MEETING HINTIKKA'S CHALLENGE TO PARACONSISTENTISM

WALTER CARNIELLI
State University of Campinas — UNICAMP

Abstract. Jaakko Hintikka, in a series of talks in Brazil in 2008, defended that IF ("independence-friendly") logic and paraconsistent logic are, in a sense, very similar. Having sketched the proposal of a new paraconsistent system, he maintains that several achievements of IF logic could be reproducible in paraconsistent logic. One of the major difficulties, left as a challenge, would be to formulate some truth conditions for this new paraconsistent first-order language in order to make IF logic and paraconsistent logic more inter-related. My proposal is that this would demand an innovative game-theoretical semantic approach to paraconsistentism, but also that the syntax of the paraconsistent "Logics of Formal Inconsistency" can model the internal logic of Socratic *elenchi*. I aim to discuss these, and other points posed by Hintikka, as challenges and opportunities for paraconsistentism, paraconsistent logics and IF logics, as well as to raise some criticisms on Hintikka's view about paraconsistency.

Keywords: Independence friendly logic, paraconsistency, Socratic elenchi.

1. What IF logic is not

Independence friendly (IF) logic, as Jaakko Hintiikka defends in several places and in Hintikka 2010, is not any alternative or "nonclassical" logic, but replaces first-order logic by fixing some of its flaws. One of these flaws, according to Hintikka, is that usual quantification theory is unable to express dependence or independence from a variable x bounded by a quantifier $Q(x)$ with respect to another variable y bounded by another quantifier $Q'(y)$. A satisfactory logical syntax ought to be able to express all such logically possible patterns of dependence and independence between variables, but usual quantification does not, since it just specifies that the scope of one quantifier would be included in the scope of the other. Appealing to generalized quantifiers is not better, as the entire theory of quantifiers, generalized from the classics, suffers from analogous deficiency.

The key is to understand how nested quantifiers are connected or dependent. If, for instance, quantifiers Q_1x and Q_2y are nested, as in:

(1) $$(Q_1x)(Q_2y)A(x,y)$$

Principia **13**(3): 283–97 (2009).
Published by NEL — Epistemology and Logic Research Group, Federal University of Santa Catarina (UFSC), Brazil.

the intended meaning is that the scope of $Q_2 y$ is included in the scope of $Q_1 x$; inclusion, however, is just a particular kind of relation (in this case, antisymmetric and transitive, and not necessarily reflexive). But, as much as in relational models of modal logic suggest, why should we be bound to just this relation? Why no relation at all? IF logic aspires to remedy this frailty, but setting quantifiers free. This independency is syntactically expressed by means of a slash notation, $Q_2 y/Q_1 x$, meaning that the quantifier $Q_2 y$ occurring within the scope of $Q_1 x$ is not necessarily bound by just an "inclusion" (linear) relation.

Henkin (1961) provided the first syntactic and semantic analysis for independent quantifiers; for instance

$$\forall x \exists y (\exists z / \forall x) A(x, y, z).$$

The slash means that $\exists z$ is outside the scope of $\forall x$, which appears a previous quantifier (and also outside the scope of $\exists y$). Henkin already noted that Skolem functions could express quantifier unboundedness, for instance in the case of this example, as

$$\exists f \exists g \forall x A(x, f(x), g(z)).$$

As Cook and Shapiro recognize in (1998), IF first-order logic can indeed be seen as a natural extension of first-order logic (not necessarily second-order logic) with enormous expressive resources. Indeed, some important metatheoretical results independent of traditional first-order logic such as the axiom of choice and König's lemma are logical truths of IF logic. Expressing other concepts like infinity and equicardinality, which need higher-order logics, is also possible in IF logic.

Hintikka's observation that games can be assigned to formulas was somehow preceded by Paul Lorenzen in the sixties, but perhaps the most significant finding by Hintikka was that one can read Skolem functions as winning strategies in such games. Starting from game-theoretical semantics (GTS) for traditional first-order logic (with the usual "verifiers" and "falsifiers") a semantics for IF logic can be obtained by allowing games with *imperfect* information. Since falsity is defined as the existence of a winning strategy for the falsifier, not all IF first-order sentences are either true or false, and the law of excluded middle is not validated in the GTS for IF logic. In this respect we already attain an obvious connection between IF logic and paracomplete logics, the dual of paraconsistent logics (cf. Loparić & da Costa 1984). Paracomplete logics do not validate the law of excluded middle, and it sounds amazing that IF logicians have overlooked this 25-year-old idea.

