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Review of *Philosophical Logic: An Introduction to Advanced Topics**

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This book serves as a concise introduction to some main topics in modern formal logic for undergraduates who already have some familiarity with formal languages. There are chapters on sentential and quantificational logic, modal logic, elementary set theory, a brief introduction to the incompleteness theorem, and a modern development of traditional Aristotelian Logic: the "term logic" of Sommers (1982) and Englebretsen (1996). Most of the book provides compact introductions to the syntax and semantics of various familiar formal systems. Here and there, the authors briefly indicate how intuitionist logic diverges from the classical treatments that are more fully explored.

The book is appropriate for an undergraduate-level second course in logic that will approach the topic from a philosophical (rather than primarily mathematical) perspective. Philosophical topics (sometimes very briefly) touched upon in the book include: intuitionist logic, substitutional quantification, the nature of logical consequence, deontic logic, the incompleteness theorem, and the interaction of quantification and modality. The book provides an effective jumping-off point for several of these topics. In particular, the presentation of the intuitive idea of the

* George Englebretsen and Charles Sayward, *Philosophical Logic: An Introduction to Advanced Topics* (New York: Continuum International Publishing Group, 2013), 208 pages.

incompleteness theorem (chapter 7) is the right level of rigor for an undergraduate philosophy student, as it provides the basic idea of the proof without getting bogged down in technical details that would not have much philosophical interest. This chapter would serve as a strong basis for an in-class discussion of the philosophical significance of the result, especially if the book were supplemented with other readings that explore such matters. Similarly for the discussion of quantification and modality: the chapter clearly presents the problem of using a fixed-domain semantics (since intuitively it seems that different objects may exist at different possible worlds) and it proposes a standard variable-domain semantics to fix this problem. Again, the technical presentation of the two systems, together with some brief philosophical remarks, set the stage for a more complete philosophical exploration (appropriately supplemented with other readings that discuss the philosophical aspects of the problem in more detail).

The other topics that the authors take up similarly set the stage for philosophical discussion, but will require somewhat more filling-in for philosophical purposes. For example, when covering the portion of the book that deals with substitutional quantification (pp. 59-60), one would likely want to supplement the text with some examples and discussion of philosophical motives for analyzing some instances of quantification in English as substitutional. Or, when discussing the portions of the book that deal with intuitionist logic (especially in the Introduction), one would want to supplement the book with a discussion of the constructivist philosophy of mathematics or anti-realist views that typically form the philosophical basis for intuitionist logic. The discussion of "logical form" at the outset of the book (pp. 4-5) is quite compact, and would benefit from some discussion of how one might distinguish between logical constants and other expressions. The chapter on elementary set-theory (chapter 6) would benefit from a presentation of the idea that the ZF set theory (partially) presented is often thought to provide an intuitive way of thinking about sets (the "iterative" conception) that is the main modern alternative to the inconsistent naïve conception. In each of these examples, the material presented is technically proficient, and in that sense could form the starting-point for a philosophical exploration. But the text itself does not indicate what the philosophical issues are. An effective course focused on philosophy would thus need to supplement the discussion in the book.

In such a short book (around 200 pages), the authors have had to pick and choose among possible topics. As a result, the book perhaps understandably does not address several topics of philosophical interest: many-valued logic, secondorder logic, free logic, tense logic, epistemic logic, relevance logic, counterfactuals, the logic of indexicals and demonstratives, generalized quantifiers, or different approaches to definite descriptions. I would note, however, that a discussion of descriptions might have fit nicely into the chapter on quantifier logic; as it stands, that chapter covers just the standard semantics of quantifier logic that is normally covered in a first course in formal logic. One could easily cover descriptions, however, using the chapter on quantifier logic as a starting point.

A unique and interesting aspect of the text is that it extensively covers Aristotelian logic, including *modal* Aristotelian logic. The authors even spell out "bridging rules" that allow one to translate sentences from the language of standard quantifier logic into their language for term logic (and vice versa). Students interested in the historical roots of modern formal logic will be well served by this portion of the book.

I should emphasize that the book is not an introduction to *mathematical* logic (as in, e.g., Enderton 2001 or Mendelson 2009). Most notably, the book does not cover metalogical results other than soundness and completeness for sentential logic, whereas a class on mathematical logic would normally cover additional results such as completeness for quantifier logic, the compactness theorem, and the Lowenheim-Skolem theorem. Furthermore, the book only briefly touches on matters related decidability, does not introduce the concept of mathematical induction, and does not explain different approaches in proof-theory: axiomatic vs. natural deduction systems, for example. The chapter on set-theory proves that sets are never equinumerous with their powersets, but does not explain the significance of this for understanding the infinite (the concept of transfinite numbers and the concept of cardinality are not introduced). For these reasons, a second course in logic from a mathematical perspective will find the book to be too limited in scope.

Many novices will struggle with the terse writing style in the more technical parts of the book. For example, the proof of the completeness theorem for sentential logic will, in my estimation, not be accessible to (at least many) undergraduates. In particular, the authors often assume that it is clear how one proposition follows from another, even though they do not always spell out in "baby step detail" exactly how the proposition follows. To give just one example, without any further remarks, the authors inform the reader (33) that from these two propositions:

Proposition 1 For any set A of SL sentences and SL sentence ϕ: A $\vdash \phi$ if and only if it is not the case that A $\cup \{\neg\phi\}$ is d-consistent.

Proposition 2 For any set of SL sentences A and SL sentence ϕ, A $\models \phi$ if and only if A $\cup \{\neg\phi\}$ is semantically inconsistent.

this proposition follows:

Proposition 3 A $\models \phi$ only if A $\vdash \phi$ if and only if A $\cup \{\neg\phi\}$ is semantically inconsistent only if A $\cup \{\neg\phi\}$ is d-inconsistent.

While this is true, and even obvious, there are many beginning undergraduate philosophy students, unused to the language of mathematical proof, who will need help with this sort of claim. In particular, many undergraduate philosophy students would need someone to at least explain that the first two propositions spell out equivalences that allow us to get the third by substituting equivalents. This sort of remark can “grease the wheels” for undergraduates; the present book typically does not provide such assistance. If one is teaching beginners, they will need additional help at every stage to understand the key proofs.

In some cases, the technical material misses the chance to introduce standard terminology. For example, the authors do not introduce the reader to the terminology of *maximal consistent sets*, as is standard in proofs of completeness (nor do the authors mention various important figures in the development of the proof, such as Lindenbaum or Henkin). And (another example) in the discussion of Gödel’s proof, the authors use but do not refer to the successor function as such. And they never use the term ‘arithmetization’.

The book most likely to compete with this one on the market for textbooks that serve “philosophically-oriented second courses in logic” is Ted Sider’s (2009) *Logic for Philosophy*. Sider’s book provides more thorough coverage of all the topics I have mentioned (aside from term logic) and uses undergraduate-friendly, philosophically engaged prose throughout. It also treats some technical material in a more rigorous fashion than does the present work. Nevertheless, if what is wanted is a very compact, convenient presentation of some central themes in philosophical logic, presented in a way that sets the stage for further discussion in class, Englebretsen and Sayward’s book will serve that purpose well.

References

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