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TRINITY AND CONSISTENCY

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P. T. Geach has argued that it is impossible to demonstrate that the doctrine of the Trinity is consistent. I try to show why—on a common understanding of the notion of consistency—his reasoning is flawed and why, on Geach's own principles, one should expect that if the doctrine of the Trinity is true then it will be possible to prove that the doctrine is consistent, and it will be possible to do this in a way that makes no appeal to the truth of any religious doctrine.

> Of course I did not and do not claim to understand the inner life of God; but the doctrine of the Trinity, as Newman pointed out, can be stated in a small number of theses, each of which can be grasped and held with real assent. Proving that this is a consistent set of theses is quite another thing, and I made no claims to have such a proof.… A general proof of consistency, which would at once enable one to dismiss all such attacks, is certainly not going to be available to mortal man. If the propositions in which the doctrine of the Trinity is formulated could be proved to be a consistent set, then a fortiori each one would have been proved possibly true. But as regards the inner life and mutual relations of the Divine Persons, there is no difference between possible and necessary truth.… Since in this realm possible and necessary truth coincide, proving the possible truth of Trinitarian theses would mean proving their truth, which is certainly not possible to mortal man.—Peter Geach[1]

If the reasoning in this passage is correct, Geach has established a very important claim in philosophical theology, viz., that there can be no proof that the doctrine of the Trinity is consistent. In the first section below I will try to show that the reasoning in this passage is faulty—or, at least that it is so on a very common understanding of the notion of consistency.

The following section will discuss a method that one might try to use to demonstrate the consistency of the doctrine of the Trinity. As we shall see Geach's own principles seem to dictate that if the doctrine of the Trinity is true then a consistency proof—one which makes no appeal to the truth of any religious doctrines—is possible. The final section will reevaluate the above argument using alternative accounts of consistency.

Section One: Geach's argument criticized

Geach's argument relies on the following two assumptions, which will be my focus.

(1) If a set of propositions is consistent then each proposition in the set is possibly true.[2]

(2) If a proposition concerns the inner life and mutual relations of Divine Persons, then it is possibly true if and only if it is necessarily true.

In presenting his argument Geach does not explicitly define the notion of consistency. Typically in logic a set of propositions is said to be consistent just in case it is not possible to deduce from it an explicit contradiction, i.e., a proposition of the form (P & ~P). For the time being I will employ this notion of consistency. (Later I will consider the argument using alternative notions of consistency.) In terms of assumptions (1) and (2), Geach's argument can be stated as follows. If the doctrine of the Trinity is consistent, then by (1) each proposition in the doctrine is possibly true, and thus by (2) each proposition in the doctrine is necessarily true. But propositions that are necessarily true are true. Given this reasoning we could convert a proof of the consistency of the doctrine of the Trinity into a proof of the truth of the doctrine. But, Geach holds, the latter is impossible for mortal man.[3] It follows that mortal man cannot prove the consistency of the doctrine.

For this argument to work (1) and (2) must be true on a single fixed reading of the notion of possibility. I will claim that they are not. But first I want to consider a particular reading that might make the argument look plausible and explain why the argument fails on that reading. Suppose that (1) and (2) are read as involving metaphysical possibility. A proposition expresses a metaphysical possibility if it describes a way things could have been.[4] When it comes to the inner life and mutual relations of Divine Persons, it might seem that there is only one way things could have been; there are no contingent matters in this realm (even if there exists no God at all, on this line of thought, that is not a contingent matter). If this is correct, then in this realm metaphysical possibility and metaphysical necessity coincide, and thus (2) holds. It may also seem that (1) holds when read as concerning metaphysical possibility, but that is not the case: consistency does not guarantee metaphysical possibility. A couple of examples should help to make this clear. One could hold a consistent theory that claims that, say, gold is a compound of aluminum and phosphorus. The theory would of course contain a falsehood, but it need not lead to a contradiction. The fact that "Gold is a compound of aluminum and phosphorus" is a consistent proposition does not show that it expresses a metaphysically possibility. In fact it expresses a metaphysically impossibility: any compound of aluminum and phosphorus, no matter how much it resembled gold, just would not be gold. For a second example consider arithmetic (the theory of the natural numbers under the operations of successor, addition and multiplication). In the realm of arithmetic there is "no difference between possible and necessary truth"; there are no contingent truths of arithmetic. At least this is the case if "possible" and "necessary" are understood as expressing metaphysical possibility and necessity. Note however that the consistency of an arithmetical proposition does not show that it is possibly true in this sense. From Gödel's incompleteness theorem it follows that for any consistent, decidable set S of true first order sentences in the language of arithmetic there will be some sentence A in the language of arithmetic such that neither A nor ~A is deducible from S.[5] From this it follows that both A and ~A are consistent with S, and thus also consistent on their own. Yet one of the pair will express a metaphysically necessary truth of arithmetic, and the other will express a metaphysical impossibility.

