THE MOST FAMOUS EQUATION

Dan Bruiger, May 2023

Abstract: The mass-energy relationship does not derive logically from Special Relativity, since it is not a function of relative velocity. It might be a function of acceleration. 'Mass' is re-examined as a stand-alone concept.

I. Mass

Newton had based his assumption of absolute space principally on his "bucket experiment," which made it clear that motion involving acceleration (e.g., centrifugal force) is not merely relative. That is, it makes a real difference whether the water is turning inside the bucket or the bucket is turning around the water. Later, Mach suggested that what makes this difference is all the other matter in the universe. If the water is not spinning relative to the stars, it will not be displaced up the side of the bucket through centrifugal force. According to Mach, an objective, preferred frame of reference is supplied by the distribution of mass in the universe. Presumably, that could change with time and could vary with local density and therefore with direction in space. If masses are not distributed evenly, then the inertia of a moving object would depend on its location. Furthermore, since any frame of reference is necessarily material, its own gravitation would influence the motion it is supposed to measure and would contribute to the measured inertia. Since there is no location completely free of forces, the very definition of an inertial system is at risk. An object is unaccelerated if no force acts on it; but we can know that no force acts on it only because we see that it is unaccelerated.

As a measure of the objective "amount of matter," mass should not depend on the observer's state of motion. But perception is relative to the observer in the absence of an absolute point of view. Mass can be measured locally, for example by weighing with a balance or spring scale, or alternatively by accelerating with a spring scale. But it can only be determined from a distance through inferences based on the interactions of the object with other things, which involve perceived changes in its motion. The motion observed *does* depend on the state of motion of the observer, and thus the perceived mass also depends on it.

As a key factor in dynamics, mass is always paired with a variable of motion.[1] (Contemporaries of Newton had criticized the circularity of the definitions of force and mass, which applies to momentum and kinetic energy as well.) Like momentum, kinetic energy is a conjoint effect of mass with changing position, and the latter depends on the observer's state of motion. In effect, mass serves as a coefficient of velocity in momentum, of acceleration in force, and of the cumulative result of acceleration in kinetic energy. In all cases, what is actually measurable from a distance involves the product of the paired variables, not just mass per se. From a distance, inertial mass has meaning only as a coefficient of velocity or acceleration, since motion is what is actually observable from a distance. In other words, as a stand-alone concept, distant mass cannot have the same meaning as mass that is locally measurable.

II. The *Specialness* in Relativity

Einstein's 1905 paper[2] became known as the theory of Special Relativity (SR), though it is actually a theory of invariance. His overarching concern was a program to preserve the classical structure of physics, such that the laws of physics should be the same for all observers. SR involves the special

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[1] I.e., force = ma; momentum = mv; kinetic energy = $mv^2/2$.

[2] "On the Electrodynamics of Moving Bodies"

case when observers must contend, by means of the Lorentz transformations, with constant relative motion. The general case extended to include acceleration—that is, non-inertial systems.

For inertial systems, relativistic effects are symmetric and mutual in the line of sight. Like length contraction and time dilation, all relativistic effects in SR must be mutual between uniformly moving frames. This includes *increase of inertia or mass*.[3] Like length contraction and time dilation, the relativistic increase of inertial mass with speed is a symmetric effect between observers: each perceives the other's inertia or momentum to have increased, since they are functions of velocity as well as mass. Hence, in a situation such as a particle in an accelerator, the relativistic increase of mass in SR cannot be an ontologically "real" or "objective" change in what is asymmetrically considered the moving object. The motion between particle and observer (i.e., the accelerator) is relative and mutual, when constant velocity is concerned. However, of course, no observer occupies the framework of the particle. The observer who uses a linear accelerator to measure speeding particles may claim to occupy the rest frame and that the *particle* increases in mass; but there is no observer claiming the point of view of the moving particle, from which the accelerator symmetrically also must appear to gain in mass![4] If there is objective (that is, asymmetric) increase of inertial mass of particles in high-energy experiments, it must be due to acceleration rather than uniform velocity.[5]