Although Hintikka seemed to believe that no compositional semantics could be given to IF logic (cf. Hintikka 1996), Hodges (in 1997a; see also Hodges 1997b) showed that games can be replaced by compositional semantics, in which a formula is interpreted by a set of sets of assignments. Such sets are called *teams* by Abramsky & Vänänen 2008, where it is shown that Hodges' construction is not *ad hoc*, but is just a particular case of a more general algebraic semantics connected — not surprisingly — to intuitionistic implication. We shall return to this point later on below.

2. IF logic defining new paraconsistent logics

Hintikka claims that IF logic suggests several questions concerning paraconsistent logic, although they have a completely different initial motivation. Taking into account the failure of *tertium non datur* in IF logic and the failure of the Principle of *Pseudo-Scotus* or Principle of *Ex Contradictione Sequitur Quodlibet* in paraconsistent logic, the independence-friendly and the paraconsistent programs can be compared up to duality. Hintikka proposes that, if we translate the role of basic truth-values in IF logic (recalling that not all first-order sentences in IF logic are either true or false, and thus some can be seen as indefinite) we will gain a new first-order paraconsistent logic:

old truth-values	*translated truth-values*
true or indefinite	true
false or indefinite	false
true	true but not false
false	false but not true
indefinite	true and false

To honor the town of Paraty, where the meeting that inspired his paper was held, Hintikka dubbed this new logic *paratyconsistent logic*, which is obtained as a semantical translation of IF logic. Several questions are prompted by this new logic:

(1) How to define the corresponding *syntactical* translation from IF logic which would axiomatize paratyconsistent logic?

This enterprise is in principle not unfeasible, since the true sentences of paratyconsistent logic are obtained from the "not false" sentences in IF logic, which are known to make a recursively enumerable collection. However, it is doubtful that such a recursive axiomatization would grant completeness to paratyconsistent logic with respect to the intended semantics, since IF logic itself is not characterizable by any recursive axiomatization.

But even before tackling the syntactical side of this new paraconsistent logic, its semantics have problems waiting to be clarified. As in any paraconsistent logic, interpreting negation is challenging. A quotation in Hintikka 2010, section 7, adduces a particular point:

> Especially basic are questions concerning negation. What is the natural treatment of negation in paratyconsistent logic? Should the negation that is in fact used in paraconsistent logics be interpreted as the contradictory negation or as some kind of stronger (dual) negation? Some of the problems listed above can only be solved by introducing a second negation into one's logic. How can this be done in the context of paraconsistent logic?

This difficulty, however, has been already faced by paraconsistent logic with a relatively high degree of success: the *possible-translations semantics*, introduced almost two decades ago in Carnielli 1990 (re-worked in Marcos 1999 and Carnielli 2000).

Possible-translations semantics make extensive use of a *translation* between a logic **L**1 and a logic **L**2: a translation is a mapping $* : \mathbf{L1} \longrightarrow \mathbf{L2}$ such that, for every set $\Gamma \cup \{\alpha\}$ of **L**1-formulas,

$$\Gamma \vdash_{\mathbf{L1}} \alpha \text{ implies } \Gamma^* \vdash_{\mathbf{L2}} \alpha^*.$$

Here, α^* denotes $*(\alpha)$ and Γ^* stands for $\{\gamma^* : \gamma \in \Gamma\}$. If 'implies' is replaced by 'iff' in the definition above, then $*$ is called a *conservative translation*. A general account on translations and their generalizations (transfers and contextual translations) is done in Carnielli, Coniglio & D' Ottaviano 2009. It should be remarked that it is this idea of translation, albeit in a less formal way, which underlies the shift from IF logic to paratyconsistent logic explained above.

Now, possible-translations semantics deal in a very natural way with the question of handling two negations (or, in general, several connectives of a similar kind) in a same logic. Consider, for instance, the following three-valued matrices, where T and t are the designated values:

$\wedge$	T	t	F
T	t	t	F
t	t	t	F
F	F	F	F

$\vee$	T	t	F
T	t	t	t
t	t	t	t
F	t	t	F

$\rightarrow$	T	t	F
T	t	t	F
t	t	t	F
F	t	t	t

	$\neg_1$	$\neg_2$	$\circ_1$	$\circ_2$
T	F	F	t	F
t	F	t	F	F
F	T	t	t	F

Such tables give a precise semantical interpretation for the logic **mbC**, a simple and decent logic of the family of the "Logics of Formal Inconsistency" studied in Carnielli, Coniglio & Marcos 2007. The truth-value t may be interpreted as 'true by default', or 'true by lack of evidence to the contrary', and T and F are the familiar 'true' and 'false' truth-values. As it can be seen by inspecting the tables, the matrices for conjunction, disjunction and implication are free from the value T, so the compound sentences are always either frankly false (i.e., receive the truth-value F) or 'true by default'.