So Geach's argument fails if it is understood to involve metaphysical possibility and necessity. Might another notion of possibility be found on which (1) and (2) hold true together? I think that under plausible assumptions concerning validity and the logical structure of simple subject-predicate propositions, and under the assumption (which I shall make for the course of the argument) that the doctrine of the Trinity is true, it is not difficult to see that the answer is negative. If (1) and (2) hold on a fixed reading of possibility they will take us from the consistency of a proposition about the inner life and mutual relations of the Divine Persons to the necessary truth of that proposition. So to show that (1) and (2) cannot hold on a single reading of possibility it will be enough to show that there is a consistent proposition (that is, a proposition from which an explicit contradiction cannot be deduced) that concerns this subject matter and is false. To do this I will add a single false proposition to the doctrine of the Trinity and show that from this proposition there cannot be a deduction of a proposition of the form (P & ~P). I will proceed in stages. First I will add a partially interpreted sentence to the doctrine and consider two ways of completing the interpretation of the sentence. One complete interpretation of the sentence will yield a true proposition concerning the inner life of God, and the other will yield a false proposition concerning the inner life of God. It will be argued that if an explicit contradiction could be derived from the false proposition, then an explicit contradiction could also, per impossible, be derived from the true proposition. Hence the false proposition must be consistent.

Consider the partially interpreted sentence "a is unbegotten", where "a" is an uninterpreted proper name. On the first (complete) interpretation let "a" be a name referring to the first person of the Trinity; on this interpretation "a is unbegotten" is a true proposition.[6] On the second interpretation let "a" be a name referring to the second person of the Trinity; here we have a false proposition. Suppose that there is a deduction of an explicit contradiction (P & ~P) from the proposition "a is unbegotten" (second interpretation). Thus the argument from "a is unbegotten" (second interpretation) to (P & ~P) is valid. Then, unless there is a hidden difference between the logical structure of the proposition "a is unbegotten" (second interpretation) and the proposition "a is unbegotten" (first interpretation), the corresponding argument (under the first interpretation) from "a is unbegotten" to (P & ~P) will also be valid.[7] Of course if a proper name were (in Russell's terminology) a disguised description, or if a proper name somehow stood in for a cluster of descriptions, there could be significantly different logical structures associated with "a is unbegotten" under the two interpretations, and this could result in there being a deduction of an explicit contradiction from one but not the other. For example, suppose that proper names are really disguised descriptions. If "a" was short for "the begotten Divine Person" then from "a is unbegotten" we could deduce the contradiction "a is begotten and a is not begotten". But if "a" was short for "the unbegotten Divine Person who begets another" then from "a is unbegotten" we could not deduce the contradiction "a is begotten and a is not begotten"

I will assume however that it is incorrect to treat proper names as disguised descriptions or disguised clusters of descriptions, and that the criticisms Kripke raises against these views are substantially correct.[8] Furthermore I contend that on the most plausible account of proper names there is no relevant difference in the logical structure of "a is unbegotten" under the two interpretations that would allow the deduction of a sentence of the form "P & ~P" in one case but not allow a formally parallel deduction of a sentence of the form "P & ~P" in the other case. On what is perhaps the most common account of proper names, a proper name is held to be logically simple—it has no associated logically complex structure. On such an account a deduction of an explicit contradiction from "a is unbegotten" on one interpretation will allow a formally parallel deduction of an explicit contradiction on the other interpretation. Since "a is unbegotten" is true on the first interpretation, it follows that there can be no deduction of an explicit contradiction from "a is unbegotten" under either interpretation.

