

# Wittgenstein and the Status of Contradictions

Louis Caruana, Philosophy Faculty, Gregorian University

In the *Remarks on the Foundation of Mathematics*, we find Wittgenstein exclaiming: 'Contradiction. Why this *one* bogey? This is surely very suspicious.' (Wittgenstein, 1967, III 56). During one of his early discussions with members of the Vienna circle, he is claimed to have made the following prediction: 'I am prepared to predict that there will be mathematical investigations of calculi containing contradictions and people will pride themselves on having emancipated themselves from consistency too.' (Goldstein 1989, p. 540).<sup>1</sup>

Some readers of Wittgenstein follow Wrigley in viewing him as proposing a radical conventionalist philosophy of contradictions – not only of contradictions but of practically everything (Wrigley 1980, Wright 1980, Review 1977). According to this view, the only way of making sense of his remarks on contradictions is to assume that he endorsed a conventional idea of necessity. Any instance of necessary truths in Wittgenstein's later writings is, according to this account, a result of something a group of people set up. It is a feature of a linguistic practice that owes its existence purely to a conscious or unconscious congruence of a way of behaving. Mathematics, according to this view, is essentially a game whose rules are conventional.

The prediction of Wittgenstein regarding contradictions should therefore be seen as a conjecture about ways of acting, about agreed tastes. For instance, at one point, Wittgenstein describes how contradictions that are evident in what people do and say do not always produce catastrophic effects. He states that 'people do not get into those troubles' (Wittgenstein 1976, 288). According to people like Wrigley, the reason that no trouble arises is simply that, in some situations, people have adopted certain conventions. According to such conventions, contradictions are not as problematic as they are in the typical language-games used by logicians and mathematicians.

It is worthwhile highlighting the link between Wittgenstein's views on contradictions and his more general views on mathematics. The former views are rightly seen as a subset of the latter. Commentators like Wrigley are probably applying the alleged Wittgensteinian conventionalism in mathematics to the particular case of contradictions. To take the bull by the horns therefore, one needs to see whether Wittgenstein's conventionalism in mathematics is indeed as radical as Wrigley suggests.

Consider the case of a mathematical proof. What constitutes a proof? The reply attributed to Wittgenstein is in line with his fundamental position on rule-following. According to this, the ultimate source of normativity in rule-following is located in the community as it engages in a given practice. For him therefore, it seems natural to hold that a mathematical proof is neither a novel application of established rules, nor a discovery of hidden mathematical structures. It is a creative way of changing a common inclination among the community of

---

<sup>1</sup> Wittgenstein's remarks bearing on contradiction are scattered in a number places. For example Wittgenstein 1967, II 78, 81, 82; III 57-60; III 87; V 12, 13, 21, 26, 28. For further details, see Priest, Routley, Norman 1989; Priest and Smiley 1993; Priest 1998.

specialists (Wittgenstein 1967, IV 30, VII 9; 1979 p. 185-6). This change of inclination involves the act of convincing others of something they had never imagined before. The mathematician originator of a new mathematical proof or concept has the task of doing something that will convince his colleagues that what he does is essentially supplying a proof. For example, when mathematicians allegedly proved that denumerable infinity,  $\aleph_0$ , is a *number*, they went through a procedure that convinced their colleagues that they have indeed supplied a proof. The proof itself is part of the novelty.

Not everything is allowed, however. In fact, for this particular example, Wittgenstein thinks mathematicians are mistaken whenever they use the expression ‘the number of elements in an infinite series’ as if it had a clear meaning. He thinks that the use of  $\aleph_0$  as a number is an inappropriate grammatical inclination. His judgement here shows that he did not consider the criterion for acceptance or rejection of novelty a simple democratic process of establishing the major trends in the various communities of mathematicians. There is an important aspect that distinguishes him from the pure conventionalist. For him, the criterion for acceptance or rejection of novelty is the legitimacy of subtle changes in the meaning of central symbols used. A community of mathematicians cannot just decide for itself what meaning certain symbols or phrases have. Meaning, as manifested in the use of words, is like a net reaching out beyond any one community.<sup>2</sup>

