Primary Literature

1. P.M. Petroff, A. Lorke, A. Imamoglu: Epitaxially self-assembled quantum dots. Phys. Today 54, (5), 46–52 (2001).
2. B.D. Josephson: The discovery of tunnelling supercurrents. Rev. Mod. Phys. 46, 251–54 (1974).
3. P.W. Anderson: Absence of diffusion in certain random lattices. Phys. Rev. 109, 1492 (1958).
4. K. von Klitzing: Nobel lecture: The quantized Hall effect. Rev. Mod. Phys. 58, 519–31 (1986).
5. K.C. Schwab, M.L. Roukes: Putting mechanics into quantum mechanics. Phys. Today 7, 36 (2005).
6. E. Cornell, C.E. Wieman: Nobel Lecture: Bose–Einstein condensation in a dilute gas, the first 70 years and some recent experiments. Rev. Mod. Phys. 74, 875 (2002).
7. S. Haroche: Entanglement, decoherence and the quantum/classical boundary. Phys. Today 51, (7), 36–42 (1998).
8. M. Arndt, K. Hornberger, A. Zeilinger: Probing the limits of the quantum world. Phys. World 18, 35 (2005).

Secondary Literature

9. J. Imry: *Introduction to Mesoscopic Physics*. Oxford University Press, Oxford (1997).
10. Y. Yamamoto: *Mesoscopic Quantum Optics*. Wiley-Interscience, New York (1999).
11. I. O. Kulik: *Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics*. Springer, Amsterdam (2000).
12. E. Joos, H.D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, I.O. Stamatescu: *Decoherence and the Appearance of a Classical World in Quantum Theory*. Springer, Berlin (2003).
13. D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, M. Aspelmeyer: *Optomechanical entanglement between a movable mirror and a cavity field*. Phys. Rev. Lett. 98, 030405 (2007).

Metaphysics of Quantum Mechanics

Craig Callender

Quantum mechanics, like any physical theory, comes equipped with many metaphysical assumptions and implications. The line between metaphysics and physics is often blurry, but as a rough guide, one can think of a theory's metaphysics as those foundational assumptions made in its interpretation that are not usually directly tested in experiment. In classical mechanics some examples of metaphysical assumptions are the claims that forces are real, that inertial mass is primitive, and that space is substantival. The distinctive feature of these claims is that they are all rather far removed from ordinary tests of the theory. Newton defended all three of the above claims at one time or other, whereas Mach attacked each one; however, both scientists agreed on enough of the formalism and its connection to experiment

to predict (e.g.) the same periods for given pendulums. What they disagreed about were the ingredients necessary to use classical mechanics to explain and understand the world.

Controversy engulfed the metaphysics of classical mechanics soon after its origin. Newton's idea of forces proved extremely contentious among the scientists of his time. Although metaphysical assumptions need not be controversial, quantum mechanics is also no stranger to metaphysical dispute. If anything, here the situation is more undecided because the theory was born with two different formalisms (Heisenberg's ► matrix mechanics, wave functions) and no clear interpretation. Heisenberg [1] originally offered a merely instrumental understanding of his formalism (later he opted for an interpretation employing discontinuous quantum jumps), whereas Schrödinger [2] viewed his theory as having physical content: it described, he thought, the evolution of continuous matter waves. The formalisms subsequently proved to be equivalent, but the metaphysical pictures could hardly have been more different. Soon thereafter, Bohr's ► complementarity thesis took shape, ► Heisenberg's uncertainty principle was discovered, and Born provided a ► probabilistic interpretation of the wavefunction. The combination of these three theses formed the essential core of the so-called Copenhagen interpretation. Associated especially with Bohr [3], the Copenhagen interpretation is itself the subject of active interpretation [10], and few advocates of the theory agree on all of the theses commonly associated with it. (See ► Born rule; Consistent Histories; Nonlocality; Orthodox Interpretation; Schrödinger's Cat; Transactional Interpretation). Nevertheless, if correct, it makes dramatic metaphysical assumptions. These include the ideas that measurement brings into being the measured property as opposed to revealing it, that there is a "complementarity" between dynamic and kinematic aspects of the world, and that all properties of atoms are inherently contextual – that is, irreducibly relative to a measuring apparatus.