III. Mass-energy equivalence

The relativistic increase of mass with linear velocity in SR (which supplied the reasoning for Einstein's derivation of $E=mc^2$) is not the same phenomenon as the conversion of internal energy to external energy (or vice-versa), with which it is often confused. Relativistic increase of mass due to velocity is a mutual epistemic effect that should not be confused with the equivalence of mass and energy expressed in "the most famous equation." They *are* often conflated, however, perhaps because Einstein used SR to argue for the equivalence.[6] Yet, he himself was never satisfied with his several proofs of the formula, which were never without problems pointed out by critics.[7] In proposing mass-energy equivalence, he was speculating intuitively about the internal energy of the atom. Little was known at the time about the atom's internal dynamics, nor was much thought given to potential logical inconsistencies in the basic concepts involved.[8] The formula quantifies the equivalence, but sheds no light on the actual physical processes responsible in the "transformation" of mass to energy or vice-versa. The formula suggests (and Einstein hinted) that the *entire* mass of any object could be converted to energy, and that any form of energy could be converted into some

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[3] According to Einstein, such effects would also include temperature: "Thus, the temperature of a moving system is always lower… than with respect to a reference system that is at rest relative to it." [Doc47] This too should be mutually perceived. Two observers in constant relative motion should each perceive the other's temperature as lower, if it could be measured from a distance (for example spectroscopically).

[4] While the accelerator could theoretically be weighed, by definition the moving particle cannot.

[5] Non-linear motion in a cyclotron would be accelerated by definition, even if angular speed is constant.

[6] "Indeed, it is difficult to find a scientific equation whose ontological implications have been misunderstood so widely and in so many ways." [Marc Lange "The Most Famous Equation" The Journal of Philosophy, Vol. 98, No. 5 (May, 2001), pp. 219-238]

[7] Cf. Hecht, E. (2011) American Journal of Physics, 79, 591-600; also Moylan, P., Yan, L. X., & Gironda, M. (2021). "On the Controversy over the Logical Correctness of Einstein's First Paper on Mass-Energy Equivalence." Advances in Historical Studies, 10, 21-33.

[8] For example, if electrons have mass, and mass is internal (i.e., kinetic) energy, then electrons must have internal energy. Does that mean electrons are not "fundamental" but consist of moving parts with kinetic energy? And of what do *those* parts consist (ad infinitum)? Or, is there such a thing as "pure energy," some potential energy other than energy that is ultimately kinetic? *Are* there indivisible ("fundamental") particles, or are all composite? Such questions are modern versions of the ancient conundrum of the one and the many.

sort of object with mass. This unrealistic generalization ignores actual physical mechanisms for the (partial) release of energy, such as nuclear fission and fusion.

Einstein proposed a "rest energy" E_0 in the frame of the observer considered at rest.[9] Since that is by definition *not* kinetic energy of something moving as a whole with respect to an external frame, it must be its "internal energy," (whatever that would turn out to mean), or else a potential energy in some field.[10] The dependency of mass on internal energy is a profound truth, as is the idea that rest mass represents some form of internal energy—for example, vibration inside an atom. In contrast, *relativistic mass* is an observational effect of motion relative to the observer.[11]

A mere difference of perspective between two frames of reference is not the same as a true change of kinetic energy—by loss or gain of radiation, for example.[12] A change of perspective, from viewing the moving thing as a whole to considering its internally moving parts, is not a real change in the system. Einstein's argument does not in itself establish the connection between radiant energy and mechanical inertia.[13] The equivalence of mass and energy is an empirical fact verified in atomic physics; "internal energy" within the atom *is* converted to "external" kinetic energy of resulting particles of decay or else radiant energy. But this fact does not follow from SR as it is supposed to in Einstein's several papers on mass-energy equivalence.[14]

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[9] That is, "rest energy" and "rest mass" are equivalent for an object *at rest* in the observer's frame of reference. This is not the same situation as the apparent (increase of) mass from the point of view of frameworks in relative motion. Cf. L. B. Okun "The Einstein formula: $E_0 = mc^2$. 'Isn't the Lord laughing?'" arXiv: 0808.0437v1 {physics.hist-ph} Aug 2008.

[10] A problem with "energy" as a unified concept is that it disregards specific measures in differing contexts (for example, radiant vs kinetic vs potential).