The novelty (with respect to traditional semantics) is that there are two distinct interpretations for the negation connective $\neg$ and for the consistency operator $\circ$. This is understandable from the idea that propositions can be valuated in *multiple scenarios*. We may think that propositions are either plain false, or are evaluated as t or T. The first situation concerns a true-by-default proposition, which is treated as a false proposition by the negation $\neg_2$ (since $\neg_2 t = t$) and as true by $\neg_1$ (since $\neg_1 t = F$). On what concerns the consistency operator $\circ$, the first interpretation $\circ_1$ only considers as consistent the 'classical' values T and F, while $\circ_2$ considers that nothing is consistent (i.e., $\circ_2$ assigns F to every truth-value) . This can be viewed as two "logical worlds", one endowed with $\wedge, \vee, \rightarrow, \neg_1$ and $\circ_1$ and other with with $\wedge, \vee, \rightarrow, \neg_2$ and $\circ_2$. However, these two "logical worlds" engender combined valuations, according to which three-valued interpretation is assigned to the connectives by the translations.

This collection of truth-tables, which we call $\mathscr{M}_0$, will be used to give the desired semantics for **mbC**. Now, considering the algebra $For^0_{\mathscr{M}}$ of formulas generated by $\mathscr{P}$ over the signature of $\mathscr{M}_0$, let's define the set $\textsc{Tr}_0$ consisting of all functions $*: For^\circ \longrightarrow For^0_{\mathscr{M}}$ subjected to the following clauses:

$(tr0)$ $p^* = p$, if $p \in \mathscr{P}$;

$(tr1)$ $(\alpha \# \beta)^* = (\alpha^* \# \beta^*)$, for all $\# \in \{\wedge, \vee, \rightarrow\}$;

$(tr2)$ $(\neg\alpha)^* \in \{\neg_1\alpha^*, \neg_2\alpha^*\}$;

$(tr3)$ $(\circ\alpha)^* \in \{\circ_1\alpha^*, \circ_2\alpha^*, \circ_1(\neg\alpha)^*\}$.

We say the pair $\mathrm{PT}_0 = \langle \mathscr{M}_0, \textsc{Tr}_0 \rangle$ is a *possible-translations semantical structure for* **mbC**. If $\vDash^0_{\mathscr{M}}$ denotes the consequence relation in $\mathscr{M}_0$, and $\Gamma \cup \{\alpha\}$ is a set of formulas of **mbC**, the associated PT-consequence relation, $\models_{\mathrm{PT}_0}$, is defined as:

$$\Gamma \models_{\mathrm{PT}_0} \alpha \ \text{ iff } \ \Gamma^* \vDash^0_{\mathscr{M}} \alpha^* \ \text{ for all translations } * \text{ in } \textsc{Tr}_0.$$

We will call a *possible translation* of a formula α any image of it through some function in $\textsc{Tr}_0$.

Not only **mbC** is sound and complete with respect to such possible-translations semantics, but all logics C_n in the well-known da Costa's hierarchy can be characterized by such semantics in terms of copies of a three-valued logic ($LFI1$, or equivalently, $J3$, see Carnielli 2000).

Although specially devised to deal with paraconsistency, possible translations semantics can be also applied to several other logical systems; but in connection with paraconsisent logics it offers a solution to the difficult problem of algebraizing the logics C_n by means of a new concept of algebraization, as argued in Bueno-Soler & Carnielli 2005.

In (2010), Hintikka claims that a perfect correlation between IF logic and paraconsistency would only be achieved if paraconsistent logicans were prepared to give up compositionality:

> . . . the semantics of IF first-order logic is unavoidably noncompositional. This non-compositionality is due to the fact that the force of a quantifier is affected by its dependence and independence of the quantifiers in whose scope it occurs. (Compositionality is equivalent with semantical context-independence.) Now to the best of my knowledge non-compositional semantics for paraconsistent logic has not been seriously contemplated. Hence something has to be fundamentally changed if in semantical treatments of paraconsistent logic paraconsistent logicians want to use the bridge provided by paratyconsistent logic for the purpose of reaching a self-applicable truth condition and for the wider purpose of enabling a paraconsistent language to serve as its own metalanguage. In a general theoretical perspective, having to give up compositionality is likely to be the deepest methodological change needed for paraconsistent logicans to make use of the kinship of paraconsistent logic with IF logic.