Stepping back from its history, we see that the basic ontology of the quantum world is very much undetermined. Thanks to the infamous measurement problem [7,8] we have an extra layer of assumptions that might be called metaphysical – although in another sense these assumptions are simply the ordinary claims of any physical theory. The reason for this extra layer is that one must first solve the measurement problem and then provide the best interpretation of that solution. Experiment cannot yet decide among these theories, and in some cases, never will. Thus the choice of solution is not directly tested in experiment, nor are some of assumptions made by any given solution. The metaphysics of quantum mechanics thus hangs on both a particular solution to the measurement problem and then the best interpretation of that solution. (For measurement problem, see ► Bohmian mechanics; Measurement theory; Modal Interpretation; Objectification; Projection Postulate).

Working in the Schrödinger formalism, the measurement problem arises from the (1) linearity of the equation evolving the wave function, and (2) the claim that the ► wave function or quantum state is representationally complete – that is, that there are properties of kind A in the world if and only if the quantum state is in an eigenstate of the operator $\mathfrak{R}$ believed to represent that property. If linear dynamical evolu-

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tion of the quantum state is uninterrupted, then the ▶ superpositions of microscopic states necessary for quantum predictions will evolve into superpositions of macroscopic states. And if the quantum state offers a complete representation of what there is, then the systems described by these macroscopic superpositions do not have any definite measurable properties. Since measurements seem to have determinate outcomes, we appear to have an inconsistency between the theory and experience.

Putative solutions to this problem fall naturally into three classes. The first class consists of theories (sometimes dubbed "hidden variable theories") denying that the quantum state is representationally complete. In addition to the wavefunction evolving according to some linear equation, there are posited what J.S. Bell [7] calls "beables" (as opposed to ▶ observables) and a dynamics for these beables. Beables are the basic ontology of the theory. In classical electromagnetism, they are the electric and magnetic fields; in Newtonian mechanics, the beables are the particles. In quantum mechanics, typically particle or field ontologies are posited. The ontology is dualistic: interpreted realistically, there are both beables and wavefunctions in the world. The best-known version of this kind of reaction was first discovered by de Broglie but later developed by Bohm [5]. According to this theory, there are in addition to wavefunctions particles with always-determinate trajectories evolving in three-dimensional space, governed by an equation that is a function of the system's wavefunction. Even within a solution in this class one finds varying metaphysical pictures [7, 12]. One can find deterministic and indeterministic Bohm theories, particle and field-based theories, theories that treat ▶ spin as a beable and ones that do not – even theories that do not treat fermions as beables. Some believe the wavefunction is part of reality, others that it is nomological, and still others treat it instrumentally.

The second class of solutions are unified in their claim that the evolution of the quantum state is not always linear. So-called "collapse" theories state that upon measurement there is an instantaneous ▶ wavefunction collapse from a superposition to an eigenstate (when the state is expanded in the relevant basis for the observable being measured). Proposals for what triggers this collapse include the "classicality" of the device (some Bohrians – although perhaps not Bohr [10]), non-physical minds (Wigner) [11], and in more recent theories, such as GRW [4] (after Ghirardi, Rimini and Weber), certain thresholds being reached in the system's mass density or particle number.

Again, even within one class of putative solutions, we find a diverse array of possible metaphysical assumptions. In some theories the wavefunction represents an objective part of reality, in others our state of knowledge. Even within a particular solution, say, ▶ GRW [4], there are a variety of metaphysical pictures available. In one especially radical interpretation of GRW, there is nothing but a sometimes-collapsing wavefunction evolving in 3N-dimensional state space, where N is the number of "constituents" of the system. On this view, 3-dimensional objects like us are aspects of the universal wavefunction that have grown "clumpy" in $3N$-dimensional configuration space. According to the "mass density" theory, there is a continuous distribution of mass throughout spacetime, and the mass density at a point is a function of the wavefunction. Yet according to the "flash ontology" the-

ory, the basic ontology is one of primitive spacetime events that are the loci of GRW collapses [11].

