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**Advances in Argumentation in AI**

**Guest Editors**
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Costanza Larese

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## ARTICLES

# A PWK-style Argumentation Framework and Expansion

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**Abstract**

In this article we consider argumentation as an epistemic process performed by an agent to extend and revise her beliefs and gain knowledge, according to the information provided by the environment. Such a process can also generate the suspension of the claim under evaluation. How can we account for such a suspension *phenomenon* in argumentation process? We propose: (1) to distinguish two kinds of suspensions – critical suspension and non-critical suspension – in epistemic change processes; (2) to introduce a Paraconsistent Weak Kleene logic (PWK) based belief revision theory which makes use of the notion of topic to distinguish the two kinds of suspensions previously mentioned, and (3) to develop a PWK-style argumentation framework and its expansion. By doing that, we can distinguish two kinds of suspensions in an epistemic process by virtue of the notion of topic.

We are highly grateful to Marcello D'Agostino, Fabio Aurelio D'Asaro, and Costanza Larese for their remarkable efforts in editing this issue. The friendly interaction and helpful advice provided by Fabio Aurelio D'Asaro made the publishing process pleasant. We would like to express our gratitude to the two anonymous referees of this issue for their insightful remarks and suggestions, which significantly improved the quality of the article. Massimiliano Carrara was partially funded for this research by the CARIPARO Excellence Project (CARR_ ECCE20_ 01): *Polarization of irrational collective beliefs in post-truth societies. How anti-scientific opinions resist expert advice, with an analysis of the antivaccination campaign* (PolPost). Wei Zhu was funded by the Postdoctoral Stipend from the Fritz Thyssen Foundation, Germany, for the project (Ref. 40.22.0.015PH)*Suspension and Computational Errors in Paraconsistent Weak Kleene Belief Update.*

**Keywords:** Suspension; Abstract Argumentation; Paraconsistent Weak Kleene logic (PWK); PWK Belief Revision; *Off-topic* Interpretation in PWK

# 1 Introduction

There is a close connection between belief revision and argumentation.[1] Here we consider one specific aspect of argumentation where this connection is viewed as an epistemic process performed by an agent to improve her beliefs and gain knowledge by acquiring some new information from the external environment: the suspension of the claim under evaluation.[2]

Such an aspect, suspension, is characterized by the absence of belief and disbelief concerning a proposition $\phi$.[3] We consider it as a state of absence of judgements on propositions or arguments in the reasoning process. Moreover, following [16] we propose to distinguish two kinds of suspensions: *non-critical* suspension and *critical* suspension. When an agent neither believes nor disbelieves (or reject) certain information, such a suspension is non-critical. It is non-critical because the agent can still form a judgment or continue to process an argument as long as she gains more information from her environment. As we are going to see, such a kind of suspension can be modeled through the standard AGM model for belief revision.[4] Instead, a critical suspension occurs when an agent gains some *irrelevant*, *meaningless*, *off-topic* and even *malicious* information from the environment. This suspension cannot be held in the subsequent epistemic process, and should be filtered and set apart from the argumentation process.[5]

To better understand the two cases of *suspension* consider the following analogy with non-critical and critical errors in computation. In a computational program, a non-critical error stops the computation program partially, and this error can be fixed in the subsequent computation process. Instead, a critical error stops the program completely and this error cannot be fixed.[6] One can see the two types of suspension in terms of the two types of computational errors: *non-critical suspension* corresponds to the non-critical error, whereas *critical suspension* corresponds to the critical error.

[1] See e.g. [24], [36], [6], [8], and [3]. For a survey of argumentation theory (specifically in Artificial Intelligence), see e.g. [11], [33], and [20].

[2] See [3] where belief revision and argumentation are related and compared as two formal approaches to model reasoning processes.

[3] Some recent views consider it as a question-directed (or inquisitive) attitude [25, 26, 27].

[4] See e.g. [1].

[5] For a further discussion see §4.1 below.

[6] On this kind of computational errors see e.g. [32].

Now, in general a belief revision process, as modeled in AGM, uses classical propositional logic as its background logic. However, classical propositional logic assumes that each propositional variable has a classical truth-value – i.e. true or false. Hence, it excludes the possibility that an agent's belief state permanently stops because some propositions fail to obtain a truth-value. Moreover, it assumes that each proposition is on-topic. But in the case of a critical suspension an agent stops reasoning because it obtains some meaningless, off-topic information. This problem suggests us to change the background logic of the current belief revision theory and to make it able to filter the information so as to prevent potential critical errors from occurring during the belief revision process. Thus, in this article we develop an expansion of AGM theory based on a Paraconsistent Weak Kleene logic (PWK), where the third value of PWK is read as *off-topic*, and we conceive a PWK-style argumentation framework that is capable of distinguishing the two kinds of above mentioned suspensions in argumentation.

The present paper is organized as follows. In §2 we introduce the PWK logic and the *off-topic* interpretation of its non-classical truth-value, **u**. Also, we discuss how such an interpretation can be used to account for two kinds of suspensions occurring in argumentation: critical suspension and non-critical suspension. In §3 we present PWK belief revision (PWK-BR). In §4, we put forth a PWK abstract argumentation framework and its expansion, which is capable of distinguishing critical suspension and non-critical suspension. Finally, in §5 we make some concluding remarks.

## 2 PWK and the Off-topic Interpretation

In this section we will introduce two of the main elements we will need to develop our proposal: the Paraconsistent Weak Kleene logic and the off-topic interpretation of its non-classical truth value, i.e. **u**.

Traditionally, Kleene's three-valued logics divide into two families: strong and weak.[7] Weak Kleene logics, WK3, originate from weak tables (see table 1, below). Arguably, the two most important WK3 are **(author?)** [13][8] and [29]'s logics (B and H, respectively), which differ in the designated values they take on.[9] B assumes that classical truth is the only value to be preserved by valid inferences. H includes

[7]See [31].

[8]Translated in [14].

[9]There is an increasing interest in WK3. To give some examples, [19] develop sequent calculi for WK3, [34] introduce a cut-free calculus (a hybrid system between a natural deduction calculus and a sequent calculus) for PWK, [17] explores some connections between H and Graham Priest's *Logic of Paradox*, LP, and [18] focus on logical consequence in PWK.

also the non-classical value among the designated ones. Thus, it turns out that H, or better, PWK, is the paraconsistent counterpart of B. Precisely, PWK corresponds to the so-called Halldén's *internal logic*, that is a logic that includes the standard propositional connectives, but cannot express the meaningfulness of its own statements. Instead, Halldén's *external logic* extends PWK with a unary connective that allows to build statements such as "$\phi$ is meaningful".[10] In what follows, we will use PWK. Thus, let us briefly introduce it.