This requirement for "the deepest methodological change needed for paraconsistent logicans to make use of the kinship of paraconsistent logic with IF logic" seems strange in view of the above mentioned compositional semantics given by Hodges to IF logic. The challenge reduces, perhaps, to whether the possible-translations semantics is compositional. But considering that the "Principle of Compositionality" (or "Frege's Principle") asserts that the meaning of a complex expression is fully determined by the meanings of its constituents and by its structure, it is unequivocal that possible-translations semantics are compositional. The only caveat is that the meaning of the constituent expressions is also determined by the translations, which are part of the rules used to combine constituent expressions. So, this challenge seems to be easily dissolved by no-show: on the one hand, the semantics of IF first-order logic does not need to be unavoidably non-compositional, and on the other hand the semantics of paraconsistent is indeed compositional.

3. Interrogative games for paraconsistent and IF logics

Hintikka criticizes paraconsistent logic by its "weak spot": its interpretation. And blow his own horn on proudly speaking on the impact of IF logic as "the best thing that could have happened to paraconsistent logic".

> But what on earth can it mean for a proposition to be true and false? If Cain tells in a cross-examination that it is both true and false that he killed Abel, he might be cited for contempt of the court. And it does not seem to be any less strange to say that a proposition and its negation can both be true. The existing discussions of paraconsistent logics do not yield a satisfactory unique account of the concrete down-to-earth meaning of self-contradictory but yet not disprovable propositions. And it would seem that it will be equally difficult to give a fully operationalized account of how a proposition and its negation can both be true.

There is a big distinction, however, between "a proposition to be true and false" (a situation which never concerned paraconsistency) and another situation where a proposition and its negation can both be true. Hintikka conflates the two situations, claiming that the problem of interpreting propositions of paraconsistent logic that are both true and false corresponds to the problem of interpreting the propositions in an IF language that have the truth-value "indefinite". But this "indefinite" value of IF logic, if it really means that a proposition is both true and false, does not appeal to paraconsistency.

He is basically right when he talks about game theoretical semantics. Not, as he puts it, because "the game theoretical semantics for IF logic seem to provide paraconsistent logic with the concrete semantical interpretation it has been missing", but due to the fact that we can give a *dual* game theoretical semantics to certain paraconsistent logics with respect to IF logic.

The semantical games that interpret IF logics emerge from the intuition that the truth of a sentence S means the existence of a winning strategy for the "verifier" in a semantical game $G(S)$ related to S, so to try to determine the truth of S is to determine whether such a winning strategy exists or not.

Such games are questioning or interrogative games, deeply distinct from semantical games that see the notion of truth as equivalent to the notion of winning a game. Interrogative games are knowledge-seeking procedures, according to Hintikka 2004, ch. 13, and are central on Aristotle's theory and practice of scientific and philosophical argumentation. Aristotle was committed to two-person interrogative games (not dissimilar to the Socratic *elenchus*) in his early proposal for a scientific method, not only in the *Topics*, but in both the *Analytics*. Dialectical games are at the origin of logic (cf. Hintikka 2007, cf. also Kapp 1942): Aristotle became the founder of deductive logic by just studying predictable answers.

I completely agree with Hintikka (cf. Hintikka 2010) that the problem of coping with contradictory information belongs to interrogative games (not to semantical games or to the formal games of theorem-proving). But he says more:

> A different question which is not even controversial is that the logic of interrogative games should be paraconsistent in the sense of admitting situations where one's prima facie information involves contradictions.

But how could contradictions be dealt with by means of strategic rules? As much as games that interpret IF logics emerge from the intuition that the truth of a sentence S means the existence of a winning strategy for the "verifier" in a game $G(S)$ related to S, the games that would interpret paraconsistent logics would amount to the existence of a *non-losing* strategy for the "falsifier",[1] as in the famous Tic-Tac-Toe game: any of both players cannot lose if playing strategically, but cannot win either if the adversary is also playing strategically. Of course, every game ends in a tie if both players are playing strategically. So, to determine (from the game-theoretical viewpoint) the truth of S in a paraconsistent logic (in particular, the "paratyconsistent") is to determine whether such a non-losing strategy exists or not. This does not exclude, I believe, the interest on deductive rules of inference.

4. Paraconsistency and the *elenchus*

In the dialectical games, as exemplified in dialogues of Plato with Socrates, a proponent of a thesis A has to reply to a series of questions from an opponent whose task is to show, by driving the proponent to an *elenchus*, that his thesis A is part of an inconsistent set of premises. The interpretation that the opponent (a role usually undertaken by Socrates), when conducting the dialogue to an *elenchus*, is actually arguing for the truth of $\neg A$ is convincingly rejected by Castelnérac & Marion:

> The point of dialectical games is therefore that the proponent of a thesis A has to pass a *test of consistency*: A has to fit into a set of beliefs Γ held by its proponent in such a way that the addition of A to Γ does not render it inconsistent. If and when the opponent unveils an inconsistency, it is shameful for the proponent.