The third class of solutions tries to explain away the mismatch between macroscopic superpositions and experience by neither supplementing the wavefunction description of the world nor interrupting its linear evolution. Originally developed by Everett [6], advocates of the so-called relative-state interpretation claim that our experience supervenes upon macroscopic superpositions in a way that is more complicated than one normally thinks. According to the "many worlds" version, quantum measurements literally split the world into two or more mini-worlds – one corresponding to each possible measurement outcome. The most interesting versions of Everettian theories, however, do not add anything to the wavefunction but instead discover different observers as emergent from complex relations encoded in the wavefunction of the world [13]. It is hardly necessary to say that the metaphysical implications of this view for our conception of ourselves, the external world and probabilities—to name just three topics – are quite dramatic.

Finally, it is worth mentioning that there is a very different group (e.g. [9]), inspired by Bohr that treats quantum mechanics instrumentally. These thinkers consider the wavefunction to be solely an epistemic device that gives observers information about the probabilities of finding various outcomes. Collapse of the wavefunction is viewed as merely the modification of one's subjective credence in light of new information. Because the wavefunction does not represent a genuine state of a real physical system, and these theorists are silent about what the information is information about, the theory offers no physical picture of the world.

In general, no matter the solution to the measurement problem, we expect any non-instrumental version of quantum mechanics to provide answers to various metaphysical questions. Is the wavefunction epistemic or ontological? What is the basic ontology (i.e. beables) of the theory? Do we live in ▶ Hilbert space or four-dimensional spacetime? What is the mechanism responsible for the non-local quantum correlations? What is the interpretation of the probabilities given to us by Born's rule? Do measurements create or reveal the measured properties? Answers to these questions will hang on both the best solution to the measurement problem and the best interpretation of that solution. It is important not to confuse these two issues. For instance, it is commonly said that quantum mechanics implies that atoms don't have determinate trajectories; but strictly speaking, these conclusions follow only from some versions of some interpretations. The original Bohm theory is an empirically adequate (for non-relativistic phenomena) counterexample to this claim, for instance.

The same warning applies to what is one of the most vexed metaphysical questions surrounding quantum mechanics, the question of determinism. (▶ Indeterminism and determinism in quantum mechanics) A physical theory is deterministic if, roughly, given a complete state of the universe at any one time, a unique past and future follow. With suitable assumptions classical mechanics is deterministic. With the advent of quantum mechanics, many of the theory's founders famously declared that determinism was "dead". The Schrödinger evolution of the wavefunction is deterministic; however, the collapse of the wavefunction is stochastic, so the

full theory is indeterministic. Quantum mechanics proved, they thought, that "God plays dice". However, as we have just seen, this claim is interpretation-dependent. There are plenty of no-collapse interpretations of quantum mechanics, e.g. Everett, Bohm, and some versions of these are deterministic. The question of whether "God plays dice" is still open.

(See Consistent histories, Ignorance interpretation, Ithaca Interpretation, Many Worlds Interpretation, Modal Interpetation, Orthodox Interpretation, Transactional Interpretation).

Interestingly, the many interpretations of quantum mechanics illustrate why the line between metaphysics and physics is sometimes blurry. Given current technology, there is no way to experimentally decide between, say, a Wignerian collapse theory ("human consciousness causes collapse ▶ Wigner's friend") and one or more versions of GRW ("reaching a threshold of particle number in the system makes collapse likely"). But in principle these theories do issue different predictions for some observables. In this sense, the metaphysics of today may be the physics of tomorrow. In addition, even before any crucial experiment is performed—and it is not clear that there ever will be such between certain pairs of interpretations—we see that science can have a real bearing on these metaphysical disputes. Scientists value more than good predictions. They also prize simplicity, unification, consilience and other theoretical virtues. Even if there is no test between two given interpretations, there may be good reasons to adopt one over another. One interpretation may possess a symmetry others do not, resolve a problem others cannot, or uniquely extend to a promising new theory (say, some version of ▶ quantum gravity).