Geach of course has a theory of proper names that is at odds with the one just sketched. It might be helpful to see how this example would be handled using Geach's theory. Unlike Kripke's account, in which proper names have no descriptive content, Geach holds that a proper name carries as part of its sense a criterion of identity, where the criterion of identity determines what the name is a name for.[9] Note that Geach does not hold that the criterion of identity associated with a proper name uniquely singles out the reference of the name. For example, if "King" and "Fido" are names for dogs they are associated with the same criterion of identity (same dog) even if they differ in reference (i.e., they pick out different dogs). Geach holds that a proper name does not carry any further descriptive sense beyond supplying a criterion of identity. To handle this feature of Geach's account of proper names we add to our example the stipulation that both interpretations associate the name "a" with the criterion of identity for being the same divine person, otherwise the interpretations remain the same; i.e., on the first interpretation "a" refers to the first person of the Trinity and on the second interpretation "a" refers to the second person of the Trinity. On Geach's treatment of proper names there will not be a difference in the logical structure of "a is unbegotten" under the two interpretations that would allow the argument from "a is unbegotten" to (P & ~P) to be valid under one interpretation of "a" but not under the other. Thus my earlier argument still holds. If "a is unbegotten" is consistent under one reading it is also consistent under the other reading. But of course "a is unbegotten" is true—and thus consistent—under the first interpretation.

Thus we see that under a plausible account of proper names—either the account that treats proper names as logically simple or Geach's own account of proper names—there can be consistent yet false propositions about the "the inner life and mutual relations of Divine Persons". If that is correct then Geach fails in his attempt to argue that it is impossible for mortal man to prove the consistency of the doctrine of the Trinity. In my criticism I have relied on a particular reading of the notion of consistency. The question arises whether under an alternative reading of the notion of consistency Geach's argument is sound. I will return to this question in the final section, but for the moment I will remain focused on the notion of consistency with which we began in order to reassess the question of whether (on this understanding of consistency) it is possible to give a proof of the consistency of the doctrine of the Trinity.

Section Two: Is a general proof of consistency possible?

Surprisingly, some of Geach's other claims would seem to imply that there can be a "<u>general</u> proof of consistency" for the doctrine of the Trinity. As Geach notes, the doctrine of the Trinity can be stated in a small number of theses. Let $p_1, \ldots, p_n$ abbreviate such a set of theses. One who thinks that the doctrine is inconsistent (and has studied a little logic) will then hold that the argument "$p_1, \ldots, p_{n-1}$, <u>ergo</u> not-(p_n)" is valid. Geach tells us:

> Truth cannot clash with truth. If the doctrines of the Faith are true, they can conflict neither with one another nor with truths in some other domain. An argument that purports to show such a conflict must thus be regarded as no proof but a fallacy; and any fallacy can in principle be exposed by producing some unexceptionable counterexample, an argument of the same form in which the premises are uncontroversially true and the conclusion is uncontroversially false. And the logic needed will just be ordinary logic, universally accessible and acceptable, not logic accessible only to 'baptised reason', whatever that may be.[10]

On Geach's account then there will be a counterexample to the argument "$p_1, \ldots, p_{n-1}$, <u>ergo</u> not-(p_n)". It will be an argument "$p_1^*, \ldots, p_{n-1}^*$, <u>ergo</u> not-(p_n^*)" which is "of the same form in which the premises are uncontroversially true an the conclusion is uncontroversially false." Of course in that case $p_1^*, \ldots, p_n^*$ will be of the same form as $p_1, \ldots, p_n$ (the theses that make up the doctrine of the Trinity) and each of the p_i^*'s will be uncontroversially true. This will provide a "<u>general</u> proof of consistency, which would at once enable one to dismiss all such attacks" on the consistency of the doctrine. Note that this is just how one would expect a defense of the consistency of the doctrine of the Trinity to go. One would produce a set of propositions that is formally parallel to the doctrine of the Trinity and in which each proposition is clearly true.[11] Peter van Inwagen in fact has a well known paper, based on Geach's treatment of relative identity, in which he basically attempts just such a proof of consistency.[12]

Section Three: Alternative concepts of consistency

In assessing Geach's argument for the claim that the consistency of the doctrine of the Trinity cannot be proven I have fixed upon a particular understanding of the notion of consistency—viz., a set of propositions is consistent provided that an explicit contradiction cannot be deduced from the set. Call this the logical notion of consistency and say that a set of propositions is logically consistent if it meets this criterion. Since Geach does not provide a definition of consistent set of propositions in the passage cited at the beginning of the paper, one might wonder whether (a) Geach may have used a nonequivalent notion of consistency, or (regardless of Geach's understanding of consistency) whether (b) Geach's argument could be reformulated using a different notion of consistency so that it is sound and significant under the alternative understanding of consistency. In this section I will take up these two concerns, starting with (a).