They cite, as an example, Charmides and Critias with their high opinions on themselves, and Critias becoming silent at Charmides (169e) because of feeling ashamed of having fallen upon an inconsistency.

The dialogical games are a kind of modern hair of dialectical games, which inherits a dialogue ability to provide a semantics for several logical systems. A dialogical formulation of paraconsistency was proposed in Rahman & Carnielli 2000 which permits the building of paraconsistent and paracomplete systems (systems rejecting *tertium non-datur*) for propositional and first-order logic, with their corresponding tableaux (see remarks, criticisms and proposals for furthering in Van Bendegem 2001).

Dialectical games and dialogical games can be seem to be symmetrically opposite, as defended in Castelnérac & Marion. In dialectical games the Proponent must first commit himself to a thesis A, to which the Opponent adduces, with the Proponent's agreement, a new hypothesis B which leads to an absurdum. In dialogical logic, the situation is dually symmetrical: the Opponent is first committed to a thesis A. Although this may add force to the above proposal of a "non-losing strategy" for games that would interpret paraconsistent logics, I cannot completely discern the relevance of such dual symmetry. For the purpose of this discussion it is important to show how paraconsistent logics can indeed establish that the Opponent argues for invalidity in dialectical games, as argued in Castelnérac & Marion.

In fact, if the Proponent commits himself to further theses (say, in conjunct B, for simplicity) and at a final step of the dialogue the Opponent is able to show the inconsistency of the $\Gamma = \{A, B\}$, and then inferring A from B are, as a matter of fact, two distinct inferential moves in paraconsistent logics.

By borrowing the interpretation of the *elenchus* in Castelnérac & Marion:

1. The Proponent assert a thesis, A, which the Opponent (Socrates) considers false and targets for refutation.

2. Socrates brings further premises into agreement, say collectively referred to as B, which Socrates intends to use.

3. Socrates then argues (and the Proponent agrees), that B entails $\neg A$.

4. Socrates then claims that he has shown that $\neg A$ is true and A false.

The main criticism against this interpretation by Vlastos (1994, 11), in short, is (using logic notions not available to the Greeks, but in a benevolent sense) that if $\Gamma = \{A, B\}$ is inconsistent, then $A, B \vdash \perp$, where $\perp$ is the symbol for absurdity or contradiction (*falsum*). This is in line with the fact that in usual logic systems inconsistency and contradictions are indistinguishable, but if $A, B \vdash \perp$ then $B \vdash \neg A$, as the intended interpretation maintains; however, it also holds that $A \vdash \neg B$. This is clearly incoherent with any claim of having shown A to be false and $\neg A$ true.

It is not difficult to be convinced, indeed, that the Socratic method does not concern justifying truth claims, but rather testing for logical coherence or consistency. In favor of Vlastos program, it has to be said that in Vlastos 1982, 711, where commenting on the difference of what he calls "indirect elenchus" versus the "standard elenchus" by the Socrates of Plato's earlier dialogues, he does not only point out that the former mode of argument is a potent instrument for exposing inconsistency within the interlocutor's beliefs, but at page 714 of Vlastos explicitly say:

> Success in elenctic argument need not show that one's own beliefs are consistent; it may show only that the opponent's efforts to probe their inconsistencies have been blocked by one's superior dialectical skill. Socrates could hardly be unaware of these hazards.

But if this is Socrates' main instrument of philosophical investigation, why should we not devote more attention to the minimal logic which supports Socratic reasoning, instead of committing Socrates to all the formidable arsenal of full "classical" logic?

The logic **mbC**, briefly explained at Section 2, permits us to express the main point underlying the *elenchus* starting from the consistency operator $\circ$. Indeed:

1. The proponent asserts a thesis, A
2. Socrates procures further premises, say B
3. Socrates then argues (with the Proponent's agreement), that $\{A, B\}$ is inconsistent; in our meticulous logic **mbC**, $A, B \vdash_{mbC} \neg \circ (A \wedge B)$, but from this it does not follow neither $\neg A$ nor $\neg B$;
4. Socrates has then driven the Opponent to an *elenchus* without any commitment to $\neg A$ being true, and far less to $\neg B$ being true.

An important point is that, in (3), the fact that $A, B \vdash_{mbC} \neg\circ(A \wedge B)$ does not entail in **mbC** neither $\neg A$ nor $\neg B$ can be *proven* through the possible-translation semantics for **mbC**.

Although the consistency operator $\circ$ of **mbC** is very elementary, further properties of consistency(e.g., propagation of consistency through disjunction) can be added by just attaching new axioms; details can be found in Carnielli, Coniglio & Marcos 2007 and in Carnielli & Marcos 2002. The logic **Ci**, another member of the family of the "Logics of Formal Inconsistency", is obtained from **mbC** by adding (cf. Theorem 102 in Carnielli, Coniglio & Marcos 2007) the following axiom schemas:

(ci) $\neg\circ\alpha \rightarrow (\alpha \wedge \neg\alpha)$,

(cf) $\neg\neg\alpha \rightarrow \alpha$.