## 2.1 PWK

The language of PWK is the standard propositional language, $L$. Given a nonempty countable set $\mathsf{Var} = \{p, q, r, \dots\}$ of atomic propositions, the language is defined by the following Backus-Naur Form:

$$\Phi_L ::= p \mid \neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid \phi \supset \psi$$

We use $\phi, \psi, \gamma, \delta \dots$ to denote arbitrary formulas, $p, q, r, \dots$ for atomic formulas, and $\Gamma, \Phi, \Psi, \Sigma, \dots$ for sets of formulas. Propositional variables are interpreted by a valuation function $V_a : \mathsf{Var} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ that assigns one out of three values to each $p \in \mathsf{Var}$. The valuation extends to arbitrary formulas according to the following definition:

**Definition 2.1** (Valuation)**.** A valuation $V : \Phi_L \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ is the unique extension of a mapping $V_a : \mathsf{Var} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ that is induced by the tables from Table 1.

Table 1 provides the full *weak tables* from **(author?)** [31, §64], that obtain "by supplying [the third value] throughout the row and column headed by [the third value]".[11] Note that in PWK, like in the others WK3, negation works like in Strong Kleene logics, whereas conjunction and disjunction work differently. The way **u** transmits is usually called *contamination* (or *infection*), since the value propagates from any $\phi \in \Phi_L$ to any construction $k(\phi, \psi)$, independently from the value of $\psi$ (here, $k$ is any complex formula made out of some occurrences of both $\phi$ and $\psi$ and

[10] Notice that [29] calls $C_0$ what we call PWK.

[11] It is clear by table 1 that $\wedge$ and $\supset$ can be defined in terms of $\neg$ and $\vee$ as usual, namely $\phi \wedge \psi = \neg(\neg\phi \vee \neg\psi)$ and $\phi \supset \psi = \neg\phi \vee \psi$. Nevertheless, we prefer to introduce them all as primitives for the sake of clarity.

| $\phi$ | $\neg\phi$ |
|---|---|
| **t** | **f** |
| **u** | **u** |
| **f** | **t** |

| $\phi \vee \psi$ | **t** | **u** | **f** |
|---|---|---|---|
| **t** | **t** | **u** | **t** |
| **u** | **u** | **u** | **u** |
| **f** | **t** | **u** | **f** |

| $\phi \wedge \psi$ | **t** | **u** | **f** |
|---|---|---|---|
| **t** | **t** | **u** | **f** |
| **u** | **u** | **u** | **u** |
| **f** | **f** | **u** | **f** |

| $\phi \supset \psi$ | **t** | **u** | **f** |
|---|---|---|---|
| **t** | **t** | **u** | **f** |
| **u** | **u** | **u** | **u** |
| **f** | **t** | **u** | **t** |

**Table 1:** Weak tables for logical connectives in $\Phi_L$

whatever combination of $\vee$, $\wedge$, $\supset$, and $\neg$). To better capture the way **u** works in combination with the other truth-values, let us introduce the following definition:

**Definition 2.2.** For any $\phi \in \Phi_L$, *var* is a mapping from $\Phi_L$ to the power set of Var, which can be defined inductively as follows:

- $var(p) = \{p\}$,
- $var(\neg\phi) = var(\phi)$,
- $var(\phi \vee \psi) = var(\phi) \bigcup var(\psi)$,
- $var(\phi \wedge \psi) = var(\phi) \bigcup var(\psi)$,
- $var(\phi \supset \psi) = var(\phi) \bigcup var(\psi)$.

Then, the following fact expresses *contamination* very clearly:

**Fact 2.1** (Contamination)**.** For all formulas $\phi$ in $\Phi_L$ and any valuation $V$:

$$V(\phi) = \mathbf{u} \quad \textit{iff} \quad V_a(p) = \mathbf{u} \text{ for some } p \in var(\phi).$$

The logical consequence relation of PWK is defined as preservation of non-false values – i.e. the designated values are both **u** and **t**. In other words:

**Definition 2.3.** $\Gamma \vDash_{\mathsf{pwk}} \Delta$ iff there's no interpretation $\mathbb{I}$ such that:

$$\mathbb{I}(\phi) \neq \mathbf{f} \text{ for all } \phi \in \Gamma \text{ and } \mathbb{I}(\psi) = \mathbf{f} \text{ for some } \psi \in \Delta.$$

PWK is reflexive and transitive. It is also monotonic (i.e. if $\Gamma \vDash_{\mathsf{pwk}} \Delta$ then $\Gamma \cup \{\alpha\} \vDash_{\mathsf{pwk}} \Delta$), but given the behaviour of conjunction in the premise side, PWK has a 'non-monotonic flavour', in the sense that, for example, $p \vDash_{\mathsf{pwk}} p$ but $p \wedge q \nvDash_{\mathsf{pwk}} p$. Further, note that the inclusion of all the atoms of a premise set $\Gamma$ in a conclusion set $\Delta$ guarantees that if $\Gamma \vDash_{\mathsf{cl}} \Delta$ then $\Gamma \vDash_{\mathsf{pwk}} \Delta$, where $\vDash_{\mathsf{cl}}$ is the classical consequence relation.

## 2.2 Off-topic Interpretation for u

Recently, the third value **u** of WK3 – initially understood as *nonsense*, *meaninglessness* or *undefined* – has been studied in more depth. A recent proposal by [10] suggests to read **u** of WK3 as *off-topic*. More specifically: Beall proposes to "[...] read the value 1 not simply as *true* but rather as *true and on-topic*, and similarly 0 as *false and on-topic*. Finally, read the third value 0.5 as *off-topic*" [10, p. 140]. [12] Thus, *What is a topic?* is arguably a crucial question for his proposal. Unfortunately, **(author?)** [10] is silent about that. But we can make some assumptions and develop his proposal in order to make it suitable for our purposes.

We assume that topics can be represented by sets.[13] We use bold letters for topics, such as **s**, **t**, etc. $\subseteq$ is the inclusion relation between topics, so that $\mathbf{s} \subseteq \mathbf{t}$ expresses that **s** is included into (or is a subtopic of) **t**.[14] Given that, we define a *degenerate* topic as one that is included in every topic. Also, we define the overlap relation between topics as follows: $\mathbf{s} \cap \mathbf{t}$ iff there exists a non-degenerate topic **u** such that $\mathbf{u} \subseteq \mathbf{s}$ and $\mathbf{u} \subseteq \mathbf{t}$. Further, it is assumed that every meaningful sentence $\alpha$ comes with a *least* subject matter, represented by $\tau(\alpha)$. $\tau(\alpha)$ is the unique topic which $\alpha$ is about, such that for every topic $\alpha$ is about, $\tau(\alpha)$ is included into it.

