points where `This point is *F*er than the standard' is true and points where it is not true. In hard indeterminacy it is between points where this statement is true and points where it is false. So the transition in soft indeterminacy is not so abrupt. Furthermore, in type (c) soft indeterminacy, `This point is *F*er than the standard' becomes less and less true as we move down through the zone of indeterminacy. This makes the transition even softer. In this respect type (c) is the least paradoxical type of soft indeterminacy.

10 Other views

Many authors have described indeterminacy in a way that implies or suggests it is hard. Remember that in Section 4 I defined a comparative `*F*er than' to have hard indeterminacy if and only if there is more than one point in a chain such that it is not *F*er than the standard and the standard is not *F*er than it. I explained in Section 4 that none of these points can be equally as *F* as the standard, and I explained in Section 3 that if there is only one point in a chain such that it is not *F*er than the standard and the standard is not *F*er than it, then it will be equally as *F* as the standard. So `*F*er than' has hard indeterminacy if and only if there is a point in the chain such that it is not *F*er than the standard, and the standard is not *F*er than it, and it is not equally as *F* as the standard. But most authors take it for granted that indeterminate comparatives will satisfy a condition like this. For instance, Christopher Peacocke argues that `redder than' is indeterminate, and says of two colour-patches that `neither is red to a greater degree than the other, nor are they equally red'.[8] Joseph Raz says that when *A* and *B* are incommensurable it is generally `false that of *A* and *B* either one is better than the other or they are of equal value'.[9] When goods are incommensurable, I myself have said `neither alternative will be better for the person and they will not be equally good for her either'.[10] I could find many more examples. All these authors seem to imply indeterminacy is hard.

But I doubt if many of them are seriously attached to this view. For one thing, there is a way of reading most of their statements that makes them consistent with soft indeterminacy. When Peacocke says that neither object is red to a greater degree than the other, and nor are they equally red, I read him as meaning it is false that either object is red to a greater degree than the other and false that they are equally red. But he could also be read as saying it is not true that either object is red to a greater degree than the other, and not true that they are equally red. This allows the possibility that these statements are neither true nor false, which is consistent with soft indeterminacy. (Raz, though, deliberately rules out this alternative reading.) More important, these authors have not realized that hard indeterminacy in a comparative is incompatible with vagueness. I am sure that, once they realize it, they will have no inclination to believe in hard indeterminacy any more. I do not expect much controversy about my claim that indeterminacy is normally soft.

There is a tradition in economics – inspired by Amartya Sen – of representing certain comparatives by means of strict partial orderings without vagueness.[11] For instance, Sen himself uses a strict partial ordering to represent `more unequal than' (applied to distributions of income),[12] and in another place to represent `socially preferred to', which I take to mean simply `better than'.[13] This seems to imply hard indeterminacy. But actually I do not think Sen necessarily has hard indeterminacy in mind. Let us look at one of his arguments.[14]

Suppose we have to compare various alternative arrangements for society, where some are better for some people and others better for others. To judge which are better than which, we have to weigh some people's good against others'. Suppose we cannot do this weighing precisely, but we can do it roughly. There is a range of weights we might give to each person; we should use weights from this range, but it is not precisely determined which. If, for each person, we picked one weight from her range, so we had a set of weights, one for each person, that set would determine a complete ordering of the options. If we picked a different set of weights, that would determine a different complete ordering. Suppose we go through all the combinations of possible weights; each will determine a complete ordering. Now let us say one option is `clearly above' another if and only if it is above the other in all these orderings. `Clearly above' is then a strict partial ordering without vagueness, and Sen takes it to represent betterness. Does this mean he thinks `better than' is not vague?

I can see no evidence for that. Sen says one option is better than another if it is clearly above the other, but I doubt he would insist one option is not better than another if it is not clearly above the other. If neither of two options is clearly above the other, he does not tell us his views about their relative goodness.

If he had to tell us, it would be most natural for him to adopt soft indeterminacy. His explanation of how betterness comes to be a partial ordering is close to supervaluation theory. When applied to alternative arrangements for society, we can say that `better than' is vague because it is indeterminate how we should weigh together the good of different people. Within the appropriate range, each set of weights constitutes a sharpening of `better than'. Supervaluation says it is true that one option is better than another if and only if it is true in all sharpenings. Apparently Sen agrees. Supervaluation also says it is true that one option is *not* better than another if and only if *that* is true in all sharpenings. Sen does not state a necessary condition for one option not to be better than another, but it would be natural for him to agree to this too. If he did, he would have adopted soft indeterminacy. When Sen says betterness is a partial order, I do not think that is meant to be a complete account of betterness. If the account were consistently completed, it would amount to soft indeterminacy.