The upshot of the line of reasoning I am taking is that pure conventionalism, of a completely arbitrary kind, is a mistaken characterisation of Wittgenstein’s philosophy of mathematics. If this is a mistaken characterisation of his philosophy of mathematics, it is very probably a mistaken characterisation of his discussion of contradictions as well. It would be very odd for him to lapse into radical conventionalism only for the question of contradictions. The problem however is: if not pure conventionalism, then what? Pure conventionalism seems to be the easy way out. What I propose is to examine the possibility of building on the framework suggested by Wittgenstein towards a *non-conventional* version of the logic of contradiction. The suggestion that Wittgenstein himself would have liked to develop his views in this direction gains some support from some scattered comments of his, the most significant of which perhaps is the following:

‘But you can’t allow a contradiction to stand: Why not? We do sometimes use this form in our talk, of course not often – but one could imagine a technique of language in which it was a regular instrument. It might for example be said of an object in motion that it existed and did not exist in this place; change might be expressed by means of a contradiction’ (Wittgenstein 1967, V 8).

‘Of course not often,’ he says. This is because the major threat from admitting contradictions is that of explosion. Standard logic cannot handle inconsistency. We define a theory as a set of axioms together with all their logical consequences. We can therefore say that a theory is inconsistent when it contains at least one single proposition together with its

---

<sup>2</sup> It is a grammatical mistake to say that ‘has  $\aleph_0$  elements’ is equivalent to the ordinary phrase ‘has  $p$  elements’ (Frascolla 1994, chap 3). This non-equivalence itself however is a matter of semantics, and thus embedded on the practice of the community. Any practice is always somewhat constrained. Some constraints are conventional; others can be called non-conventional. These latter are due to how the world is and determine what Wittgenstein calls ‘natural history’ (Wittgenstein 1953, § 415). Cf. Caruana 2000, chapter 7. Normativity and rule-following is discussed in more detail in Caruana 2003.

negation. From an inconsistent theory, one can deduce any proposition whatsoever.<sup>3</sup> Consequently, one says that standard logic is *explosive* in the presence of inconsistency. Inconsistency therefore is not a matter of degree. One instance of inconsistency within a theory destabilises the system completely. Is this, however, all there is to say as regards contradictions? If the answer is yes, then Wittgenstein's remarks can be seen as musings about an impossible wonderland. If, however, the answer is no, then perhaps some valuable conclusions can be drawn by following his lead.

The main problem is to see how to avoid the explosive situation described above. The suggestion I will be examining is the following. It is possible to talk intelligently about a situation containing a contradiction if one devises a way of 'containing' the contradiction. That will ensure that it does not spill over to disrupt the entire universe of discourse. This may be called a 'containment account' of contradictions. If I show that such an account is plausible, I will also be thereby showing that Wittgenstein's prediction is not to be discarded as deviant non-sense but as the seed of fruitful creativity.

The best way to arrive at a plausible containment account, is perhaps to start at the logical studies of dialectic carried out by Rescher and Brandon (Rescher and Brandon 1980; see also Haack 1982; Denyer 1989). Rescher starts by determining a crucial distinction to be made in any serious attempt at dealing with contradictions. This distinction is the one between what may be called hard ontological inconsistency, involving inconsistency in the very nature of things, and what may be called soft inconsistency, which refers to inconsistency easily explainable by reference to the variety of perspectives. Hence, when  $P \ \& \ \sim P$  is a soft inconsistency, it is quickly seen as no inconsistency at all. Digging just under the surface of things reveals that here one is dealing simply with the situation where  $P$  is the case for one particular perspective while  $\sim P$  is the case for another perspective. The inconsistency can be resolved by adjusting the meaning of the words used in expressing  $P$ . When, however,  $P \ \& \ \sim P$  is a hard inconsistency, no amount of digging deeply will be enough to neutralise the situation.