Primary Literature

1. W. Heisenberg: The Physical Content of Quantum Kinematics and Mechanics. Zeitschrift fur Physik **43**, 172 (1927)
2. E. Schrödinger: *Collected Papers on Wave Mechanics* (Chelsea, New York, 1982)
3. N. Bohr: *Atomic Theory and the Description of Nature* (Cambridge University Press, Cambridge, 1934)
4. G.C. Ghirardi, A. Rimini, T. Weber: Unified Dynamics for Microscopic and Macroscopic Systems. Physical Review **D 34**, 470 (1986)
5. D. Bohm: A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables. I & II.' Physical Review **85**, 166, Physical Review, **85**, 180 (1952)
6. H. Everett, H.: Relative State Formulation of Quantum Mechanics. Review of Modern Physics **29**, 454–462 (1957)

Secondary Literature

7. J.S. Bell: *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, New York, 1987)
8. D.Z. Albert: *Quantum Mechanics and Experience* (Harvard University Press, Cambridge, MA, 1992)

9. J. A. Wheeler: Information, Physics, Quantum: The Search for Links, In *Complexity, Entropy and the Physics of Information* (Addison-Wesley, Redwood City, 1990), edited by W. Zurek.
10. J. Faye: Copenhagen Interpretation of Quantum Mechanics, In *The Stanford Encyclopedia of Philosophy (Summer 2002 Edition)*, edited by E. Zalta, URL = http://plato.stanford.edu/archives/sum2002/entries/qm-copenhagen/
11. G.C. Ghirardi: Collapse Theories, In *The Stanford Encyclopedia of Philosophy (Fall 2007 Edition)*, edited by E. Zalta, URL=<http://plato.stanford.edu/archives/fall2007/entries/qm-collapse/>
12. S. Goldstein: Bohmian Mechanics, In *The Stanford Encyclopedia of Philosophy (Fall 2007 Edition)*, edited by E. Zalta, URL=<http://plato.stanford.edu/archives/fall2007/entries/qm-bohm/>
13. J. Barrett: Everett's Relative-State Formulation of Quantum Mechanics, In *The Stanford Encyclopedia of Philosophy (Spring 2003 Edition)*, edited by E. Zalta, URL=<http://plato.stanford.edu/archives/spr2003/entries/qm-everett/>

Mixed State

Peter Mittelstaedt

The most general state of a proper quantum system S with ▶ Hilbert space $\mathcal{H}_S$ is given by a self-adjoint positive operator with trace 1, i.e. by an operator

$$W_S = W_S{}^+ \geqslant 0 \text{ with tr } \{W_S\} = 1.$$

It can be shown that these positive trace class 1 operators form a convex set $\mathcal{T}_1{}^+(\mathcal{H}_S)$ [1].

Two kinds of states must be distinguished. If W_S is idempotent, i.e. $W_S = W_S{}^2$, then W_S is a *pure state* given by an projection operator $P[\varphi]$ where $\varphi \in \mathcal{H}_S$ is an element of $\mathcal{H}_S$. If, however, $W_S \neq W_S{}^2$, then W_S describes a *mixed state*. As any self-adjoint operator, a *mixed state* W_S can be decomposed according to its spectral decomposition

$$W_S = \sum_i w_i \, P[\mathcal{M}_i]$$

with real numbers w_i such that $0 \leqslant w_i \leqslant 1$ and projection operators P (▶ projection), which project on subspaces $\mathcal{M}_i$ of $\mathcal{H}_S$. It must be emphasised, however, that the decomposition is not uniquely defined, since there are many other, non-orthogonal decompositions of W_S. If, in addition, the operator W_S has a degenerate spectrum, there are also infinitely many orthogonal decompositions.

There are two kinds of mixed states of S given by an operator $W_S = \sum_i w_i \, P[\varphi_i]$ with $0 \leqslant w_i \leqslant 1$, which are distinguished by their preparation.

(a) Mixture of states

Assume that a preparation apparatus does not work completely accurately and prepares systems with states $\varphi_1, \varphi_2, \varphi_3 \ldots$, say, with *a priori* probabilities $p_1, p_2, p_3 \ldots$, which depend on the construction of the apparatus. In this case, any