[12] Interestingly, a similar proposal comes from [21] and [22] where it is provided an informational semantics for three values, in which **u** is interpreted as *informationally indeterminate*.

[13] This is a natural assumption. As discussed in **(author?)** [30], topics are represented by sets in all the main approaches you can find in the literature. In this paper we take no position with respect to what exactly a topic is, that is whether a set of sets of proposition (a partition of the logical space), a set of objects, etc. We just set some constraints about how topics behave and how they relate to sentences.

[14] The inclusion relation, $\subseteq$, is usually taken to be reflexive, so that every topic includes itself.

Thus, we say that $\alpha$ is *exactly* about $\tau(\alpha)$.[15] But $\alpha$ can also be *partly* or *entirely* about other topics: $\alpha$ is entirely about $\mathbf{t}$ iff $\tau(\alpha) \subseteq \mathbf{t}$, whereas $\alpha$ is partly about $\mathbf{t}$ iff $\tau(\alpha) \cap \mathbf{t}$.

Next, we assume the following conditions concerning how topics behave with respect to the logical connectives:

1. $\tau(\phi \wedge \psi) = \tau(\phi) \cup \tau(\psi)$.
2. $\tau(\phi \vee \psi) = \tau(\phi) \cup \tau(\psi)$.
3. $\tau(\neg\phi) = \tau(\phi)$.

As shown in **(author?)** [15, §2], from these assumptions we can also prove that the topic of a complex sentence boils down to the union of the topics of its atomic components.

Further, not only do sentences have a topic, but also sets of sentences do. More in detail, we have the following:

**Definition 2.4.** Given a set $S$ of sentences of $\Phi_L$, i.e. $S \subseteq \Phi_L$, the topic of $S$, that is $\tau(S)$, is such that $\tau(S) = \bigcup\{\tau(\phi) \mid \phi \in S\}$.

Then, since both theories and arguments can be represented by sets of sentences, we can legitimately speak about their topics: the topic of an argument (or theory) is the union of all and only the (least) topics of each of its sentences. Thus, as for sentences, given any argument $A$ we say that: $A$ is exactly about $\tau(A)$; $A$ is entirely about $\mathbf{t}$ just in case $\tau(A) \subseteq \mathbf{t}$; and $A$ is partly about $\mathbf{t}$ just in case $\tau(A) \cap \mathbf{t}$. Moreover, as shown by **(author?)** [15, Corollary 2.2], what a set of sentences $S$ is about boils down to the union of what the atomic components of each claims in $S$ are about: that is, $\tau(S) = \bigcup\{\tau(p) \mid p \in var(S)\}$, where $var(S)$ is the set of all and only the atomic variables occurring in the sentences that belong to $S$.

Finally, let us set a reference (or discourse) topic, $\tau_R$, that is the topic that one or more agents discuss/argue about. Then, a sentence $\phi$, or an argument $A$, or a theory $T$ are off-topic with respect to $\tau_R$ iff $\tau(\phi), \tau(A), \tau(T) \nsubseteq \tau_R$ – i.e. iff $\phi$, $A$ and $T$ are not entirely about $\tau_R$. Given such a regimentation of the notion of topic and Beall's off-topic interpretation of $\mathbf{u}$, our aim now is to use them to get an argumentation framework based on PWK.

[15] Throughout this paper, when we talk about the topic of a sentence we mean its least topic. In case we want to refer to one of its topics that is not the least one, we will make it clear.

### 2.3 Off-topic and Critical/Non-critical Suspensions

In §3 we integrate the off-topic interpretation of **u** into a PWK belief revision theory, based on the standard AGM model. But before we do that, it is important to point out the reason behind the development of our framework. Such an integration allows us to distinguish two kinds of suspensions that may occur in an epistemic process of change of beliefs. Since an argumentation can be represented by a set of sentences, in line with Definition 2.4 we assume that an agent's argumentation process has a topic – i.e. the reference topic.

Let's take an example. Suppose that an argumentation process is about the topic represented by the question "How many stars are there?". An argument like "There is an infinite number of stars in the universe because it is infinite in space" is an on-topic one in the argumentation process, which should participate in the argumentation process. However, an argument like "Alice is in wonderland because I read about it in a book" is an off-topic one in the argumentation process, which should be filtered and set apart from the argumentation process. Let us make an example to show how an off-topic argument can be harmful to the reasoning process. Suppose there are three arguments in the argumentation process whose topic is "How many stars are there":

(1) "100 stars are in the sky"

(2) "Alice is in wonderland or there are no stars in the sky"

(3) "Alice is not in wonderland"

If we do not set apart off-topic arguments from on-topic ones, from (2) and (3) we can derive "there are no stars in the sky", which is in conflict with (1). If we set apart (2) and (3) from the argumentation process as off-topic arguments, we can derive that "100" is the conclusion.[16]

Given a reference topic, an epistemic agent's argument can be either on-topic or off-topic with respect to it. If the argument is off-topic, we get a critical-suspension of the conclusion of the argumentation process. In other words, the epistemic agent assigns **u** to the claim that is meant to be the conclusion of the argument at stake. If it is on-topic, the conclusion might be believed, disbelieved or non-critically suspended, depending on how the argument works and on there being other good arguments attacking such conclusion – i.e. depending on the argumentation framework in which the epistemic agent performs her argumentation process. In particular, a conclusion is non-critically suspended just in case it generates a contradiction, that

[16] We express our gratitude to a referee who proposed this example.

is if we can draw both that conclusion and its negation from our set of beliefs. In that case, the suspension is non-critical in the sense that the claim under evaluation is neither believed nor disbelieved, and it remains available to be processed in a further argumentation process where new (on-topic) information is acquired.

# 3 A PWK Based Belief Revision Theory

The next step is to enter belief-revision. This is the process through which an ideal rational agent revises her own beliefs to get an ever-improving understanding of the world, i.e. a better representation of it. How does this process work? There is a well-known formal account that gives a model of it: the AGM theory. Here, we aim at developing a different version of belief revision: a PWK based belief revision theory (PWK-BR). Now, since our PWK-BR is based on (and can be seen as an expansion of) the AGM theory, let us start by quickly introducing AGM.

## 3.1 AGM Theory

Among all the belief revision theories, AGM theory is widely recognized as a milestone. It was initialized by [1] and soon developed by [28]. The main question of AGM belief revision theory is: in order to accommodate new information which is contradictory to an agent's own beliefs, how to get rid of the inconsistency as well as minimizing the information loss? To solve this problem, a worked out formal epistemology of belief revision theory is required. Basically, such a theory needs consider the following essential components, which are: a formal representation of epistemic states; a classification of the epistemic attitudes; an account of the epistemic inputs and a classification of epistemic changes; and a criterion of rationality. Thus, the main framework of AGM theory can be listed as follows:

1. An agent's belief state is formalized as a belief set $\Theta$, which is closed under the consequence operation $Cn$. Since AGM theory adopts classical propositional logic, $Cn$ is $\vDash_{\mathsf{cl}}$ in this regard. Specifically, the definitions of the consequence operation and belief set are as follows.