[11] Cf. Carl G. Adler "Does mass really depend on velocity, dad?" Amer, J. of Physics 55 (8) Aug 1987, p740ff: "It is internal kinetic energy that counts toward inertia not (to paraphrase Einstein) *mere translational kinetic energy of the body as a whole…* Elsewhere Einstein states explicitly that the mass of a body is nothing else than the energy possessed by the body *as judged from a coordinate system moving with the body."* [i.e., at rest—(original italics)] Cf. also Kevin Brown "Einstein on the Inertia of Energy" https://www.mathpages.com/home/kmath600/kmath600.htm]: "the total energy E of a body consists of two parts, intrinsic and extrinsic. The intrinsic part of a body's energy arises from internal degrees of freedom, and does not depend on the speed of motion of the overall object, whereas the extrinsic part of a body's energy is the part that does depend on the overall motion of the body… Since, by definition, the internal energy of the object doesn't depend on the speed of the object, *it is the same regardless of which system of reference we use.*" (italics added) However, the latter point is debatable, since each moving component could be regarded separately as moving with respect to an observer, if it could be observed at all. Moreover, relativistic effects should vary according to the component of motion toward or away from the observer. Thus, relativistic mass should vary with actual direction of motion, as was initially considered in the distinction between 'longitudinal' and 'transverse' mass.

[12] The internal energy of a system affects its inertial mass as a whole and how it moves as a whole in relation to an external reference frame. But even potential energy is a quantity that refers ultimately to motion. Internal energy might be measured indirectly through temperature (which too refers ultimately to motion). Thus, the temperature of a moving system would appear lower to an external observer than to an observer in the moving system. But this effect too must be mutually perceived, in contrast to the increase of inertia that would result from heating an object, which increases its "internal" energy.

[13] Cf. Kevin Brown, *op cit*: "It's true that [Maxwell's] equations already imply the relation $E = pc$, where E is the energy and p is the momentum of an electromagnetic wave, and hence if we insert the classical definition of momentum $p = mc$ we get $E = mc^2$ (as had already been noted previously by others, such as Poincare and Thompson), but this doesn't really establish any connection between radiant energy and mechanical inertia." To insert classical momentum for electromagnetic momentum already assumes the equivalence the formula is supposed to show.

[14] Hecht (2011) op cit. Einstein shows that the initial and final energies (after emission of radiant energy) differ *in the moving frame* by the amount $E/\sqrt{(1- v^2/c^2)}$, which he claims is an objective (non-symmetric) fact. However, in SR this is logically a symmetrical mutual effect, claimed equally by an observer in either frame of reference.

IV. Why does *c* appear in the mass-energy equivalence?

From an epistemic point of view, neither mass nor energy are *substances* that can be "inter-converted"; rather they are *measures* of observable interactions. Perhaps these measures are "complementary" in some sense (e.g. as quantum variables are, or as mental and physical descriptions are). In any case, one is entitled to wonder why c^2 appears as the conversion factor between mass and energy. Why does c appear at all in $E=mc^2$, and why is it *squared*?

It makes sense that expressions of the mass-energy relationship derived from electromagnetic theory would involve c. But electromagnetic mass is neither inertial or gravitational mass. In terms of concepts of mass, electrodynamics remains distinct from classical dynamics. Nor does E in the equation explicitly represent kinetic energy in particular. While the parallel with the formula for kinetic energy ($K = mv^2/2$) is suggestive, the mass-energy relationship seems to derive from a consideration of the speed of transmission of disturbances (waves) in material media—applied, in this case, to electromagnetic waves in the ether. The square of that speed is equal to the ratio of the elastic constant of the medium to its density. That in turn derives from equations describing the periodic motion of oscillators.[15]

V. Conclusion

Einstein derived matter-energy equivalence with arguments based on SR. These arguments slide tacitly from an epistemic interpretation to an ontological one. While the formula $E_0=mc^2$ may be accurate, its theoretical dependence on SR remains dubious, at least if the clock hypothesis is true.[16] Or, to put it differently, the mass-energy relationship does not derive logically from SR, since it is not a function of relative velocity. It might be a function of acceleration, which suggests not a mutual effect (as in SR) but an interaction of moving matter with some objective field or ether. While much has been written about the evolving concepts of mass and energy, the confusions around the derivation of "the most famous equation" suggest that these fundamental concepts can still profitably be explored from an epistemic point of view.

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[15] See: Max Born *Einstein's Theory of /Relativity* Dover, 1962, p114-15 and p185.

[16] The *clock hypothesis* is the assumption that the rate at which a clock is affected by time dilation does not depend on its acceleration but only on its instantaneous velocity. [Wikipedia: time dilation]