A standard world view, such as the one of the *Tractatus*, and of many systematic philosophers since Aristotle, is characterised by two laws. The first, the law of excluded middle, holds that, of a pair of propositions  $P$  and  $\sim P$ , *at least* one obtains, a fact classically expressed by the phrase *tertium non datur*. The second law, that of contradiction holds that, of a pair of propositions  $P$  and  $\sim P$ , *at most* one obtains. These two laws suggest that a non-standard world corresponding to what we find in the later writings of Wittgenstein may be one where either the first law fails or the second law fails or both.

---

<sup>3</sup> Starting from the premise  $P$  that  $A \ \& \ \sim A$ , one can see how any other proposition follows because of the following simple argument. For all  $A$ ,  $A \Rightarrow (A \ \text{or} \ B)$ . From  $P$ , however, we hold also that  $\sim A$ .  $(A \ \text{or} \ B) \ \& \ \sim A \Rightarrow B$ . Therefore we can conclude that  $B$ , for any  $B$  whatsoever. For the classical treatment of contradiction see Aristotle *Metaphysics* Book  $\Gamma$  : 'A thing cannot at the same time be and not be' (996b30). This formulation is related to the being of a thing, rather than to what one can say about the thing. Following Aristotle, many have considered the principle of contradiction, or of non-contradiction, depending on the way one looks at it, as valid for all being. In Aristotelian metaphysics, it is considered a primary principle, an indispensable tool, because it allows the apprehension of being in its intrinsic intelligibility (e.g. St. Thomas Aquinas, *Summa Theol.* 1a 2ae 94,2). Since it excludes its own negation, it cannot be denied, but neither can it be derived. There are indications that Aristotle wasn't ignorant of the explosive nature of even a single inconsistency in one's belief system. For instance, he claims that 'if words have no meaning, our reasoning with one another, and indeed with ourselves, has been annihilated; for it is impossible to think of anything if we do not think of one thing' (*Metaphysics*, 1006b 8-10). See also: Dancy 1975.

Rescher calls the first type of inconsistent world schematic. This world is indeterminate at some points, that is, for some points, one can hold neither  $P$  nor  $\sim P$ . There is nothing to say as regards these points. For instance, if we have a world in which it can be neither raining nor not raining, then it would be a schematic world. The second type of inconsistent world, called simply by Rescher an inconsistent world, is one in which the law of contradiction does not hold. Hence here for at least one pair of propositions  $P$  and  $\sim P$ , both propositions hold together simultaneously. If our world were such that it could be both raining and not raining at the same time, it would have been what Rescher calls an inconsistent world.

One must recall here that such worlds are underdetermined and overdetermined respectively not because of our epistemological limitations. They are so because of a characteristic intrinsic to them. Because of this important point, I will avoid Rescher's vocabulary, which is suggestive of epistemological limitations on the part of the knower. I will call the first world a *perforated world*, in the sense that it contains holes that are neither  $P$  nor  $\sim P$ . The same kind of danger can arise with Rescher's terminology regarding the second kind of world, the one he simply calls inconsistent. The major characteristic of this world is that some parts or points of it are overdetermined, they can be both  $P$  and  $\sim P$ . What I suggest here is to call this kind of world a *superimposed world*. It may be helpful to think of such a world as being folded over at some edges, the crease representing precisely where both  $P$  and  $\sim P$  hold.

To work out a plausible containment account of contradictions, one needs to be clear at the very start on one important point. This concerns our way of expressing ourselves. In all our talk, we are obliged to keep away from self-contradiction. This holds also when we attempt to talk about contradictions. What needs to be done, therefore, is to distinguish between the truth of  $P$  in our world, the world of our discourse, and the truth of  $P$  in some other world, a possible world, either of the perforated type or of the superimposed type. This expresses the possibility of speaking intelligently about a world  $w$  in which, for some  $P$ ,  $P$  is true in  $w$  and also  $\sim P$  is true in the same  $w$ . This ensures that there is no explosion of the type considered above. Explosion occurs when we hold that, in some world, it is true that  $(P \ \& \ \sim P)$ . The perforated or superimposed worlds under consideration are such that we can hold that neither  $P$  nor  $\sim P$  are true for some  $P$ , or that both  $P$  and  $\sim P$  are true for some  $P$ . The ontological status of  $P$  and that of  $\sim P$  are considered independent issues, so as to block the inference from  $P$  and  $\sim P$  to the conjunction  $(P \ \& \ \sim P)$ . In this way, the contradiction is contained. The task of containment is being carried out by the complexity of the ontology determining the possible worlds under consideration.