Geach has developed a broad and interesting account of consistency which has applicability to theoretical judgment, belief, command, counsel, and practical reason.[13] Our focus will be on the consistency of a set of propositions. In his essay "Why Logic Matters," Geach's explains propositional inconsistency as follows:

> … any inconsistency except the flat inconsistency of contradictories has itself to be explained in terms of valid inferences from a set of premises, leading both to a conclusion and its contradictory.[14]

This account of inconsistency appears to be equivalent to the one I have employed,[15] and it is typical of the way Geach has developed the notion of consistency over the years.[16] Furthermore, Geach introduces the version of his argument for the unprovability of the consistency of the doctrine of the Trinity as it appears in the article "Nominalism" (see note 1) with the words:

> Modern logicians like Church and Godel have shown what a severe requirement the demand for a general consistency proof is; as regards many theories, the demand is demonstrably unreasonable. And as regards the doctrine of the Trinity in particular we can see that a demand for a consistency proof could never be satisfied.

The notion of consistency employed by these modern logicians is equivalent to that of the nondeducibility (in a given system) of an explicit contradiction.[17] For these reasons I believe that I have given a fair representation of the notion of consistency as it appears in Geach's argument.[18]

But, as noted above, the question arises whether Geach's argument could be reformulated using a different notion of consistency, and if so whether such an argument might yield important insights concerning the doctrine of the Trinity. As a first attempt, one might be inclined to relax the notion of consistency and say that a set of propositions is <u>conceptually consistent</u> if the conjunction of its members is not analytically false.[19] I do not think that replacing the notion of consistency in the first premise by conceptual consistency will rescue Geach's argument. Consider again the two readings of "<u>a</u> is unbegotten" (on the first reading "<u>a</u>" refers to the first person of the Trinity, and on the second reading "<u>a</u>" refers to the second person of the Trinity). If proper names have no descriptive content (beyond possibly conveying a criterion of identity) then if either reading of "<u>a</u> is unbegotten" is not analytically false, then neither reading is analytically false. But on the first reading the proposition is true and thus not analytically false. Thus the second reading provides a false but conceptually consistent proposition about the inner life of God. So Geach's argument will not work on this reading of "consistency."

Another approach would be to interpret "consistent" to mean "jointly possible," so that a set of propositions is consistent if and only if it is possible for all the propositions in the set to come out true together. On such a reading the first premise will of course be trivially true.[20] Whether the second premise is true will depend on exactly what is meant by "possible". If by "possible" we mean "conceptually possible" (= not analytically false), then the second premise fails: in the earlier example "a is unbegotten" is not analytically false under the second reading, and yet it is not a necessary truth.

Another way to understand "consistent" to mean "jointly possible" is in terms of metaphysical possibility: a set of propositions is consistent in this sense if and only if there is some possible world in which all the propositions in the set are true together.[21] On this reading the first premise is trivially true, and the second premise makes the plausible claim that what is true in any possible world concerning "the inner life and mutual relations of Divine Persons" is also true in all possible worlds. So on this reading the argument purports to show that no human can prove that there is a possible world in which the doctrine of the Trinity is true (for such a proof could be converted into a proof of the doctrine of the Trinity is actually true).

I have no objection per se to this argument,[22] though I believe that its usage of the term "consistent" engenders confusion in two ways. First, it is simply a different notion of consistency than we normally use, either in everyday life or in philosophical discussion. To take an example, if someone held that water is really an element and that there are atoms (in the sense in which Democritus used the term) of water, then that person would hold a theory that is not true with respect to any possible world, and yet we would not have any basis for holding his theory to be inconsistent (as we normally use the term).

Second, discussions of the consistency of the doctrine of the Trinity are in general squarely concerned with the logical or conceptual notion of consistency. To many people the doctrine of the Trinity appears to imply contradictory propositions—e.g., that there is exactly one God and that there is not exactly one God but three Gods; or that the Father and the Son are the same and yet they are not the same. If there could be a general proof of the logical or conceptual consistency of the doctrine of the Trinity, that would of great importance to philosophical theology. This is simply a different question from whether we could prove that there is a metaphysically possible world in which the doctrine of the Trinity is true.