But what of the hard-liner who believes one person's good can never be weighed against another's to determine what is better overall? Suppose this person thinks one option is better than another if and only if it is better for someone and no worse for anyone. Between any pair of options that are not related in this way, suppose she thinks it false that either is better than the other or that they are equally good. She thinks the relation I called earlier `Pareto-better than' is actually the whole of `better than', the comparative of `good'. It is a strict partial ordering, and let us suppose once again it is not vague. This person unequivocally thinks that `better than' has hard indeterminacy.

My answer to this person is that her position is too implausible to be correct. I could understand someone who thinks there is no such thing as goodness, viewed from a neutral perspective, so that no option could ever be better than another. I mean: plain better, rather than, say, better for a particular person. But our hard-liner recognizes the existence of goodness; she simply thinks it can never be determined by weighing one person's good against another's. Now imagine some piece of good fortune could befall either of two people. Suppose neither of them has any particular entitlement to it. One would scarcely benefit from it at all because she has already received her fill of good fortune. The other would benefit tremendously; the good fortune would lift her from grinding poverty to a comfortable and enjoyable life. Would it be better for the first to receive the good fortune or the second? If you do not believe in goodness, you will think this question meaningless. But if you recognise goodness, you cannot plausibly deny the question has an answer: the second person. So, if goodness exists, then sometimes it can be determined by weighing together the good of different people. Once you acknowledge that, you will have to recognize borderline cases where it is indeterminate precisely how different people's good should be weighed. You will have to acknowledge vagueness in `better than', and that makes it softly indeterminate.

11 Conclusion

So indeterminate comparatives are softly indeterminate, and that includes `better than'. What conclusions can we draw? I shall talk about betterness only.

One is that the commonest formulations of indeterminateness are incorrect or at least misleading. When it is indeterminate which of two things is better, people commonly say that, of two options, neither is better than the other and they are not equally good either. This suggests it is false that either is better than the other, and false that they are equally good. But in a standard configuration, when a point in the zone of indeterminacy is compared with the standard, it is not false that it is better than the standard, and it is not false that the standard is better than it. Indeed, there are grounds for saying that either one is better than the other, or they are equally good. These grounds come from supervaluation theory.

In Section 8, where I introduced supervaluation, I showed that some of the sharpenings of a particular comparative must be fully determinate. But I could not rule out on logical grounds the possibility that some of them might have hard indeterminacy. But by now I think I can plausibly rule it out on other grounds. I have not found any indubitable cases of hard indeterminacy even amongst artificial comparatives, and I very much doubt any exist amongst natural comparatives. So it seems reasonable to doubt they exist amongst the sharpenings of vague comparatives. If so, all sharpenings of `better than' must be fully determinate. In that case, I explained in Section 8 that, for every point in a chain, either it is better than the standard, or the standard is better than it, or it is equally as good as the standard.

When it is indeterminate which of two things is better, their goodness is in a sense incomparable. But the conclusion I have just drawn makes it clear that in other senses it is not incomparable. Even if you reject this conclusion of supervaluation theory, just recognizing that soft indeterminacy is a sort of vagueness should make you recognize a sort of comparability. When it is indeterminate which of two things is better, it is not true that one is better than the other, but it is also not false. Furthermore, if the indeterminacy is type (c), then it is true to some degree that one is better than the other. And if one of the things improves a little, it will then be more true that it is better than the other. All these are facts about the things' comparative goodness.

Notes

1. This corresponds to condition C2 on page 10 of Adam Morton's `Hypercomparatives'.
2. See Morton, `Comparatives and degrees'.
3. `Hypercomparatives'. Morton himself deals with the degrees to which predicates are satisfied, but not with degrees of *truth*.
4. Morton assumes we shall eventually reach a level where the indeterminacy is hard, but I am not convinced by his arguments. I think we may reach a level where there is no indeterminacy, however.
5. See Dummett, `Wang's paradox', and Fine, `Vagueness, truth and logic'.
6. See Fine, `Vagueness, truth and logic', pp. 284–5.
7. But see Williamson, *Vagueness*, pp. 154–6, and the references there.
8. `Are vague predicates incoherent?', p. 135.
9. `Value incommensurability', p. 119.
10. *Weighing Goods*, pp. 136–7.
11. Another example is Charles Blackorby's `Degrees of cardinality and aggregate partial orderings'.
12. *On Economic Inequality*, Chapter 3.
13. *Collective Choice and Social Welfare*, Chapters 7 and 7*.
14. The one from *Collective Choice and Social Welfare*.

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