One may object here by drawing attention to the futility of such a consideration, noticing perhaps that the perforated or superimposed possible worlds described above are just fictions, perhaps logically interesting, but empirically non-existent. To counter such an objection, I will mention some interesting developments that may result from the above emancipation of a contained inconsistency. I will mention the three that readily come to mind.

Firstly, the above 'containment account' of contradictions can have an interesting application in understanding certain developments in the history of natural science. Some irresolvable riddles in physics and chemistry have been bypassed precisely by assuming that reality is sometimes too deep or complicated to be neatly charted by our theoretical descriptions. One may say that, when chemists try to describe the structure of the benzene ring as an intermediate form, explained in the theory of mesomerism, or of that of Pauling's resonance theory, their attitude shows a tendency to accept a perforated ontology. They are

saying that the molecule has neither one structure nor its opposite. They may certainly hope to resolve the problem later. Their attitude of perseverance with the theory as it is, however, shows that the contradiction is considered contained, and not left to contaminate and explode the entire edifice. Other examples of an attitude favouring an inconsistent ontology, this time a superimposed one, include the one taken by early quantum physicists as regards the status of the electron, both a particle and a wave.<sup>4</sup>

There remains however the crucial questions whether the actual world is perforated or not, and whether the actual world is superimposed or not. When facing a theoretical impasse, such as the clash between the particle description and the wave description, should one conclude that a discovery has been made about the world, namely that it is superimposed precisely at that spot? Or should one take the opposite view, and conclude that, *because* of the unacceptable theoretical contradiction, one needs to confess one's ignorance and admit that the full description of that area of the real is still to be discovered? It is certainly a mark of light-headedness to go about spotting perforations and superimpositions every time there is disagreement. If, however, the scientific theories under consideration can prove their mettle elsewhere, as in the spectacular case of Quantum Mechanics, then a stubborn contradiction may be taken to mean the discovery of a previously unconceived of aspect of the world we live in.

The second possible application of the containment-account of contradictions concerns a classic debate in the area of philosophy of religion, namely the one about God's limitations on creating the world. Does God's omnipotence mean that God can even create a square circle? Or is logic a limitation on God's power? Thomas Aquinas defended a position according to which logic reflects ontology so well that limits on logic are equivalent to limits on what could be (*Summa Theol.* I, 25, 3). He concludes that God cannot do the impossible. This, for Aquinas, is not a limitation on God's part. René Descartes, on the other hand, argues that the power of God cannot have any limits, not even the limits set by the law of non-contradiction. Omnipotence goes beyond what is graspable by our finite minds (*Lettre au P. Mesland*, 2 Mai 1644; cf. Hughes 1995, 114-151). As is evident from this paper, Wittgenstein's line of thinking could be developed in a way favouring Descartes' attitude. It leaves some space, as it were, between logic and ontology. His line of thinking seems to lead to the claim that, when theologians refer to God by the term *mysterium tremendum*, they are using these terms precisely to keep the door open for a creation whose nature can go beyond our expectations, open even for the possibility that our world may in fact turn out to be perforated or superimposed.

In this context, one may be tempted to refer to Wittgenstein's famous mention of God in dealing with the question whether the group 7777 occurs or does not occur in the infinite decimal expansion of  $\pi$  (Wittgenstein 1953, § 352). We often say that God sees the entire expansion of  $\pi$  but we do not. Hence, we conclude that the group 7777 either occurs or does not occur, and that there is no third way. This, however, is mistaken. Wittgenstein wants to use this example to make a point about propositions. According to him, the mistake lies in the