I will end with a final remark on the relationship between the concerns of modern logicians with consistency proofs and the question of whether there can be a proof of the consistency of the doctrine of the Trinity. Geach is of course quite right to point out that the work of modern logicians has "shown what a severe requirement the demand for a general consistency proof is; as regards many theories, the demand is demonstrably unreasonable." Philosophers should be cautious about demanding proofs of consistency. On the other hand it is generally recognized that, for example, the constraints on proving the consistency of a given formalization of arithmetic arise from peculiar features of the formalization involving its complexity. We have little reason to expect that the same troubles will arise with respect to demonstrating the consistency of the doctrine of the Trinity. As far as I can see, Geach's argument does not give us any further reason to think that the logical consistency of the doctrine of the Trinity cannot be proven.[23]

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NOTES

[1] Peter Geach, Truth and Hope (Notre Dame: University of Notre Dame Press, 2001), 41. Essentially the same argument is found in Geach, "Nominalism," Logic Matters (Berkeley: University of California Press, 1972), 293, which originally appeared in Sophia 3 (1964), 3-14; see p. 6 for the argument.

[2] In his writings Geach generally uses the term "proposition" (uncapitalized) to mean "a form of words in which something is propounded, put forward for consideration" (Logic Matters, 255; see also, Reference and Generality, Third Edition (Ithaca and London: Cornell University Press, 1980), 51) and not as a term to refer to abstract propositional content. Thus Geach uses "proposition" in roughly the way many 'analytic' philosophers now use the term "statement". Geach objects to the latter usage because in ordinary usage "statement" appears to imply assertoric force, whereas a proposition (in Geach's sense) or a statement (in the specialized sense often used by philosophers) can appear unasserted, e.g., as a clause in a conditional. Unfortunately Geach's terminology has not caught on among philosophers. In any case when Geach speaks of the "consistency of a set of propositions", this should be taken as equivalent to what many analytic philosophers mean by "consistency of a set of statements". Throughout this paper I will use the term "proposition" in conformity with Geach's usage.

[3] Though Geach does not tell us in this passage why mortal man cannot prove that the doctrine of the Trinity is true, elsewhere he discusses Aquinas' reason for holding that natural reason cannot inform us about the distinction of the Divine Persons; see P.T. Geach, The Virtues

(Cambridge: Cambridge University Press, 1977), 78-79. Briefly, it is because, except for the incarnation, the "external works and deeds of God are common to all three persons". This means that when we reason from effects in the world back to God as the cause we cannot make distinctions among the Divine Persons.

4. The expressions "metaphysical possibility" and "metaphysical necessity" come from Saul Kripke, Naming and Necessity (Cambridge, Massachusetts: Harvard University Press, 1980), which is based on Kripke's lectures given in 1970 at Princeton University. The clarification Kripke brought to modal concepts of course came well after the original appearance of Geach's argument in 1964 (see note 1).

5. I am simplifying somewhat to avoid bringing in too many technicalities.

6. Recall that we are using the term "proposition" in Geach's sense in which a proposition is "a form of words in which something is propounded".

7. The interpretation of "a" will differ in the two arguments. If "a" occurs in the conclusion then the conclusion will be differently interpreted in the two arguments. Nonetheless under either interpretation the conclusion will be an explicit contradiction.

8. For the purposes of my argument it is not actually necessary to accept Kripke's full criticism of accounts of proper names that treat proper names as disguised descriptions or clusters of descriptions. It is enough to hold that at least sometimes proper names are logically simple (they do not have a descriptive content as part of their sense), and that the name "a" as is occurs in my example is logically simple.

9. Reference and Generality, 68-71.

10. Truth and Hope, 40.

[11.] Even if the set of propositions formally parallel to those used to express the doctrine of the Trinity was merely consistent, that would still imply the consistency of the doctrine of the Trinity. In the passage quoted above Geach claims that "any fallacy can in principle be exposed by producing some unexceptionable counterexample, an argument of the same form in which the premises are uncontroversially true and the conclusion is uncontroversially false." Whether or not there will always be a counterexample with uncontroversially true premises and an uncontroversially false conclusion is an issue we need not pursue.