   **Definition 3.1** (Consequence Operation $Cn$)**.** A consequence operation on a language $\mathcal{L}$ is a function $Cn$ that takes each subset of $\mathcal{L}$ to another subset of $\mathcal{L}$, such that:

*(a)* $A \subseteq Cn(A)$.

*(b)* $Cn(A) = Cn(Cn(A))$.

*(c)* *If* $A \subseteq B$*, then* $Cn(A) \subseteq Cn(B)$.

**Definition 3.2** (Belief Set $\Theta$)**.** $\Theta$ is a set of sentences. It is a belief set if it is closed under $Cn$. That is, $\Theta = Cn(\Theta)$.

2. An agent has three kinds of epistemic attitudes, which are: belief, disbelief and suspension. Suspension in fact is not an attitude but a lack of attitude, called non-attitude. For writing convenience, we call it is an attitude. These attitudes are exclusive and exhaustive. Hence, a sentence is believed, disbelieved or kept in suspension.

3. In AGM, an epistemic input is regarded to be external, in terms of a new sentence from the environment.

4. Three basic kinds of epistemic change operators are expansion, contraction, and revision. Since the aim is to model the process of belief-revision, some operations on $\Theta$ representing the belief changes can be defined. In the AGM account, there are three: expansion ($+$), contraction ($-$), and revision ($*$).

   - Expansion models the addition of a belief, say $\alpha$, when nothing is removed: $\Theta$ is replaced by $\Theta + \alpha$, that is the smallest logically closed set containing both $\Theta$ and $\alpha$. Thus, $\Theta + \alpha = \{\beta : \Theta \cup \{\alpha\} \vDash \beta\}$, where $\vDash$ denotes the selected consequence relation.
   - Contraction models the removal of a belief. This is not just to delete $\alpha$ from $\Theta$. Since the result must be logically closed, we may have to delete other things as well. From $\Theta$ we get $\Theta - \alpha$, that is a set such that $\Theta - \alpha \subseteq \Theta$ and that $\alpha \notin \Theta - \alpha$, but this change can be accomplished in different ways – i.e. there are many sets $\Theta - \alpha$ satisfying these conditions. The AGM account does not give an explicit definition of contraction but gives a set of axioms that $\Theta - \alpha$ must satisfy, the so-called basic AGM postulates.
   - Finally, revision models the addition of a belief to $\Theta$ when other sentences have to be removed to ensure that the resulting set of beliefs, $\Theta * \alpha$, is consistent. As for contraction, also revision has been axiomatically characterized.

5. The rational criterion of AGM belief revision theory is the principle of information economy, which requires an agent to accommodate new information and at the same time to minimize the loss of the original beliefs. This criterion is resulted from the fact that data are valuable. It is better to preserve as much data as possible. To ensure this criterion, AGM postulates are developed to regulate the performance of the belief change operators.

Let us now turn to our different belief revision theory: PWK-BR.

## 3.2 Belief States in PWK-BR

Differently from the AGM belief set, in PWK-BR an agent's belief state concerns a topic. An agent's epistemic attitude toward a given proposition $\alpha$ depends on whether $\alpha$ is on-topic or off-topic with respect to a given reference topic – i.e. the topic of the argumentation process she is performing. If $\alpha$ is on-topic, the agent can believe it, disbelieve it, or keep it in non-critical suspension. If $\alpha$ is off-topic, the agent would keep it in critical suspension. Non-critical suspension and critical suspension are two exclusive attitudes:

1) If $\alpha$ is in a non-critical suspension, $\alpha$ is still available to be believed or disbelieved by the agent in a subsequent process of belief revision triggered by new information.

2) If $\alpha$ is off-topic – i.e. it is a piece of irrelevant information –, then it should be isolated from the current belief change process and kept in critical suspension, with no chance to change its belief-status, unless the reference topic is changed.

Let us then define a belief state in PWK-BR:

**Definition 3.3** (Belief State in PWK-BR)**.** An agent's belief state is a triple $\langle\Theta, \Delta, \Sigma\rangle$. $\Theta$, $\Delta$ and $\Sigma$ are all sets of propositions of $\Phi_{\mathcal{L}}$ (i.e. $\Theta$, $\Delta$, $\Sigma \subseteq \Phi_{\mathcal{L}}$), such that:

*a)* a belief set is defined as $\Theta = \{\alpha : \Theta \vDash_{\mathsf{pwk}} \alpha, \alpha \in \Phi_{\mathcal{L}}\} \setminus \{\alpha : \alpha \text{ is off-topic}, \alpha \in \Phi_{\mathcal{L}}\}$, that is $\Theta$ is PWK-logically closed and does not have any off-topic proposition as member;

*b)* a non-critical suspension set is defined as $\Delta \subseteq \{\beta : \beta \text{ is on-topic}\}$ and $\Delta = \Delta \bigcup \{\neg\beta \mid \beta \in \Delta\}$, for any $\beta \in \Delta$;

*c)* a critical suspension set is defined as $\Sigma \subseteq \{\gamma : \gamma \text{ is off-topic}\}$;

*d)* are exclusive, but not necessarily exhaustive: $\Theta \cap \Delta = \Theta \cap \Sigma = \Delta \cap \Sigma = \varnothing$ and $\Theta \cup \Delta \cup \Sigma \subseteq \Phi_L$.

## 3.3 Expansion, Contraction and Revision in PWK-BR

In PWK-BR, expansion, contraction and revision are three operations that take both a belief state and a proposition as input, and output a new belief state. Specifically, we define such operators as follows:

**Definition 3.4** (Expansion $\oint^{+}$)**.** The expansion of a belief state $\langle\Theta, \Delta, \Sigma\rangle$ with respect to a new proposition $\phi$ is represented by an operator defined from $\langle\langle \mathscr{P}(\Phi_L), \mathscr{P}(\Phi_L), \mathscr{P}(\Phi_L)\rangle, \Phi_L\rangle$ to $\langle \mathscr{P}(\Phi_L), \mathscr{P}(\Phi_L), \mathscr{P}(\Phi_{\mathcal{L}})\rangle$, such that:

$$\oint^{+}(\langle\Theta, \Delta, \Sigma\rangle, \phi) = \begin{cases} \langle\Theta + \phi, \Delta, \Sigma\rangle & \textit{if } \phi \textit{ is on-topic,} \\ \langle\Theta, \Delta, \Sigma \cup \{\phi\}\rangle & \textit{if } \phi \textit{ is off-topic.} \end{cases}$$

where + is the AGM-expansion.[17]

**Definition 3.5** (Contraction $\oint^{-}$)**.** The contraction of a belief state $\langle\Theta, \Delta, \Sigma\rangle$ with respect to a new proposition $\phi$ is represented by an operator defined from $\langle\langle \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}})\rangle, \Phi_{\mathcal{L}}\rangle$ to $\langle \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}})\rangle$, such that:

$$\oint^{-}(\langle\Theta, \Delta, \Sigma\rangle, \phi) = \begin{cases} \langle\langle\Theta, \Delta\rangle - \phi, \Sigma\rangle & \textit{if } \phi \textit{ is on-topic,} \\ \langle\Theta, \Delta, \Sigma\rangle & \textit{if } \phi \textit{ is off-topic.} \end{cases}$$

where − is the AGM-contraction.