It is to be noted that axiom **(ci)** establishes that the negation of consistency implies contradiction, while axiom **(cf)** is just the reduction of negations.

Now one may define an *inconsistency* connective $\bullet$ by setting $\bullet\alpha := \neg\circ\alpha$. It can be shown that $\neg\circ\alpha$ and $(\alpha \wedge \neg\alpha)$ are equivalent in **Ci**; moreover, **Ci** has some (restricted) forms of contraposition which were missing in **mbC**, such as $\alpha \rightarrow \circ\beta \vdash \neg \circ \beta \rightarrow \neg\alpha$.

But we can go a bit further. The logic **Cio**, another logic of formal inconsistency, is obtained (see Carnielli, Coniglio & Marcos 2007, 71) by adding the following axiom schemas to **Ci**:

(co1) $(\circ\alpha \vee \circ\beta) \to \circ(\alpha \wedge \beta)$;

(co2) $(\circ\alpha \vee \circ\beta) \to \circ(\alpha \vee \beta)$;

(co3) $(\circ\alpha \vee \circ\beta) \to \circ(\alpha \to \beta)$.

Now, in **Cio** the law of "spreading consistency", $\circ\alpha \to \circ(\alpha \wedge \beta)$, holds well; together with the above contraposition law of **Ci** one thus easily derives:

$$\vdash_{Cio} \neg \circ (\alpha \wedge \beta) \to \neg \circ \alpha$$

which, from the *elenchus*

$$A, B \vdash_{Cio} \neg \circ (A \wedge B)$$

gives:

$$A, B \vdash_{Cio} \neg \circ A$$

but since

$$\neg \circ A \vdash_{Cio} \neg A$$

we in fact obtain

$$A, B \vdash_{Cio} \neg A,$$

of course, at the cost of *assuming* a lot of logical laws. It is worth noting that all such laws are classically valid, but it is clearly excessive to assume all of them in the elenchic derivation. What seems to be conspicuous is that we *do not* need to assume all the machinery of "classical" logic in order to logically express the *elenchus*. But it is really relevant to make clear that we are not supposing any extravagant variety of logic, but just a careful fragment.

The fact that this logical fragment is able to express elenchic derivations, by itself raises serious doubts on an indulgently accepted tradition: 'classical" logic does not seem to coincide with the logic of the classics!

Of course, we also obtain in our analysis the undesired $A, B \vdash_{Cio} \neg B$, which only shows how excessive the extra laws are. It is also to be remarked that what one derives is $\neg A$ for a *paraconsistent* negation $\neg$. This is *not* classical negation yet; in order to derive $\sim A$, where $\sim$ has all properties of classical negation, we need a further assumption on the degree of consistency (or 'entrenchment', as in Castelnérac & Marion) of B. Only in a situation where one can secure this information on B such a derivation is possible. In our setting, this means precisely assuming or granting $\circ B$. The fact that this additional information is necessarily extra-logic is argued by Hintikka himself in Hintikka 2010:

> No logic in a genuine sense of the term can provide strategies for deciding which putative information to reject or not to reject in a context of actual material inquiry.

But we are left with a conceptual problem: in the "Logics of Formal Inconsistency', as we have seen, contradiction needs not to be identified with inconsistency, and neither is consistency necessarily identifiable with non-contradiction (though they *may* coincide). When not coincidental, what does consistency (represented by the operator $\circ$) really mean?

Well, many attributes can be associated with joint denial without being represented by contradictory pairs. In a procedure, for instance, "duration" and "end" are contrary ideas –they cannot hold together, but a procedure may has not been started yet, and not finished. So they are contrary, but not contradictory. Or 'all courage is endurance of the soul' and 'some kind of courage is not endurance of the soul' which, again, cannot be both true but can be both false. To suppose that the notion of contrary subsumes the notion of contradictoriness seems to be in line, for instance, with Anton 1987; in this way contradictory attributes are just a subset of contrary attributes, and "inconsistency" may apply to contrary attributes as well, not only to contradictory ones. When driven to an elenchus, the proponent may be contemplating an inconsistent set of premises in the sense of contrary premises, a situation which is not acceptable, but which is not identified as a contradiction.

I do not mean that inconsistency is reducible to contrariness, just that it encompasses contradictoriness. Back to our logical discussion, assuming $\circ(A \wedge B)$ may mean, for instance, to assume that A and B are not contraries.

