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Book Reviews

Ernst Schröder, **Parafrasi Schröderiane. Ovvero: Ernst Schröder, Le operazioni del Calcolo Logico**. Original German text with Italian translation, commentary and annotations by Davide Bondoni, LED Edizioni, Milan, 2010, pp. 208, 15,5 × 22 cm, ISBN 978-88-7916-474-0.

1. Schröder, Peirce and Frege

This book is a translation of Ernst Schröder's *Der Operationskreis des Logikkalkuls*[1] ("The Circle of Operations of the Logical Calculus") first published in 1877, a booklet of 37 pages on equational algebraic logic influenced by Boole and Grassmann, and considered by A. N. Whitehead in his *Universal Algebra* of 1898 (p. 115) as an "important pamphlet". The book under review contains the original German text and the first (to my knowledge, the only) Italian translation of Schröder's book, as well as translations of some important related papers, in seven appendices (numbered A to G): a note by Schröder himself ([9]), a famous review by Adamson ([1]), Ellis ([5]), a short paper on the calculus of logic by Cayley ([4]), an excerpt from Boole (pages 146–149 of [2]), an excerpt from Peirce [8]) and a posthumous article by Frege ([6]).

The note by Schröder ([9]) clarifies his own ideas on the logicization of the calculus, his emphasis on the principle of duality, and his explanations of the fundamental operations, including the so-called inverse operations (subtraction and division). Other translated papers refer to the work of Schröder and are concerned with a problem by Boole and its solution by Schröder, also discussed by Peirce and by Frege.

[1] This term is found with different spellings: *Logikkalculs*, *Logikkalkuls* and *Logikkalküls*, as in the famous Doctoral dissertation by K. Gödel in the University of Vienna of 1929, *Über die Vollständigkeit des Logikkalküls*.

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An additional chapter by the book's editor discusses the relations between the analytical development of a function and Taylor's series, and the book ends with a list of the main results to be found in the *Operationskreis des Logikkalkuls*, as well as a fully annotated bibliography which completes the main references and corrects several errors regarding mistakenly cited papers and books.

The book under review, which contains many original comments and notes, is a very valuable tool for understanding the thought of the logicians that were preparing, in the second half of the nineteenth century, the revolutionary development of mathematical logic as an independent discipline in the twentieth century. It is also a very relevant tool for all who intend to compare the original German text of Schröder with a Romance language: Bondoni's work is a very useful resource not only for those who read Italian, but also for those readers whose mother tongue is Spanish, French, Portuguese or even English (if we keep in mind that two thirds of English vocabulary has Latin origins).

Schröder taught number theory, trigonometry and higher analysis at the Karlsruhe University, as well as algebra and the theory of functions, but apparently not much logic. This may help explain his holistic interest in logic as conjoined with algebra and analysis. Not only the *Der Operationskreis des Logikkalkuls* did inaugurate Schröder's logical work, but it also helped to disseminate his logical notation: several logicians adopted the Peirce-Schröder notation, and important parts of logic such as Zermelo's axioms and the Löwenheim (later, Löwenheim-Skolem) theorem were originally expressed in such a notation. With regard to logical notation, as Davide Bondoni himself observes, Schröder's notation is not a formal language like the one in the *Begriffsschrift*, but a *Zeichensprache*, a language of signs, and, I may add, constructs a linguistic scenario with much more capacity for heuristics and interpretation. This fact, I believe, helps us to assess the vocation of Schröder's notation for problem solving.

Peirce and Schröder never met, and their correspondence was unfrequent and often interrupted. While Peirce admired Schröder overall, he sometimes criticized Schröder in harsh terms in private correspondence, and even in public, as he did in a Harvard Lecture of 1903, cf. [7] p. 208:

> The most striking thing in his first volume is a fallacy. His mode of presentation rests on a mistake and his second volume which defends it is largely retracted in his third [and] is one big blunder. There are

some very fine things in his third volume and his posthumous volume I hope will contain still better. He was a growing man.

Schröder, on the other hand, apparently venerated Peirce, and has in some ways contributed to the myth of the "American Aristotle", but he did not have the same regard towards Frege. In his critical review of the *Begriffsschrift* he says that Frege had not created a *lingua characteristica*, but had at most contributed to the creation of a *calculus ratiocinator*, which, he believed, was also the goal of his own system in *Der Operationskreis des Logikkalkuls*. Part of Frege's reply in [6] is due to this provocation.