**Definition 3.6** (Revision $\oint^{*}$)**.** The revision of a belief state $\langle\Theta, \Delta, \Sigma\rangle$ with respect to a new proposition $\phi$ is represented by an operator defined from $\langle\langle \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}})\rangle, \Phi_{\mathcal{L}}\rangle$ to $\langle \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}}), \mathscr{P}(\Phi_{\mathcal{L}})\rangle$, such that:

[17] $\mathscr{P}$ denotes a power set, which applies to all its occurrences in this article.

$$\oint^{*}(\langle\Theta,\Delta,\Sigma\rangle,\phi)=\begin{cases}\langle\langle\Theta,\Delta\rangle*\phi,\Sigma\rangle & \textit{if } \phi \textit{ is on-topic,}\\ \langle\Theta,\Delta,\Sigma\cup\{\phi\}\rangle & \textit{if } \phi \textit{ is off-topic.}\end{cases}$$

where $*$ is the AGM-revision.

All the AGM postulates can be preserved in PWK-BR. Therefore, PWK-BR counts as an extension of AGM theory. This is ensured by the following theorem, the proof of which can be found in **(author?)** [16]:

**Theorem 3.1.** AGM postulates agree with the PWK-BR.

*Proof.* According to the definitions, AGM operators are adopted to deal with the on-topic part of PWK belief change. $+$, $-$, and $*$ are embedded into $\{\oint^{+},\oint^{-},\oint^{*}\}$. As long as AGM operators follow AGM postulates, $\{\oint^{+},\oint^{-},\oint^{*}\}$ do as well. Therefore, AGM postulates, which regulate $\{\oint^{+},\oint^{-},\oint^{*}\}$, also support this PWK belief change framework based on $\{\oint^{+},\oint^{-},\oint^{*}\}$. □

# 4 PWK Abstract Argumentation Framework and Expansion

## 4.1 Motivating Ideas

In this section we put forward our proposal to account for suspension in a PWK-based argumentation process. Our main considerations are as follows.

First, suspensions should be analyzed and identified in an argumentation theory. Given that (1) both belief revision and argumentation theory are important approaches in knowledge representation to formalize epistemic processes, and that (2) suspensions are identified and distinguished in a PWK belief revision theory, suspensions can be considered in argumentation theory just as they are considered in belief revision theory (recall the discussion in §2.3). One distinction between a belief revision process and an argumentation process lies in their starting points. A belief revision process assumes a consistent set of propositions, while an abstract argumentation process starts with a set of arguments related by binary attack relations. We put forth two suggestions regarding the two different types of suspensions. 1) A non-critical suspension in an abstract argumentation framework occurs when all arguments in the framework are self-attacking. For instance, an argument is self-attacking if its conclusion contradicts one of its premise. (See [9, 4, 12] for recent discussions about self-attacking arguments.) This is problematic because self-attacking arguments cannot be used to justify any other argument. To address this issue, the attack relations connected to these self-attacking arguments should be removed, except for their own self-attack loop. When an argumentation process is suspended in this way, no conflict-free subset of the framework exists. As a result, there are no admissible, grounded, ideal, preferred, or stable extensions in the framework. 2) A critical suspension in an abstract argumentation framework occurs when certain arguments are irrelevant to the topic being discussed in the argumentation process. The reason why a critical suspension in an abstract argumentation framework is important is that it can use up the computational resources and lead the argumentation process to arrive at an incorrect conclusion. In this case, the attack relations of these off-topic arguments should be set apart from the argumentation process.

Second, to analyze suspensions in an argumentation process we can take [16]'s proposal as a plausible approach. It analyzes suspensions in an epistemic change process on the basis of PWK logic with Beall's off-topic interpretation and AGM theory. Similarly, we can consider two kinds of suspensions in an argumentation process by relying on the notion of topic. As discussed in §2.3, our assumption is that an argumentation process has a topic – the reference topic – corresponding to a set of answers to a specific question. Any off-topic epistemic inputs would result in a major interruption of the argumentation process because it is important that it stays on topic without introducing any unrelated information.

Third, it is a feasible task to account for suspensions in argumentation theory by bringing together Dung's abstract argumentation theory and PWK-BR. Dung's argumentation theory and AGM theory have been integrated in a whole comprehensive framework corresponding to the AGM-style abstract argumentation theory (see e.g. [5, 6, 7, 8]). Thus, we claim that a PWK-style abstract argumentation framework can be developed in a similar way from PWK-BR and Baumann, Brewka and Linker's works.

Last, a PWK-style abstract argumentation framework is worth investigating. It is not just an aimless technical integration of all the previously mentioned works, but an integrated view that enables us to account for different kinds of suspensions in argumentation. Since suspension is an important *phenomenon* actually occurring in argumentation processes, the development of a PWK-style argumentation framework is a worthwhile enterprise.

## 4.2 PWK Abstract Argumentation Framework

Given the discussions above, we propose a PWK abstract argumentation framework that has a topic $t$ on the basis of Dung's abstract argumentation framework and some recent proposals concerning integrating Dung's abstract argumentation theory with AGM theory.[18] Let's start by outlining some fundamental definitions of argumentation frameworks before defining a PWK abstract argumentation framework.

**Definition 4.1** (Argumentation framework AF [2])**.** An argumentation framework AF is a pair $\langle \mathsf{Ar}, \mathsf{att} \rangle$ in which Ar is a finite set of arguments and $\mathsf{att} \subseteq \mathsf{Ar} \times \mathsf{Ar}$.