The are lots of misunderstanding, possibly by lack of attention to the relevant literature. In Pietarinen 2002, for instance, when referring to the well-known Jaskowski's problem A. Pietarinen, perhaps unpremeditatedly, says that: "This problem [of finding a logic deserving the label name of paraconsistent] has proved elusive thus far, as the received paraconsistent logics fall short of having a satisfactory semantic explication."

Forms of dialectical (and dialogical) games are, of course, played today, and they can solve informational conflicts and provide new information. In a criminal investigation, when interviewing suspects, a detective will only know that one of them is lying if an inconsistency (what we may call a tripartite *elenchus*) arises. The reason Hintikka in (2010: 12) tags such useful information as an "illusion" is disconcerting, for the least:

> Perhaps the illusion that paraconsistent logics could tell us something about the rejection of information is due to an uncertainty about the meaning of negation in paraconsistent logic.

5. How far can we go? Mind the gap!

The rapprochement between paraconsistency and independence-friendly modes of thinking is due to Hintikka's perspicuity, but the issue raises many more questions than he is perhaps willing to admit. It may be true (see quotation above) that the impact of independence-friendly logic is one of the best things that could have happened to paraconsistent logic, but the reciprocal is also worthy of admission.

The paraconsistency program seems to be broader than the independence-friendly movement: certain dialogical games have a paraconsistent character, as shown in Rahman & Carnielli 2000, and certain dual IF logic, as the 'paratyconsistent' logic sketched by Hintikka, will be paraconsistent.

The insistence of non-compositional semantics as a "semantical solution" for paraconsistency does not seem to be very fruitful, as argued at Section 2. Game theoretical semantics (emphasizing non-losing strategies, rather than winning strategies) can be given to paraconsistent logics, or at least to 'paratyconsistent' logic, but paraconsistent logics have other semantics available, as the possible-translations semantics.

If the internal logic of interrogative games (so important in Hintikka's view on Aristotle's theory) should be paraconsistent as he admits, the analogous roles between the verifier and falsifier in game semantics, and the Proponent and the Opponent in dialectical games, does not seem to give any definite reason to dismiss "the formal games of theorem-proving" (Hintikka 2010: 12). On the contrary, the theorem-proving machinery of the Logics of Formal Inconsistency, which represent a good majority of paraconsistent logics, permits accurate expression of the internal logic of an *elenchus*.

A consequence of narrowing the gap between paraconsistent and independence-friendly logics is that difficult questions are posed for both sides, and the conciliation also deeply defies theorists of IF logic as well as of paraconsistency. Borrowing from Hintikka: what else could logicians hope for?

References

Abramsky, S. & Väänänen, J. 2009. From IF to BI: a tale of dependence and separation. *Synthese* **167**(2): 207–30.

Anton, J. P. 1987. *Aristotle's Theory of Contrariety*. Lanham: University Press of America.

Bueno-Soler, J. & Carnielli, W. A. 2005. Possible-translations algebraization for paraconsistent logics. *Bulletin of the Section of Logic* **34**(2): 77–92.

Cameron, P. & and Hodges, W. 2001. Some Combinatorics of Imperfect Information. *Journal of Symbolic Logic* **66**(2): 673–84.

Carnielli, W. A. 1990. Many-valued logics and plausible reasoning. *Proceedings of the Twentieth International Symposium on Multiple-Valued Logic*. Charlotte, NC: IEEE Computer Society, 328–35.

———. 2000. Possible-translations semantics for paraconsistent logics. In Batens, D. et al. (eds.) *Frontiers of Paraconsistent Logic: Proceedings of the I World Congress on Paraconsistency*, Logic and Computation Series. Baldock: Research Studies Press, King's College Publications, 149–63.

Carnielli, W. A. & Marcos, J. 2002. A Taxonomy of C-Systems. In W. A. Carnielli; M. E. Coniglio; I. M. L. D'Ottaviano (eds.) *Paraconsistency: the Logical Way to the Inconsistent*. (Lecture Notes in Pure and Applied Mathematics, v. 228.) New York: Marcel Dekker, 1–94. Preprint available at *CLE e-Prints* **1**(5).
`http://www.cle.unicamp.br/e-prints/abstract_5.htm`

Carnielli, W. A.; Coniglio, M. E.; Marcos, J. 2007. Logics of formal inconsistency. In D. Gabbay and F. Guenthner (eds.) *Handbook of Philosophical Logic*, v. 14, 2nd. ed. Springer, 15–107. Preprint available at *CLE e-Prints* **5**(1).
`http://www.cle.unicamp.br/e-prints/vol_5,n_1,2005.html`

Carnielli, W. A.; Coniglio, M. E.; D'Ottaviano, I. M. L. 2009. New dimensions on translations between logics. *Logica Universalis* **3**(1): 1–18.

