Teaching Truth-Functional Conditionals

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One of the beautiful things about symbolic logic is that one can learn a lot through the symbolism (e.g., one can easily appreciate scope ambiguities in English because they are resolved right on the surface when sentences are translated into a standard symbolic notation). Unfortunately, when one goes on to develop a truth-functional account of the propositional operators and then tries to apply that account to conditionals used in ordinary speech, things are liable to seem artificial and even counter-intuitive to the student. Why, they will ask, should a conditional turn out to be true if its antecedent is false? And isn't it bizarre to say that "If I'm on the moon then I'm not on the moon" is true rather than false (and a contradiction)? To answer these sorts of difficulties requires a careful philosophical account of the truth-functional and non-truth-functional usage of the conditional. Yet even when such an account is given, a genuine grasp of the matter is likely to elude most students, so it is not easy to decide just how much class time can fruitfully be used to explore this issue.

With these thoughts in mind I have developed a short extra-credit project for students who may want to pursue the issue a little more deeply. The assignment (a copy of which is printed below) takes the following format:

(1) The distinction between truth-functional and non-truth-functional operators is reviewed.

(2) An argument is made for holding that in English there is both a non-truth-functional and a truth-functional usage of the conditional, the latter having the same truth table as the symbolic operator "→". The argument for recognizing a truth-functional use of the English conditional is a variation on one developed by P. T. Geach in his very insightful chapter on "Hypotheticals" in *Reason and Argument* (University of California Press, 1976).

(3) A set of problems is given in which the student is required (among other things) to construct a parallel argument for recognizing a non-truth-functional and a truth-functional usage of "unless". To complete these problems the student will need a good grasp of the argument in (2).

The problems are intended to be relatively easy for anyone who understands the argument in (2). Depending on the level of the class one might want either (a) to give the class a relatively quick sketch of truth-functional and non-truth-functional operators and a rough account of the truth-functional rendering of conditionals and leave the details of the presentation to the extra credit project, or (b) to present parts (1) and (2) to the class in detail and leave part (3) as extra credit. The handout for the extra-credit project follows:

The Logic of "If ... then ..." and "unless"

Truth-functional vs. non-truth-functional operators

In our symbolic language we use the symbols "&", "∨", "→", "↔" and "~" as *truth-functional operators*. Such operators can be given truth tables. So, for example, "&" has the truth table:

A	B	(A & B)
T	T	T
T	F	F
F	T	F
F	F	F

If we are told the truth values for A and for B we can determine the truth value for (A & B). We speak of the truth value of (A & B) as being a *function* of the truth values of A and of B. In ordinary English too we can sometimes recognize expressions that work as truth-functional operators. So, for example, when we use "and" to join together a pair of sentences it normally works as a truth-functional operator and can be translated by "&".

Some expressions we will call *non-truth-functional operators*. Consider the word "because". Like the word "and," "because" can be used to join together a pair of sentences to form a new sentence. Take the two sentences, S: "Sam sought shelter" and B: "Betsy breathed". These can be joined to form new sentences using either "and" or "because":

Sam sought shelter and Betsy breathed.

Sam sought shelter because Betsy breathed.

The first of these uses a truth functional operator "and" (which can be translated in our symbolic language by "&"). But the word "because" is not truth functional. We can see this if we try to form a truth table for the two sentences.

S	B	S and B	S because B
T	T	T	?
T	F	F	F
F	T	F	F
F	F	F	F

We cannot fill in the first row of the table for "S because B" since merely being told that "S" and "B" are true does not allow us to infer the truth value of the whole sentence "S because B"; we need additional information: Why did Sam seek shelter? Was it because Betsy breathed? We say that the truth value of "S because B" is not a truth function of the truth values of S and B.

Problem 1: Classify the following as truth-functional or non-truth-functional: "although", "and then", "however", "before".

A non-truth-functional usage of "if...then..."

It would seem that there is a non-truth-functional usage of "if ... then ...". Suppose that someone says:

If <u>you throw that liquid on the fire</u> then <u>there will be an explosion</u>.