[12.] Though perhaps van Inwagen's examples are merely consistent (see the previous note); they could in any case easily be modified to make them actually true. See Peter van Inwagen, "And Yet They are Not Three Gods but One God" in Thomas V. Morris (ed.), Philosophy and the Christian Faith (Notre Dame: University of Notre Dame Press, 1988), 241-78. I do not know how far Geach would accept van Inwagen's formalization of his (Geach's) account of relative identity, nor can I discuss here whether van Inwagen succeeds. For Geach's own treatment of the doctrine of the Trinity in terms of his theory of relative identity see his "Aquinas" in G.E.M. Anscombe and P.T. Geach, Three Philosophers (Ithaca and London: Cornell University Press, 1961), 118-20. See also my “The Doctrine of the Trinity and the Logic of Relative Identity,” Religious Studies 25 (June 1989), 141-52. For recent criticisms and a survey of pertinent literature see Michael C. Rea, "Relative Identity and the Doctrine of the Trinity," Philosophia Christi (forthcoming).

[13.] See for example Chapter Three of Truth and Hope; Chapter Two of Reason and Argument, and "Imperative Inference," Analysis 23 (Supplement, January 1963), 37-42 (Reprinted in Logic Matters, 278-85).

[14.] "Why Logic Matters" in H. D. Lewis (ed.), Contemporary British Philosophy, 4th Series (London: Allen and Unwin, 1976), 98.

[15.] If from a set of propositions an explicit contradiction can be deduced then the set of propositions is certainly inconsistent according to Geach's criteria. On the other hand, suppose that a set S of propositions is inconsistent according to Geach's criteria. Then either one of two cases hold. In the simple case S already contains a pair of contradictory propositions (i.e., we have "the flat inconsistency of contradictories"). Otherwise S doesn't contain a pair of contradictory propositions, but from S a pair of contradictory propositions can be inferred. Either way from S a pair of contradictory propositions validly follow, and from a pair of contradictory propositions an explicit contradiction will follow. To see how, on Geach's account, we may go from the fact that a set S of propositions leads to contradictory conclusions to the claim that S validly implies an explicit contradiction, we may reason as follows. Let S be a set of propositions from which a pair of contradictory propositions follow and let A be an arbitrary proposition. Then the same pair of contradictory conclusions which validly follow from S also validly follow from $S \cup \{\sim A\}$, and also from $S \cup \{A\}$. Thus applying the rule reductio ad absurdum (as set out in Reason and Argument, 67) we may infer that S implies both A and ~A, from which we conclude that S implies the explicit contradiction (A & ~A).

[16.] For similar accounts see "Imperative Inference," 38 (pp. 280-81 in the Logic Matters reprint) which originally appeared in 1963; Reason and Argument (1976), 8-9; and Harry A. Lewis (ed.), Peter Geach: Philosophical Encounters (Dordrecht: Klewer Academic Publishers, 1991), 99-100.

[17.] Gödel also employs the notion of ω-consistency, but that need not concern us here.

[18.] In addition it should be noted that in the discussion following the passage quoted at the beginning of this paper Geach again alludes to the work done in modern logic on consistency proofs, and he identifies consistency with freedom from contradiction; see Truth and Hope, 41-43. This is a further indication that the notion of consistency used in his argument is the one which has been of concern to modern logicians, i.e., the logical notion of consistency used in the first two sections of this paper.

19. This definition only applies to finite sets of propositions, which is all that concerns us here. Though this is a common (and I believe perfectly legitimate) notion of consistency, it is doubtful that Geach would find it acceptable, given his doubts about the notion of analyticity. See, for example, Geach, Reason and Argument, 73-74.

20. I will assume that the notion of possibility used to explain consistency is the same as the notion used throughout the remainder of the argument.

21. A referee suggested that I consider whether Geach may have intended to use the notion of consistency in accordance with this definition. Another reader has suggested that I consider a notion of consistency in which a set S of propositions is consistent if and only if no contradiction follows from S together with other metaphysically necessary propositions. Where S is finite (as in the case of the doctrine of the Trinity) this notion of consistency comes to the same thing as the 'joint possibility' notion of consistency under consideration. This can be shown as follows. Case 1: S is jointly possible. Thus since all the members of S can be true together, a contradiction cannot follow from the members of S together with propositions that are necessarily truth. Case 2: S is not jointly possible. Let P be the conjunction of the members of S. Since S is not jointly possible, ~P is true is all possible world, and thus ~P is a necessary truth. From S together with the necessary truth ~P we may deduce the explicit contradiction (P & ~P).

22. Here it may be worth noting that if I am wrong in my interpretation of Geach's writing and he really intended to use the notion of consistency argue along the lines currently under consideration, then my previous objections do not apply to his argument.

23. For helpful comments on an earlier draft I would like to thank the referees and the editor of this journal.