Castelnérac, B. & Marion, M. (to appear) Arguing for inconsistency: dialectical games in the academy. In G. Primiero and S. Rahman (eds.) *Acts of Knowledge: History, Philosophy and Logic*. London: College Publications.

Cook, R. & Shapiro, S. 1998. Hintikka's Revolution: "The Principles of Mathematics Revisited." *Brit. J. Phil. Sci* **49**: 309–16.

Hintikka, J. 1996. *Principles of Mathematics Revisited*. Cambridge: Cambridge University Press.

———. 2007. Socratic Epistemology: Explorations of Knowledge-Seeking by Questioning. New York: Cambridge University Press.

———. 2004. Analyses of Aristotle. Dordrecht: Kluwer Academic Publishers.

———. 2010. IF Logic Meets Paraconsistent Logic. In W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano (eds.) *The Many Sides of Logic*. London: College Publications, 3–13.

Hintikka, J.; Halonen, I.; Mutanen, A. 1999. Interrogative logic as a general theory of reasoning. In R. Johnson and J. Woods (eds.) *Handbook of Applied Logic*. Dordrecht: Kluwer Academic Publishers.

Hintikka, J. & Sandu, G. 1996. Game-theoretical semantics. In Johan van Benthem and Alice ter Meulen (eds.) *Handbook of Logic and Language*. Amsterdam: Elsevier.

Hodges, W. 1997a. Compositional semantics for a language of imperfect information. *Logic Journal of the IGPL* **5**: 539–63.

———. 1997b. Some strange quantifers. In G. R. J. Mycielski and A. Salomaa (eds.) *Structures in Logic and Computer Science*. Vol. 1261 of Lecture Notes in Computer Science. Springer, 51–65.

Kapp, E. 1942. *Greek Foundations of Traditional Logic*. New York: Columbia University Press.

Loparić, A. & da Costa, N. C. A. 1984. Paraconsistency, paracompleteness and valuations. *Logique et Analyse* **106**: 119–31.

Marcos, J. 1999. Semânticas de Traduções Possíveis (Possible-Translations Semantics, in Portuguese). Master's thesis, State University of Campinas (Unicamp), Campinas.

http://www.cle.unicamp.br/pub/thesis/J.Marcos

Pietarinen, A. 2002. Some Useful Social Metaphors in Logic. In H. R. Arabnia and Y. Mun (eds.) *Proceedings of the International Conference on Artificial Intelligence*. CSREA Press, 1278–82.

Rahman, S. & Carnielli, W. A. 2000. The dialogical approach to paraconsistency. *Synthese* **125**(1–2): 201–32.

Van Bendegem, J. P. 2001. Paraconsistency and dialogue logic: critical examination and further explorations. *Synthese* **127**(1–2): 35–55.

Vlastos, G. 1982. The Socratic Elenchus. *The Journal of Philosophy* **79**(11): 711–14.

———. 1994. *Socratic Studies*. Cambridge: Cambridge University Press

WALTER A. CARNIELLI
Centre for Logic, Epistemology and the History of Science – CLE
and Department of Philosophy
State University of Campinas – UNICAMP
P.O. Box 6133
13083-970 Campinas, SP
Brazil
walter.carniell@cle.unicamp.br

Resumo. Em uma série de seminários e conferências no Brasil em 2008, Jaakko Hintikka, em uma série de palestras no Brasil em 2008, defendeu que a "IF-lógica" ("independence friendly logic") e a lógica paraconsistente são, em certo sentido, bastante similares. A partir do esboço de um novo sistema paraconsistente, ele afirma que várias potencialidades da IF-lógica podem ser reproduzidas na lógica paraconsistente. Uma das grandes dificuldades, deixada como um desafio, seria a formulação de condições de verdade para esta nova linguagem paraconsistente de primeira ordem, com vistas a garantir uma maior inter-relação entre esta e a IF-lógica. Argumento que tais condições de verdade não somente sugerem uma abordagem inovadora em semântica de jogos para a paraconsistência, mas também que a partir deste ponto de vista as "Lógicas da Inconsistência Formal" podem modelar a lógica interna dos elencos socráticos. Pretendo discutir estes e outros pontos levantados por Hintikka, tomando-os como desafios e oportunidades para o paraconsistentismo e para a IF-lógica, bem como levantar algumas críticas sobre a visão de Hintikka sobre a paraconsistência.

Palavras-chave: IF-lógica, paraconsistência, elencos socráticos.

Notes

[1] There are some evidences that Hintikka would agree to this suggestion, which I proposed during his seminars at UNICAMP in 2008.