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Imagine that this conditional is spoken in a normal context, e.g., someone is about to throw a liquid on the fire and the sentence is spoken as a warning. Suppose also that the person does not throw the liquid on the fire and there is no explosion, i.e., the antecedent and consequent are both false. Is the conditional true? We do not have enough information to answer this question, since the answer largely depends on information we have not been given, e.g., was the liquid gasoline or water? Since merely knowing the truth value of the antecedent and consequent is not enough to determine the truth value of the conditional we say that we have a non-truth-functional usage of "if ...then ...". This usage of the conditional is very common, so we may draw the following conclusion:

(1) There is a very common usage of "if... then..." that is not truth-functional.

A truth-functional-usage of "if... then ..."

Note that in our symbolic language the operator "→" is truth-functional by definition. That is, we have defined "→" to have the following truth table:

A	B	(A → B)
T	T	T
T	F	F
F	T	T
F	F	T

The question arises whether there is a truth-functional usage of the English expression "if ... then ..." which agrees with the above table for "→". An argument will be given that there is. One fact we appeal to is the following:

(2) If the antecedent of any normal "if ...then..." statement is true and the consequent is false, then the whole conditional is false.

This fact tells us that any normal usage of "if ... then ..." at least agrees with the second row of the truth table for (A → B), as shown below:

A	B	(A → B)	If A then B
T	T	T	?
T	F	F	F
F	T	T	?
F	F	T	?

What we need to show is that there is a normal usage of "if ... then ..." that also agrees with the table for (A → B) in rows 1, 3, and 4. Since rows 1, 3, and 4 are the rows in which (A → B) is true, what we need to show is that the truth of (A → B) implies the truth of "if A then B" (for some normal usage of "if ... then ..."). In other words we need to show that (for some normal usage of "if ... then ...") the following argument is valid:

<u>(A → B)</u>
∴ If A then B

In our search for a normal usage of "if ... then ..." for which this is a valid argument we will rely on a couple of facts, the first of which is:

(3) The following is a valid argument:

(A → B)
<u>A</u>
∴ B

Problem 2: Using a truth table prove that the argument in (3) is valid.

The second fact we will need is:

(4) There is a normal usage of "if...then..." for which the following rule, **conditionalization**, holds. Whenever an argument of the form:

p
<u>*q*</u>
∴ *r*

is valid, so is the argument:

<u>*p*</u>
∴ *if q then r*

The truth of (4) should be obvious. If I know that from the premises *p* and *q* I can infer *r*, then I can infer from the premise *p* alone that ***if*** *q then r*. Let us apply this rule by substituting "(A → B)" for "p", "A" for "q", and "B" for "r". Then conditionalization tells us that if the argument:

(A → B)
<u>A</u>
∴ B

is valid, then so is:

<u>(A → B)</u>
∴ If A then B

Fact (3) tells us that the first argument is valid, so it follows that the second one must also be valid, which is just what we wanted to show. From this it follows that whenever (A → B) is true, so is "If A then B". Thus, since rows 1, 3 and 4 of the truth table for (A → B) are all true, rows 1, 3 and 4 of the truth table for "if A then B" must also be true. Since we already saw that the second row of the tables for (A → B) and "if A then B" agree (i.e., they are false), the truth tables exactly match. Thus there is a normal usage of "if ... then ..." which is truth functional and has the following truth table:

A	B	(A → B)	If A then B
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

The logic of "unless"

A very similar argument would seem to show that "unless" has a truth-functional usage that matches the following truth table for "∨".

A	B	(A ∨ B)
T	T	T
T	F	T
F	T	T
F	F	F

The argument relies on three claims that parallel (2), (3) and (4), viz.:

(5) If both components of an "unless"-statement are false, then the statement as a whole is false.

(6) The following argument is valid:

(A ∨ B)
~B
∴ A

(7) There is a normal usage of "unless" for which the following rule holds. Whenever an argument of the form:

p
~q
∴ *r*

is valid, so is:

p
∴ *r unless q*

Problem 3: Discuss the plausibility of (5) and (7) and use a truth table to prove the argument in (6) is valid.

Problem 4: Use (5), (6) and (7) to construct an argument that there is a usage of "A unless B" that has the same truth table as (A ∨ B). Your argument should parallel the argument just given to show that there is a truth functional usage of "if ... then ..." which has the same truth table as the "→".

Problem 5: Can you develop an example in which "unless" is not used truth functionally?