---

<sup>4</sup> My point may also be applied perhaps to the attitude expressed in the Bohr interpretation of Quantum Mechanics as regards the superposition of the wave-function before collapse. A completely instrumentalist interpretation of Quantum Mechanics obviously avoids all such questions by stating that there is no reality at all to talk about. I am avoiding this way of interpreting the formalism, because it seems an extreme position difficult to integrate with the rest of natural science. On chemistry, see Hooykaas 1999, pp. 257-262.

hidden assumption that infinity is a quantity, which is false. The question 'Can God know all the places for the expansion of  $\pi$ ?' is senseless because of this assumption. In this section of the *Philosophical Investigations*, Wittgenstein is neither talking about God, nor about what we can say about God's power. He is just uncovering the limits of the very meaning of 'proposition'. For him, the principle that every proposition is either true or false partially *defines* what a proposition is. We must recall, however, that the very definition of 'proposition' has a meaning that is itself embedded in human practice. Our assent to the laws of non-contradiction and excluded middle show our respect for certain practices, we call inexorable.<sup>5</sup> The upshot is that, in the line of argument followed in this paper, Wittgenstein's point about 7777 in the expansion of  $\pi$  is not of direct relevance. We can still hold that his views on contradiction favour Descartes' position, namely that God's omnipotence goes beyond what is graspable by our finite minds.

The third possible application of the containment-account of contradictions concerns the intersubjective aspect of knowledge. The limited emancipation of contradictions as described above can open up new ways of understanding how living with a collective set of beliefs containing some contradictions is not a paralysing handicap. The political and social repercussions of this idea could find an echo in the way a pluralistic society may find grounds for its cohesion. It is customary to assume that cultural cohesion is exclusively dependent on a stock of truths common to all. If people concentrate on what they know in common and forget the differences in opinion, peace and harmony are guaranteed. What is knowable, however, may turn out to be far more complicated than previously admitted by any single sub-group of society. The collective set of all beliefs adhered to at any one time will certainly be inconsistent at various points. The containment account of contradictions may be useful here in viewing such a belief-set less problematic for everyday living than it used to be viewed before.

There is, however, a danger here. It lies in the possibility of arguing that the refractive character of belief-systems and of value-systems is to be accepted as it is. This is equivalent to holding that belief-systems need no deeper elaboration, no further reflection, by the various groups that propose them. Those who hold this view often argue that the objective aspect of the world to which such discourse aspires to be responsible is either a pure fiction, which leads us back to pure conventionalism, or is hopelessly perforated or superimposed. So why should one feel obliged to reach out to others, to engage in conversation with those who have another viewpoint? The drive towards unity and towards clearer criteria for better living is thus undermined. If this were true, society would have no other course to take than the one leading towards epistemological fragmentation, or cultural apartheid. This danger should be kept in mind, because a heavily perforated world, just as a heavily superimposed one, is as good as no world at all.<sup>6</sup> Not everything, therefore, that results from the emancipation of contradictions is desirable.

Is this what Wittgenstein had in mind when discussing contradictions? I am not in a position to give a definite answer to this question. In the quotation mentioned above, he calls a contradiction 'a possible regular instrument' in some language-games. One may ask: an

---

<sup>5</sup> Cf. Wittgenstein 1967, I, § 118. P.M.S. Hacker insists that Wittgenstein, in *Philosophical Investigations* § 352, was not impugning the validity of the law of excluded middle. In my view, this comment is correct, but does not say anything about Wittgenstein's views on contradiction elsewhere in his other writings. Cf. Hacker 1990, pp. 377-379.

<sup>6</sup> Cultural fragmentation seems to be the social correlate of logical explosion.

instrument for what? If my development outlined so far is acceptable, the word could be a useful instrument for a number of things. In some language-games, especially the ones constrained by exaggerated logical precision, a contradiction is used to show a danger of explosion, a dead-end. It is here used to constrain the language-game, as a kind of traffic warden: one can go wherever one likes as long as this point is avoided. It can also be used to indicate failure in latching onto the real world. In other language-games, however, contradiction has a much less hostile role. It is used as an instrument to indicate that the ontology being presupposed may be of a certain complex kind. It can be used to indicate that there is more to life than what can be expressed in precise algorithmic trees, or to indicate that rationality goes beyond ratiocination.