Given an argumentation framework $\mathsf{AF} = \langle \mathsf{Ar}, \mathsf{att} \rangle$ and $\mathsf{Args} \subseteq \mathsf{Ar}$, for any arguments $a, b \in \mathsf{Ar}$, $(a, b) \in \mathsf{att}$ is to be read as "$a$ attacks $b$"; $a$ attacks Args iff there is $b \in \mathsf{Args}$ such that $(a, b) \in \mathsf{att}$; Args attacks $a$ iff there is $b \in \mathsf{Args}$ such that $(b, a) \in \mathsf{att}$; Args attacks $\mathsf{Args}' \subseteq \mathsf{Ar}$ iff there are $a \in \mathsf{Args}, b \in \mathsf{Args}'$, such that $(a, b) \in \mathsf{att}$.

**Definition 4.2** (PWK abstract argumentation framework)**.** A PWK abstract argumentation framework is a triple $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t \rangle$ where Ar is a finite set of abstract arguments, $\mathsf{att} \subseteq \mathsf{Ar} \times \mathsf{Ar}$ is the attack relation, and $t$ is a set of topics, such that:[19]

1. For any arguments $a, b \in \mathsf{Ar}$, $a$ attacks $b$ if $(a, b) \in \mathsf{att}$.
2. $a \in \mathsf{Ar}$ is an on-topic argument if $a$'s topic belongs to $t$ – i.e., $\tau(a) \in t$.
3. $b \in \mathsf{Ar}$ is an off-topic argument if $b$'s topic does not belong to $t$ – i.e., $\tau(b) \notin t$.

Let us explain some assumptions concerning the definition above. First of all, we take every argument $a \in \mathsf{Ar}$ to be an abstract atomic argument, which means we do not assume any specific structure on such arguments. This is in line with [23]. As [3] remarks:

> While the word *argument* may recall several intuitive meanings, abstract argumentation frameworks are not (even implicitly or indirectly) bound to any of them: an abstract argument is not assumed to have any specific structure but, roughly speaking, an argument is anything that may

[18] See [1, 23, 6, 8]

[19] We will use the symbol $\mathscr{PAF}$ to denote the set of all PWK argumentation frameworks.

> attack or be attacked by another argument, where, again, no specific meaning is ascribed to the notion of attack.
>
> [3, p. 12]

In an abstract argumentation framework, the arguments are indeed abstract, which means that they can be adapted to suit different theories about arguments. We take the notion of argument in this abstract way in our framework.

Next, we have made an important assumption for the PWK abstract argumentation framework: that is, any argument $a \in \mathsf{Ar}$ is about only one unique topic. The reason for making such an assumption is intuitive: we want to keep the idea simple enough to be understood. This is helpful for us to clarify our ideas. Indeed, such an assumption limits the possibility that $a$ can be about several different topics. We will confine our discussion to this limited scope in this article.

In order to express being "off-topic", we assume a set of topics, $t$, in a PWK abstract argumentation framework, which is a collection of abstract single topics. For a similar reason, we do not assume that $t$ has any specified structures to express the connections between topics. As any argument corresponds to one topic, it either belongs to $t$, or does not belong to $t$. For brevity, we use $a \in_t t$ to denote that an argument $a$'s topic is included in $t$. In other words, $a \in_t t$ if and only if $\tau(a) \in t$.[20] Hence we can express what an on-topic argument is. That is, $a$ is on-topic of $t$ iff $a \in_t t$; otherwise $a$ is off-topic.

Last, we preserve [23]'s abstract relation $\mathsf{att}$ in the Definition 4.2: we do not assume any specific meaning to the notion *attack*. It just has a form of a binary relation between arguments and does not embody any form of evaluation ([35]). In a PWK abstract argumentation framework, $\mathsf{att}$ can be between any arguments, regardless of their being on-topic or off-topic.

Let us make an example concerning Definition 4.2.

**Example 4.1.** Let a PWK argumentation framework be $\langle \mathsf{Ar}, \mathsf{att}, t \rangle$, where $\mathsf{Ar} = \{a, b, c, d\}$, $\mathsf{att} = \{(a, b), (b, d), (d, a)\}$, and $a, b, c \in_t t$, $d \notin_t t$.

This example shows a PWK argumentation framework that has four arguments $a, b, c, d$ and a set of topics $t$, where $a, b, c$ are on-topic arguments and $d$ is an off-topic argument. $a$ attacks $b$; $b$ attacks $d$; $d$ attacks $a$.

[20] $\in_t$ can be specified in different ways, according to a specific theory of topic. For example, it can be defined by a set of judgment rules that recognize whether an argument belongs to the set of topics $t$ or not.

Next, given the discussion in the previous section, we can establish a classification for two types of suspension within a PWK framework $\langle \mathsf{Ar}, \mathsf{att}, t\rangle$.

**Definition 4.3** (Suspension)**.** Let $\langle \mathsf{Ar}, \mathsf{att}, t\rangle$ be a PWK argumentation framework.

1. A suspension is classified as non-critical if all the on-topic arguments in $\mathsf{Ar}$ attack themselves, meaning that for any $a \in \mathsf{Ar}$ and $a \in_t t$, $(a, a) \in \mathsf{att}$.
2. A suspension is classified as critical if certain arguments in $\mathsf{Ar}$ deviate from the argumentation topic, that is, there exists $b \in A$, such that $b \notin_t t$.

What are the outcomes resulting from these two classifications of suspension? To understand this better, we can define the extensions of a PWK framework.

**Definition 4.4** (Extension)**.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ and $\mathsf{Args} \subseteq \mathsf{Ar}$.

1. $\mathsf{Args}$ is a conflict-free on-topic extension if and only if $\mathsf{Args}$ does not attack itself and for any $a \in \mathsf{Args}$, $a$ is on-topic. That is, $(a, b) \notin \mathsf{att}$ and $a \in_t t$ for all $a, b \in \mathsf{Args}$.
2. $\mathsf{Args}$ is an admissible on-topic extension if and only if $\mathsf{Args}$ is a conflict-free on-topic extension and $\mathsf{Args}$ defends all its elements.
3. $\mathsf{Args}$ is a complete on-topic extension if and only if $\mathsf{Args}$ is a conflict-free on-topic extension and the set of on-topic arguments defended by $\mathsf{Args}$ is equal to $\mathsf{Args}$.
4. $\mathsf{Args}$ is a preferred on-topic extension if abd only if $\mathsf{Args}$ is an admissible on-topic extension and for no admissible on-topic extension $\mathsf{Args}'$, $\mathsf{Args} \subseteq \mathsf{Args}'$.
5. $\mathsf{Args}$ is a grounded on-topic extension if and only if $\mathsf{Args}$ is the minimal complete on-topic extension. That is, $\mathsf{Args}$ is an complete on-topic extension and there is no complete $\mathsf{Args}' \subseteq \mathsf{Ar}$, such that $\mathsf{Args}' \subseteq \mathsf{Args}$.
6. $\mathsf{Args}$ is a stable on-topic extension if and only if $\mathsf{Args}$ is a complete on-topic extension that attacks any on-topic argument in $\mathsf{Ar} \setminus \mathsf{Args}$.