2. Mistakes, but fruitful ideas

Although popularly held as the work which helped to improve and to clarify Boole's ideas, a closer look at *Der Operationskreis des Logikkalkuls*, made more palatable by this Italian translation, reveals a certain sloppiness in the way Schröder derives his conclusions and states his main results. For instance, Theorem 14, page 37 in the German text ("Entwicklung eines Ausdrucks", on page 98 of the Italian translation), asserts that:

Theorem 2.1 (Theorem 14). *Any class B can be expressed in a linear and homogeneous way by means of any other class A in the form* (i) $B = (X \cap A) \cup (Y \cap -A)$ *where X and V are indeterminate* (*i.e., variables*) *that can be even* [*or also*] *equated to $\varnothing$ or V.*

Schröder starts his analytical development with $B = (B \cap A) \cup (B \cap -A)$ (in his original notation, $b = b.a + b.a_1$) and then proceeds by a strange method of "generalization": since setting $X = Y = B$ makes the equation i satisfied, he concludes for the general case.

But this of course only works if X contains B, or under some other conditions. In particular, X and Y cannot be simultaneously empty, for instance.

The proof of the important Theorem 20 (page 45 in the German text, page 112 of the Italian translation) is also faulty. The theorem states the following:

Theorem 2.2 (Theorem 20). *The equation*

$$(X \cap A) \cup (Y \cap -A) = \varnothing$$

is completely equivalent to

$$X \cap Y = \varnothing \text{ and } A = (U \cap -X) \cup Y,$$

where U is an arbitrary class.

Although the Lemma used in one of the sides of the proof is defective, the theorem is true if one reads it as "where U is an arbitrary class in a specified collection of classes" instead of "where U is an arbitrary class". Indeed, it is sufficient to take U in the collection $\mathfrak{C} = \{A - Z : Z \subseteq Y\}$.

In this way one still gets a general and useful theorem:

Theorem 2.3 (Theorem 20). *Given A, X and Y, the equation*

$$(X \cap A) \cup (Y \cap -A) = \varnothing$$

is completely equivalent to

$$X \cap Y = \varnothing \text{ and } A = (U \cap -X) \cup Y,$$

where U is an arbitrary class in a specified collection of classes.

Indeed, it is enough to take $U \in \mathfrak{C}$; it is easy to see that $U \in \mathfrak{C}$ is infinite if A and Z are infinite classes.

It may be interesting to note at this point that this theorem is easily proven by means of the polynomial ring calculus (cf. [3]), which is a method inspired in the heritage of Boole-Schröder.

An unusual characteristic of Bondoni's approach is also the aesthetic features connecting mathematics and poetry, which he considers exemplified in Schröder's work, specially in the iteration of functions. The book under review ends with an original appraisal of the Taylor series in the work of Schröder, in a chapter titled *Serie di Taylor e sviluppo booleano*. Bondoni criticizes Schröder's recovering of the Boolean developments of a logical form in a Taylor series (power series). Schröder's attempt is a bit confuse, but it is certainly more acceptable in the light of Boole's ideas on discrete differentiation and integration in his "Calculus of Finite Differences" (Boole Press, 2008). It seems clear that Schröder's intuitions on infinite series influenced his later views on infinite sums ($\sum$) and products ($\prod$) as, respectively, universal ($\forall$) and existential ($\exists$) quantifiers. But Schröder was again careless with regard to the ways in which quantifiers and statement connectives interact.

In [11], for instance, C. Thiel reports on an error in Schröder's exposition of the distribution of a quantifier preceding a conditional (Schröder‘s "Subsumption"), an error, according to [11, p. 172–173], only partially corrected by Schröder later on:

> There remains the question why errors like those we have analyzed could be committed by such an outstanding logician as Ernst Schröder. If I may venture upon a conjecture, it seems to me that the treatment of quantification within the algebra of logic, i.e. in the framework of a logic of classes and of propositions, barred or at least impeded a clear insight into the intricate matter, and that it was the deductive approach with its explicit concern for conditionals and implications that made a clarification possible. Obviously this conjecture is rooted in the assumption that Schröder would not have committed his errors if he had been forced to obtain the pertinent formulae by a step-by-step („lückenlos" in Frege's sense) deduction starting from perspicuous quantificational axioms — and such a conjecture does, of course, not admit of any "proof".

It is not hard to conjecture that the inattentive way Schröder dealt with quantifiers would explain his incautious use of quantification in *Der Operationskreis des Logikkalkuls*. This is, however, the risk of being a pioneer, and this book by Davide Bondoni contributes much to our appreciation of the spectacle of the birth of modern logic. It is not only a truly recommendable book, but a book to be imitated.

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