My overall aim was simply to suggest how a plausible 'containment-account' of inconsistency can be sketched out in line with some of Wittgenstein's remarks. This is one possible direction to take. There may be others.

## References

- Aristotle, *Metaphysics*, trans. W.D. Ross, in: *The Basic Works of Aristotle*, R. McKeon (ed.), New York: Random House, 1941, pp. 681-934.
- Caruana, Louis, 'Realism and Rule-following', in: *Normatività, Fatti, Valori*, a cura di R. Egidi, M. dell'Utri, M. De Caro, Analisi Filosofiche Quodlibet, Macerata, 2003, pp. 143-152.
- , *Holism and the Understanding of Science*, (Ashgate New Critical Thinking in Philosophy), Aldershot, UK: Ashgate, 2000.
- Dancy, Jonathan, *Sense and Contradiction: a Study in Aristotle*, Dordrecht and Boston, 1975.
- Denyer, N., 'Dialethism and Trivialisation', *Mind* 98 (1989), pp. 259-63.
- Descartes, René, 'Lettre au P. Mesland – 2 Mai 1644', in: *Oeuvres de Descartes – Correspondance, tome IV*, C. Adam & P. Tannery (eds.), Paris: Vrin, 1972, pp. 110-120.
- Frascolla, P., *Wittgenstein's Philosophy of Mathematics*, London: Routledge, 1994. 635.
- Goldstein, L., 'Wittgenstein and Paraconsistency' in: *Paraconsistent Logic: Essays on the Inconsistent*, G. Priest, R. Routley, J. Norman (eds.), München: Philosophie Verlag GmbH, 1989, pp. 540 - 562.
- Haack, Robin, 'Wittgenstein's Pragmatism', *American Philosophical Quarterly* 19, 1982, pp. 163 -171.
- Hacker, P.M.S., *Wittgenstein, Meaning and Mind – an analytic commentary on the Philosophical Investigations, volume 3*, Oxford: Basil Blackwell, 1990.
- Hooykaas, R., *Fact, Faith and Fiction in the Development of Science*, Dordrecht: Kluwer, 1999.
- Hughes, Gerald, *The Nature of God*, London: Routledge, 1995.
- Priest, G. 'What's so bad about Contradictions?' *Journal of Philosophy* 95 (1998) 410-26.

- Priest, G. and Smiley, T., 'Can contradictions be true?' *Proceedings of the Aristotelian Society, Supplementary Vol. 68* (1993), 17-54.
- Priest, G., Routley, R., Norman, J., (eds.), *Paraconsistent Logic: Essays on the Inconsistent*, München: Philosophie Verlag 1989.
- Rescher, N. & Brandom, R., *The Logic of Inconsistency: a Study in non-standard possible-world Semantics and Ontology*, Oxford, Basil Blackwell, 1980.
- Review of Lectures on the Foundations of Mathematics, *Philosophical Quarterly* 27 (1977) pp. 370 - 371.
- Thomas Aquinas, *Summa Theologica*, Latin text and English translation, introductions, notes, appendices and glossaries, London: Blackfriars, 1964-1981.
- Wittgenstein, L., (1953), *Philosophical Investigations*, trans. G. E. M. Anscombe, Oxford: Blackwell.
- (1967), *Remarks on the Foundations of Mathematics*, ed. G.H. von Wright, R. Rhees and G.E.M. Anscombe, trans. G.E.M. Anscombe, 3rd edn, Oxford: Blackwell.
- (1976) *Lectures on the Foundations of Mathematics, Cambridge 1939*, ed. C. Diamond, Harvester, Hassocks.
- (1979), *Wittgenstein's Lectures, Cambridge 1932-1935*, A. Ambrose (ed.), Oxford: Blackwell.
- Wright, C., *Wittgenstein on the Foundations of Mathematics*, Duckworth, London, 1980.
- Wrigley, M., 'Wittgenstein on inconsistency', *Philosophy* 55 (1980), pp. 471-484.