**Lemma 4.1.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ be a PWK argumentation framework. It is considered to be in a non-critical suspension if and only if there does not exist any conflict-free on-topic extension for $\langle \mathsf{Ar}, \mathsf{att}, t\rangle$.

*Proof.* From the left to the right: when $\langle \mathsf{Ar}, \mathsf{att}, t \rangle$ is kept in a non-critical suspension, then for any $a \in \mathsf{Ar}$ and $a \in_t t$ such that $(a, a) \in \mathsf{att}$. As a result, it is impossible to include any such $a$ in a conflict-free on-topic $\mathsf{Args} \subseteq \mathsf{Ar}$, because of the presence of $(a, a) \in \mathsf{att}$. This implies that there are no conflict-free on-topic extensions possible for $\mathsf{Ar}$. From the right to the left: when $\langle \mathsf{Ar}, \mathsf{att}, t \rangle$ does not contain any conflict-free on-topic extensions, then for any $a \in A$ and $a \in_t t$ such that the singleton set $\{a\}$ is not conflict-free. Therefore, it follows that $a$ attacks itself, meaning that $(a, a) \in \mathsf{att}$. □

This outcome is comprehensible because if every on-topic argument within an argumentation framework attacks itself, then it becomes impossible to draw any conclusions from them. Non-critical suspension are considered problematic, because it makes the argumentation framework uninformative by undermining all of the arguments and limiting its ability to provide rational conclusions. Therefore, it is important to prevent non-critical suspension to ensure that the argumentation framework remains informative.

**Lemma 4.2.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t \rangle$ be a $\mathsf{PWK}$ argumentation framework. Let any argument $a \in \mathsf{Ar}$ such that $a \notin_t t$, then there does not exist any conflict-free on-topic extension.

*Proof.* The proof is evident. In case there are no on-topic arguments within the framework, it is impossible to have any conflict-free on-topic extensions. □

Compared to non-critical suspensions, critical suspensions can be less apparent if we do not evaluate whether an argument is on-topic or off-topic. Lemma 4.2 shows that if all arguments in the framework are under critical suspension, then it becomes impossible to have any conflict-free on-topic extensions, thereby undermining the framework.

**Lemma 4.3.** Any abstract argumentation framework $\langle \mathsf{Ar}, \mathsf{att} \rangle$ can be extended to a $\mathsf{PWK}$ abstract argumentation framework $\langle \mathsf{Ar}, \mathsf{att}, t \rangle$ by specifying a set of topics $t$ and a membership relation $\in_t$ between an argument and $t$.

*Proof.* Let $\langle \mathsf{Ar}, \mathsf{att} \rangle$ be an abstract argumentation framework and $t$ be a set of topics. $\langle \mathsf{Ar}', \mathsf{att}', t \rangle$ is derived from $\langle \mathsf{Ar}, \mathsf{att} \rangle$ if $\langle \mathsf{Ar}', \mathsf{att}', t \rangle$ satisfies the following conditions:

1. $\mathsf{Ar}' = \mathsf{Ar}$ and $\mathsf{att}' = \mathsf{att}$;

2. for any argument $a \in \mathsf{Ar}'$, $a$ is on-topic if $\tau(a) \in t$; otherwise, $a$ is off-topic.

Since $\langle \mathsf{Ar}', \mathsf{att}', t\rangle$ satisfies Definition 4.2, it is a PWK argumentation framework, which extends $\langle \mathsf{Ar}, \mathsf{att}\rangle$ by specifying a set of topics $t$. □

To see this lemma clear, let us make the following example that shows a PWK argumentation framework generated by specifying a set of topics $t$. As we discuss before, we try to keep $t$ as simple as possible, and thus we do not specify a method that generates a $t$. In the following example, $t$ is generated by selecting some topics from the arguments' topics. This is not necessarily the only way to generate a $t$.

**Example 4.2.** Let an abstract argumentation framework be $\langle \mathsf{Ar}, \mathsf{att}\rangle$, where $\mathsf{Ar} = \{a, b, c, d\}$, $\mathsf{att} = \{(a, b), (b, c), (d, d)\}$. Let $t = \{\tau(a)\} \cup \{\tau(b)\}$. Then a PWK abstract argumentation framework is $\langle \mathsf{Ar}, \mathsf{att}, t\rangle$, where $a, b \in_t t$ are on-topic. $c, d$ are on-topic if $\tau(c), \tau(d) \in_t t$.

Note that we do not delete any argument and any attack relations to derive a PWK abstract argumentation framework from any abstract argumentation framework. We just add a set of topics $t$ that distinguishes on-topic arguments from off-topic ones.

## 4.3 PWK Argumentation Expansion

To expand a PWK argumentation framework, we use the method described in [6] and define a kind of $\sigma$-kernel that makes constrains on the attack relation by removing from $\mathsf{att}$ certain attack relations that are related with off-topic arguments. To do that, let us introduce the extension-based semantics, and the definition of $\sigma$-kernel, which is a sub-framework of $\langle \mathsf{Ar}, \mathsf{att}\rangle$ that meets specific requirements regarding $\mathsf{att}$.

According to [6, 8], given any $\mathsf{AF} = \langle \mathsf{Ar}, \mathsf{att}\rangle$, $\sigma$ is a function that assigns to $\mathsf{AF}$ a set of sets of *arguments* denoted by $\sigma(\mathsf{AF}) \subseteq 2^{\mathsf{Ar}}$. Generally, there are six basic kinds of $\sigma$ extensions: conflict-free, admissible, complete, preferred, grounded, and stable extensions. For example, a conflict-free extension of $\mathsf{AF}$ is $cf(\mathsf{AF})$: $\mathsf{Args} \in cf(\mathsf{AF})$ iff for all $a, b \in \mathsf{Args}$, $(a, b) \notin \mathsf{att}$. Following this idea, we can define an on-topic extension for $\mathsf{Paf}$ as follows.

**Definition 4.5** (On-topic extension)**.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ be a PWK abstract argumentation framework and $\mathsf{Args} \subseteq \mathsf{Ar}$. Then, an on-topic extension for $\mathsf{Paf}$, $on(\mathsf{Paf})$, is such that $\mathsf{Args} \in on(\mathsf{Paf})$ iff for any $a, b \in \mathsf{Args}$, $a, b \in_t t$.

As a result, $on(\mathsf{Paf})$ is a set of any subset of $\mathsf{Ar}$ that contains only the on-topic arguments. Next, we can define a $\sigma$-kernel, $k(\sigma)$, from $\mathscr{PAF}$ to $\mathscr{PAF}$ by removing certain attack relations from a $\mathsf{Paf}$, such that $\mathsf{Paf}^k(\sigma) = \langle \mathsf{Ar}, \mathsf{att}^{k(\sigma)}, t\rangle$. In particular, let us define a $t$-kernel, namely $k(t)$, which removes the attack relations from $\mathsf{Paf}$ that are related with off-topic arguments. Before that, let us define the following relations between any PWK argumentation frameworks.

**Definition 4.6.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ and $\mathsf{Paf}^* = \langle \mathsf{Ar}^*, \mathsf{att}^*, t\rangle$ be two PWK argumentation frameworks that have the same set of topics $t$.

*1.* $\mathsf{Paf} \subseteq_t \mathsf{Paf}^*$ if and only if $\mathsf{Ar} \subseteq \mathsf{Ar}^*$, $\mathsf{att} \subseteq \mathsf{att}^*$.

*2.* $\mathsf{Paf} =_t \mathsf{Paf}^*$ if and only if $\mathsf{Paf} \subseteq_t \mathsf{Paf}^*$ and $\mathsf{Paf}^* \subseteq_t \mathsf{Paf}$.

**Definition 4.7** ($k(t)$)**.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ be a PWK abstract argumentation framework, and $k(t)$ is an $t$ kernel function, such that $\mathsf{Paf}^{(k(t))} = \langle \mathsf{Ar}, \mathsf{att}^{k(t)}, t\rangle$ and $\mathsf{att}^{k(t)} = \mathsf{att} \smallsetminus \{(a, b) \mid a \notin_t t \text{ or } b \notin_t t\}$.

Next, we shall define a set of on-topic models for a PWK argumentation framework $\mathsf{Paf}$, called $k(t)$-models ($Mod^{k(t)}$). A model of a $\mathsf{Paf}$ is an argumentation framework related to $\mathsf{Paf}$ that satisfies certain conditions.

**Definition 4.8.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ be a PWK argumentation framework. The set of $k(t)$-models of $\mathsf{Paf}$ is defined as $Mod^{k(t)}(\mathsf{Paf}) = \{\mathsf{Paf}^* \mid \mathsf{Paf}^{k(t)} \subseteq_t \mathsf{Paf}^{*k(t)}\}$.

To understand Definition 4.8 better, let us take an example.

**Example 4.3.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t\rangle$ be a PWK argumentation framework, where $\mathsf{Ar} = \{a, b, c\}$, $\mathsf{att} = \{(b, a), (b, c)\}$, and $a, b \in_t t$. Then $\mathsf{Paf}^* = \langle \mathsf{Ar}^*, \mathsf{att}^*, t\rangle \in Mod^{k(t)}(\mathsf{Paf})$, where $\mathsf{Ar}^* = \{a, b, c\}$, $\mathsf{att}^* = \{(b, a), (c, b)\}$, and $a, b \in_t t$.

Next, we consider how a PWK abstract argumentation framework $\langle \mathsf{Ar}, \mathsf{att}, t \rangle$ expands itself with respect to another framework $\langle \mathsf{Ar}^*, \mathsf{att}^*, t^* \rangle$ argumentation framework under a same topic. We denote such an operator as $+^{k(t)}$.

**Definition 4.9.** Let $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t \rangle$ and $\mathsf{Paf}^* = \langle \mathsf{Ar}^*, \mathsf{att}^*, t^* \rangle$ be two PWK abstract argumentation frameworks. A function $\mathsf{Paf} +^{k(t)} \mathsf{Paf}^*$ is a $k(t)$-expansion if and only if $Mod^{k(t)}(\mathsf{Paf} +^{k(t)} \mathsf{Paf}^*) = Mod^{k(t)}(\mathsf{Paf}) \cap Mod^{k(t)}(\mathsf{Paf}^*)$ and $t = t^*$.

Definition 4.9 constrains the expansion of PWK argumentation to a specific set of topics, represented by $t = t^*$. As a result, the set of topics remains the same after the expansion. Moreover, the attack relations that are related to only on-topic arguments are preserved. After the expansion, the off-topic arguments are still considered off-topic, under the same set of topics $t = t^*$.

**Theorem 4.1.** For any PWK argumentation framework $\mathsf{Paf} = \langle \mathsf{Ar}, \mathsf{att}, t \rangle$ and $\mathsf{Paf}^* = \langle \mathsf{Ar}^*, \mathsf{att}^*, t^* \rangle$, there exists an $\mathsf{Paf}' = \langle \mathsf{Ar}', \mathsf{att}', t' \rangle$, such that $Mod^{k(t)}(\mathsf{Paf}') = Mod^{k(t)}(\mathsf{Paf} +^{k(t)} \mathsf{Paf}^*)$ if $t = t^* = t'$. *Moreover, if* $Mod^{k(t)}(\mathsf{Paf} +^{k(t)} (\mathsf{Paf}^*) \neq \varnothing$, *then* $\mathsf{Paf}'^{k(t)} = \mathsf{Paf}^{k(t)} \cup \mathsf{Paf}^{*k(t)}$.

*Proof.* According to Definition 4.9, for any $k(t)$ expansion, $Mod^{k(t)}(\mathsf{Paf} +^{k(t)} \mathsf{Paf}^*) = Mod^{k(t)}(\mathsf{Paf}) \cap Mod^{k(t)}(\mathsf{Paf}^*)$ and $t = t^*$. Therefore, if the intersection $Mod^{k(t)}(\mathsf{Paf}) \cap Mod^{k(t)}(\mathsf{Paf}^*) \neq \langle \varnothing, \varnothing, t \rangle$. Then for any $\mathsf{Paf}^\circ \in Mod^{k(t)}(\mathsf{Paf}) \cap Mod^{k(t)}(\mathsf{Paf}^*)$, $Mod^{k(t)}(\mathsf{Paf}^\circ)$ equals to $Mod^{k(t)}(\mathsf{Paf} +^{k(t)} \mathsf{Paf}^*)$. □

Two PWK argumentation frameworks can be incorporated through PWK argumentation expansion under the same set of topics. Such expansion is different from a PWK belief set expansion operation, because in PWK belief set expansion, the off-topic sentences are collected in $\Sigma$. However, for a PWK argumentation framework expansion, all the off-topic arguments are kept in the argument set $\mathsf{Ar}$ in an isolated way: that is, the attack relations between any off-topic argument and any on-topic argument are deleted to avoid the off-topic arguments from a argumentation process which is around a topic.

## 5 Concluding Remarks

In this article we have presented the basic elements of a PWK-style argumentation framework that extends the abstract argumentation framework and makes a distinction between two kinds of suspension. the AGM belief revision model with two kinds of suspension. What is next? In future works we would like to have a *precise* model to distinguish whether an argument is on-topic or off-topic, by using a game-theoretic semantics, as we have done in other works [15].

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